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TEACHING OF MATHEMATICS IN THE PRIMARY SCHOOLS:

A COMPARATIVE STUDY: BOPHUTHATSWANA AND LEBOWA

by

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DECLARATION

I declare that the dissertation for the degree of MASTER OF EDUCATION at the University of the North hereby submitted by me, has not previously been submitted by me for a degree at this or at any other university, and that it is my own work in design and in execution and that all material contained therein has been duly recognized.

Mokhaba.
M B MOKHABA

DEDICATION

This work is dedicated to:

1 Sentsho Caizer, my father and Mmapula Freda, my mother who are the source of inspiration in my life;

2 My beloved wife, Mmabona Anna, and my son Bojelo Abel.

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SUMMARY

The writer of this dissertation, as a teacher of Mathematics and also a senior sub-examinor of Standard 8 Mathematics, discovered in the execution of his duties that a large number of the pupils fail Mathematics in the examinations. It also became clear to the researcher that the nature of the errors committed by the pupils at Standard 8 level reveals that the pupils were not well grounded in the primary school Mathematics. Consequently the title of this dissertation is "Teaching of Mathematics in the primary schools: a comparative study: Bophuthatswana and Lebowa".

A detailed study of Piaget and the methods of teaching and learning Mathematics was made. In his works, Piaget mentioned that the number concept plays an important role in the study of Mathematics. He also mentioned the concepts which are necessary in the study of Mathematics, namely, the concepts of number, time, space, length, area, volume, weight and substance. Piaget also gave a detailed description of the child's cognitive development.

The teacher in performing his task must make use of audio-visual aids, methods which will promote pupil activity in the classroom, Mathematics learning centres, Mathematics

laboratories and games in Mathematics. These techniques may help to make teaching more effective and also improve the pupils' performance in Mathematics.

The standard of work of the pupils from Bophuthatswana was determined through the application of the "Scholastic Achievement Tests in Arithmetic - Standard 4" of the Human Sciences Research Council. Standard 5 pupils from ten schools in Bophuthatswana wrote the above-mentioned tests as they have already covered the Standard 4 syllabus of Mathematics. These ten schools were chosen from three inspection circuits in Bophuthatswana. The sample schools included small and big schools as well as urban and rural schools.

The results of the "Scholastic Achievement Tests in Arithmetic - Standard 4" revealed that the achievements of the pupils was below expectation. Pupils from urban and rural schools performed almost equally poor. The poor performance was not confined to a particular inspection circuit or circuits. The three inspection circuits performed almost equally poor.

Five of the ten schools wrote the "Diagnostic Mathematical Tests" of the Human Sciences Research Council. The purpose of these tests was to determine the problem areas. The tests covered natural numbers, vulgar fractions and decimal fractions. The results of the diagnostic tests revealed that the pupils performed slightly better when dealing with natural numbers. In the case of the vulgar fractions and decimal fractions the performance of the pupils was below expectation.

The results of the "Scholastic Achievement Tests in Arithmetic - Standard 4" of the pupils from Bophuthatswana were compared with those of the pupils from Lebowa. In these tests the Bophuthatswana group performed slightly better than the Lebowa group.

The results of the "Diagnostic Mathematical Tests" of the pupils from Bophuthatswana were also compared with those of the pupils from Lebowa. In Tests 5 and 6 the Bophuthatswana group performed slightly better than the Lebowa group. In Tests 7, 8 and 9 the Lebowa group performed slightly better than the Bophuthatswana group.

The possible factors which might have contributed to the poor performance of the pupils are the domestic milieu of the black child, the development of the black child's intellectual potential, Mathematics in everyday life of the black people, formation of mathematical concepts, medium of instruction and teachers' qualifications in Mathematics.

After a careful study of the possible factors which might have contributed to the poor results of the pupils it was recommended that underqualified Mathematics teachers should be helped to improve their qualifications in Mathematics. They should also be helped by the appointment of a subject advisor in each inspection circuit. During their initial training, students should be introduced to methods which promote pupil activity. It is also recommended that Mathematics textbook writers should pay special attention to the black child's numeration system and the language problem.

OPSOMMING

Die navorser is 'n Wiskunde onderwyser en is ook 'n senior hulp-eksaminator van Standerd 8 Wiskunde. In die uitvoering van sy werk, het hy besef dat baie kinders Wiskunde in die eksamen druip. Die aard van die foute wat deur die kinders begaan word, dui op 'n moontlike gebrekkige onderrig in die laerskool. Gevolglik het die navorser "Teaching of Mathematics in the primary schools: a comparative study: Bophuthatswana and Lebowa" as veld van ondersoek gekies.

Die werke van Piaget en die metodes van onderrig in Wiskunde is goed bestudeer. In sy werke dui Piaget die gevare aan van die onderrig van gevorderde werk in Wiskunde voordat die volgende basiese konsepte met insig bemeester is; die konsepte van getal, tyd, ruimte, lengte, oppervlakte, volume, massa en materie. Hy het ook 'n breedvoerige beskrywing van die kind se intellektuele ontwikkeling gegee.

Gedurende die onderrig sessies moet die onderwyser gebruik maak van hulpmiddels, aktiwiteitsmetodes, Wiskunde leersentrum sowel as die Wiskunde laboratorium. Verder moet Wiskunde spele aangemoedig word. Hierdie tegnieke kan help om die onderrig van Wiskunde meer effektief te maak en die prestasie van die kinders te verbeter.

Om die standaard van die werk van die kinders van Bophuthatswana in die algemeen te bepaal, is die "Scholastic Achievement Tests in Arithmetic - Standard 4" van die Raad vir Geesteswetenskaplike Navorsing toegepas. Standerd 5 leerlinge van tien skole in Bophuthatswana is getoets omdat hulle alreeds die Standerd 4 Wiskunde leerplan afgehandel het. Hierdie tien skole is gekies uit drie inspeksiekringe van Bophuthatswana. Die monster het klein en groot skole, sowel as skole van plattelandse en stedelike gebiede ingesluit.

Die resultate van die "Scholastic Achievement Tests in Arithmetic - Standard 4" was nie bevredigend nie. Die kinders van die stedelike skole het net so swak soos die kinders van die plattelandse skole presteer. Verder is ook gevind dat die prestasies van die verskillende skole nie veel verskil het nie. Die swak prestasies was nie tot sekere inspeksiekringe beperk nie.

Vyf van die tien skole het die "Diagnostic Mathematical Tests" van die Raad vir Geesteswetenskaplike Navorsing geskryf. Die doel van hierdie toetse was om die probleemareas te identifiseer. Die toetse het bewerkinge met natuurlike getalle, gewone breuke en desimale breuke gedek. Die resultate van die diagnostiese toetse dui aan dat die kinders beter presteer het in die bewerkinge met natuurlike getalle. Die kinders het swak presteer in die bewerkinge met gewone breuke en desimale breuke.

Die resultate van die "Scholastic Achievement Tests in Arithmetic - Standard 4" van die monster van die kinders van Bophuthatswana was vergelyk met dié van 'n monster van kinders van Lebowa. In hierdie toetse het die Bophuthatswana groep beter presteer as die Lebowa groep.

Die resultate van die "Diagnostic Mathematical Tests" van 'n monster van kinders van Bophuthatswana was ook vergelyk met dié van 'n monster van kinders van Lebowa. In Toetse 5 en 6 het die Bophuthatswana groep beter presteer as die Lebowa groep. Die Lebowa groep het egter beter presteer as die Bophuthatswana groep in Toetse 7, 8 en 9.

Faktore wat moontlik die prestasies van die swart kinders kon beïnvloed het, is die huislike milieu van die swart kind, ontwikkeling van die swart kind se intellektuele potensiaal, Wiskunde in die alledaagse lewe van die swartman, vorming van Wiskunde konsepte, medium van onderrig en onderwysers en hulle kwalifikasies.

Na 'n studie van die faktore wat die prestasies van die swart kinders kon beïnvloed het, is dit aanbeveel dat die ondergekwalifiseerde onderwysers gehelp moet word om hulle kwalifikasies in Wiskunde te verbeter. Hulle moet ook gehelp word deur die benoeming van 'n vakadviseur in elke inspeksiekring. Gedurende die opleiding van die studente aan die onderwyskolleges moet hulle aktiwiteitsmetodes geleer word. Dit word ook aanbeveel dat die skrywers van Wiskunde handboeke kennis moet neem van die swart kind se metodes van tel en sy getalsbegrip en dat die probleem van die onderrigmedium weer aandag moet geniet.

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CHAPTER 1

INTRODUCTORY ORIENTATION

1.1 INTRODUCTION

It seems as if the pupils in the primary schools of Bophuthatswana generally perform badly in Mathematics. A large number of them fail Mathematics every year in their examinations. This problem is also found in the middle as well as the high schools.

Indications are that the above-mentioned problem starts at the primary school level and continues at the middle and high schools. This may indicate that the teaching of Mathematics in the primary schools of Bophuthatswana needs attention.

From birth, the child is exposed to a world in which number plays a significant role. It is all around him and forms part of the culture in which he lives. It therefore follows that there must be a harmonious development between the teaching of Mathematics in the primary schools of Bophuthatswana and the development of the child's number concept. This is true because the growth of mathematical understanding develops step by step with each forward move depending upon the consolidation of previous experience. In this regard Piaget maintains that the initial foundations of number are laid down in the sensori-motor period, when the child, through maturation and experience, eventually develops the concept of the permanence of objects and the ability to distinguish among various objects (McNally, 1977, p.103).

1.2 DEFINITION OF TERMS

1.2.1 What is a problem?

The word problem in this research refers to areas of difficulties in the teaching-learning situation of the subject Mathematics in the primary schools of the Republic of Bophuthatswana. Areas of difficulties may be experienced in different sections of Mathematics.

1.2.2 What is Mathematics?

The Oxford Dictionary defines Mathematics as a science of space and number in the abstract. Now in this context the emphasis falls inter alia on the "number" part of the definition. Hence the teaching of Mathematics should keep pace with the development of the number concept. Apart from the number concept there are other concepts to be considered, such as the concept of space, the concept of length and measurement, the concepts associated with area and volume, the concept of substance and the concept of weight (Van den Berg, 1978, pp.22-23).

But many of these definitions of Mathematics are either too narrow to be comprehensive, or too wide to be of value, but it may be argued that almost every area of human endeavour needs at some stage or other to consider such attributes as quantity, order or structure. It is, therefore, reasonable and economical to study these attributes as a separate discipline, and this study, suitably abstracted, is known as Mathematics (Gardner et al, 1973, p.11).

From the above definition the basic characteristics of Mathematics may readily be derived. In spite of recent developments in primary schools, Mathematics is still basically an abstract subject, since the elements which it manipulates are themselves abstract, for example, "three" cannot exist concretely except as three books or three cars. Mathematics is also precise and ordered since its fundamentals themselves have that quality. These characteristics are what makes Mathematics difficult but rewarding, both to teach and to learn.

Mathematics deals with such topics as quantity, order, pattern and structure. It may be used as a powerful tool in the investigation of other subjects or themes but by its very nature transcends these and should not be limited in this way (Gardner, et al, 1973, p.11)

Today, Applied Mathematics is the mainspring of our civilization, for we are becoming increasingly conscious that "to measure is to know." (Sandford, 1930, p.1).

1.2.2.1 <u>The nature of Mathematics</u>

Closely connected with the question what Mathematics is, is the endeavour to determine the nature of Mathematics. The understanding of the nature of Mathematics is of paramount importance in the teaching of Mathematics, as our teaching of the subject will be influenced and determined by our understanding of the nature of the subject as such.

Mathematicians are not in agreement as to the exact nature of their subject. This is clearly illustrated by Professor T van der Walt in his inaugural address at the Rand Afrikaans University, who said: "Die vraag na die aard van Wiskunde is miskien meer gestel as enige ander vraag van grondliggende belang, en nog meer antwoorde is daarop gegee. Wiskunde is die wetenskap van vanselfsprekende waarhede; wiskunde is die studie van abstrakte strukture; wiskunde is die maak van noodwendige gevolgtrekkings; wiskunde bestaan uit alle volsinne van die vorm 'As p, dan q'; wiskunde is wat wiskundiges doen; hierdie is maar 'n paar voorbeelde uit die arsenaal van antwoorde." (Van den Berg, 1978, pp.36-37).

The uncertainty as regards the nature of Mathematics compelled educationists to search for the solution. A possible solution may be found in the functional aspect of Mathematics. It is the "doing" aspect of the subject that should be emphasized (Van den Berg, 1978, p.37).

There are too many intelligent children who fail to understand Mathematics. This implies that educationists will have to re-examine, not only their methods of instruction, but also the nature of Mathematics. Even in this respect, the educationist must see the nature of Mathematics from the point of view of a teacher who wishes to bring his pupils to a true understanding of the basic mathematical concepts which they have to apply in their everyday life (Van den Berg, 1978, p.38).

From the above discussion it is clear that an over-emphasis on the axiomatic-deductive approach both in the way Mathematics is taught and subject matter offered can lead to over-formalised teaching and dampen the creative aspect of Mathematics. This form of teaching has a direct bearing on the learning of pupils. Pupils may resort to parrot-learning, which may be indicative of learning without understanding (Van den Berg, 1978, p.39).

Another problem area is the medium of instruction, namely English. It is maintained that "... the tendency to parrot answers without much thought or understanding among black pupils may be the direct result of having to learn through the medium of a second language, a strange language." It follows that when Mathematics is seen as an abstract subject, language attainment will necessarily influence the development of mathematical concepts (Van den Berg, 1978, p.39). The language problem will be discussed in detail at a later stage.

1.2.2.2 What is the use of Mathematics?

The uses of Mathematics according to Van den Berg (1978, p.21) can be broken down into the following five facets:

i) To enable pupils to cope with mathematical situations they may be faced with in everyday life.

ii) To help pupils to develop logical habits of thoughts and systematic, concise expressions.

iii) To develop in the pupils an interest in, an appreciation of, and a love for the unchanging nature of the laws of numbers.

iv) To prepare the pupils to perform calculations which may be needed in other school subjects or in further studies.

v) To develop in the pupils an ability to make calculations accurately and rapidly.

1.2.3 Teaching

The Oxford Dictionary defines the term teaching as the act of imparting knowledge or skills. In this context the term means the act of imparting mathematical principles and methods to pupils.

1.2.4 Bophuthatswana

Bophuthatswana is one of the independent states in Southern Africa. It is a country which obtained its independence from the Republic of South Africa. It is a country for the Batswana, who were in the Republic of South Africa, and Setswana is the official language of this country. This research will be conducted at a representative sample of schools of Bophuthatswana.

1.2.5 Primary schools

According to the report of the National Education Commission - Education for Popagano (Lekhela, 1978, p.14), primary schools are schools which offer tuition from Grade I to Standard 4. This means that this research will look into the problems connected with the teaching of Mathematics in some of the above-mentioned classes.

1.3 STATEMENT OF THE PROBLEM

At present Mathematics is becoming more important because other subjects such as Economics, Chemistry and Physics are better understood through the help of Mathematics. It has become very important that Mathematics must be well taught and understood at the primary school level. In other words, pupils must be taught Mathematics in such a way that it becomes pleasant and interesting to them. Ineffective teaching of Mathematics may not only be due to its difficulty; but endeavours such as changing the classroom environment by grouping the pupils, promoting individual work, introducing a variety of audio-visual aids, introducing games, understanding the intellectual development of our pupils, mastering the subject matter may help pupils to understand Mathematics (Wheeler, 1950, p.23).

Other factors may contribute toward poor performance of pupils at the primary school level. Cultural background of the pupil, teaching methods applied at school, rote learning which may be encouraged at the expense of learning with insight, may also have an influence on the achievements of the children.

The following questions may also indicate problem areas which will have to be investigated: is the black pupil's formation of mathematical concepts different from that of his European counterpart? Is Setswana, which is the mother tongue in Bophuthatswana, sufficiently developed as far as the vocabulary is concerned for expressing the number concept as clearly as in English or Afrikaans? (Van den Berg, 1978, p.2). In this research an attempt will be made to find answers to the above-stated questions.

In this research the emphasis will be on the cognitive aspect of the child because the development of the abstract mathematical concepts is mainly the function of the intellect. Be that as it may, the conative and the affective aspects will be made use of whenever the need arises (Van den Berg, 1978, p.2).

In a research undertaken by J A T Wentzel in Lebowa primary schools, it was found that pupils performed badly in Mathematics. In this research, the researcher would like to establish the standard of performance of the pupils in the primary schools of Bophuthatswana in Mathematics. Thereafter, the researcher will compare the results of the research in Lebowa with that of Bophuthatswana.

1.4 AIM OF THIS RESEARCH

The aim of this research is to firstly determine the standard in Mathematics of the pupils and secondly, to determine why they fail Mathematics in the primary schools of Bophuthatswana. After identifying possible reasons for the pupils' under-achievement in Mathematics, the researcher intends putting forward some recommendations.

The following aspects will be covered in this study:

i) Research to determine the standard of Mathematics in the Standard 4 classes of Bophuthatswana primary schools.

ii) To determine whether the problem lies in the present syllabus.

iii) To determine which factors may have an influence on the achievements of the pupils.

iv) To compare the findings of this research with those of the research carried out in Lebowa primary schools by Wentzel.

1.5 SCOPE OF THE RESEARCH

The scope of this research will be confined to the problems encountered in the teaching of Mathematics in the primary schools of Bophuthatswana. That is, to point out whether the performance is poor or not. If it can be established that the performance is in deed poor, attempts will be made to pinpoint possible causative factors and the area or areas of difficulty in the Mathematics syllabus. Thereafter, recommendations will be made to help solve the problem.

This research will start with an intensive study of literature which will include the following:

a) A detailed study of research already carried out on the black child's mathematical ability.

b) A detailed study of the works of Jean Piaget, which deals with:

i) The development of the number concept.

ii) The concept of space.

iii) The concepts of length and measurement.

iv) The concepts associated with area and volume.

v) The concept of substance.

vi) The concept of weight.

vii) The developmental stages of the child.

c) A detailed study of literature which treats the following aspects in Mathematics:

i) Methods of learning Mathematics.

ii) Methods of teaching Mathematics.

iii) Games in Mathematics.

iv) Mathematical laboratory.

v) Audio-visual aids in Mathematics.

d) Collection of data from the Bophuthatswana primary schools through the help of:

i) Application of standardized tests: Scholastic Achievement test in Arithmetic, Standard 4.

ii) Diagnostic Mathematical Tests.

iii) Questionnaires.

e) Interpretation and analysis of the results of this research.

f) Comparison of the findings on the primary schools of Bophuthatswana and those of the primary schools of Lebowa.

1.6 METHOD OF RESEARCH

This research is undertaken with the assumption that pupils perform badly in Mathematics at the primary school level in Bophuthatswana because Mathematics is not taught properly and in some instances Mathematics is taught by mathematically under-qualified teachers. The aim of this research is firtsly therefore, to identify areas of difficulty and to recommend ways of improving the teaching of Mathematics. Secondly, the aim of this research is to compare the performance of pupils in Mathematics in the primary schools of Bophuthatswana with the primary schools of Lebowa.

Consequently the method of research will be as follows:

a) A review of literature on the topic with the aim of bringing this research in line with the research already carried out in the field of Mathematics education.

b) Interviews with inspectors, teachers of Mathematics and principals of schools which will be used as samples with the aim of gaining insight into the problems connected with the teaching of Mathematics.

c) A standardized test, namely, Scholastic Achievement Test in Arithmatic Standard 4, will be given to the pupils with the aim of determining their standard in Mathematics.

d) Diagnostic tests to determine specific problem areas.

e) A questionnaire will be sent to the teachers of Mathematics of the selected schools with the aim of determining the difficulties they encounter in teaching Mathematics, and to invite suggestions from them.

f) The annual report of the Department of Education of Bophuthatswana of 1982 will be consulted in order to obtain the following information:

i) Pupil-teacher ratio as in 1982.

ii) Teachers and their qualifications.

g) The research will end up with a comparison of the performances of pupils in the primary schools of Bophuthatswana with the performances of pupils in the primary schools of Lebowa as follows:

i) Comparison of the results of the standardized test - Scholastic Achievement Test in Mathematics, Standard 4.

ii) Comparison of diagnostic test results.

iii) Interpretation of the results of the standardized tests.

iv) Interpretation of diagnostic tests.

1.7 FORMULATION OF THE HYPOTHESIS

The researcher maintains that pupils perform badly in Mathematics in the primary schools of Bophuthatswana. This problem is also present in the middle schools (classes - Standards 5-7) and even in the high schools (classes - Standards 8-10) of Bophuthatswana (Lekhela, 1978, p.17). The solution to this problem may be the identification of the problem areas in the primary schools coupled with the correct approach to the teaching of Mathematics.

To achieve his aim, the researcher must look into the qualifications of teachers, their methods of teaching, available facilities and the role played by the medium of instruction.

The researcher's opinion is that, should teachers improve their knowledge of Mathematics, employ games in their teaching methods, make use of a mathematics laboratory and specialize in their teaching; they may be able to improve their pupils' performance in Mathematics.

1.8 RESEARCH INTO THE POSSIBLE REASONS FOR THE FINDINGS

Possible reasons for poor achievements as indicated by completed research, will receive the necessary attention.

1.9 FINDINGS AND RECOMMENDATIONS

The findings will be analyzed and recommendations made.

In chapter 2, we are going to focus our attention on the stages of cognitive development of the child, learning and teaching strategies appropriate to primary school Mathematics.

CHAPTER 2

PIAGETIAN COGNITIVE DEVELOPMENT RESEARCH AND MATHEMATICAL EDUCATION

Different authors give different views on the stages of development of the child. For the purpose of this research it will suffice to confine ourselves to Piaget's stages of development. In this Chapter, Piaget's stages of development will be discussed in detail. Thereafter the educational implications of his stages of development will also be discussed.

2.1 INTRODUCTION

Piaget's brilliant observations and analyses provide a framework for understanding the child's overall strategies in thinking during the pre-school period. In his view, cognitive or intellectual activities are adaptive. They function in the individual's adaptation to his environment. Intelligence is seen as an aspect of biologival adaptation, of coping with the environment, organizing and re-organizing thought and action. Thus adaptation involves an interaction between the processes of assimilation and accommodation. Assimilation refers to the fact that the child relates what he perceives to his existing knowledge and understanding (Mussen, et al, 1969, p.303).

The psychological basis of Mathematics teaching can only be understood by making a study of the cognitive development of the child as he progresses from infancy to maturity. The greatest contribution to our knowledge in this field has undoubtedly come from Jean Piaget. Piaget, assisted by his collaborators, has presented us, on the one hand, with a highly comprehensive theory of intellectual development, and, on the other hand, with a mass of experimental data which confirms his theory.

2.2 BASIC MATHEMATICAL PRINCIPLES

Piaget maintains that in order to understand Mathematics there are a number of mathematical concepts which the child must grasp. These concepts according to Van den Berg (1978, pp.22-23) are:

i) The concept of number.

ii) The concept of capacity.

iii) The concepts of substance, weight and volume.

iv) The concept of length.

v) The concept of area.

vi) The concept of time.

2.2.1 Development of the number concept

The search for an understanding of the development of number begins where the search for the origins of the child's intellectual development begins, in the sensori-motor stage. The initial foundations of number are laid down in the sensori-motor stage when the child through maturation and experience eventually develops the concept of the permanence of objects and the ability to distinguish among various objects (McNally, 1977, p.104).

At the same time as the ability to classify is developing, the child begins to concern himself with order and series. Through experience, he notes that one toy car is bigger than another or that a box is smaller than another one. This ability to order objects according to size is also an important element in the development of the concept of number.

Initially, the child does not ask the question "How many?", but simply wants to know to whom the object belongs. Gradually, by matching, he comes to realize that the number of elements in one set is more, less or the same as those in another as he compares. He has moved from a concern with the quality of objects to a rudimentary concern with quantity, an important aspect of which is the ability to regard each element as a unit (McNally, 1977, p.104).

To effect a discussion of how cardinal and ordinal numbers develop, one must look at the following problem. "Four books plus three books is how many books?" (McNally, 1977, p.104). First of all, it means that four (4) is the symbol that has been ascribed to the set of four objects irrespective of what these elements are. Four (4) in this example happens to stand for four books, but it can stand for the set of any four elements. This is the aspect of number which is known as cardinal number. The four (4) may also refer to the fourth position in a series of objects. This is the aspect of number which is known as ordinal number (McNally, 1977, p.109).

One point of importance in this regard is that although children can count before they can conserve numerical equivalence, this facility is no guide to a child's understanding of the cardinal property of number

(McNally, 1977, p.105).

The child comes to understand numbers through maturation and experience. This enables him to co-ordinate the logical operations, thereby providing a frame of reference necessary for the integration of the cardinal and ordinal properties of number (McNally, 1977, p.105).

Piaget notes that children in the sensori-motor stage constantly make a mistake of one unit and he explains this inability to co-ordinate cardinal and ordinal aspects of number in the following way (Piaget, 1964, p.111):

i) The development of the ability to conserve, including reversibility, as it applies to the co-ordinated concepts of classification, ordering and one-to-one correspondence, is fundamental to the understanding of numbers as a logical system.

ii) Classification in the sense of being able to group objects according to their similarities and differences is an important precursor to the understanding of number.

iii) Matching and sorting of objects are important experiences for children in the pre-operational stage because they lead to the opportunity to classify, to seriate one-to-one correspondence and to order. These also lead to the formation of sets of objects and to the comparison in terms of gross quantitative judgements such as, more, less or the same.

iv) Co-ordination of cardinal and ordinal properties of number does not take place until the stage of concrete operations. Research suggests that cardination and ordination develop in parallel fashion.

v) Understanding the processes of addition and subtraction depends upon conservation and reversability.

It has been found that pre-operational children lack conservation of number. Conservation is the awareness of the invariance of material despite certain transformations (Keats et al, 1978, p.27). The following is an example of a test item used in research. The child is presented with two rows of five plastic squares each, arranged in one-to-one correspondence. The tester says: "Here are two rows of plastic squares. This one can be yours and this one can be mine. Do we both have the same number of squares or do you have more squares or do I have more squares?" (both the same). The tester spreads out one row, or arranges it in a circle, or bunches it up. "Now do you have more or do I have more or do we both have the same number?" (I have more) (Keats, et al, 1978, p.28).

The pre-operational child is influenced by the perceptual features of the stimulus configuration. He may see that one row is now spread out, so he believes that there must be more squares in it. He does not realize that if the squares were moved back again, that is reversing the movement, there would be the same number as before. When he developes the idea of reversibility, his thoughts becomes truely operational.

Conservation of number is usually attained around six or seven years of age, but the level of general intelligence (Goodnow and Bethon, 1966, pp.573-581), socio-economic status (De Lacey, 1970, pp.293-304) and contact with Western culture (Heron and Simonsson, 1969, pp. 281-292) have been found to accelerate the rate of development. Conservation is a fundamental requirement in the understanding of number, for without it, a child cannot match sets by pairing to establish equivalent sets and may have difficulty in addition (Piaget, 1964, pp.8-9).

2.2.2 The concept of capacity

This concept is generally referred to in the research literature by the term "quantity" (Keats, et al, 1978, p.28), referring to the amount of material that a vessel contains. At the intuitive stage the child does not realize that the quantity of a liquid remains constant if it is poured from one container to another of a different shape. If two similar beakers have an equal quantity of water in them, the child will usually agree that they have the same amount of water, but if the contents of one is poured into a tall thin beaker or a short wide beaker, he would say that the new beaker contains more or less than the original untouched one. Children often say that because it is now higher in the tall thin beaker, it must have more in it. They are influenced by the perceptual impression, and do not reason logically. They lack conservation of quantity.

2.2.3 The concepts of substance, weight and volume

Another aspect of quantity is referred to in the research literature as conservation of "substance" or of "mass" (Keats et al, 1978, p.29). Here the emphasis is on the amount of matter in an object. The material used in the studies is plasticine or clay. It can be changed easily from a ball into a snake, a pancake, a cross, etc. Pre-operational children believe that such transformations change the amount of plasticine.

The method used to determine the conservation of matter, weight and volume is extremely simple. The child is handed a ball of modelling clay, together with a lump of the same material, from which he is asked to make another ball "as big and as heavy as the first." Once he is satisfied that the two balls are identical, the demonstrator changes the shape of one of them by pulling it out into a coil, a roll or sausage shape, by flattening it into a disc or by cutting it up into pieces. He asks the child if the two objects still have the same weight, quantity of matter, or volume. The child is expected to justify all his answers, so that it is possible to determine not only whether or not he accepts the idea of conservation, but also how he substantiates and elaborates it (Piaget and Inhelder, 1974, p.4).

This method has shown that the elaboration of conservation passes through three stages whose chronological appearance suggests that they must represent three distinct stages of intellectual development. The child becomes convinced that a change in the shape of the ball produces no changes in its weight, when previously he was certain that the weight varies with each deformation (Piaget and Inhelder, 1974, p.4).

The clay ball experiment reveals the successive nature of the elaboration of the principles of the conservation of matter, weight and volume. In this development four major stages are distinguished. During the first of these, up to about the age of seven or eight, the child does not grasp the conservation of substance, weight or volume. During the second, from the age of about eight to ten years, he grasps the conservation of substance but not those of weight and volume. During the third, from the age of ten to between eleven and twelve, he grasps the conservation of matter and weight but not yet that of volume. Finally, during the fourth stage, starting from eleven or twelve years, he grasps all three types of conservation (Piaget and Inhelder, 1974, p.5).

2.2.4 The concept of length

A young child is shown two sticks placed parallel and he agrees that they are equal in length. When one stick is moved a centimetre or more in the direction in which it points, he will say that they are no longer equal in length. The pre-operational child measuring with a hand-span would say that an object is longer than his pen, when in reality it is equal. He fails to align the pencil and object accurately. He does not understand the meaning of measurement (Keats, et al, 1978, p.29).

2.2.5 The concept of area

The following example of a test item measures conservation of area. Two pieces of equal-sized green cardboard are presented and described as paddocks of green grass. A toy cow is placed on each and the child agrees they would have the same amount to eat. Then five small blocks representing houses are placed on each paddock in such a way that the child is aware each gets an equal number. On one paddock the houses are placed in line, adjacent to one another, but on the other they are scattered haphazardly. The position of the houses gives the child an illusion of more grass on the first than on the other. The child is asked if the cows still have the same amount to eat, or if one has more than the other. The child in the pre-operational stage who lacks conservation of area, will be unable to understand the meaning of area, even though he may be able to count the number of squares in a rectangle (Keats, et al, 1978, pp.29-30).

2.2.6 The concept of time

2.2.6.1 The sequence of events

To determine time, we must appeal to causal operations, that is, establish a chain between causes and effects by explaining the latter in terms of the former. Time is inherent in causality (Keats, et al, 1978, p.29).

The child's conception of time begins with an examination of the way in which children link two events into a simple causal chain, for instance the motion of falling objects. The child is presented with photographs of a falling body at various phases of its descent chosen at random and asked to put these into the right order. This technique demonstrates the non-intuitive way in which children group time sequences. Before the age of seven or eight, children are not yet capable of reasoning about several possibilities at the same time. In other words, they lack the power of operational reversibility needed for the selection of various possible orders, whereas eight-year-olds can make use of that power and thus re-construct true and irreversible events (Piaget, 1969, p.6).

2.2.6.2 Duration

The child's conception of duration is a test of his grasp of the order of events. A closer analysis of duration will corroborate the view that time concepts emerge as motions become increasingly co-ordinated. In effect, during the first stage the very concept of duration or time intervals is so imprecise that if a child is asked to state whether the time it takes the water to run from I_1 to I_2 is longer, shorter than or the same as the time it takes from II_1 to II_2, he not only fails to realise that the two are equal because he sees the level in II rising more rapidly than in the level drops in I. Also he fails to grasp that velocity is inversely proportional to time (Piaget, 1969, p.35).

During the second stage the child discovers this inverse relationship and begins to appreciate that time intervals may be divorced from speed or distance, but is still unable to co-ordinate these intervals. Thus he continues to hold that the duration I_1, I_2 is longer than the duration II_1 II_2, simply because the water level drops more slowly in I than it rises in II. Finally, at the third stage, the child succeeds in correlating durations with the correct order of events (Copeland, 1974, pp.168-169).

2.2.6.3 Succession of perceptible events

Children have great difficulty in reconstructing the correct succession of even so simple a series of events as the flow of a liquid. This is because two distinct problems are involved: Firstly, the construction of the correct order after the event and secondly, the correct perception of this order while the event is actually taking place (Keats, et al, 1978, p.30).

Concerning the second problem, two bodies starting side by side, moving with the same speed and stopping (a) successively and (b) simultaneously, could be presented to the child. At each of the three stages the child would have no difficulty in establishing the succession or simultaneity of the stopping points. To analyse the child's idea of temporal succession, bodies moving at different velocities could be introduced (Piaget, 1969, p.83).

The child may be presented with two small figures or mechanical snails moving across a table, either at different but continuous velocities, or else by fits and starts, each start being accompanied by a rap on the table. In this case there can be no failure to perceive the synchronism of the two runs or the order of succession or simultaneity of the final stopping points (Copeland, 1974, p.169).

The result obtained with these experiments fit into three stages. During the first stage successions and durations remained undifferentiated from distances. During the second stage the initial intuitions slowly became differentiated because simultaneity was recognised independently of positions or velocities or because duration was understood to be inversely proportional to velocity. During the third stage the subjects became capable of applying the technique of operational grouping to all the relations involved, and went on to construct a coherent system involving both durations and successions (Piaget, 1969, pp.85-86).

2.2.6.4 Simultaneity

The children may be presented with the experimental setup under the following new conditions: the two figures I and II are set off together from the same starting line (A_1 and A_2), move in the same direction and stop together, but I is moved more quickly than II, with the result that they come to rest at a distance of some 3cm to 4cm from each other, I stops in C_1 while II stops in B_2 (Piaget, 1969, p.98).

According to their answers, the subjects can be fitted into three stages. At the first stage, they fail to grasp the simultaneity of the two end points, often even that of the starting points and also the fact that the two figures move for the same length of time. They argue that I takes longer than II because it goes further or more quickly and may think that II stops first because it covers a smaller distance. During the second stage they still deny the simultaneity and equality of the two synchronous durations, but maintain that II goes on for a longer time because it moves slower or else discover the simultaneity of the motions but still deny the equality of the synchronous durations. A very few subjects under certain conditions, grasp both the equality of the synchronous durations and also the simultaneity of the starting and end points. Later in this stage these various advances begin to become co-ordinated. During the third stage, finally, simultaneity and the equality of synchronous durations are appreciated and correlated (Copeland, 1974, p.169).

2.2.6.5 The equalization of synchronous durations and the transitivity of equal time relations

Even when the child appreciates that two bodies start and stop simultaneously, he does not necessarily conclude that their motions are of equal duration. An experiment which explains this behaviour may be conducted as follows: A large vessel is allowed to empty through an inverted and tapering tube yielding two identical jets of water. The water is collected in small bottles or glasses of different shapes and dimensions. A single tap controls both of the Y-tube so that the water can clearly be seen to start and stop running simultaneously. If the two bottles are of the same shape and dimensions, the water will rise to the same level in both. In that case the equality of the synchronous durations is invariably recognized. On the other hand, if the two bottles do not have the same shape, and equal quantities do not rise to the same level, children at the first two stages will deny the equality of the time of flow (Piaget, 1969, p.121).

2.2.6.6 The colligation of durations and the transitivity of unequal time relations

As soon as the child discovers the equality of synchronous durations, he becomes capable of grasping the transitivity of equal durations or of equal quantities of water. He therefore realises that if A = B, and B = C, then A = C (Copeland, 1974, p.170).

A simple experimental technique may be adopted to illustrate this principle. The subjects are presented with a reservoir discharging water through a Y-tube and with ten bottles of increasing capacity, A, B, C,...J. These bottles are of various shapes so that it is impossible to judge their capacity and the time needed to fill them. Two bottles are chosen at random and the subject is asked, firstly, which of the two will be filled more quickly? Secondly, he has to supply the reason. Thirdly, whether it will take more or less time to fill than the other. Having obtained the answers, the subject may be presented with the following two problems. Firstly, he may be asked to arrange three or four botles in increasing order of filling time, and secondly, he may be asked about the transitivity of colligations: If $A < B$ and $B < C$ as demonstrated by the experiment, is $A < C$ or $A > C$? (Piaget, 1969, pp. 141-142).

By their answers, all the subjects can be fitted into three stages. During the first stage they will be incapable of arranging three bottles by the method of comparing two at a time, and of making any logical deductions as to the durations. During the second stage they will have learned to compare two bottles at a time, but fail to co-ordinate different pairs of bottles while at the third stage they will be capable not only of constructing seriations or colligations, but also of grasping the resulting transitivity (Keats, et al, 1978, p.30).

2.2.6.7 Measurement of time

The fundamental postulate on which all time measurement is based is the existence of motions that take the same time to recur under the same conditions. This interpretation of the isochronism of repeated actions involves a vicious circle because in order to ascertain the isochronism of a given motion, one have to measure its duration by means of other motions whose isochronism depends on measurements which postulate it. The vicious circle becomes opened up as the coherence and range of the results increases and as the postulate of isochronism becomes transformed into a principle of conservation of velocities (Copeland, 1974, pp.170-171).

The following type of experiments of the measurement of time could be undertaken: The child could be given a large sand-glass, 45cm high, and such that successive sand levels could readily be distinguished. The lower part of the sand-glass, in which the sand collects, is marked so as to obviate any kind of equivocation. The upper part bears three gradations: a white line, 3/4 way up, a green line, 1/2 way up and a blue line 1/4 up, corresponding to equal and successive moments. The child may be introduced to the principle of time measurement by being asked to correlate various stages in his own work, for example, transferring small marbles from one jar to another, with the arrival of the sand at the white, green and blue lines. He is also asked to compare his work or measurements at different speeds with the flow of the sand (Piaget, 1969, p.177).

During the first stage the child thinks the sand runs more or less rapidly according to the speed of the work or motion whose duration he is asked to time. During the second stage the child grasps the idea of isochronism and the conservation of velocities. During the third stage the child succeeds in transforming the spatial units into time units (Piaget, 1969, p.195).

2.3 PIAGET'S STAGES OF COGNITIVE DEVELOPMENT

According to Piaget, action, rather than perception, is the primary source of knowledge (Rosskopf, et al, 1974, p.2). To know objects, one has to modify them in some way - for instance, by changing their positions. The division into developmental stages is therefore based on the character of the actions that link the subject to the surrounding world.

2.3.1 The sensori-motor stage (from birth to two years)

The sensori-motor stage cannot be classified as a stage of thought. Real thinking, according to Piaget, does not exist until the child reaches the concrete operational stage. Real thinking for Piaget is logical (McNally, 1977, p.14).

Another feature of the development of thought is the gradual progression from a situation where the environment is overwhelmingly in control, that is, to a situation where the thought processes respond to the logic of the situation and judgements are made on the logical relationships existing among the objects, statements, events, and not in terms of what appears to be (McNally, 1977, p.14).

The sensori-motor stage can be divided into six sub-stages.

2.3.1.1 General assimilation sub-stage (0-one month)

The point of departure of development should not be sought in the reflexes conceived as simple isolated responses, but in the spontaneous and total activities of the organism. There are relatively fixed and predictable reflexes embedded in this total activity, but they can be viewed as a differentiation of this global activity. Some of these reflexes are developed by exercises instead of remaining unchanged or atrophying and are the points of departure for the development of schemes of assimilation (Piaget & Inhelder, 1969, p.6).

The new born child possesses the basic reflexes of sucking, grasping, crying and the movement of parts of the body. Initially the child sucks at anything, regardless of what it is and will grasp automatically any object which touches the palm of his hand. This substage is called the general assimilation sub-stage because in the first few weeks of life the infant is merely exercising already existing schemes. Thus it can be said that the infant assimilates all environmental events into the existing reflexive schemas in an undifferentiated way (McNally, 1977, p.15).

Gradually the infant modifies his schema as accommodation force him to adapt and he becomes adept at distinguishing between satisfying and non-satisfying objects (McNally, 1977, p.15). His behaviour begins to exhibit the first signs of direction as he seeks and searches for the nipple. This pattern of behaviour was not present at birth. At this stage the child is not yet engaged in intentional behaviour.

2.3.1.2 Primary circular reaction sub-stage (one to four months)

By the end of sub-stage one the infant is able to distinguish in a rudimentary way between objects. He will reject, if hungry, an object placed in his mouth which does not produce milk. This pattern of progressive integration and development of schemas is continued in this sub-stage with the co-ordination of such schemas as thumb sucking and moving the head in the direction of sound (McNally, 1977, pp.15-16).

A circular reaction is one in which the completion of the responses pattern or sequence is the cue for its repetition and can be seen to operate when the child accidentally causes an interesting or satisfying effect and seeks to reproduce and maintain this satisfying state of affairs (McNally, 1977, p.16). An example of this is when the child moves his hand across his field of vision and later repeats it. It is called a primary circular reaction because it is centred on the infant's body.

In this sub-stage the child is beginning to move his attention to the environment although the environment is still in control. Although the child's actions seem to reflect a sort of magical belief in causality without any material contact, his use of the same means to try to achieve different ends indicates that he is on the threshold of intelligence (Piaget & Inhelder, 1969, p.8).

2.3.1.3 Secondary circular reactions sub-stage (four to eight months)

The most interesting aspect of the development of sensori-motor actions during the first year of the child's life is that it not only leads to elementary learning experiences which are the source of simple habits on a level where intelligence, strictly speaking, is not yet observed, but it also provides a continuous series of intermediaries between habitual and intelligent reactions. Thus after the reflex sub-stage, 2.3.1.1, and the sub-stage of the first habits, 2.3.1.2, a third sub-stage, 2.3.1.3, introduces the next transitions after the beginning of co-ordination between vision and prehension - around four and a half months on the average (Piaget & Inhelder, 1969, p.9).

The circular reactions are now directed to objects in the vicinity of the child as well as to his own body. They are secondary reactions because they involve the co-ordination of activities which are no longer reflexive. Thus the reaching, grasping and listening activities are co-ordinated into a higher order schema such as pulling a ribbon on a toy. During this sub-stage, progress is made toward the permanence of objects. He will make an attempt to look for an absent object, but he does not perservere long (McNally, 1977, p.16).

As the infant becomes more adept at the co-ordination of sucking, hitting, grasping, pulling, seeing, etc., he begins to move things about and at this stage an elementary concern for the relation of objects to one another develops. At this stage the infant begins to become concerned with the positions of objects around him as well as the actions he can apply to them. Despite progress with respect to object permanence, the child remains egocentric during this sub-stage (McNally, 1977, pp.16-17).

2.3.1.4 Co-ordination of secondary schemas sub-stage (eight to twelve months)

The child is moving towards the end of the first year of life and the co-ordination and development of schemas provide further advances. The concept of the permanence of objects is now fairly well established and the infant comes to see objects other than himself as "causes" (McNally, 1977, p.17). At this sub-stage the child develops intentional relationships and the child begins to anticipate events (Elkind, 1976, p.85).

In this sub-stage we observe more complete acts of practical intelligence. The subject sets out to obtain a certain result, independent of the means he is going to employ, for example, obtaining an object that is out of reach or has just disappeared under a piece of cloth or a cushion. The instrumental acts appear only later and they are seen from the outset as means, for example, seizing the hand of an adult and moving it in the direction of the unreachable object, or lifting the screen that masks the hidden object (Piaget & Inhelder, 1969, pp.10-11).

The infant is now able to examine objects more effectively, turning them around as he moves them closer and further away. Through such behaviour the infant comes to realize that the objects remain the same no matter what position they are in. This can be seen clearly when a feeding bottle is presented to the infant the wrong way round and he quickly turns it (McNally, 1977, p.17).

2.3.1.5 Tertiary circular reactions (twelve to eighteen months)

Tertiary circular reactions get their name from the fact that at this stage the child actively experiments in order to investigate the properties of objects.

In the course of the fifth sub-stage, 2.3.1.5, which makes its appearance around eleven or twelve months, a new ingredient is added to the foregoing behaviour, the search for new means by differentiation from schemes already known. An example of this is what is called the "behaviour pattern of the support" (Piaget & Inhelder, 1969, p.11). An object is placed on the rug out of reach of the child. The child, often trying in vain to reach the object directly, may eventually grasp one corner of the rug, and then, observing a relationship between the movements of the rug and those of the object, gradually pulls the rug in order to reach the object

(Piaget & Inhelder, 1969, p.11).

During this sub-stage, the infant is making considerable progress with respect to the permanence of objects. If he sees where an object is hidden, he will search for it there (Piaget & Inhelder, 1969, p.11).

2.3.1.6 Beginning of thought (eighteen to twenty-four months)

Finally, a sixth sub-stage 2.3.1.6 marks the end of the sensori-motor period and the transition to the pre-conceptual stage. During this stage the child becomes capable of finding new means not only by external or physical groping, but also by internalized combinations that culminate in sudden comprehension or insight (Piaget & Inhelder, 1969, pp.11-12).

At this sub-stage there is a further development in the child's concept of the permanence of objects. The child is now able to search for and find objects which are hidden. This means that the child is able to keep in mind an image of an object and to realize that no matter where the object is, it still exist. This superior ability to represent objects mentally has a significant effect upon the child's development of motions of causality (McNally, 1977, p.18).

2.3.2 Pre-operational stage (two to four years)

The structure of the pre-operational stage is considered by Piaget to be function logic (Piaget & Inhelder, 1969, p.141). Piaget considers that this function logic differs from the operations which characterize the concrete operational stage in that the former lacks reversibility. This is consistent with the priority given by Piaget to the concept of reversibility in all his accounts of cognitive processes.

This stage is sometimes called the symbolic period. By symbolic function is meant the ability to represent something such as an object, event or conceptual schema by what Piaget refers to as a signifier. This can be language, a mental image or a symbolic gesture (McNally, 1977, p.20).

During the pre-operational stage the child exhibits the following important characteristics:

2.3.2.1 Formation of mental symbols

For Piaget the formation of mental symbols can be traced directly to imitation which begins in the sensori-motor stage. Piaget gives the example of a sixteen months old girl who sees a playmate become angry, screams, and stamp her foot. The child has not seen this happen before, but an hour or two after the departure of the playmate, the child imitates the scene, laughingly. Piaget refers to this as deferred imitation which he believes constitutes the beginning of representation (Piaget & Inhelder, 1969, p.146).

One very important aspect of the formation of mental symbols is the meaning that is attached to the words. As the child grows older the symbolic function will involve both symbol - what the object means to the child - and the word.

2.3.2.2 Play

By symbolic play, as the name suggests, Piaget refers to the way in which young children use an object or situation to stand for another while playing. Play is then largely assimilation while imitation is largely accommodation (Piaget & Inhelder, 1969, p.57).

During this stage symbolic play is concerned primarily with affective conflicts (Piaget & Inhelder, 1969, p.57).

2.3.2.3 Drawing

In his work on symbolic function, Piaget draws attention to drawing, which, like symbolic play, gives the child pleasure and also assists him to interpret the world by imitating the real. For Piaget, drawing is imitative accommodation (Piaget & Inhelder, 1969, p.63).

Initially at two to two and a half years, the child merely scribbles and this is purely play. Until about eight or nine years the child's drawing is realistic in intention. The child at first draws what he knows about people or objects before he is able to draw what he sees (McNally, 1977, p.24).

2.3.2.4 Language

The child is first able to use words in the sensori-motor stage, but does not use words in a symbolic way. The meaning of words to children of this stage is not constant.

The child begins in the sensori-motor stage by using one word sentences. By the end of the second year he has progressed to two word sentences. From two to four years the use of language expands rapidly and the child gradually masters the rules of syntax while his understanding of the spoken word also increases. During this stage the child gradually attains the ability to use words to stand for absent objects, events, actions and wishes (Piaget & Inhelder, 1969, p.86).

2.3.2.5 Reasoning

A child of two or three years cannot reason either inductively or deductively, but, instead, reasons transductively (McNally, 1977, p.26). In transductive reasoning, the child goes from particular to particular without any apparent logical connection. An example of this was given by Piaget. He reports that on an afternoon when Lucienne did not take a nap she said: "I haven't had my nap so it isn't afternoon." (McNally, 1977, p.26). At this stage thinking is related to the wishes of the child as can be seen from the above example. The child's desires distort his thinking and also the child's reasoning is aimed at achieving a personal goal.

Transductive reasoning prevents the child from forming true concepts. He is unable to distinguish between "all" and "some" (Piaget & Inhelder, 1969, p.89).

2.3.3 The intuitive stage (four to seven years)

Although the child is capable of distinguishing between reality and the symbol, the distinction becomes blurred at times. For much of the time he lives in a world of his own. Piaget describes his language as egocentric because its main purpose is not communication with other persons (Piaget & Inhelder, 1969, p.84). Although he is learning to discriminate between objects and persons in his immediate surroundings, his classifications are still very primitive. His reasoning is dominated by what he perceives. Piaget refers to this period as the stage of intuitive thought (Hyde, 1970, p.26).

At this stage his thought has developed to the stage where he can give reasons for his actions and for his beliefs. He is unable to classify at a higher level, for example, sorting coloured shapes either by colour or shape. Language too progresses rapidly. Thought at this stage is restricted by two things, namely, immediate perceptions and by the fact that he is unable to keep in mind more than one relation at a time (McNally, 1977, p.27).

Quite early in his use of language, the child begins to use words like more or less, the same as, taller or shorter than, etc. and also the number words (Gourlay, 1967, p.312). The child does not use them in a constantly and logical fashion as Piaget has demonstrated in the following experiment (Piaget, 1964, p.6).

The child was presented with two jars A and B, filled with the same amount of water. On being questioned the child said that there was the same amount of water in the jars. If the contents of jar A was poured into the long thin jar B, the child at the intuitive stage would either say:

a) There was more water in B than in A, because B is taller; or

b) there was less water in B than in A, because B was thinner.

The child at the intuitive stage is dominated by what he sees. He has not yet built up in his mind the cognitive organization of the older child who can apply this organization to correct immediate sense impressions. The child at the intuitive stage reacts simply to what he sees and not always consistently. Sometimes his answer will be determined by the lengths of the jars and sometimes by the width (Piaget, 1964, p.8).

The jars-experiment brings out two of the important characteristics of the intuitive stage, namely:

i) That the child can go back and forward in his thought in carrying out these operations - this is what Piaget terms "reversibility"; and

ii) in so doing, the quantities or numbers involved remain unaltered - what Piaget calls "conservation" (Gourlay, 1967, p.413).

2.3.4 Concrete operational stage (seven to eleven years)

In this stage actions have become internalized thought operations. These are sufficiently co-ordinated to make logical thinking possible although the child's reasoning is still imperfect at this stage. Piaget describes it as concrete because it only operates in a specific situation whereas later on he will be able to generalize from one situation to another. Perception still plays a part in his reasoning, but he depends less on it. He is most successful in a situation where he is able to manipulate actual objects and materials in the course of reasoning about them. For instance, in the case of conservation tests, he can by using his own actions, return the deformed ball of plasticine to its original shape. He has achieved reversibility (Hyde, 1970, p.27).

Logical thought emerges when a certain basic stock of concepts have been acquired and when these concepts have been organized into coherent systems. The concepts which figure in operational thought are called "operations" because they are internalized responses. Three kinds of concepts, according to Berlyne, are of particular importance (Berlyne, 1965, pp.183-184):

2.3.4.1 Classes

The concept of a class or the operation of classification is an internalized version of the action of grouping together objects recognized as similar.

2.3.4.2 Seriation

Seriation consists in arranging elements according to increasing or decreasing size. There are intimations of this operation on the sensori-motor level when the child of one and a half or two years builds, for example, a tower of two or three blocks whose dimensional differences are immediately perceptable. Later, when the subject must seriate ten markers whose differences in length are so small that they must be compared two at a time, the following stages are observed: (i) firstly, the markers are separated into groups of two or three, each seriated with itself but incapable of being co-ordinated into a single series; (ii) secondly, a construction by empirical grouping in which the child keeps rearranging the order until he finally recognizes he has it right; (iii) lastly, a systematic method that consists in seeking first the smallest element, then the smallest of those left over, and so on. In this case the method is operatory, for a given element is understood in advance to be simultaneously larger than the preceding elements and smaller than the following elements (Piaget & Inhelder, 1969, pp.101-102).

2.3.4.3 Number

Number results primarily from an ignoring of differential qualities which renders every individual element equivalent to any other element as one orange is equivalent to one tree or equivalent to one person as far as number is concerned. Once this is established, sets are classifiable according to inclusion: $1 < (1+1) < (1+1+1)$, etc. They are also seriable, and the only way to count them apart and not to count the same element twice in these inclusions is to serialize them in space or time, for example, one and then another one, and so on (1→1→1→ ; etc.). Number therefore appears as a synthesis of seriation and inclusion (Piaget & Inhelder, 1969, p.105).

The number system is the joint product of classification and ordering. The number 6, for instance, depends on the operation of grouping 6 objects together to form a class and that of placing 6 between 5 and 7 in the sequence of natural numbers.

Systems of operations are called "groupings" and their stability depend on their having four properties. Unless these properties are present, the relations between the elements of a group will change as attention is directed to different parts of them. According to Berlyne the properties are (Berlyne, 1965, pp.184-185):

2.3.4.3.1 Closure

Any two operations can be combined to form the third operation, for example, 2+3=5; all men and all women = all human adults.

2.3.4.3.2 Reversibility

For any operation there is an inverse operation which cancells it, for example, 2+4=6, but 6-4=2; all men and all women = all human adults, but all human adults except women = all men.

2.3.4.3.3 Associativity

When three numbers are to be combined, it does not matter which two are combined first, for example, (2+3)+4=2+(3+4); Jane, Mary and Helen, or Helen, Mary and Jane.

2.3.4.3.4 Identity

There is a "null operation" formed when any operation is combined with its inverse, for example, 2-2=0; I travelled 10km to the North and I travelled 10km to the South = I find myself back where I started.

2.3.5 Abstract or formal operational stage (twelve years and upwards)

The cognitive structures which the child acquires at the concrete operational stage develop out of the child's handling of concrete material and then internalizing the operations which he performed on these materials (Gourlay, 1967, p.415).

Gradually the concrete aids needed by the child in his thinking become redundant and he becomes capable of what Piaget regards as "logical thinking" (Hyde, 1970, p.27). This development is greatly assisted by his increasing mastery of language. Other developments take place, for example, his moral values, which previously depended very heavily on the attitudes and precepts of his parents, become organized into autonomous systems (Hyde, 1970, p.27).

Children of this stage are known as adolescents. According to Piaget, one of the most striking characteristics of the adolescent is his interest in theoretical problems not related to everyday realities, and his facility for elaborating abstract theories. All of them have systems and theories that transform the world in one way or another. Having reached the stage of formal thought, the adolescent is able to draw conclusions from pure hypotheses and not merely from actual observation. This explains why formal thinking represents so much more difficulty and so much more mental work than concrete thought (Piaget & Inhelder, 1969, pp.130-131).

According to Piaget, personality, which has also been developing achieves its final form in adolescence. Personality results from submitting oneself to some kind of discipline. Its strength lies in a combination of personal autonomy and social co-operation. It cannot mature until the intellectual conditions of adolescence exist (Piaget & Inhelder, 1969, p.131).

The adolescent is constantly meditating about society. He wants to reform society. He has nothing but disdain or disinterest for the real society he condemns. As he comes into contact with the reality of the world in which he lives, he gradually adapts to it and disequilibrium gives way to equilibrium (Piaget & Inhelder, 1969, p.131).

2.4 EDUCATIONAL IMPLICATIONS OF PIAGET'S WORK

Piaget's work may have educational implications for the teaching of Mathematics. His theory of intellectual development may have the following educational implications:

2.4.1 Teachers should become thoroughly familiar with Piaget's theory of intellectual development so that they may be aware of how their pupils organize and synthesize ideas. Teachers may gain extra insight if they analyze their own thinking, since they may discover that in some situations they operate at a concrete rather than at an abstract level.

2.4.2 If possible, the teacher must assess the level and the type of thinking of each pupil in his class. The teacher should ask individual pupils to perform experiments similar to some of Piaget's experiments and evaluate the pupils' explanations of their reactions.

2.4.3 The teacher should keep in mind that learning through activity and direct experience is essential. Audio-visual teaching aids are necessary for pupils to learn on their own.

2.4.4 The teacher should arrange situations to permit social interaction so that pupils may learn from one another. To achieve this, the teacher should group able and unable pupils together. Homogeneous groupings are not preferred to heterogeneous grouping.

2.4.5 The teacher should plan his learning experiences to take into account the level of thinking attained by an individual or group. The teacher should encourage the pupils to classify things on the basis of a single attribute before he exposes them to problems that involve two or more attributes. The teacher should give his pupils chance to explain answers so that he can be aware of their level of thinking.

2.4.6 The teacher should keep in mind that the pupils may be influenced by egocentric speech and thought. He should also make each pupil aware of the fact that everyone has the same conception of a specific word. If confusion becomes apparent or if a pupil becomes impatient about his or her failure to communicate, the teacher should request an explanation in different terms. If still it does not yield the desired results, the teacher may ask several children to explain their conception of the problem.

2.4.7 In conclusion, Piaget's theory therefore lends considerable support to the activity methods in the teaching of Mathematics in the primary schools.

2.5 THE MATHEMATICAL ABILITY OF THE BLACK CHILD

One of the tasks of this research is to look into the black child's mathematical ability. Furthermore, it must look into the cultural background of the black child, with the aim of looking for factors which seem to hinder him ro realize his innate intellectual potentialities. The role which is to be played by the school is to be determined. Is the school able to develop the black child's mathematical abilities in a meaningful way or not? If not, what are the factors which impede the realization of this goal?

In order to determine the contribution expected from the school to develop the black child's mathematical abilities, the cultural milieu of the child must be investigated (Van den Berg, 1978, p.5).

2.5.1 The traditional milieu of the black child

The traditional domestic environment of the black child is characterized by its poverty in terms of Western standards. The size of the traditional black hut is well-known. One room often serves as kitchen, sitting room and bedroom. The occupants normally sleep on the ground. The commodities are few and inadequate. Domestic articles such as tools, books, magazines, pictures, furniture or toys are conspicuously absent (Groenewald, 1976, p.15).

Apart from the physical domestic environment of the black child, simplicity is also a characteristic of his daily way of life. The tasks which he has to perform and the situations confronting him require no particular intellectual effort. The truely traditionally orientated black child grows up in a relatively unchanging world. With regard to their daily pattern of living, the boys are attuned to rural matters. Herding goats, sheep and cattle is their main task from the age of six to seven. Simple weapons and tools such as the assegai, war axe, clay pot, basket, hoe and pick-axe developed from their experience with the world. For the younger generation to continue to exist they must acquire the same knowledge, master the same skills, contract the same attitudes and orient themselves in the same way as their ancestors (Groenewald, 1976, pp.15-16).

The westernized black child lives in a complete house which has bedrooms, diningroom and kitchen. Many household implements are present. In most cases there is a radio and perhaps a television set in the house. He attends school and some even attend Sunday schools and church services. Briefly stated, the present-day black child seems to lead a life of Western culture.

The most important aim of the education of the black child is to introduce him to the habits and customs of the tribe into which he was born. The education of the child is directed at his integration in a preponderantly static pattern of life. From the moment of his birth the child is a member of a group and the group or community exerts a strong formative influence on him. The demands of the tribal ethos are expressed on traditional folklore, its proverbs and tales, in the system of reverential restraints or taboos, in the current social comment on the actions of the children and the educational abilities of the parents (Groenewald, 1976, p.16).

In this westernized cultural environment the educational aim is different from that of the traditional child. His aim of education is to realize his innate potentialities and to express them in all spheres of life, be it on the economic sphere, social sphere, religious sphere or political sphere. The tribal norms are kept by some families which observe them up to now.

No written language existed in the original traditional black society. The history, traditions and knowledge accumulated through the years were preserved by narrative repetition. The accumulated knowledge of the tribe was assimilated in stories, legends, proverbs and riddles and is handed down to posterity by oral repetition. A good memory is therefore held in high regard. Tribal stories are learnt and memorized by the black child through repetition (Groenewald, 1976, pp.17-18).

Presently, the black culture has its own written language. Important events and traditional culture appear in writing. There are authors who write novels, dramas, short stories and poetry. Early in the primary schools the medium of instruction is the mother tongue but the medium of instruction from Standard 3 is English. There seems to be the tendency to rely on rote learning. The possible reason may be the medium of instruction. Therefore the pupils seem to incline to memorization due to lack of understanding.

The traditional black child accepts the absolute authority of the adult (Van den Berg, 1978, p.7). This type of obedience which is expected from him may dampen his initiative, originality and creativeness.

Apart from the fact that black mothers, as a result of numerous obligations, do not have enough time to devote to their children and are unable to provide the necessary material conditions which promote intellectual development, it is also true that the traditionally oriented black parents are not adequately developed to do justice to their children in this respect. The parents are not really able to activate and stabilize the child's interest and to enlarge his horizons. Curiosity is not encouraged and intellectual activation by drawing the child's attention to new aspects in their environment is inadequate. Neither are illiterate parents of school-going black children able to provide indispensable assistance with school work (Groenewald, 1976, p.19).

Maternal love for and interest in the child, which in Pedagogics and Psychology have been confirmed as of the utmost importance for creating a climate beneficial to intellectual development, are prominent and intense during the first two years of life of the black child, but after weaning, which, as a rule, takes place traumatically, there is often a visible decrease in the attention parents devote to the child. As a rule the elder daughters then begin to take over the mother's obligations concerning the care of the baby (Groenewald, 1976, p.20).

The westernized mother as such does not give her child much of her time as most parents are working in the industries, mines, or perform professional work or serve as domestic servants. The child is left in the care of his grandmother, a domestic servant or attends a creche. The present-day school-going children do have a chance of receiving help from their parents as most of their parents are literates.

2.5.2 The realization of the black child's intellectual potential within his traditional cultural milieu

A child forms his impressions of and attitudes toward materials in his environment from the age of three. The crucial age for the development of the child's intellectual potentialities is between the ages of ten months and eighteen months, and the child's future competence is determined by the mother's actions during this period (Van den Berg, 1978, p.8). These first impressions and attitudes form the basis for the child's future intellectual development. In the case of a poor home environment the intellectual development of the child may be retarded (Van den Berg, 1978, p.8).

The attitudes of the child's parents and his elders may also dampen his intellectual growth. Few intellectual demands are made on the black child. Supersition plays an important role in the lives of black people (Van den Berg, 1978, p.8). These factors inhibit the natural curiosity of the black child and may dampen his intellectual development.

The black people's emphasis on the child as a member of a group as opposed to the encouragement of individuality may also impede the child's intellectual development. Other factors which may influence the development of the intellectual potentialities are the religious ceremonies that are performed according to tribal traditions and the importance of witchcraft which is widespread (Van den Berg, 1978, p.9).

2.5.3 Mathematics in the lives of the traditional black people

Mathematics through the ages has been developed by trade among various civilizations. Once the economic activity among a group of people develops, then the need for more sophisticated ways of counting and computing is created (Van den Berg, 1978, p.9).

Since the primitive tribal black society had in the past only a subsistence economy, little contact with other people took place. Contact with other people means the transmission of knowledge and especially ways of conducting trade.

The traditional black people had had no need for a more involved system of computation. Consequently the following aspects characterize the numeration system of the traditional tribal black (Van den Berg, 1978, pp.10-14):

a) Setswana's numeration system has ten as its primary base. This is so because counting is practised as a one-to-one correspondence with the fingers.

b) Gesture counting is used simultaneously with the spoken language. Similarly, the Setswana words for the numerals are derived from the gestures made to illustrate the numerals. For example:

Numeral in Setswana	Derivation	Finger gesture
nngwe	state of being alone	extend left small finger
pedi	raise second finger	extend left ring finger
tharo	raise third finger	extend three fingers
nne	raise fourth finger	extend four fingers
tlhano	all the fingers united	extend five fingers
thataro	take the right thumb	extend right thumb
supa	to point	extend right thumb and index finger
robedi	bend two	extend three fingers on the right hand
robong	bend one	extend four fingers on the right hand
some	cause to stand	extend all fingers

c) Since the black words for the numerals are in many cases derived from the finger gestures used to illustrate them, the large numbers become long and unwieldly descriptions. For example, in Setswana the number 3 333 is spoken of as "ketetharo-kgolotharo-sometharo tharo."

The abstract cardinal number did not exist in the traditional tribal black people's language.

The number was always coupled to some specific object, for example, four stones. Large numbers were never used by the traditional black before his contact with Europeans.

d) In the traditional rural setting no currency of any kind was used before the arrival of the Europeans. Trade transactions took place through barter. As trade increased some goods came to be used as currency, for example, cattle, shells and salt.

Scientific investigations showed clearly that the black child, because of his orientation towards the concrete, unlike the white child, does not develop a more analytical and systematic way of thought. The development of thought to abstract levels is hampered (Duminy, 1968, p.70).

The aforementioned concrete directedness of blacks is clearly illustrated by the traditional numeration. Grammatically the number always has to correspond with the kind of object that is being counted. The following example illustrates the roundabout way of naming larger numbers. In Setswana, for example, the number 122 is named as follows: Kgolo-somepedi-pedi. In other words, they make use of the expanded notation in counting. This presupposes that the black child should have a clear idea of place value as seen from the above example.

From the way of counting of the black child it can be deduced that he lacks the concept of cardinal number. It is for educationists who draw the syllabus of primary school Mathematics to take this point into consideration. The syllabus should state it very clearly that the pupils should be given exercises which will develop the concept of cardinal number.

Authors of primary school Mathematics textbooks should be aware of the black child's numeration system. This will help them to include enough exercises which may help to develop the black child's concept of cardinal number.

It often happens that the teacher of the primary school Mathematics prescribes a textbook written for the white children. Such textbooks may not be able to develop the black child's concept of cardinal number. The white child counts differently. He has a clearer idea of cardinal number compared with the black child. The following example illustrates the way of counting of white children. In English, for example, the number 50 is named as fifty. In other words, they make use of the cardinal number.

The black child needs more exercises to help him develop his concept of cardinal number compared with the white child who needs more exercises to help him to understand the place value. This indicates that a textbook which is good for the black child is not necessarily good for the white child at primary school level. These two groups of children have different problems which should be catered for.

e) The meaning of time and space. In his research on the formation of mathematical concepts in young children, Piaget found that the concepts of time and space were fundamental to the understanding of Mathematics (Piaget, 1974, p.42). For the black people living in a traditional society, nature itself is the clock. The day is reckoned according to its significant events. These people are mostly pastoral farmers and the periods of the day are given names that correspond to the main activities connected with pastoral farming (van den Berg, 1978, p.13).

Distance is reckoned in terms of the number of days it takes a man to walk from one place to another. The rising of the sun is the event affecting the whole community. It is immaterial whether the sun rises at 05h00 or 07h00. When a person says that he will meet another one at sunrise, it does not matter whether the meeting takes place at 05h00 or 07h00, as long as it is during the general period of sunrise (van den Berg, 1978, p.14).

2.5.4 Examples of mathematical ability among black people

After a study of the traditional cultural milieu of the black man, the question arises as to the most effective ways of helping the black children to master Mathematics. Regardless of the fact that these children learn through the medium of a second language, English, and despite the primitiveness of the black man's traditional way of life, it is true that he possesses some mathematical ability (van den Berg, 1978, p.15).

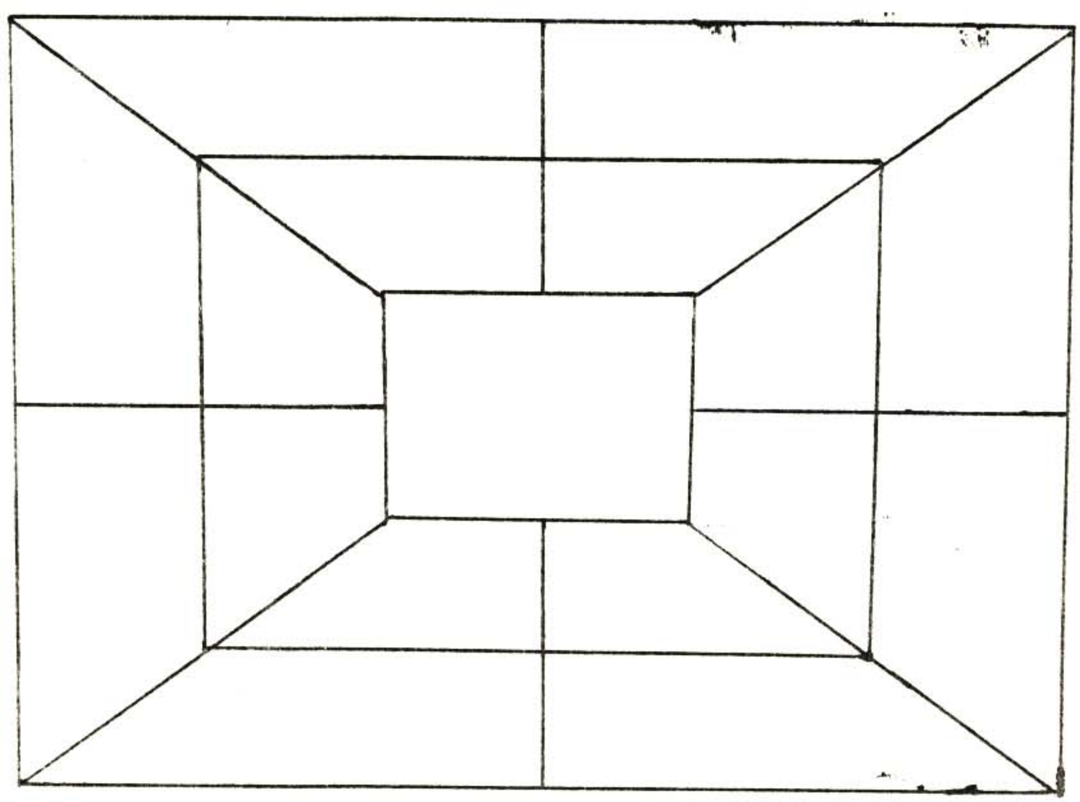
There are several indications of mathematical ability among the black people which lead one to expect that with proper teaching and correct approaches, black children should experience no more difficulties with Mathematics than other children. The following are some examples of mathematical ability among the black people:

i) The black people have a remarkable flair for producing beautiful and intricate geometrical patterns. This flair is evident in the painting of houses, in the carving and decorating of masks, sculptured figures and in the decoration of simple domestic objects. The traditional black people have a marked preference for round huts (van den Berg, 1978, p.16).

   The black women take great pride and pleasure in weaving intricate geometrical patterns with coloured beads. Embroidery is done by the women, while carving is done by men.

ii) Another aspect of mathematical ability which the black people are capable of is the recognition of number patterns, especially as they occur in games. Games form an integral part of the child's education. One of the games which is played on the board, ground or stone, has the following pattern (van den Berg, 1978, p.17):

Figure 1



Two players take part, each with twelve stones, each set of objects having different colours. Each player in turn places a stone at an open intersection. The stones are put in such a way that a line of three stones is formed. If one player succeeds in this he removes one of his opponent's stones. Stage two begins when both players have placed all their stones on the board. The stones are then moved in turns with the aim of getting three stones in a row. The third stage of the game comes when one of the players is left with three stones. This player is allowed to move to any open intersection on the board. When a player's number of stones is reduced to two, he has lost the game (van den Berg, 1978, pp.17-18). This game requires quick thinking and much skill.

2.5.5 Summary

It seems as if the black child has mathematical ability and intuition. It seems that with the correct approach in the schools, black pupils may develop a level of competency in mathematical proficiency.

2.6 TEACHING AND LEARNING MATHEMATICS IN PRIMARY SCHOOLS

To be successful, a teacher needs subject knowledge as well as teaching knowledge. Many primary school teachers may have a problem with subject knowledge. In South Africa, for example, entry to the teacher training course was possible for candidates with a Standard 8 in Mathematics could enrol for this course. Such teachers may have little understanding and little liking for Mathematics, but they will be responsible to lead pupils in gaining their early mathematical experiences.

Even someone who has not specialized in Mathematics have some notion of the key place which Mathematics occupies in our modern civilization. It has long ranked in its own right as one of the great realms of human intellectual achievements. It also equips us with some of our most powerful practical tools, and it underlies the whole build-up of modern science and our technological world (Land, 1966, p.20).

As the result of their teaching methods, teachers may be turning Mathematics into a difficult subject for most pupils. This is in fact the view of many present-day mathematicians who would want to overhaul our methods of teaching Mathematics. They believe that at the teen-age level, when the large mathematical vistas could begin to open up, far too many pupils have already been spoilt by years of dislike for Mathematics (Land, 1966, p.22).

Teaching method is very important in whatever subject we happen to teach. Our understanding of how pupils learn complex skills or subject matter over long periods of time is intuitive. Learning and teaching are very complex cognitive processes. We do not have a sound psychological theory and straight forward ways of describing learning (Du Boulay, 1977, p.52). Be that as it may, we may find some good points if we use games, audio-visual aids and teaching strategies, which will improve our teaching of Mathematics and make the learning of Mathematics a pleasure for our pupils.

2.6.1 Games in Mathematics

Often the primary school Mathematics tonic may be sweetened by providing a dash or two of mathematical games. Mathematical games are often viewed as motivating and reinforcing and perhaps sometimes as vehicles for initial development of concepts. They seem to give the ordinary Mathematics class a touch of innovation as well as a spirit of excitement and the novelty of the mathematical games not only yield educational promise but also present new instructional problems (Sovchik & Meconi, 1978, p.340).

Some games are based on mathematical structures and playing the game involves an experience of the structure. Sometimes in order to play the game well, one must analyse the structure (Fielker, 1971, p.11).

Even when a game does not involve identifiable Mathematics, it may feel mathematical. This is perhaps because strategies are developed, involving ideas of cause and effect (implication), or systematic ways of exploring a variety of possibilities (programming), or assessing which line of action is better (probability). From these an overall strategy may be possible. Chess, for example, involves the strategies of "If I take the pawn, he will take my bishop.", and the overall strategy of building up an attacking or defensive position on the board (Fielker, 1971, p.11).

To put two pupils to play a game of snakes and ladders, and afterwards ask them to write down the numbers of all the squares that could possibly be occupied after one throw each or two throws each, leads directly to overtly mathematical thinking (Gardner, et al, 1973, p.100).

2.6.1.1 Simple versus complex games

In recent years games have been increasingly used to help pupils learn Mathematics. A bewildering array of games confronts the teacher as he pages through instructional materials catalogue or Mathematical journals. However, as (Pratscher et al, 1982, p.397) noted, the fragmented nature of research on the cognitive effects of games of Mathematics has resulted in a lack of useful systematized information on the use of games.

2.6.1.1.1 Advantages of simple games

One obvious advantage of a simple game is that its rules are easy to learn. Thus less time needs to be spent explaining rules to the pupils and more time can be spent in playing the game. If the game is simple enough for the pupils to read and learn the rules on their own, the game can be incorporated into a learning centre (Pratscher, et al, 1982, p.397).

In simple games the rules of the game are usually not intricately interwoven with the mathematical content of the game. Thus simple games tend to be more easily adaptable to a variety of content. War and Bingo are examples of simple games that can be used to supplement instruction in a wide variety of mathematical topics (Pratscher, et al, 1982, p.397).

2.6.1.1.2 Advantages of complex games

Complex games tend to be more interesting and more challenging than simple games. Hence, the continuing popularity of games like chess and monopoly. Related to this is the fact that in many complex games winning is more a matter of skill than luck. Instructional advantages can be taken of this fact if the rules of the game ensure that mathematical skill is a greater factor in winning than luck, thus providing motivation for the player to increase his mathematical skill. In many instructional situations there is a place for an element of luck in a game. Since this element of luck allows the less able pupil also a chance to win, it maintains his interest in playing the game (Pratscher, et al, 1982, p.397).

The possibility to incorporate combinations of mathematical content, higher order cognitive tasks and applications of concepts and skills in complex games, is another advantage (Gardner, et al, 1973, p.100).

2.6.1.2 Some general principles on games

A number of general principles can be deduced from the preceding discussion of the relative advantages of simple and complex games. Firstly, complex games must be played a number of times to justify the time spent on explaining the rules and to allow pupils time to learn the rules well so that they can concentrate on the Mathematics involved in the game. Simple games can be used at most twice. If used more frequently, the pupils may lose interest.

Secondly, care must be taken to explain the rules of a game clearly and thoroughly. Examples should be used liberally, and the basic moves in the game should be demonstrated. Pupils should be watched as they begin to play the game to make sure they understand and follow the rules. Good rules are designed to enhance the instructional value of the game and should be followed (Gardner, et al, 1973, p.100).

Thirdly, in creating either simple or complex games, the rules must be made as easy to understand and follow as possible. The smaller the memory load caused by the rules, the better. Supplying written rules or incorporating game instructions on a game board can help the pupils to concentrate on learning Mathematics rather than remembering rules (Sovchik & Meconi, 1978, p.340).

Fourthly, the proportion of the game playing time that pupils are actually interacting with Mathematics must be made as great as possible. Time spent dealing cards, moving pieces, deciding what move to take, and, especially waiting for the other players to move is time not spent learning Mathematics (Fielker, 1971, p.16).

2.6.1.2.1 Dynamic principle

Preliminary as well as structured games must be provided as necessary experiences from which mathematical concepts can eventually be deduced as long as each type of game is introduced at the appropriate time (Dienes, 1964, p.44).

Introductory games should be played with concrete materials because the pupils are still young. More complicated games can gradually be introduced to challenge the pupil's thinking ability.

2.6.1.2.2 Constructivity principle

In the structuring of the games, construction should always precede analysis, which is almost altogether absent from pupils learning until the age of twelve (Dienes, 1964, p.44).

2.6.1.2.3 Mathematical variability principle

Concepts involving variables should be learnt by experiences involving the largest possible number of variables (Dienes, 1964, p.44).

2.6.1.2.4 Perceptual variability principle

To allow as much as possible for individual variations in concept formation, as well as to induce pupils to gather the mathematical essence of an abstraction, the same conceptual structure should be presented in the form of as many perceptual equivalents as possible (Dienes, 1964, p.44).

2.6.1.3 Competition in games

Large group games, by their very nature, tend to emphasize a competitive environment. An individual game, on the other hand, can restrict the competitive arena to a comparison of previous performance levels by an individual pupil. Some competition is inevitable in our society.

But, values like personal integrity and co-operation also need to be developed. Uncontrolled competition can often yield a best and brightest syndrome where only the elite win at Mathematics. We need to be careful with competitive mathematical games so as to avoid discouraging the losing pupil. In this case, the pupil experiencing difficulty with Mathematics is discouraged by a succession of losses, each loss making it more difficult for the pupil to have confidence in his mathematical abilities (Sovchik & Menoni, 1978, p.342).

2.6.1.4 Evaluation of mathematical games

The following list of questions seems to be important when evaluating the quality of mathematical games (Sovchik & Meconi, 1978, p.340):

i) Are the instructions easy to understand?

ii) How clear are the scoring procedures?

iii) Can a player win often on the basis of chance?

iv) Is the game designed for large groups, small groups, or an individual?

v) Does the game carefully focus on the Mathematics objective in need of reinforcement?

vi) Is the game enjoyable to play?

viii) Does the game provide success potential for varying levels of mathematical abilities?

ix) How do pupils check if they are unsure of the correctness of a response?

2.6.1.5 Examples of mathematical games

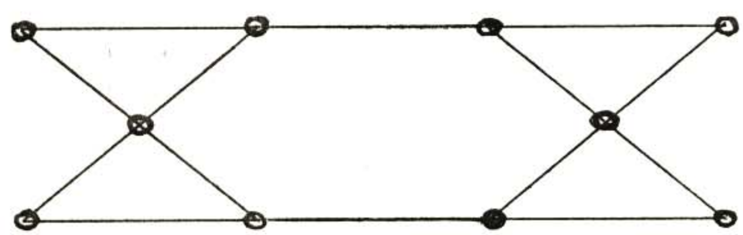
There are quite a good number of mathematical games which are suitable to be played by primary school pupils. Most of those games are found in Tahta's booklet, Pegboard games, 1967. The following games have been chosen as examples:

i) Exago
ii) Jump-ball
iii) The queen's game
iv) Eight-marker puzzle
v) Bean-bag toss
vi) Jewelbox game
vii) Add-a-trial

2.6.1.5.1 Exago

Instead of the usual grid for noughts and crosses, use the layout of points on an isometric pegboard as shown by the figure below. Coloured pegs are used to show when a move has been made. Each of two players takes it in turns to insert a peg of his own colour on the board. The first to have three pegs in a row wins (Fielker, 1971, p.12).

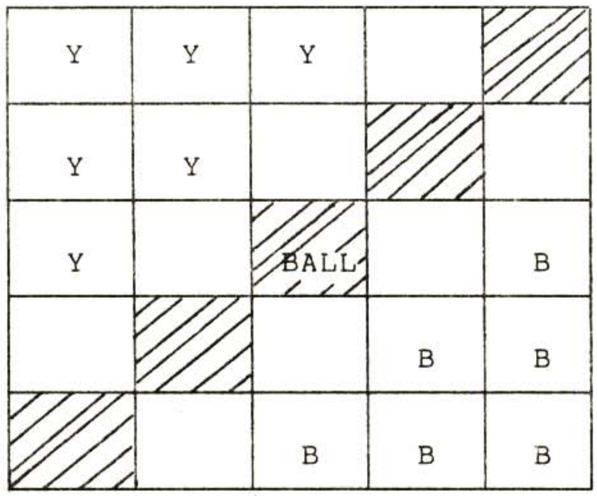
Figure 2



2.6.1.5.2 Jump-ball

This game can be played with pegs on a pegboard or with counters on square paper. Blue and yellow counters are placed as shown in the diagram, and a "ball" is placed in the centre square. Moves are along lines of squares, not diagonally and not backwards. The object is to move the ball into one's own territory, that is, anything other than shaded squares (Fielker, 1971, p.12).

Figure 3



The "ball" is moved by being jumped over; the "ball" is then moved back one square in the direction from which the jumper came. A jump must start in a square adjacent to the "ball" or an apponent's piece, and can continue in a straight line into the next empty square over any number of the opponent's pieces. Pieces jumped over are removed (Fielker, 1971, p.12).

When the "ball" is at the edge of the opponent's ground, the "ball" may be forced from the edge by jumping a piece over the "ball" and over the edge, losing the piece (Fielker, 1971, p.12).

In the event of the "ball" finishing on neutral territory, the winner is the player with more pieces left (Fielker, 1971, p.12).

A large board of 7 X 7 squares is better than the 5 X 5 squares if the number of pieces remain the same (Fielker, 1971, p.12).

2.6.1.5.3 The queen's game

Two sets of distinguishable counters each containing 15 of the same kind, "queens", and one different, "king", are placed as shown on an 8 X 8 square board. The "kings" are in the same column (Fielker, 1971, p.12).

Figure 4

Q	Q	Q	K	Q	Q	Q	Q
Q	Q	Q	Q	Q	Q	Q	Q
Q	Q	Q	Q	Q	Q	Q	Q
Q	Q	Q	K	Q	Q	Q	Q

Moves are as for the "king" and "queen" in chess. A "king" may move into any adjacent square; a "queen" may move through any number of empty squares in a straight line parallel to the side of the squares or diagonally. A move may not finish on a square occupied by one of the player's own pieces, but it may finish on a square occupied by an opponent's piece, which is then taken and removed from the board. The winner is the player who takes his opponent's "king" (Fielker, 1971, p.13).

2.6.1.5.4 Eight-marker puzzle

The eight-marker puzzle is a variation of the ancient game of checkers, but it is played on just one row of squares. Playing the game helps pupils develop problem-solving ability and gaining strategy analysis (Spaulding, 1975, p.246).

Instructions: Draw a row of nine squares. Markers must be of two different colours. Four of them may be red and the other four may be blue. Place the markers on the squares as shown in the figure, leaving the middle square empty. "R" indicates a red marker and "B" a blue one (Spaulding, 1975, p.246).

Figure 5

R	R	R	R		B	B	B	B

The aim of this game is to interchange the red and the blue markers, following these rules (Spaulding, 1975, p.247):

i) The red markers can only be moved to the right, the blue ones only to the left.

ii) A marker may be slid to an adjacent empty square if the square lies in the proper direction.

iii) A marker may jump, in the proper direction, a marker of the opposite colour if the next square beyond it is empty.

Another problem-solving technique that is useful in finding solutions in simpler cases is to find all possible moves at each point in the puzzle before the next move is made. It is easier to make the right choice if one determines all possible alternatives first.

2.6.1.5.5 Bean-bag toss

In a game of bean-bag toss players take turns tossing a bean-bag in a grid similar to the one below in order to see who can reach a score of 100 or more by adding the numbers obtained in each toss. In this game, a player can win on a basis of chance. The chance factor is not always bad. In order for all pupils to experience success, games should have a reasonable balance of chance and mathematical skill (Sovchik & Meconi, 1978, p.341).

Figure 6

13	6	7	14
5	1	2	8
12	4	3	9
16	11	10	15

The physical skill, for example, the ability to throw very well, will help the player to win. The mathematical skill in this game is the correct addition of numbers. This game will help the player to improve his ability of addition.

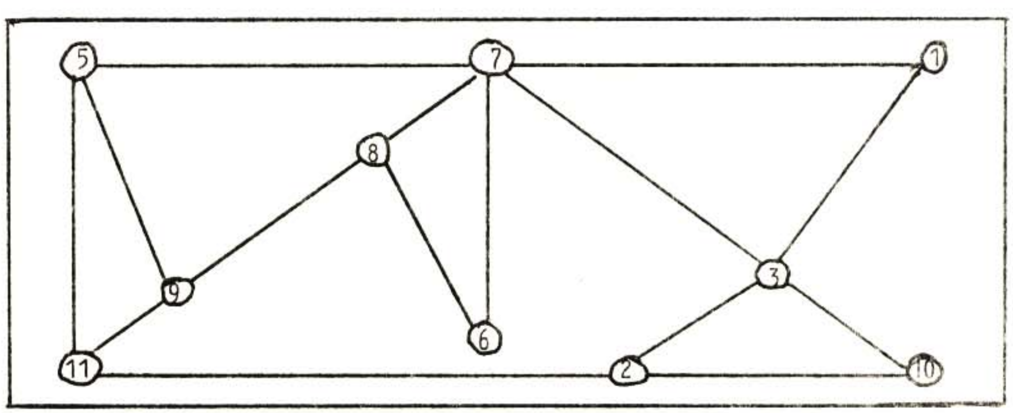
2.6.1.5.6 Jewelbox game

If one wants pupils to experience a challenge, when addition is involved, a more appropriate game would be the jewelbox game. In order to open the jewelbox, the proper buttons on the combination lock must be pushed. The rules indicate that you have to (Sovchik & Meconi, 1978, p.341):

Start from 1 and end up with 11 and that only numbers joined by lines may be added in succession and they may only be added once and that the sum of the numbers added must be 50.

In this game pupils must give careful thought to the addition facts in order to win and to open the box. They must also try combinations to see if they have attained their goal.

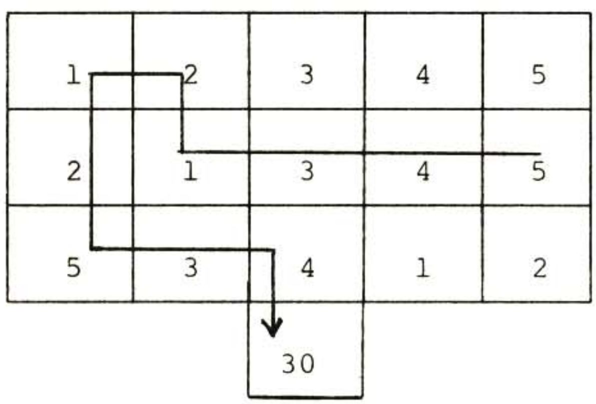
Figure 7



2.6.1.5.7 Add-a-trial

A task which encourages pupils to look for and investigate patterns through simple addition is encountered in the game of add-a-trial. The object of this game is to form a continuous trial of numbers leading to 30. One such series is shown, 5 + 4 + 3 + 1 + 2 + 1 + 2 + 5 + 3 + 4 = 30. Several such series exist (Fielker, 1971, p.18).

Figure 8



2.6.2 Audio-visual aids in Mathematics

According to some psychologists the basis of all learning is experience. Usually learning is best effected by concrete, direct, firsthand experience. Teachers are often unable to give pupils firsthand experiences and resort to the use of words, both written and spoken.

However, most teachers realize that the use of words alone cannot and will not provide vivid learning experience. They are constantly searching for methods and materials which will make lessons meaningful. With a wise selection and use of a variety of audio-visual materials, experiences can be provided which may develop understanding and enhance thinking (Lincoln, 1980, p.3).

2.6.2.1 Simple teaching aids

Numerous teaching aids used in the mid-twentieth century still have a place in our classrooms and can be used beneficially. A number of these simple teaching aids will now be discussed briefly.

2.6.2.1.1 The textbook

The Mathematics textbook is a major factor in determining which Mathematics topics are taught and how they are taught. A textbook has often dictated the scope, the sequence, and even the pace of the Mathematics programme. Thus, the textbook is a powerful means of determining whether new topics are brought into the schools or whether the old Mathematics is maintained. This is all in addition to its basic function as a learning tool in the classroom. Its importance increases when instruction is inadequate. However, the Mathematics curriculum should not be determined by the textbook, rather the textbook should be selected on the basis of prior curriculum decision (Wilson, 1980, p.93).

Since the Mathematics textbook has such a powerful influence in the Mathematics classroom, it may be an invaluable tool depending upon the discretion with which it is handled. Too often, it is over-used. The greater variety of new textbooks available and the continuing development of new school Mathematics curriculum make frequent change of textbooks necessary. Some forward-looking schools respond to this need by using more than one textbook for every class, with a new textbook added every two or three years (Lowery & Leonard, 1978, p.393).

a) Advantages of a good textbook

In modern education the textbook is indispensable, but it must be used correctly as a teaching aid. The school textbook has the following advantages (Conradie, 1979, p.11):

i) It is not complicated to use and it is relatively cheap.

ii) While a teacher is using it, individual adaptations can be made. A reader can stop at any time and later resume reading in order to facilitate understanding.

iii) Parts of a textbook can be glanced at in a cursory fashion to gain a general impression.

iv) A textbook constitutes a permanent record and its contents used repeatedly without further cost.

v) Facts can be given compactly and precisely.

vi) A textbook acts as a guide for both pupil and teacher. It serves as a unit and at the same time lends continuity to the learning situation.

To offer these advantages a textbook must satisfy certain conditions. The subject matter must be presented clearly, systematically and at the level of the learner. Easily readable language must be used and difficult work must be simplified by using illustrations. The textbook must contain information that is up to date (Lincoln, 1980, p.5).

b) The dangers of "textbook teaching"

Most criticism of textbooks is aimed at their frequent poor design and improper use. The school textbook has the following disadvantages (Erickson & Curl, 1972, pp.83-85):

i) Some textbooks, unfortunately, are not exciting to read because they are inadequately or irrelevantly illustrated, and they are rigidly organized sequentially or topically, lending themselves too easily to assignment and memorization of details without understanding.

ii) According to Dale (1969, p.668) the textbook skims over the surface of many ideas, it covers the ground but does not uncover it.

iii) The textbook often tells the reader much more about a subject than he wants to know.

iv) Textbooks lend themselves to lock-step teaching methods.

v) Teachers assume that most of the content of the course is included in the textbook and that all the pupils can read and understand it at the same rate.

vi) Textbooks often fail to stimulate inquiry, discovery and problem-solving.

vii) The process of publication takes too long, and textbooks are frequently out of date.

c) Programmed textbooks

A programmed textbook is a book designed in such a way that it guides the pupil by a series of short steps to understand the material being presented. These short steps are in the form of brief expository statements and questions, to which the reader responds by writing the answers in spaces provided. The reader checks his answers against those supplied by the book. The questions provide the learner with immediate knowledge of his progress. In some programmes, called branch programmes, incorrect answers may direct the learner to remedial work, but the most important function of the question-and-answer process is the deeper involvement of the reader in the learning activity. He is virtually forced by the questions to take responsibility for learning (Lowery & Leonard, 1978, pp.393-396).

These programmed textbooks are being advocated to supplement classroom teaching because they require each pupil to react to each question. In this way he is led to discover a generalization. In addition, answers are given to each question or problem so that correct answers are reinforced and errors are corrected (Johns & Rising, 1967, p.376).

Programmed textbooks also are recommended because they permit independent study at whatever tempo the individual wishes to proceed. This seems an ideal way of providing for individual differences. The slow learner progresses at a rate appropriate to him, while the talented may be accelerated (Conradie, 1979, p.61).

Programmed textbooks have also their negative effects as shown by research. It is sometimes monotonous for pupils, especially slow learners, to work independently day after day writing answers to questions. To the superior pupil, the questions also seem trivial and time consuming (Conradie, 1979, p.63).

The unsatisfactory experiences with programmed textbooks do not mean that they are not useful. Rather, it means that different ways of using these textbooks must be devised. Classroom discussion, experimentation and group activities need to be combined with programmed textbooks. Perhaps their major role is to supplement instruction by presenting remedial work or enrichment of topics (Coppen, 1971, p.215).

A particularly attractive use of programmed textbooks and the related audiotapes or videotapes is to provide assistance to pupils who have been absent. This use alone should encourage teachers to familarize themselves with available programmes so that they may assign specific sections to pupils who miss lessons (Coppen, 1971, p.215).

2.6.2.1.2 The chalkboard

Within the past years, blackboards have come to be known as chalkboards, because they are no longer produced only in the black colour.

The chalkboard, since its inception, has been one of the most commonly used visual materials in the classroom and laboratory. Many teachers have developed unusual skill in chalkboard presentations, while others have neglected to practice even the simple techniques necessary to make satisfactory presentations. Since the chalkboard is usually available in every classroom, teachers should plan and practice chalkboard presentations much the same as they plan and practice other teaching techniques. The teacher who can vitalize the lesson through good, clear, well-proportioned illustrations will be a more successful teacher (De Kieffer & Cochran, 1962, p.21).

The chalkboard is one of the quickest and easiest - and often the only means of illustrating an important point. It is immediately available for both the teacher and the pupil to use in visualizing subject matter. It can add variety and contrast to teaching. It can be erased quickly and new material added as the lesson progresses (Erickson & Curl, 1972, p.103).

a) <u>Simple chalkboard techniques</u>

The following simple techniques should prove helpful in improving the teacher's chalkboard presentations (Dale, 1969, p.312):

i) Do not confine the material to small detailed drawings; work in large patterns.

ii) Test the size of your writing to be sure that it is legible from the last row of desks in the classroom. If the teacher's writing is not easily read, he should print.

iii) The teacher should plan a neat, orderly presentation.

iv) The teacher should develop one point at a time.

v) When talking at the chalkboard, the teacher should stand to one side of the chalkboard so that all the pupils could see. He should use a pointer if necessary.

vi) The teacher should erase or cover distracting material on the chalkboard so that pupils will concentrate on the material being presented.

b) <u>Factors to consider in using the chalkboard</u>

Preparing materials on the chalkboard in advance of the class period often saves the time of both teacher and pupil. In certain instances the teacher may wish to make the sketch during the class if the drawing is of the developmental type. A series of drawings on the chalkboard in view of the entire class may have a distracting influence and make the presentation less effective.

Some teachers cover chalkboard drawings with a wrapping paper and then uncover the material one step at a time to give the full impact of the illustration. A confused series of drawings on the chalkboard may be so distracting to the pupils as to be of little value. Other devices found to be of great value in chalkboard presentations are prepared patterns, the grid technique and chalkboard drawing instruments (Coppen, 1971, p.68).

The prepared pattern method utilizes holes punched at intervals in the chalkboard to establish outlines on often used objects. The grid technique is used to transfer a small picture to the chalkboard; horizontal and vertical grids are drawn over the picture and the grid is enlarged to a different scale on the chalkboard. This technique is often used by teachers who cannot make satisfactory freehand sketches (Wittish & Schuller, 1962, pp.57-58).

2.6.2.1.3 The bulletin-board

The bulletin-board can be used to create interest in the subject matter, provided that the material displayed is changed regularly and is meaningful. To achieve this the teacher must continuously give attention to it. If time does not permit this, then a pupil or small group of pupils, be made responsible for it (Erickson & Curl, 1972, p.100).

The following hints on the use of this teaching aid can be used as guidelines (Conradie, 1979, p.14):

i) Plan the display around a central theme.

ii) Decide on the design and composition and then make a rough sketch of the display on a piece of paper.

iii) Keep the display as simple as possible. Use a few well chosen pictures rather than too much material which may be confusing.

iv) Use a title that attracts attention. A question or play on words sometimes has this effect.

v) The writing must be clear and visible from a distance.

vi) Plan in advance; prepare a tentative time-table for a term or year, but allow for possible new subjects.

vii) Keep a file with readily available good material.

viii) The displays must be discussed with the class and the teacher must draw the pupils' attention to the display. It can supplement the teacher's efforts.

2.6.2.1.4 The flannel-board

The flannel-board is an exceptionally handy aid for display and demonstration purposes. It can be any size and is covered with flannel or felt. If the back of the material to be displayed has a piece of felt or sandpaper glued to it, then the material can be pressed onto the board (Erickson & Curl, 1972, p.106).

The flannel-board has the advantage that the material can be prepared before use and then stored after the lesson for future use. If a lesson is well prepared, then the flannel-board saves time during the course of the lesson (Erickson & Curl, 1972, p.106).

It is sometimes incorrectly assumed that the flannel-board is only suitable in the primary schools. In actual practice, the flannel-board can be used effectively in any presentation in which a systematic step by step demonstration is needed. There is no reason why it should not be used in the lecture rooms at tertiary institutions. For systematic presentations it is a very handy aid (Coppen, 1971, p.99).

2.6.2.1.5 Pictures

Today as never before, pictures are available from a wide variety of sources such as newspapers, magazines, brochures and books. For classroom use, drawings, paintings, cartoons, illustrations, photographs and stereographs provide teachers with a wealth of effective, inexpensive materials that are simple to procure and easy to use (De Kieffer & Cochran, 1962, p.26).

Pictures help to provide motivation for pupils and bring interest to the learning situation. They assist in developing language, art and creative expressions such as storytelling, dramatization, reading, writing, painting, drawing, and help pupils interpret and remember the content of accompanying textual material. Social understanding can also be developed through their use. In many situations they provide the best source materials for study and investigation. Pictures add variety to teaching for they can be used with individuals, small work groups, or when projected, with even the largest classes (Johnson & Rising, 1967, p.412).

a) Using pictures

Pictures, like all other teaching aids, should be selected and used with specific purposes in mind. They do not teach and must be integrated with particular lessons. It is not wise to use too many pictures at the same time; select a few key pictures which will best develop understanding (Wittish & Schuller, 1962, p.81).

Pictures may be used by individual pupils for picture reading exercises. They may also be used for special reports, bulletin-board displays or as for further reading and study (Wittish & Schuller, 1962, p.81).

Small groups can also make good use of pictures. Pictures can be collected or selected by pupils to be used for special interests. Pictures can be used as a basis for group reports and as reference materials for study and research.

The meanings and understanding which pupils gain from pictures differ from one pupil to another. Therefore, the teacher must take time to teach pupils how to "read" pictures (Dale, 1969, p.243).

Most teachers are familiar with the use of pictures for presenting ideas to an entire class. They can be used as background displays on the walls, as exhibit and as bulletin-board displays, as visual texts for motivation or review, or as common learning experiences in developing understanding. However, quite often pictures are too small for the entire group to use, and the opaque projector must be used to enlarge the image (Dale, 1969, p.243).

b) Selection of pictures

Pictures can be collected and selected by pupils and teachers; however, there are several general questions that should be answered before the selection process starts (De Kieffer & Cochran, 1962, p.29):

i) What are the purposes in collecting and using the pictures?

ii) Will they add to the pupils' knowledge?

iii) Why are pictures being used?

iv) How are they going to be used?

After determining the general value of the picture, it should be evaluated with some specific criteria in mind, such as (Wittish & Schuller, 1962, pp.81-87):

i) Suitability - will it contribute to the learning situation?

ii) Artistic - does it meet artistic standards?

iii) Technically correct - does it actually show what it is supposed to present?

iv) Well composed - does it have a centre of interest or is it confusing in appearance?

v) Clear in detail - are the details clear and meaningful or are there too many details?

vi) Realistic - is it truthful or has it been poorly designed?

vii) Size related - are there familiar objects in the picture so that pupils can determine relative sizes?

viii) Effective in colour or contrast - if colour is used, are the colours true and meaningful?

2.6.2.1.6 Charts and graphs

The terms charts and graphs, as used in modern education, have changed somewhat from the original usage. Formally, chart usually referred to a type of map used for navigation, an outline map exhibiting temperature variations or perhaps a sheet guiding tabular information. Present usage of the term includes a wide variety of graphic and pictorial material. The word graph was used primarily to refer to line graphs and to mathematical graphs. Current usage has expanded the term to include many variations of this teaching device (De Kieffer & Cochran, 1962, p.38).

Charts and graphs are needed in the instructional programme as a means of breaking down cold statistics into a language that can be understood by the pupil. Factual information as printed or written in statistical form is understood by the pupil but has little meaning to the pupil. Facts can often be built into charts or graphs that quickly clarify meanings and often save considerable time in the presentation (Wittish & Schuller, 1963, p.108).

When using charts and graphs teachers must always keep in mind that these materials are symbolic. The pupils need a background of information to which the graphic information can be related. He must understand the symbols used and why certain ones represent specific things. Although primary school pupils have seen charts in books and on television, the skill of reading such materials needs to be developed in the classroom. Simple pictographs can be introduced to develop the pupil's ability to understand the symbols used, the ability to read such graphs and deduce certain conclusions. Charts and graphs are a means of getting the pupil to think, compare, relate, and use factual information (Erickson & Curl, 1972, p.97).

2.6.2.1.7 Diagrams

A diagram is a drawing consisting mainly of lines and symbols illustrating in a concise way, interrelationships, a general plan, or characteristic of a process, object, area and so on. Diagrams are abstract and if used in teaching pupils, they must be carefully prepared. Diagrams are more useful if used for summaries and reviews rather than for introducing a lesson. A diagram actually represents a framework for an object or process. It provides a conceptual structure and a representation of the total organization which is necessary for clear insight. Sometimes diagrams must be used in conjunction with the original, actual object so that the visual symbol cannot be seen as representing anything but the original. It is thus advisable to start with the original and gradually work towards the diagram while reminding the pupils that the diagram is only an abstract image of the original (Conradie, 1979, p.20).

2.6.2.1.8 Overhead projectors

An overhead projector is so constructed that it throws an image over the shoulder or over the head of the person manipulating it so that the picture appears on a screen behind him (Conradie, 1979, p.17).

An overhead projector is invaluable for classroom demonstration, especially as it may be used in a lighted classroom. The teacher can face the class, talk to the pupils and throw on to the screen or wall behind him pictures to illustrate his point. Some advocates have claimed that the overhead projector will do away with the chalkboard, but this seems unlikely. No overhead projector can yet project as much material as may fill a chalkboard and the chalkboard remains essential for improvisation and rapid notes (Kent, 1969, p.51).

A number of different objects can be projected from an overhead projector. Slides and transparencies can be used and, perhaps even more important, diagrams and notes may be copied with a wax pencil on an acetate sheet and projected onto a screen as the teacher faces the class (Wittish & Schuller, 1962, p.137). In addition, the teacher may use film strips and other projected material well. These are relatively inexpensive, easy to handle and effective (Weaver & Bollinger, 1961, p.14).

2.6.2.2 Some didactical considerations on the use of simple aids

i) The use and integration of teaching aids must be planned carefully. The aim of the lesson must be determined beforehand and the specific aim of the teaching aid and its effect must then be considered. The timing of its use in the lesson must be planned as well as how it is to be introduced and used in the lesson. The role it is to play, if any, in the application of the lesson has also to be determined (Erickson & Curl, 1972, p.109).

ii) The pupils must be prepared in advance for the use of the teaching aid and what they will gain from its use must be ensured. For example, it must be ensured that they interpret correctly what they are going to hear or see. The aid should be accompanied by an assignment which needs their attention (Conradie, 1979, p.21).

iii) After using the aid establish whether the pupils really have obtained insight and give them the opportunity to respond actively (Conradie, 1979, p.21).

iv) The teacher should guard against the unplanned excessive use of audio-visual aids. Too many impressions may confuse the pupils. All pupils at school may be exposed to more than one aid, not necessarily of a visual nature, during the course of a lesson, if the lesson is well planned and the pupils are used to it (Dale, 1969, p.508).

v) It is very important that all pupils in a classroom have full opportunity to use the aid. Avoid indistinct or small examples of objects not easily seen by all pupils (Dale, 1969, p.508).

vi) An aid has no value on its own; its value depends on its use. The didactic intuition and initiative of the teacher will always be determining factors in the successful usage of an aid (De Kieffer & Cochran, 1962, p.230).

2.6.3 Teaching strategies appropriate to primary school Mathematics

The emphasis is on the strategies which may be employed in the teaching of primary school Mathematics. There are many new ideas in primary school Mathematics and most have some merit. The aim is to locate the basic features of the teaching strategies and to give teachers the opportunity to fit them into a practical scheme of teaching.

2.6.3.1 Problem-solving method

The term "problem" is commonly applied to a quantitative situation, that is, a situation involving numbers, in which the information is given in descriptive form, generally set out in sentences, and in which there is a question proposed for an answer (Bunker, 1969, p.152).

In a general sense, problem-solving encompasses a complex set of interrelationships among processes. Among those processes are the abilities to discriminate, classify, organize information, ascertain patterns, think logically, interpret, project and understand any language that may accompany the encounter of the situation (Castaneda, et al, 1982, p.22). Those appropriate for primary school pupils are classification, estimation, pattern generation, translation, trial-and-error and verification (Castaneda, et al, 1982, p.20).

The primary school teacher will find it very convenient to use the terms "problem" with the idea of a verbal description and question in mind; but his own approach to the teaching of this work must take into account the individual pupils who are to answer the question. The true problem will involve reasoning or thinking, and the difficulties encountered here will vary considerably from pupil to pupil (Kilpatrick, 1978, p.189).

2.6.3.1.1 The approach to problem-solving

It is important that all problem-solving be closely linked with experiences chosen from within the pupil's environment. In the first place, primary school pupils can build up basic number facts using concrete material or by playing a game. The number work will only become meaningful if pupils are encouraged to make up a story about the particular situation as it is in the classroom. Sometimes a story might be made up about an illustration; for example: "There are three ducks swimming in the pond and two more ducks are standing on the grass. How many ducks are there altogether?" (Polask, 1978, p.233).

Naturally, there will be a numerical answer to be given as the question is asked, but the teacher should realize the particular significance of the language side of this work. The description of the problem situation in words is the best way for a pupil to show that he really understands the meaning of a particular operation. It is this type of training, based on familiar situations, which will begin to lay the foundations for the future. The pupil must have adequate experiences in the early stages to compare later problem situations (Polask, 1978, p.239).

It will not be long before differences in ability in this work become evident. The more able pupils will be solving problems without the need to refer to concrete materials or drawings, but due to individual differences the weaker pupils will still need some concrete materials. Pupils in the junior classes of primary schools will benefit if there are frequent checks on their understanding of the computational processes. These can be done by asking pupils to give or to write a story that would explain what they are doing. Such an exercise may readily form part of the oral work which precedes written mechanical Mathematics, or may be established as a routine part of the written work associated with one or two examples chosen from the written mechanical practice exercise. For example, calculations of the form 36 + 40 + 12 + 32 are not very meaningful; but when a pupil can place this calculation into the framework of a practical activity such as finding the total number of children from four different classes going on an excursion, then there is evidence that the pupil concerned understands the meaning of the operations involved and will be able to solve other similar problems (Ellis & Alleman-Brooks, 1978, p.242).

2.6.3.1.2 Steps in problem-solving

As pupils advance to the stage at which they are encouraged and expected to solve problems presented by the teacher, it will be desirable to try to develop a systematic approach to problems. It is quite common for some pupils to rush into a solution without proper consideration of the situation as a whole, and although the more able pupils can generally work quite well without guidance, there are certain procedures which may well be followed by all pupils as they seek the solution to a mathematical problem.

Useful steps in a systematic approach to problem-solving are (Bunker, 1969, p.157):

i) Reading the entire problem carefully to get a mental picture of the whole situation and to understand what is required.

ii) Analyzing the problem situation to determine the steps needed in solving it.

iii) Setting out the solution to the problem.

iv) Checking the reasonableness and the accuracy of the solution.

The first and last of these steps are well defined and have immediate links with the language aspect and with the understanding of the problem in general. On the other hand, the second and third steps cannot be restricted to a definite set of procedures for all pupils. The teacher should pay particular attention to the individual differences of the pupils.

2.6.3.1.3 Improving ability to solve problems

Investigations on problem-solving difficulties and their apparent causes have resulted in numerous proposals for removing the difficulties and improving pupils' abilities to solve problems. The main difficulties lie in noting the relationships between the elements of the problem, and although thinking and reasoning is a complex process, the teacher must be aware of some practical methods of helping his pupils (Lester, 1982, p.94).

Developing the ability to solve problems is a cumulative process that depends on the history of problem-solving experiences of the pupil. If pupils are encouraged and guided to find solutions to problems within their limited world of experience, their interest and success in problem-solving are expected to be greatly enhanced as they move through the school years. To capture pupils' abilities to reason at an early age and to provide guidance in problem-solving activities for the purposes of both mathematical concepts and problem-solving process development (Castaneda, et al, 1982, pp.22-23) points out that the teacher must focus his attention on the following:

i) The role of problem-solving in developing mathematical concepts of pupils;

ii) selected problem situations or tasks for engaging pupils in problem-solving; and

iii) criteria designed to provide a guide to teachers and parents in formulating appropriate problem-solving situations.

Attention should also be given to developing problem-solving readiness. Unless the pupils are familiar with both the mechanical side of the processes and the meaning of the processes, they cannot apply them successfully in problem-solving. The modern emphasis on basic principles, structure and patterns in Mathematics will develop confidence in this direction and will provide a good background for problem-solving (Mervis, 1978, p.274).

Secondly, we should help pupils to develop problem-solving ways of thinking. Within the normal classroom situation, the teacher will often be able to arrange that pupils who experience difficulties in problem-solving will be catered for either in small groups or as individuals.

Billinge (1975, p.11) maintains that attention must be given to careful reading of the problems, possible dramatization of the problem and the drawing of a diagram to show the relationship between the elements involved.

There may be frequent opportunities for the teacher to use different procedures to guide pupils through to the actual computation involved in the solution of a problem, occasional lessons should be devoted to reasoning only. It is expected that the main difficulties in problems will be associated with reasoning, and so it is suggested that this aspect of problem-solving should receive special attention (Kilpatrick, 1978, p.191).

If pupils can indicate which other information is necessary before they could solve the problem, it may well be assumed that they have clear understanding of the relationships involved.

As previously noted, problem-solving is a means through which mathematical concepts can be formed and, in turn, is a goal of early and subsequent instruction in Mathematics. When problem-solving is ignored in early instruction in Mathematics, the Mathematics learned is likely to consist of memorization of number names, of the counting order of number names and rote use of number and operation names rather than conceptual learning

(Castaneda, et al, 1982, p.23).

To solve a specific problem certain activities may be necessary but the child's conceptual development will determine which actions he could take. Nelson & Kirkpatrick (1975, p.114) indicate that the action may be singular and comparatively simple; a series of actions; an accumulation of the results of a series of actions; or finding a pattern in the results of more than one series of actions.

They go on to say (1975, p.116) that the understanding of the mathematical language will help the pupil to solve the problems as he will be acting on the objects that are named in the problem, or acting on counters or blocks, that represent the objects named in the problem or making a graphic representation such as dots, marks or pictures of the objects named in the problem or using symbols for the numbers and operations to represent the action in the problem. Children may record the results of their actions graphically or with symbols.

Castaneda et al (1982, p.24) maintains that the first phase has two messages for teachers, namely, that the teacher should be aware of the action that is needed to solve the problem posed to pupils and should consciously move from the simple, single action, to the more complex actions as pupils develop problem-solving abilities; and should also be aware that as one moves along the continuum, there is increasing need for the collection and organization of facts, the strategies of trial-and-error and verification.

Castaneda et al (1982, p.25) go on further to say that the second phase also has two messages for teachers of primary school pupils, namely, that problem-solving with concrete aids is the source for the pupils' later ability to solve problems using symbols; and that teachers need to plan for pupils to build concepts if they are to be able to solve problems with symbols.

Problem-solving and the development of mathematical concepts go hand in hand. Mathematical problem-solving involves the use of mathematical concepts and symbols. Pupils with limited abilities can solve complex problems if provided with the appropriate concrete materials. Problem-solving activities for primary school pupils can therefore have as primary purpose the development of mathematical concepts.

2.6.3.1.4 Preparation of suitable problems

The previous section has considered some ways of helping pupils as they actually encounter difficulties in problem-solving. Some of these difficulties may be avoided if careful thought is given to the preparation of the problems which have to be solved.

The problems used, particularly in the early stages, should deal with pupils' own experiences and should come from their realm of interest. Teachers in the primary schools should realize that there are many opportunities for mathematical activity in and around the school. The use of these situations will do much to help the development of favourable attitudes toward Mathematics (Bunker, 1969, p.162).

Another feature which can be controlled by the teacher and which may increase pupils' confidence is the use of an understandable vocabulary. If the language does not create problems the child can give all his attention to the solution of the actual problem (Mervis, 1978, p.270).

These points relating to the preparation of problems will again remind the teacher of the importance of adapting his approach to the needs and developmental stage of the pupils. Textbooks may give a number of problems which may be quite suitable and challenging to the pupils. A textbook generally offers problems which are appropriate to a particular class. It is very difficult to accommodate the real needs or interests of each particular pupil in a specific textbook. Therefore, much responsibility is thrown upon the teacher to see that his pupils are given the right kind of problems. It is true that the modern textbook is making an effort to help the teachers but if its limitations are recognized and supplementary exercises drafted, it will provide a very useful reference for the teacher (Johnson & Rising, 1967, p.371).

2.6.3.1.5 Planning of lessons on problems

In the preceding sub-heading special emphasis was placed on the need to cater for individual differences. Now consideration is given to the ways in which lessons may be planned so that these suggestions may actually be carried out.

Most lessons on problems aim to develop problem-solving ability. It must be remembered that the nature of the work and the difficulties which will be encountered will require greater flexibility in the actual conduct of the lesson. Group work should form the basis of the organization of the majority of lessons on problems. The division of a class into two main ability groups may also enable the teacher to present his lesson more effectively (Auleta, 1969, pp-269-272).

The actual planning of this particular approach for a more able group and a slower group who will be in need of special assistance and close supervision, may be based upon the following main lesson steps (Polask, 1978, p.234)

Step 1: Revise (with the whole class).

Step 2: More able group: Pupils proceed to written practice.

Step 2: Slower group: Discussion of more problems and careful questioning by the teacher to lead in the choice of a method to solve the problem.

Step 3: More able group: Correction of work done.

Step 3: Slower group: Written practice of selected problems from those discussed in Step 2.

Step 4: More able group: Written practice.

Step 4: Slower group: Written practice; individual assistance is given as required. Correction of work done.

Step 5: More able group: Marking problems.

Step 5: Slower group: Assessment of work done.

It would be desirable for the two groups involved in this organization to solve different problems. The more able pupils will be expected to do advanced work which provides reasonable challenge while the slower group will be guided to their particular level of achievement.

It is this type of planning which may make a real contribution to the attitude of the pupils. Pupils may be willing to work with the teacher rather than against him. There will be good opportunity to stimulate the interest of the individual pupil. This may create a situation in which the teacher may be in a position to improve his pupils' problem-solving abilities.

2.6.3.1.6 Tips on planning and implementing problem-solving instruction

A teacher should not wait until research finds answers for all of the questions about problem-solving before taking some action in the classroom. Steps should be taken to help pupils become better problem solvers. Teachers have an obligation to do their best to meet the needs of their pupils in this area. Here are some tips that have proved to be useful in planning and implementing problem-solving instruction (Lester, 1982, p.95):

i) Pupils may have difficulty with a word problem because they do not understand it. If this happens, the teacher should read it to them and then have the pupils restate the problem in their own words.

ii) Pupils often interpret the meaning of problems incorrectly. The teacher has to determine whether the question was correctly understood.

iii) Pictures and diagrams which accompany a problem are often helpful, but they may sometimes be a source of confusion. The teacher must give the pupils a chance to talk about the pictures or diagrams which are associated with the problem.

iv) The teacher should try to get the pupils to estimate the answer before trying to find an exact solution.

v) The teacher should allow the pupils to act or dramatize the situation posed by a problem. This is an exceptionally useful technique with primary school pupils.

vi) Sometimes pupils seem to understand a problem but have difficulty in getting started on a solution. When this happens it is often helpful to simplify the problem by using smaller numbers or, if the problem has more than one part, do the problem one part at a time.

vii) The teacher should encourage pupils to organize their work by making tables, keeping a record of their work, looking for patterns and similar techniques.

viii) Work given on a daily or weekly basis using interesting problems for the pupils is a sensible approach.

ix) Pupils cannot develop any skill in problem-solving unless they become actively engaged in solving problems. One way of getting pupils actively involved in problem-solving is to pose problems which have relevance to their lives. Teachers should encourage pupils to bring some of their everyday problems into the classroom. The teacher and pupils may be surprised at how useful Mathematics can be in solving many of these types of problems.

2.6.3.2 The discovery method

The pupil who discovers a relationship and states it in his own original way, is proving his understanding in a way that cannot be matched by the pupil who repeats the exact words of the textbook or teacher. That which the pupil discovers for himself is much more likely to be remembered by him.

To ensure that discovery learning takes place, the teacher must create an environment that will facilitate such learning. The amount and nature of guidance to be given by the teacher, will depend on several factors such as the subject matter, available audio-visual aids, and intellectual abilities of the pupils. The difference between the knowledge which is to be imparted and which is to be discovered may be very negligible. Too much direction from the teacher may block the pupils' route to discovery; non-intervention may leave the pupils confused and aimless. Whatever the situation, the teacher must never allow the pupils to flounder too long or too helplessly. He must come to their rescue as the need arises, and encourage dialogue between himself and the pupils, and between the pupils themselves (Hutton, 1976, pp.8-9).

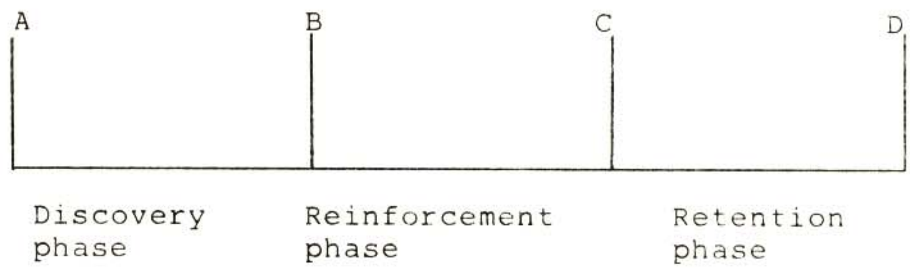
The teacher should not employ sophisticated techniques if he wishes to develop retention of learnt material. On the other hand, it is important that the pupil should experience success when learning by discovery. The effect of success is to reinforce those techniques that bring favourable results, and to foster appropriate attitudes and interest (Di Vincenzo, 1980, p.218).

How can the teacher ensure that learning by discovery is successful? Firstly, he must know something about the phases in the learning by discovery process. Secondly, he must know the different levels of understanding among pupils. Thirdly, he must know which didactic steps to use (Behr, 1971, pp.73-74).

2.6.3.2.1 The phases in the learning process

The various phases in the learning by discovery process can be represented by a straight line depicted as shown below.

Figure 9



During the period A to B, the pupil engages in various strategies such as trail-and-error, comparing, contrasting and evaluating. He is learning to discover (Shulman & Keislar, 1968, pp.13-14). At B the pupil makes the required response, with or without the teacher's help; he has discovered what was required of him. The reinforcement phase occurs in the interval between B and C. The pupil is memorizing the subject matter or improving the skill acquired at B. The teacher's participation in the pupil's learning ends at C. The teacher has no longer any control of the learning process. At any point in the time interval C to D the pupil may be tested on what he has retained (Behr, 1971, p.74).

2.6.3.2.2 The levels of understanding among pupils

For the discovery method to be successful, the teacher must recognize the different levels of understanding among his pupils. The kind of discovery the pupils are to make must be consistent with their levels of understanding. Piaget has described the following stages of development, which are relevant to primary school pupils, intuitive stage, concrete operational stage and the abstract operational stage (Piaget & Inhelder, 1969, pp.51-140).

At the intuitive stage the pupil needs concrete aids to help him make the discoveries.

At the concrete operational stage the pupil is able to formulate rules which he may be able to apply, and may no longer need to resort to concrete aids. The teacher's task at this level will be to guide pupils to make generalizations from their discoveries (Gardner, et al, 1973, p.16).

At the abstract operational stage the pupil is able to explain his findings, that is, to indicate why the rule works. Not all the pupils reach this level (Behr, 1971, p.75).

In the same classroom the teacher will find pupils falling under the three stages of development. The teacher will have to plan his discovery programmes accordingly.

2.6.3.2.3 Didactic steps to facilitate discovery learning

To facilitate discovery learning, the teacher must create a stimulating environment for the pupil to work in. There should be ample equipment, an adequate library of books and other reference materials. Investigations should be allowed to take their own course, and a great deal of flexibility in class organization, the use of time and informal social interaction be permitted. A variety of ideas provoked by the investigations should be accepted. Appraisal by the teacher must be continuous and on an individual basis. Finally, the teacher should always be near to assist pupils to organize their knowledge in a way that will help them to increase their powers of perception and invention and thus their desire to learn (Shulman & Keislar, 1968, p.115).

2.6.3.3 Inductive and deductive methods

Inductive reasoning considers available facts and draws from them a general conclusion. Inductive learning or teaching method works from several specific facts or examples toward the development of a generalization that covers all of them (Swenson, 1966, p.25).

Inductive method might be used in the development of a generalization about the addition of doubles. The pupil discovers through his experience with concrete objects that two plus two equals four, three plus three equals six, four plus four equals eight, and so on. The pupil can do addition of doubles as shown here:

$$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ +1 & +2 & +3 & +4 & +5 & +6 & +7 & +8 & +9 \\ \hline 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 \end{array}$$

The teacher who uses the deductive method begins by giving his pupils a general statement, rule or principle, and goes on to apply it to specific cases. The pupils' active participation is confined to the application of the given statement to many examples (Behr, 1971, p.70).

The deductive method is used extensively in Mathematics. The structure of Mathematics is such that the deductive method can be used in certain instances, for example, in the number system and mensuration.

The emphasis should be on helping the pupils to discover how Mathematics works. Leading pupils to discover relationships in the number system and the basic operations takes time, patience and faith in pupils' ability to figure things out for themselves. Giving adequate experiences from which pupils can draw correct generalizations and waiting for them to formulate the relation they discover may also take time, but it is time well spent.

2.7 CONCLUSION

In this chapter, the researcher has discussed the stages of cognitive development in detail. The educational implications of Piaget's theory of cognitive development were also dealt with. Furthermore, the researcher discussed learning and teaching strategies appropriate to primary school Mathematics.

In Chapter 3 we shall focus our attention on the analysis and interpretation of the results of the standardized tests. In the sample schools of Bophuthatswana, 495 Standard 5 pupils wrote the "Scholastic Achievement Tests in Arithmetic - Standard 4" and the "Diagnostic Mathematical Tests" of the Human Sciences Research Council.

CHAPTER 3

INTERPRETATION AND ANALYSIS OF THE RESULTS OF THE STANDARDIZED TESTS

3.1 THE "SCHOLASTIC ACHIEVEMENT TESTS IN ARITHMETIC - STANDARD 4"

In this chapter the researcher gives and discusses the results of the "Scholastic Achievement Tests In Arithmetic - Standard 4" of the Human Sciences Research Council. These tests were given to the pupils to determine their general standard of performance. The tests were given to 495 Standard 5 pupils at the beginning of the year. The aim was to determine the standard of performance at the beginning of Standard 5. The researcher preferred standardized tests to ordinary classtests because standardized tests are tests which are so constructed that a pupil's performance in the test enables one to compare his proficiency in the area measured by the test, with the proficiency of the average child under comparable circumstances.

Schools which were used as samples were chosen by the Circuit Education Officers of the respective inspection circuits. The schools were chosen from three different inspection circuits. Some are in urban areas and others are in rural areas. The aim was to have a sample which could be regarded as representative of the whole area.

The results of the different schools will firstly be discussed individually to find out if there are any differences in performance in the different areas. Thereafter the results of the whole group will be analized.

3.1.1 School A

Number of pupils tested: 60

3.1.1.1 Analysis of raw scores

3.1.1.1.1 Test 1. Compare row 1: Frequency table 1

Table 1

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	0	0,0	36	$\pm$60
1	3	5,0		
2	4	6,7		
3	7	11,7		
4	12	20,0		
5	10	16,7		
6	4	6,7	6-10 correct solutions 23	$\pm$ 38,3
7	6	10,0		
8	8	13,3		
9	4	6,7		
10	1	1,7		
11	0	0,0	more than 10 correct solutions 1	$\pm$ 1,7
12	1	1,7		

The main purpose of Test 1 is to test the pupil's skill in dealing with the four basic Arithmetic operations. Mechanical calculations using natural numbers and vulgar fractions constitute this test. The results of Test 1 indicates that 36 or ±60% of the 60 pupils managed to solve five or less problems out of the possible 20, while 23 or ± 38,3% solved 6 to 10 problems correctly. In other words, 98,3% of the pupils could not solve more than half of the problems. Only 1 or ± 1,7% scored more than 10 or 50% correct responses.

3.1.1.1.2 <u>Test 2. Compare row 2: Frequency table 1</u>

<u>Table 2</u>

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	0	0,0		
1	5	8,3		
2	9	15,0	44	±73,3
3	11	18,3		
4	12	20,0		
5	7	11,7		
6	3	5,0	6-10 correct solutions	
7	7	11,7		
8	4	6,7		
9	1	1,7	16	±26,7
10	1	1,7		

The purpose of Test 2 is to determine to what extent the pupils understood the concepts and to what extent they could think logically about numbers. Forty four or ±73,3% of the 60 pupils solved 5 or less problems correctly. Sixteen or ±26,7% of the pupils solved 6 to ten problems correctly. None of the group could solve more than 10 out of 20 problems correctly.

3.1.1.1.3 Test 3. Compare row 3: Frequency table 1

Table 3

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	0 6 1 15 15 12	0,0 10,0 1,7 25,0 25,0 20,0	49	±81,7
6 7 8 9 10	6 4 1 0 0	10,0 6,7 1,7 0,0 0,0	6-10 correct solutions 11	±18.3

The purpose of Test 3 is to determine the pupil's ability to solve word problems in Arithmetic. The calculations are of an extremely elementary nature. In these word problems 49 or 81,7% of 60 pupils scored 5 or less correct responses out of 20, while 11 or 18,3% scored from 6 to 8 correct responses. None of them could solve more than 8 or 40% of the problems correctly.

3.1.2 <u>School B</u>

Number of pupils tested: 55

3.1.2.1 <u>Analysis of raw scores</u>

3.1.2.1.1 <u>Test 1. Compare row 1: Frequency table 2</u>

<u>Table 4</u>

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	2 6 8 10 9 10	3,6 10,9 14,5 18,2 16,4 18,2	45	±81,8
6 7 8 9 10	2 3 1 2 1	3,6 5,5 1,8 3,6 1,8	6-10 correct solutions 9	±16,4
11	1	1,8	more than 10 correct sol-lutions 1	± 1,8

Forty five or ±81,8% of the 55 pupils solved 5 or less problems correctly while 9 or ±16,4% solved 6 to 10 problems correctly. Only 1 or ±1,8% scored above 10 or 50% correct responses.

3.1.2.1.2 Test 2 Compare row 2: Frequency table 2

Table 5

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	3 4 8 6 6 8	5,5 7,3 14,5 10,9 10,9 14,5	35	63,6
6 7 8 9 10	11 4 3 1 0	20,0 7,3 5,5 1,8 0,0	6-10 correct solutions 19	34,6
11 12	0 1	0,0 1,8	more than 10 correct solutions 1	1,8

Thirty five or 63,6% of the 55 pupils solved 5 or less problems correctly and 19 or 34,6% solved 6 to 10 problems correctly. Only 1 or 1,8% scored 10 or 50% correct solutions.

3.1.2.1.3 Test 3. Compare row 3: Frequency table 2

Table 6

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	2	3,6		
1	6	10,9		
2	10	18,2	44	±80,0
3	13	23,6		
4	9	16,4		
5	4	7,3		
6	6	10,9	6-10 correct	
7	1	1,8	solutions	
8	1	1,8		
9	1	1,8	10	±18,2
10	1	1,8		
11	0	0,0	more than	
12	0	0,0	10 correct	
13	0	0,0	solutions	
14	0	0,0	1	± 1,8
15	1	1,8		

Forty four or ±80,0% of the 55 pupils solved 5 or less problems correctly, while 10 or ±18,2% solved 6 to 10 problems correctly. Only 1 or ±1,8% scored above 10 or 50% and obtained 75% correct answers.

3.1.3 School C

Number of pupils tested: 46

3.1.3.1 Analysis of raw scores

3.1.3.1.1 Test 1. Compare row 1: Frequency table 3

Table 7

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	0 1 1 0 5 6	0,0 2,2 2,2 0,0 10,9 13,0	13	$\pm$28,3
6 7 8 9 10	10 6 5 5 2	21,7 13,0 10,9 10,9 4,3	6-10 correct solutions 28	$\pm$60,9
11 12	1 4	2,2 8,7	more than 10 correct solutions 5	$\pm$10,9

Thirteen or $\pm$28,3% of the 46 pupils solved 5 or less problems correctly and 28 or $\pm$60,9% solved 6 to 10 problems correctly. Five or $\pm$10,9% scored above 10 or 50% correct answers.

3.1.3.1.2 Test 2. Compare row 2: Frequency table 3

Table 8

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	0	0,0		
1	0	0,0		
2	1	2,2	26	±56,5
3	5	10,9		
4	11	23,9		
5	9	19,6		
6	9	19,6	6-10 correct solutions	
7	5	10,9		
8	2	4,3		
9	3	6,5	20	±43,5
10	1	2,2		

Twenty six or ±56,5% of the 46 pupils solved 5 or less problems correctly and 20 or ±43,5% solved 6 to 10 problems correctly. None of the pupils scored above 10 or 50% correct answers.

3.1.3.1.3 Test 3. Compare row 3: Frequency table 3

Table 9

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	0	0,0		
1	2	4,3		
2	4	8,7	31	±67,4
3	10	21,7		
4	10	21,7		
5	5	10,9		

6 7 8 9 10	8 2 3 0 2	17,4 4,3 6,5 0,0 4,3	6-10 correct solutions 15	±32,6

Thirty one or ±67,4% of the 46 pupils solved 5 or less problems correctly and 15 or ±32,6% solved 6 to 10 problems correctly. None of the pupils could score above 10 or 50% correct answers.

3.1.4 School D

Number of pupils tested: 36

3.1.4.1 Analysis of raw scores

3.1.4.1.1 Test 1. Compare row 1: Frequency table 4

Table 10

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	1 5 8 6 5 6	2,8 13,9 22,2 16,7 13,9 16,7	31	±86,1

6 7 8 9 10	4 0 1 0 0	11,1 0,0 2,8 0,0 0,0	6-10 correct solutions 5	±13,9

Thirty one or ±86,1% of the 36 pupils solved 5 or less problems correctly and 5 or ±13,9% solved 6 to 8 problems correctly. None of the pupils could score 9 or 45% or more correct solutions.

3.1.4.1.2 Test 2. Compare row 2: Frequency table 4

Table 11

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	0 7 6 4 7 2	0,0 19,4 16,7 11,1 19,4 5,6	26	72,2
6 7 8 9 10	4 4 0 2 0	11,1 11,1 0,0 5,6 0,0	6-10 correct solutions 10	27,8

Twenty six or 72,2% of the 36 pupils solved 5 or less problems correctly and 10 or 27,8% solved 6 to 9 problems correctly. None of the pupils scored above 9 or 45% correct solutions.

3.1.4.1.3 Test 3. Compare row 3: Frequency table 4

Table 12

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	1	2,8		
1	5	13,9		
2	3	8,3	29	±80,6
3	9	25,0		
4	7	19,4		
5	4	11,1		
6	2	5,6	6-10 correct solutions	
7	4	11,1		
8	0	0,0		
9	0	0,0	6	±16,7
10	0	0,0		
			more than 10 correct solutions	
11	1	2,8	1	± 2,8

Twenty nine or ±80,6% of the 36 pupils solved 5 or less problems correctly and 6 or ±16,7% solved 6 to 7 problems correctly. Only 1 or ±2,8% managed to score above 10 or 50% correct answers.

3.1.5 School E

Number of pupils tested: 48

3.1.5.1 Analysis of raw scores

3.1.5.1.1 Test 1. Compare row 1: Frequency table 5.

Table 13

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	1	2,1		
1	1	2,1		
2	8	16,7	34	±70,8
3	9	18,8		
4	8	16,7		
5	7	14,6		
			6-10 correct solutions	
6	5	10,4		
7	6	12,5		
8	1	2,1		
9	0	0,0	14	±29,2
10	2	4,2		

Thirty four or ±70,8% of the 48 pupils solved 5 or less problems correctly and 14 or ±29,2% solved 6 to 10 problems correctly. None of the pupils could solve 10 or 50% or more of the problems correctly.

3.1.5.1.2 Test 2. Compare row 2: Frequency table 5

Table 14

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	0	0,0		
1	1	2,1		
2	8	16,7	30	±62,5
3	6	12,5		
4	8	16,7		
5	7	14,6		

6 7 8 9 10	12 4 2 0 0	25,0 8,3 4,2 0,0 0,0	6-10 correct solutions 18	+37,5

Thirty or +62,5% of the 48 pupils solved 5 or less problems correctly and 18 or +37,5% solved 6 to 8 problems correctly. None of the pupils scored 9 or 45% or more correct answers.

3.1.5.1.3 Test 3. Compare row 3: Frequency table 5

Table 15

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	2 9 3 12 4 9	4,2 18,8 6,3 25,0 8,3 18,8	39	+81,3
6 7 8 9 10	5 3 1 0 0	10,4 6,3 2,1 0,0 0,0	6-10 correct solutions 9	+18,8

Thirty nine or 81,3% of the 48 pupils solved 5 or less problems correctly and 9 or +18,8% solved 6 to 8 problems correctly. None of the pupils could solve 9 or 45% or more of the problems correctly.

3.1.6 School F

Number of pupils tested: 54

3.1.6.1 Analysis of raw scores

3.1.6.1.1 Test 1. Compare row 1: Frequency table 6

Table 16

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	2	3,7		
1	1	1,9		
2	3	5,6	27	±50,0
3	7	13,0		
4	4	7,4		
5	10	18,5		
6	3	5,6	6-10 correct solutions	
7	11	20,4		
8	3	5,6		
9	3	5,6	23	±42,6
10	3	5,6		
11	1	1,9	more than 10 correct solutions	
12	2	3,7		
13	0	0,0		
14	0	0,0	4	± 7,4
15	1	1,9		

Twenty seven or ±50,0% of the 54 pupils solved 5 or less problems correctly and 23 or ±42,6% solved 6 to 10 problems correctly. Only 4 or ±7,4% managed to solve more than 10 or 50% of the problems correctly and 1 or 1,9%

obtained 75%.

3.1.6.1.2 <u>Test 2. Compare row 2: Frequency table 6</u>

Table 17

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	0	0,0		
1	1	1,9		
2	5	9,3	36	±66,7
3	6	11,1		
4	14	25,9		
5	10	18,5		
6	6	11,1	6-10 correct solutions	
7	7	13,0		
8	1	1,9		
9	1	1,9	17	±31,5
10	2	3,7		
11	0	0,0	more than 10 correct solutions	
12	1	1,9	1	± 1,9

Thirty six or ±66,7% of the 54 pupils solved 5 or less problems correctly and 17 or ±31,5% solved 6 to 10 problems correctly. Only 1 or ±1,9% could solve more than 10 or 50% of the problems correctly.

3.1.6.1.3 Test 3. Compare row 3: Frequency table 6

Table 18

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	1 3 8 11 13 10	1,9 5,6 14,8 20,4 24,1 18,5	46	±85,2
6 7 8 9 10	3 2 1 1 0	5,6 3,7 1,9 1,9 0,0	6-10 correct solutions 7	±13,0
11 12	0 1	0,0 1,9	more than 10 correct solutions 1	± 1,9

Forty six or ±85,2% of the 54 pupils solved 5 or less problems correctly and 7 or ±13,0% solved 6 to 9 problems correctly. Only 1 or ±1,9% could solve more than 10 or 50% of the problems correctly.

3.1.7 School G

Number of pupils tested: 48

3.1.7.1 Analysis of raw scores

3.1.7.1.1 Test 1. Compare row 1: Frequency table 7

Table 19

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	3	6,3		
1	3	6,3		
2	9	18,8	38	$\pm$79,2
3	11	22,9		
4	7	14,6		
5	5	10,4		
6	7	14,6	6-10 correct solutions	
7	0	0,0		
8	2	4,2		
9	0	0,0	10	$\pm$20,8
10	1	2,1		

Thirty eight or $\pm$79,2% of the 48 pupils solved 5 or less problems correctly and 10 or $\pm$20,8% solved 6 to 10 problems correctly. None of the pupils could solve more than 10 or 50% of the problems correctly.

3.1.7.1.2 Test 2. Compare row 2: Frequency table 7

Table 20

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	1 3 7 5 11 6	2,1 6,3 14,6 10,4 22,9 12,5	33	±68,8
6 7 8 9 10	8 5 2 0 0	16,7 10,4 4,2 0,0 0,0	6-10 correct solutions 15	±31,3

Thirty three or ±68,8% of the 48 pupils solved 5 or less problems correctly and 15 or ±31,3% solved 6 to 8 problems correctly. None of the pupils could solve more than 9 or 45% of the problems correctly.

3.1.7.1.3 Test 3. Compare row 3: Frequency table 7

Table 21

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	2 5 10 5 9 4	4,2 10,4 20,8 10,4 18,8 8,3	35	72,9

6 7 8 9 10	8 4 0 0 0	16,7 8,3 0,0 0,0 0,0	6-10 correct solutions 12	25,0
11	1	2,1	more than 10 correct solu- tions 1	2,1

Thirty five or 72,9% of the 48 pupils solved 5 or less problems correctly and 12 or 25,0% solved 6 to 7 problems correctly. Only 1 or 2,1% managed to score above 10 or 50% of the problems correctly.

3.1.8 School H

Number of pupils tested: 45

3.1.8.1 Analysis of raw scores

3.1.8.1.1 Test 1. Compare row 1: Frequency table 8

Table 22

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	3 2 9 6 7 8	6,7 4,4 20,0 13,3 15,6 17,8	35	77,8

6 7 8 9 10	1 8 0 1 0	2,2 17,8 0,0 2,2 0,0	6-10 correct solutions 10	22,2

Thirty five or 77,8% of the 45 pupils solved 5 or less problems correctly and 10 or 22,2% solved 6 to 9 problems correctly. None of the pupils could solve 10 or 50% or more of the problems correctly.

3.1.8.1.2 Test 2. Compare row 2: Frequency table 8

Table 23

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	0 4 5 9 8 7	0,0 8,9 11,1 20,0 17,8 15,6	33	73,3
6 7 8 9 10	8 2 2 0 0	17,8 4,4 4,4 0,0 0,0	6-10 correct solutions 12	26,7

Thirty three or 73,3% of the 45 pupils solved 5 or less problems correctly and 12 or 26,7% solved 6 to 8 problems correctly. None of the pupils could solve 9 or 45% or more of the problems correctly.

3.1.8.1.3 Test 3. Compare row 3: Frequency table 8

Table 24

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	1	2,2		
1	4	8,9		
2	4	8,9	39	86,7
3	13	28,9		
4	14	31,1		
5	3	6,7		
6	3	6,7	6-10 correct solutions	
7	2	4,4		
8	0	0,0		
9	0	0,0	6	13,3
10	1	2,2		

Thirty nine of 86,7% of the 45 pupils solved 5 or less problems correctly and 6 or 13,3% solved 6 to 10 problems correctly. None of the pupils could solve more than 10 or 50% of the problems correctly.

3.1.9 School I

Number of pupils tested: 52

3.1.9.1 Analysis of raw scores

3.1.9.1.1 Test 1. Compare row 1: Frequency table 9

Table 25

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	1	1,9		
1	2	3,8		
2	1	1,9	22	±42,3
3	7	13,5		
4	4	7,7		
5	7	13,5		
6	4	7,7	6-10 correct solutions	
7	6	11,5		
8	6	11,5		
9	6	11,5	27	±51,9
10	5	9,6		
			more than 10 correct solutions	
11	1	1,9		
12	2	3,8	3	± 5,8

Twenty two or ±42,3% of the 52 pupils solved 5 or less problems correctly and 27 or ±51,9% solved 6 to 10 problems correctly. Only 3 or ±5,8% could solve more than 10 or 50% of the problems correctly.

3.1.9.1.2 Test 2. Compare row 2: Frequency table 9

Table 26

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	1	1,9		
1	0	0,0		
2	10	19,2	34	±65,4
3	6	11,5		
4	11	21,2		
5	6	11,5		
6	8	15,4	6-10 correct solutions	
7	4	7,7		
8	5	9,6		
9	1	1,9	18	±34,6
10	0	0,0		

Thirty four or ±65,4% of the 52 pupils solved 5 or less problems correctly and 18 or ±34,6% solved 6 to 9 problems correctly. None of the pupils could score 10 or 50% or more of the problems correctly.

3.1.9.1.3 Test 3. Compare row 3: Frequency table 9

Table 27

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	1	1,9		
1	4	7,7		
2	6	11,5	40	±76,9
3	7	13,5		
4	13	25,0		
5	9	17,3		

6 7 8 9 10	6 5 0 0 0	11,5 9,6 0,0 0,0 0,0	6-10 correct solutions 11	±21,2
11 12	0 1	0,0 1,9	more than 10 correct solutions 1	± 1,9

Forty or ±76,9% of the 52 pupils solved 5 or less problems correctly and 11 or ±21,2% solved 6 to 7 problems correctly. Only 1 or ± 1,9% could score above 10 or 50% correct solutions.

3.1.10 School J

Number of pupils tested: 51

3.1.10.1 Analysis of raw scores

3.1.10.1.1 Test 1. Compare row 1: Frequency table 10

Table 28

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	1	2,0		
1	4	7,8		
2	11	21,6	41	±80,4
3	4	7,8		
4	10	19,6		
5	11	21,6		
6	3	5,9	6-10 correct solutions	
7	4	7,8		
8	1	2,0		
9	1	2,0	10	±19,6
10	1	2,0		

Forty one or ±80,4% of the 51 pupils solved 5 or less problems correctly and 10 or ±19,6% solved 6 to 10 problems correctly. None of the pupils could score above 10 or 50% of the problems correctly.

3.1.10.1.2 Test 2. Compare row 2: Frequency table 10

Table 29

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	5	9,8		
1	6	11,8		
2	7	13,7	46	90,2
3	8	15,7		
4	11	21,6		
5	9	17,6		

6 7 8 9 10	2 3 0 0 0	3,9 5,9 0,0 0,0 0,0	6-10 correct solutions 5	9,8

Forty six or 90,2% of the 51 pupils solved 5 or less problems correctly and 5 or 9,8% solved 6 to 7 problems correctly. None of the pupils could solve 8 or 40% or more of the problems correctly.

3.1.10.1.3 Test 3. Compare row 3: Frequency table 10

Table 30

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0 1 2 3 4 5	2 3 8 7 10 9	3,9 5,9 15,7 13,7 19,6 17,6	39	$\pm$76,5
6 7 8 9 10	7 2 2 0 1	13,7 3,9 3,9 0,0 2,0	6-10 correct solutions 12	$\pm$23,5

Thirty nine or $\pm$76,5% of the 51 pupils solved 5 or less problems correctly and 12 or $\pm$23,5% solved 6 to 10 problems correctly. None of the pupils could score above 10 or 50% correct answers.

3.1.11 Analysis of the raw scores of the whole group

Number of pupils tested: 495

3.1.11.1 Test 1. Compare row 11: Frequency table 11

Table 31

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	14	2,8		
1	28	5,7		
2	62	12,5	322	±65,1
3	67	13,5		
4	71	14,3		
5	80	16,2		
6	43	8,7	6-10 correct solutions	
7	50	10,1		
8	28	5,7		
9	22	4,4	159	±32,1
10	16	3,2		
11	4	0,8	more than 10 correct solutions	
12	9	1,8		
13	0	0,0		
14	0	0,0	14	
15	1	0,2		± 2,8%

From the group as a whole of the 495 pupils 322 or ±65,1% solved 5 or less problems correctly and 159 or ±32,1% solved 6 to 10 problems correctly. In other words, 97,2% of the pupils could not solve more than 10 or 50% of the problems correctly. Only 14 or ±2,8% managed to score above 10 or 50% and only 1 of the 14 pupils scored 15 or 75% correct answers out of the possible 20.

3.1.11.2 Test 2. Compare row 2: Frequency table 11

Table 32

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	10	2,0		
1	31	6,3		
2	66	13,3	343	69,3
3	66	13,3		
4	99	20,0		
5	71	14,4		
6	71	14,4	6-10 correct solutions	
7	45	9,1		
8	21	4,2		
9	9	1,8	150	30,3
10	4	0,8		
			more than 10 correct solutions	
11	0	0,0		
12	2	0,4	2	0,4

Three hundred and forty three or 69,3% of the 495 pupils solved 5 or less problems correctly and 150 or 30,3% solved 6 to 10 problems correctly. The majority of the pupils, 493 or 99,6%, could not solve more than 10 or 50% of the problems correctly. Only 2 or 0,4% of the pupils scored above 10 or 50% and both obtained 60% or 12 correct answers our of the possible 20.

3.1.11.3 Table 3. Compare row 3: Frequency table 11

Table 33

Raw scores out of 20	Number of pupils	Percentage of pupils	0-5 Correct solutions	Percentage
0	12	2,4		
1	47	9,5		
2	57	11,5	391	79,0
3	102	20,6		
4	104	21,1		
5	69	13,9		
6	54	10,9	6-10 correct solutions	
7	29	5,9		
8	9	1,8		
9	2	0,4	99	20,0
10	5	1,0		
11	2	0,4	more than 10 correct solutions	
12	2	0,4		
13	0	0,0		
14	0	0,0	5	1,0
15	1	0,2		

Three hundred and ninety one or 79,0% of the 495 pupils solved 5 or less problems correctly and 99 or 20,0% solved 6 to 10 problems correctly. Here 99% of the pupils could not solve more than 10 or 50% of the problems correctly. Only 5 or 1% of the pupils solved more than 50% or 10 out of 20 problems correctly. Those who scored above 10 or 50% are 5 or 1% of the total number of pupils of which only 1 managed to score 75% or 15 out of 20 correct answers.

3.1.12 Analysis of the percentile ranks of the whole group

A percentile rank indicates the position of the pupil in a sample of 100 unselected pupils of the same standard.

3.1.12.1 Test 1. Compare row 5: Frequency table 11

Table 34

Percentile ranks	Number of pupils		Percentage	
0-5	241		48,7	
6-10	80	321 365	16,2	64,9 73,8
11-15	44		8,9	
16-20	0		0,0	
21-25	50		10,1	
26-30	0		0,0	
31-35	28		5,7	
36-40	22		4,4	
41-45	0		0,0	
46-50	16		3,2	
51-55	4	130	0,8	26,2
56-60	0		0,0	
61-65	9		1,8	
66-70	0		0,0	
71-75	0		0,0	
76-80	0		0,0	
81-85	1		0,2	

Three hundred and twenty one or 64,9% of the 495 pupils fall within the range of 0-10 percentile ranks; and 365 or 73,8% fall within the range of 0-15 percentile ranks. Only 130 or 26,2% of the pupils obtained percentile ranks of 16 and above. Only 1 pupil falls within the higher percentile ranks range of 81-85.

3.1.12.2 Test 2. Compare row 6: Frequency table 11.

Table 35

Percentile ranks	Number of pupils	Percentage
0-5	173	34,9
6-10	102 275 344	20,6 ±55,5 ±69,4
11-15	69	13,9
16-20	71	14,3
21-25	45	9,1
26-30	20	4,0
31-35	0	0,0
36-40	9 151	1,8 ±30,4
41-45	4	0,8
46-50	0	0,0
51-55	0	0,0
56-60	0	0,0
61-65	2	0,4

Two hundred and seventy five or ±55,5% of the 495 pupils fall within the range of 0-10 percentile ranks; and 344 or ±69,4% of the pupils obtained percentile ranks 0-15. Only 151 or ±30,4% of the pupils fall within the percentile ranks of 16 and above. Only 2 pupils fall within the higher percentile ranks range of 61-65.

3.1.12.3 Test 3. Compare row 7: Frequency table 11

Table 36

Percentile ranks	Number of pupils	Percentage
0-5	218 391 391	44,0 78,9 78,9
6-10	173	34,9
11-15	0	0,0
16-20	53	10,7
21-25	30	6,1
26-30	0	0,0
31-35	9	1,8
36-40	2	0,4
41-45	0	0,0
46-50	5	1,0
51-55	1 104	0,2 21,1
56-60	1	0,2
61-65	0	0,0
66-70	2	0,4
71-75	0	0,0
76-80	0	0,0
81-85	0	0,0
86-90	1	0,2

Three hundred and ninety one or 78,9% of the 495 pupils fall within the range of 0-10 percentile ranks; and 104 or 21,1% of the pupils obtained percentile ranks of 16 and higher. Only 1 or 0,2% of the pupils fall within the percentile ranks of 86-90, which is the highest in Test 3.

3.1.13 Analysis of the Stanines of the whole group

The stanine scale is a nine-point scale according to which the distribution of marks of raw scores is divided into nine categories. Stanines read from 1 to 9. 1-3 is regarded as weak, 4-6 as average while 7-9 is regarded as good. The stanine obtained by the pupil indicates the position he occupies on the nine-point scale.

3.1.13.1 Test 1. Compare row 9: Frequency table 11.

Table 37

Stanines	Number of pupils	Percentage
1 2 3	171 151 365 43	34,5 30,5 ±73,7 8,7
4 5 6	100 20 129 9	20,2 4,0 ± 26,0 1,8
7	1 1	0,2 ±0,2

Three hundred and sixty five or $\pm$73,7% of the 495 pupils obtained stanines 1-3 while 129 or $\pm$26% of the pupils obtained stanines 4-6. Only 1 pupil obtained stanine 7. This means that the performance of 365 or $\pm$73,7% of the pupils is weak while the performance of 129 or $\pm$26% of the pupils is average. Only 1 or $\pm$0,2% of the pupil's performance is good.

3.1.13.2 Test 2. Compare row 10: Frequency table 11.

Table 38

Stanines	Number of pupils		Percentage		
1 2 3	108 167 140	415	21,8 33,7 28,3	±	83,8
4 5 6	74 4 2	80	14,9 0,8 0,4	±	16,1

Four hundred and fifteen or ±83,8% of the 495 pupils obtained stanines 1-3 while 80 or ±16,1% of the pupils obtained stanines 4-6. This means that the performance of 415 or ±83,8% of the pupils is weak while the performance of 80 or ±16,1% of the pupils is average. None of the pupils obtained the stanine of 7 or above. That is, none of the pupils' performance is good.

3.1.13.3 Test 3. Comapre row 11: Frequency table 11.

Table 39

Stanines	Number of pupils		Percentage	
1 2 3	116 275 53	444	23,4 55,6 10,7	89,7
4 5 6	41 7 2	50	8,3 1,4 0,4	10,1
7	1	1	0,2	0,2

Four hundred and forty four or 89,7% of the 495 pupils obtained stanines 1-3 while 50 or 10,1% of the pupils obtained stanines 4-6. This means that the performance of 444 or 89,7% of the pupils is weak while the performance of 50 or 10,1% of the pupils is average. Only 1 or 0,2% of the pupils obtained the stanine 7. In other words, 1 pupil's performance is good.

TABLE 40

FREQUENCY TABLE 1

THE COLUMN ON THE LEFT HAND SIDE INDICATES THE TESTS 1-3, PERCENTILE RANKS 5-7, AND STANINES 9-11. CORRECT RESPONSES ARE INDICATED ON THE TOP HORIZONTAL ROW

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
3	4	7	12	10	4	6	8	4	1	0	1															
5,0%	6,7%	11,7%	20,0%	16,7%	6,7%	10,0%	13,3%	6,7%	1,7%	0,0%	1,7%															
5	9	11	12	7	3	7	4	1	1																	
8,3%	15,0%	18,3%	20,0%	11,7%	5%	11,7%	6,7%	1,7%	1,7%																	
6	1	15	15	12	6	4	1																			
10,0%	1,7%	25,0%	25,0%	20,0%	10,0%	6,7%	1,7%																			
7	0	0	12	0	0	0	0	10	0	0	0	0	0	4	0	0	0	0	0	0	0	6	0	0	0	
11,7%	0,0%	0,0%	20,0%	0,0%	0,0%	0,0%	0,0%	16,7%	0,0%	0,0%	0,0%	0,0%	0,0%	6,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	10,0%	0,0%	0,0%	0,0%	0
6	9	0	0	11	0	0	11	0	0	0	7	0	0	0	0	3	0	0	0	0	0	7	0	0	0	
10,0%	15,0%	0,0%	0,0%	18,3%	0,0%	0,0%	18,3%	0,0%	0,0%	0,0%	11,7%	0,0%	0,0%	0,0%	0,0%	5,0%	0,0%	0,0%	0,0%	0,0%	0,0%	11,7%	0,0%	0,0%	0,0%	0
1	0	0	14	0	15	0	0	0	12	0	0	0	0	0	6	0	0	0	0	0	0	4	0	0	0	
1,7%	0,0%	0,0%	23,3&	0,0%	25,0%	0,0%	0,0%	0,0%	20,0%	0,0%	0,0%	0,0%	0,0%	0,0%	10,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	6,7%	0,0%	0,0%	0,0%	0
14	22	4	18	1	1																					
23;3%	36,7%	6,7%	30,0%	1,7%	1,7%																					
12	23	9	15	1																						
20,0%	38,3%	15,0%	25,0%	1,7%																						
8	41	6	5																							
13,3%	68,3%	10,0%	8,3%																							

29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
0	0	8	0	0	0	0	0	0	4	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	
0,0%	0,0%	13,3%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	6,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0
0	4	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1										
0,0%	6,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,7%										
0	0	1	0	0	0	0	0	0																		
0,0%	0,0%	1,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%																		

57	58	59	60	61
0	0	0	0	1
0,0%	0,0%	0,0%	0,0%	1,7%

TABLE 41

FREQUENCY TABLE 2

THE COLUMN ON THE LEFT HAND SIDE INDICATES THE TESTS 1-3, PERCENTILE RANKS 5-7, AND STANINES 9-11. CORRECT RESPONSES ARE INDICATED ON THE TOP HORIZONTAL ROW

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
6	8	10	9	10	2	3	1	2	1	1																
10,9%	14,5%	18,2%	16,4%	18,2%	3,6%	5,5%	1,8%	3,6%	1,8%	1,8%																
4	8	6	6	8	11	4	3	1	0	0	1															
7,3%	14,5%	10,9%	10,9%	14,5%	20,0%	7,3%	5,5%	1,8%	0,0%	0,0%	1,8%															
6	10	13	9	4	6	1	1	1	1	0	0	0	0	1												
10,9%	18,2%	23,6%	16,4%	7,3%	10,9%	1,8%	1,8%	1,8%	1,8%	0,0%	0,0%	0,0%	0,0%	1,8%												
10	0	0	9	0	0	0	0	10	0	0	0	0	0	2	0	0	0	0	0	0	0	3	0	0	0	0
18,2%	0,0%	0,0%	16,4%	0,0%	0,0%	0,0%	0,0%	18,2%	0,0%	0,0%	0,0%	0,0%	0,0%	3,6%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	5,5%	0,0%	0,0%	0,0%	0,(
4	8	0	0	6	0	0	6	0	0	0	8	0	0	0	0	11	0	0	0	0	0	4	0	0	0	0
7,3%	14,5%	0,0%	0,0%	10,9%	0,0%	0,0%	10,9%	0,0%	0,0%	0,0%	14,5%	0,0%	0,0%	0,0%	0,0%	20,0%	0,0%	0,0%	0,0%	0,0%	0,0%	7,3%	0,0%	0,0%	0,0%	0,(
10	0	0	13	0	9	0	0	0	4	0	0	0	0	0	5	0	0	0	0	0	0	2	0	0	0	0
18,2%	0,0%	0,0%	23,6%	0,0%	16,4%	0,0%	0,0%	0,0%	7,3%	0,0%	0,0%	0,0%	0,0%	0,0%	9,1%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	3,6%	0,0%	0,0%	0,0%	0,(
26	19	2	6	2																						
47,3%	34,5%	3,6%	10,9%	3,6%																						
16	12	18	8	0	1																					
29,1%	21,8%	32,7%	14,5%	0,0%	1,8%																					
18	26	5	4	1	0	1																				
32,7%	29,1%	9,1%	7,3%	1,8%	0,0%	1,8%																				

29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
0	0	1	0	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	
0,0%	0,0%	1,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	3,6%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,8%	
0	3	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,0%	5,5%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	
0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	
0,0%	0,0%	1,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	

57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83
0	0	0	0	1																						
0,0%	0,0%	0,0%	0,0%	1,8%																						
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,(

85	86	87	88
0	0	0	1
0,0%	0,0%	0,0%	1,8%

TABLE 42

FREQUENCY TABLE 3

THE COLUMN ON THE LEFT HAND SIDE INDICATES THE TESTS 1-3, PERCENTILE RANKS 5-7, AND STANINES 9-11. CORRECT RESPONSES ARE INDICATED ON THE TOP HORIZONTAL ROW

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
	1	1	0	5	6	10	6	5	5	2	1	4															
%	2,2%	2,2%	0,0%	10,9%	13,0%	21,7%	13,0%	10,9%	10,9%	4,3%	2,2%	8,7%															
	0	1	5	11	9	9	5	2	3	1																	
%	0,0%	2,2%	10,9%	23,9%	19,6%	19,6%	10,9%	4,3%	6,5%	2,2%																	
	2	4	10	10	5	8	2	3	0	2																	
%	4,3%	8,7%	21,7%	21,7%	10,9%	17,4%	4,3%	6,5%	0,0%	4,3%																	
	0	0	0	5	0	0	0	0	6	0	0	0	0	0	10	0	0	0	0	0	0	0	6	0	0	0	
%	0,0%	0,0%	0,0%	10,9%	0,0%	0,0%	0,0%	0,0%	13,0%	0,0%	0,0%	0,0%	0,0%	0,0%	21,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	13,0%	0,0%	0,0%	0,0%	0
	0	1	0	0	5	0	0	12	0	0	0	9	0	0	0	0	9	0	0	0	0	0	5	0	0	0	
%	0,0%	2,2%	0,0%	0,0%	10,9%	0,0%	0,0%	26,1%	0,0%	0,0%	0,0%	19,6%	0,0%	0,0%	0,0%	0,0%	19,6%	0,0%	0,0%	0,0%	0,0%	0,0%	10,9%	0,0%	0,0%	0,0%	0
	4	0	0	10	0	10	0	0	0	5	0	0	0	0	0	8	0	0	0	0	0	0	2	0	0	0	
%	8,7%	0,0%	0,0%	21,7%	0,0%	21,7%	0,0%	0,0%	0,0%	10,9%	0,0%	0,0%	0,0%	0,0%	0,0%	17,4%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	4,3%	0,0%	0,0%	0,0%	0
	2	11	10	16	3	4																					
	4,3%	23,9%	21,7%	34,8%	6,5%	8,7%																					
	1	17	18	9	1																						
	2,2%	37,0%	39,1%	19,6%	2,2%																						
	6	25	8	5	2																						
	13,0%	54,3%	17,4%	10,9%	4,3%																						

	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
	0	0	5	0	0	0	0	0	0	5	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	1	
%	0,0%	0,0%	10,9%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	10,9%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	4,3%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	2,2%	0
	0	1	0	0	0	0	0	0	0	3	0	0	0	0	0	0	1										
%	0,0%	2,2%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	6,5%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	2,2%										
	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2								
%	0,0%	0,0%	6,5%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	4,3%								

	57	58	59	60	61
	0	0	0	0	4
%	0,0%	0,0%	0,0%	0,0%	8,7%

TABLE 4)

FREQUENCY TABLE 4

THE COLUMN ON THE LEFT HAND SIDE INDICATES THE TESTS 1-3, PERCENTILE RANKS 5-7, AND STANINES 9-11. CORRECT RESPONSES ARE INDICATED ON THE TOP HORIZONTAL ROW

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
	5	8	6	5	6	4	0	1																			
%	13,9%	22,2%	16,7%	13,9%	16,7%	11,1%	0,0%	2,8%																			
	7	6	4	7	2	4	4	0	2																		
%	19,4%	16,7%	11,1%	19,4%	5,6%	11,1%	11,1%	0,0%	5,6%																		
	5	3	9	7	4	2	4	0	0	0	1																
%	13,9%	8,3%	25,0%	19,4%	11,1%	5,6%	11,1%	0,0%	0,0%	0,0%	2,8%																
	6	1	0	5	0	0	0	0	6	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0
%	16,7%	2,8%	0,0%	13,9%	0,0%	0,0%	0,0%	0,0%	16,7%	0,0%	0,0%	0,0%	0,0%	0,0%	11,1%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0
	8	5	0	0	4	0	0	7	0	0	0	2	0	0	0	0	4	0	0	0	0	0	4	0	0	0	0
%	22,2%	13,9%	0,0%	0,0%	11,1%	0,0%	0,0%	19,4%	0,0%	0,0%	0,0%	5,6%	0,0%	0,0%	0,0%	0,0%	11,1%	0,0%	0,0%	0,0%	0,0%	0,0%	11,1%	0,0%	0,0%	0,0%	0,C
	3	0	0	9	0	7	0	0	0	4	0	0	0	0	0	2	0	0	0	0	0	0	4	0	0	0	0
%	8,3%	0,0%	0,0%	25,9%	0,0%	19,4%	0,0%	0,0%	0,0%	11,1%	0,0%	0,0%	0,0%	0,0%	0,0%	5,6%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	11,1%	0,0%	0,0%	0,0%	0,(
	20	11	4	1																							
	55,6%	30,6%	11,1%	2,8%																							
	12	11	7	6																							
	33,3%	30,6%	19,4%	16,7%																							
	9	20	2	4	1																						
	25,0%	55,6%	5,6%	11,1%	2,8%																						

	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	5
	0	0	1																								
%	0,0%	0,0%	2,8%																								
	0	0	0	0	0	0	0	0	0	2																	
%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	5,6%																	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,(

%

TABLE 44

FREQUENCY TABLE 5

THE COLUMN ON THE LEFT HAND SIDE INDICATES THE TESTS 1-3, PERCENTILE RANKS 5-7, AND STANINES 9-11. CORRECT RESPONSES ARE INDICATED ON THE TOP HORIZONTAL ROW

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	2
	1	8	9	8	7	5	6	1	0	2																	
1%	2,1%	16,7%	18,8%	16,7%	14,6%	10,4%	12,5%	2,1%	0,0%	4,2%																	
	1	8	6	8	7	12	4	2																			
0%	2,1%	16,7%	12,5%	16,7%	14,6%	25,0%	8,3%	4,2%																			
	9	3	12	4	9	5	3	1																			
2%	18,8%	6,3%	25,0%	8,3%	18,8%	10,4%	6,3%	2,1%																			
	9	0	0	8	0	0	0	0	7	0	0	0	0	0	5	0	0	0	0	0	0	0	6	0	0	0	
3%	18,8%	0,0%	0,0%	16,7%	0,0%	0,0%	0,0%	0,0%	14,6%	0,0%	0,0%	0,0%	0,0%	0,0%	10,4%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	12,5%	0,0%	0,0%	0,0%	0
	1	8	0	0	6	0	0	8	0	0	0	7	0	0	0	0	12	0	0	0	0	0	4	0	0	0	
0%	2,1%	16,7%	0,0%	0,0%	12,5%	0,0%	0,0%	16,7%	0,0%	0,0%	0,0%	14,6%	0,0%	0,0%	0,0%	0,0%	25,0%	0,0%	0,0%	0,0%	0,0%	0,0%	8,3%	0,0%	0,0%	0,0%	0
	2	0	0	13	0	4	0	0	0	9	0	0	0	0	0	5	0	0	0	0	0	0	3	0	0	0	
9%	4,2%	0,0%	0,0%	27,1%	0,0%	8,3%	0,0%	0,0%	0,0%	18,8%	0,0%	0,0%	0,0%	0,0%	0,0%	10,4%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	6,3%	0,0%	0,0%	0,0%	0
	19	15	5	7	2																						
	39,6%	31,3%	10,4%	14,6%	4,2%																						
	9	14	19	6																							
	18,8%	29,2%	39,6%	12,5%																							
	13	26	5	4																							
	27,1%	54,2%	10,4%	8,3%																							

	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46
	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
0%	0,0%	0,0%	2,1%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	4,2%
	0	2																
0%	0,0%	4,2%																
	0	0	1															
0%	0,0%	0,0%	2,1%															

FREQUENCY TABLE 6

THE COLUMN ON THE LEFT HAND SIDE INDICATES THE TESTS 1-3, PERCENTILE RANKS 5-7, AND STANINES 9-11. CORRECT RESPONSES ARE INDICATED ON THE TOP HORIZONTAL ROW

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	2
	1	3	7	4	10	3	11	3	3	3	1	2	0	0	1												
7%	1,9%	5,6%	13,0%	7,4%	18,5%	5,6%	20,4%	5,6%	5,6%	5,6%	1,9%	3,7%	0,0%	0,0%	1,9%												
	1	5	6	14	10	6	7	1	1	2	0	1															
0%	1,9%	9,3%	11,1%	25,9%	18,5%	11,1%	13,0%	1,9%	1,9%	3,7%	0,0%	1,9%															
	3	8	11	13	10	3	2	1	1	0	0	1															
9%	5,6%	14,8%	20,4%	24,1%	18,5%	5,6%	3,7%	1,9%	1,9%	0,0%	0,0%	1,9%															
	7	0	0	4	0	0	0	0	10	0	0	0	0	0	3	0	0	0	0	0	0	0	11	0	0	0	
1%	13,0%	0,0%	0,0%	7,4%	0,0%	0,0%	0,0%	0,0%	18,5%	0,0%	0,0%	0,0%	0,0%	0,0%	5,6%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	20,4%	0,0%	0,0%	0,0%	0
	1	5	0	0	6	0	0	14	0	0	0	10	0	0	0	0	6	0	0	0	0	0	7	0	0	0	
%	1,9%	9,3%	0,0%	0,0%	11,1%	0,0%	0,0%	25,9%	0,0%	0,0%	0,0%	18,5%	0,0%	0,0%	0,0%	0,0%	11,1%	0,0%	0,0%	0,0%	0,0%	0,0%	13,0%	0,0%	0,0%	0,0%	0
	8	0	0	11	0	13	0	0	0	10	0	0	0	0	0	3	0	0	0	0	0	0	2	0	0	0	
%	14,8%	0,0%	0,0%	20,4%	0,0%	24,1%	0,0%	0,0%	0,0%	18,5%	0,0%	0,0%	0,0%	0,0%	0,0%	5,6%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	3,7%	0,0%	0,0%	0,0%	0
	13	14	3	17	4	2	1																				
	24,1%	25,9%	5,6%	31,5%	7,4%	3,7%	1,9%																				
	6	20	16	9	2	1																					
	11,1%	37,0%	29,6%	16,7%	3,7%	1,9%																					
	12	34	3	4	0	1																					
	22,2%	63,0%	5,6%	7,4%	0,0%	1,9%																					

	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	5
	0	0	3	0	0	0	0	0	0	3	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	1	(
%	0,0%	0,0%	5,6%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	5,6%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	5,6%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,9%	0
	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	(
%	0,0%	1,9%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,9%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	3,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0
	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
%	0,0%	0,0%	1,9%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,9%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0

	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	8
	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
%	0,0%	0,0%	0,0%	0,0%	3,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,
	0	0	0	0	1																						
%	0,0%	0,0%	0,0%	0,0%	1,9%																						
	0	0	0	0	0	0	0	0	0	1																	
%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,9%																	

TABLE 40

FREQUENCY TABLE 7

THE COLUMN ON THE LEFT HAND SIDE INDICATES THE TESTS 1-3, PERCENTILE RANKS 5-7, AND STANINES 9-11. CORRECT RESPONSES ARE INDICATED ON THE TOP HORIZONTAL ROW

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
	3	9	11	7	5	7	0	2	0	1																	
%	6,3%	18,8%	22,9%	14,6%	10,4%	14,6%	0,0%	4,2%	0,0%	2,1%																	
	3	7	5	11	6	8	5	2																			
%	6,3%	14,6%	10,4%	22,9%	12,5%	16,7%	10,4%	4,2%																			
	5	10	5	9	4	8	4	0	0	0	1																
%	10,4%	20,8%	10,4%	18,8%	8,3%	16,7%	8,3%	0,0%	0,0%	0,0%	2,1%																
	11	0	0	7	0	0	0	0	5	0	0	0	0	0	7	0	0	0	0	0	0	0	0	0	0	0	0
%	22,9%	0,0%	0,0%	14,6%	0,0%	0,0%	0,0%	0,0%	10,4%	0,0%	0,0%	0,0%	0,0%	0,0%	14,6%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,
	3	7	0	0	5	0	0	11	0	0	0	6	0	0	0	0	8	0	0	0	0	0	5	0	0	0	0
%	6,3%	14,6%	0,0%	0,0%	10,4%	0,0%	0,0%	22,9%	0,0%	0,0%	0,0%	12,5%	0,0%	0,0%	0,0%	0,0%	16,7%	0,0%	0,0%	0,0%	0,0%	0,0%	10,4%	0,0%	0,0%	0,0%	0,
	10	0	0	5	0	9	0	0	0	4	0	0	0	0	0	8	0	0	0	0	0	0	4	0	0	0	0
%	20,8%	0,0%	0,0%	14,4%	0,0%	18,8%	0,0%	0,0%	0,0%	8,3%	0,0%	0,0%	0,0%	0,0%	0,0%	16,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	8,3%	0,0%	0,0%	0,0%	0,
	26	12	7	2	1																						
	51,2%	25,0%	14,6%	4,2%	2,1%																						
	11	16	14	7																							
	22,9%	33,3%	29,2%	14,6%																							
	17	18	8	4	1																						
	35,4%	37,5%	16,7%	8,3%	2,1%																						

	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1								
%	0,0%	0,0%	4,2%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	2,1%								
	0	2																								
%	0,0%	4,2%																								
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	2,1%

TABLE 47

FREQUENCY TABLE 8

THE COLUMN ON THE LEFT HAND SIDE INDICATES THE TESTS 1-3, PERCENTILE RANKS 5-7, AND STANINES 9-11. CORRECT RESPONSES ARE INDICATED ON THE TOP HORIZONTAL ROW

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	2
	2	9	6	7	8	1	8	0	1																		
%	4,4%	20,0%	13,3%	15,6%	17,8%	2,2%	17,8%	0,0%	2,2%																		
	4	5	9	8	7	8	2	2																			
%	8,9%	11,1%	20,0%	17,8%	15,6%	17,8%	4,4%	4,4%																			
	4	4	13	14	3	3	2	0	0	1																	
%	8,9%	8,9%	28,9%	31,1%	6,7%	6,7%	4,4%	0,0%	0,0%	2,2%																	
	6	0	0	7	0	0	0	0	8	0	0	0	0	0	1	0	0	0	0	0	0	0	8	0	0	0	
%	13,3%	0,0%	0,0%	15,6%	0,0%	0,0%	0,0%	0,0%	17,8%	0,0%	0,0%	0,0%	0,0%	0,0%	2,2%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	17,8%	0,0%	0,0%	0,0%	0
	4	5	0	0	9	0	0	8	0	0	0	7	0	0	0	0	8	0	0	0	0	0	2	0	0	0	
%	8,9%	11,1%	0,0%	0,0%	20,0%	0,0%	0,0%	17,8%	0,0%	0,0%	0,0%	15,6%	0,0%	0,0%	0,0%	0,0%	17,8%	0,0%	0,0%	0,0%	0,0%	0,0%	4,4%	0,0%	0,0%	0,0%	0
	4	0	0	13	0	14	0	0	0	3	0	0	0	0	0	3	0	0	0	0	0	0	2	0	0	0	
%	8,9%	0,0%	0,0%	28,9%	0,0%	31,1%	0,0%	0,0%	0,0%	6,7%	0,0%	0,0%	0,0%	0,0%	0,0%	6,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	4,4%	0,0%	0,0%	0,0%	0
	20	15	1	9																							
	44,4%	33,3%	2,2%	20,0%																							
	8	18	15	4																							
	17,8%	40,0%	33,3%	8,9%																							
	9	30	3	2	1																						
	20,0%	66,7%	6,7%	4,4%	2,2%																						

	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
	0	0	0	0	0	0	0	0	0	1									
0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	2,2%									
	0	2																	
0%	0,0%	4,4%																	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	2,2%

TABLE 40

FREQUENCY TABLE 9

THE COLUMN ON THE LEFT HAND SIDE INDICATES THE TESTS 1-3, PERCENTILE RANKS 5-7, AND STANINES 9-11. CORRECT RESPONSES ARE INDICATED ON THE TOP HORIZONTAL ROW

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
	2	1	7	4	7	4	6	6	6	5	1	2															
%	3,8%	1,9%	13,5%	7,7%	13,5%	7,7%	11,5%	11,5%	11,5%	9,6%	1,9%	3,8%															
	0	10	6	11	6	8	4	5	1																		
%	0,0%	19,2%	11,5%	21,2%	11,5%	15,4%	7,7%	9,6%	1,9%																		
	4	6	7	13	9	6	5	0	0	0	0	1															
%	7,7%	11,5%	13,5%	25,0%	17,3%	17,5%	9,6%	0,0%	0,0%	0,0%	0,0%	1,9%															
	7	0	0	4	0	0	0	0	7	0	0	0	0	0	4	0	0	0	0	0	0	0	6	0	0	0	0
%	13,5%	0,0%	0,0%	7,7%	0,0%	0,0%	0,0%	0,0%	13,5%	0,0%	0,0%	0,0%	0,0%	0,0%	7,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	11,5%	0,0%	0,0%	0,0%	0,(
	0	10	0	0	6	0	0	11	0	0	0	6	0	0	0	0	8	0	0	0	0	0	4	0	0	0	0
%	0,0%	19,2%	0,0%	0,0%	11,5%	0,0%	0,0%	21,2%	0,0%	0,0%	0,0%	11,5%	0,0%	0,0%	0,0%	0,0%	15,4%	0,0%	0,0%	0,0%	0,0%	0,0%	7,7%	0,0%	0,0%	0,0%	0,(
	6	0	0	7	0	13	0	0	0	9	0	0	0	0	0	6	0	0	0	0	0	0	5	0	0	0	0
%	11,5%	0,0%	0,0%	13,5%	0,0%	25,0%	0,0%	0,0%	0,0%	17,3%	0,0%	0,0%	0,0%	0,0%	0,0%	11,5%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	9,6%	0,0%	0,0%	0,0%	0,(
	11	11	4	18	6	2																					
	21,2%	21,2%	7,7%	34,6%	11,5%	3,8%																					
	11	17	14	10																							
	21,2%	32,7%	26,9%	19,2%																							
	11	29	6	5	0	1																					
	21,2%	55,8%	11,5%	9,6%	0,0%	1,9%																					

	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	5:
	0	0	6	0	0	0	0	0	0	6	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	1	
0%	0,0%	0,0%	11,5%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	11,5%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	9,6%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,9%	0
	0	5	0	0	0	0	0	0	0	1																	
0%	0,0%	9,6%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,9%																	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0

	57	58	59	60	61	62	63	64	65	66
	0	0	0	0	2					
0%	0,0%	0,0%	0,0%	0,0%	3,8%					
	0	0	0	0	0	0	0	0	0	1
0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,9

TABLE 49

FREQUENCY TABLE 10

THE COLUMN ON THE LEFT HAND SIDE INDICATES THE TESTS 1-3, PERCENTILE RANKS 5-7, AND STANINES 9-11. CORRECT RESPONSES ARE INDICATED ON THE TOP HORIZONTAL ROW

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	2[illegible]
4	11	4	10	11	3	4	1	1	1																	
7,8%	21,6%	7,8%	19,6%	21,6%	5,9%	7,8%	2,0%	2,0%	2,0%																	
6	7	8	11	9	2	3																				
11,8%	13,7%	15,7%	21,6%	17,6%	3,9%	5,9%																				
3	8	7	10	9	7	2	2	0	1																	
5,9%	15,7%	13,7%	19,6%	17,6%	13,7%	3,9%	3,9%	0,0%	2,0%																	
4	1	0	10	0	0	0	0	11	0	0	0	0	0	3	0	0	0	0	0	0	0	4	0	0	0	[illegible]
7,8%	2,0%	0,0%	19,6%	0,0%	0,0%	0,0%	0,0%	21,6%	0,0%	0,0%	0,0%	0,0%	0,0%	5,9%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	7,8%	0,0%	0,0%	0,0%	0,[illegible]
6	7	0	0	8	1	0	10	0	2	0	7	0	0	0	0	2	0	0	0	0	0	3	0	0	0	[illegible]
11,8%	13,7%	0,0%	0,0%	15,7%	2,0%	0,0%	19,6%	0,0%	3,9%	0,0%	13,7%	0,0%	0,0%	0,0%	0,0%	3,9%	0,0%	0,0%	0,0%	0,0%	0,0%	5,9%	0,0%	0,0%	0,0%	0,[illegible]
8	0	0	7	0	11	0	0	0	9	0	0	0	0	0	6	0	0	0	0	0	0	2	0	0	0	[illegible]
15,7%	0,0%	0,0%	13,7%	0,0%	21,6%	0,0%	0,0%	0,0%	17,6%	0,0%	0,0%	0,0%	0,0%	0,0%	11,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	3,9%	0,0%	0,0%	0,0%	0,[illegible]
20	21	3	6	1																						
39,2%	41,2%	5,9%	11,8%	2,0%																						
18	21	9	3																							
35,3%	41,2%	17,6%	5,9%																							
13	26	7	4	1																						
25,5%	51,0%	13,7%	7,8%	2,0%																						

29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	
0,0%	0,0%	2,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	2,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	2,0%	
0	0	0	0	0	0	0	0	0										
0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%										
0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0,0%	0,0%	3,9%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	2,0%

TABLE 50

FREQUENCY TABLE 11

THE COLUMN ON THE LEFT HAND SIDE INDICATES THE TESTS 1-3, PERCENTILE RANKS 5-7, AND STANINES 9-11. CORRECT RESPONSES ARE INDICATED ON THE TOP HORIZONTAL ROW

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
28	62	67	71	80	43	50	28	22	16	4	9	0	0	1												
5,7%	12,5%	13,5%	14,3%	16,2%	8,7%	10,1%	5,7%	4,4%	3,2%	0,8%	1,8%	0,0%	0,0%	0,2%												
31	66	66	99	71	71	45	21	9	4	0	2															
6,3%	13,3%	13,3%	20,0%	14,4%	14,4%	9,1%	4,2%	1,8%	0,8%	0,0%	0,4%															
47	57	102	104	69	54	29	9	2	5	2	2	0	0	1												
9,5%	11,5%	20,6%	21,1%	13,9%	10,9%	5,9%	1,8%	0,4%	1,0%	0,4%	0,4%	0,0%	0,0%	0,2%												
66	2	0	71	0	0	0	0	80	0	0	0	0	0	44	0	0	0	0	0	0	0	50	0	0	0	0
13,3%	0,4%	0,0%	14,4%	0,0%	0,0%	0,0%	0,0%	16,2%	0,0%	0,0%	0,0%	0,0%	0,0%	8,9%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	10,1%	0,0%	0,0%	0,0%	0,
33	65	0	0	65	1	0	99	0	2	0	69	0	0	0	0	71	0	0	0	0	0	45	0	0	0	0
6,7%	13,1%	0,0%	0,0%	13,1%	0,2%	0,0%	20,0%	0,0%	0,4%	0,0%	13,9%	0,0%	0,0%	0,0%	0,0%	14,4%	0,0%	0,0%	0,0%	0,0%	0,0%	9,1%	0,0%	0,0%	0,0%	0,
56	0	0	101	1	104	0	0	0	69	0	0	0	0	0	53	0	0	0	0	0	0	30	0	0	0	0
11,3%	0,0%	0,0%	20,4%	0,2%	21,1%	0,0%	0,0%	0,0%	13,9%	0,0%	0,0%	0,0%	0,0%	0,0%	10,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	6,1%	0,0%	0,0%	0,0%	0,
171	151	43	100	20	9	1																				
34,5%	30,5%	8,7%	20,2%	4,0%	1,8%	0,2%																				
108	167	140	74	4	2																					
21,8%	33,7%	28,3%	14,9%	0,8%	0,4%																					
116	275	53	41	7	2	1																				
23,4%	55,6%	10,7%	8,3%	1,4%	0,4%	0,2%																				

29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	5
0	0	28	0	0	0	0	0	0	22	0	0	0	0	0	0	0	16	0	0	0	0	0	0	0	4	0
0,0%	0,0%	5,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	4,4%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	3,2%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,8%	0,
0	20	0	0	0	0	0	0	0	9	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0
0,0%	4,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,
0	0	9	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	5	0	0	0	0	0	0	1	0
0,0%	0,0%	1,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,4%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,2%	0,

57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	8
0	0	0	0	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0,0%	0,0%	0,0%	0,0%	1,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,
0	0	0	0	2																						
0,0%	0,0%	0,0%	0,0%	0,4%																						
0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,4%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,

85	86	87	88
0	0	0	1
0,0%	0,0%	0,0%	0,2%

3.2 DIAGNOSTIC MATHEMATICAL TESTS

To ascertain more specifically where the difficulty lies, one or more of a wide range of standardized diagnostic tests will be used. For example, if a pupil's progress in Mathematics seems to be much slower than may be expected, a standardized diagnostic test in Mathematics will be administered. The results of this test will indicate whether the pupil has any difficulties in the basic mathematical processes - addition, division, multiplication and subtraction (Duminy and Du Preez, 1974, p.42). If a weak spot can be identified, a programme of remedial teaching should be arranged for the pupil. In this research the diagnostic tests of the Human Sciences Research Council have been used.

3.2.1 Summary: Test 5: Basic operations ($\leq$100 000)

Number of pupils tested: 100

3.2.1.1 Subtest 1: Addition

Table 51

Item	1a	1b	1c	1d	2	3	4	5	6	7	8	9	Total
% Pupils who had the item incorrect	13	11	44	68	20	19	40	28	20	29	31	29	29,3
No of pupils who had the item incorrect	13	11	44	68	20	19	40	28	20				

In items 1a-1d the pupils were required to add natural numbers which were arranged horizontally. Item 1c, in which the place value played an important part, was solved incorrectly by 44% of the pupils. In item 1d the numbers to be added were written in words and was incorrectly solved by 68% of the pupils. This indicates that the pupils were unable to transcribe the numbers which were written in words to numerals.

In items 2-9 the pupils were required to add numbers which were arranged in columns. Pupils performed relatively better in these items compared to items 1c and 1d. The average percentage of incorrect solutions of items 2-9 is 27%. In this subtest a number of the pupils were unable to carry over correctly when adding the numbers.

3.2.1.2 Subtest 2: Subtraction

Table 52

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
% Pupils who had the item incorrect	5	18	18	16	36	73	26	71	37	49	37	36	44	47	49	41	37,7
No pupils who had the item incorrect	5	18	18	16	36	73	26	71	37	49	37	36	44	47	49	41	603

In items 1-4 the pupils were required to subtract numbers which did not involve any borrowing. The only difficulty inherent in these problems was the zero difficulty. The average percentage of incorrect solutions of items 1-4 is 14,3%. This shows that most of the pupils can subtract correctly if no borrowing is involved.

In items 5-16 borrowing was necessary. The majority of the items contained one or two zeroes. Compared to the first 4 items, pupils performed badly in these items. The average percentage of incorrect solutions is 45,5%. 73% of the pupils could not solve item 6 because it involved borrowing.

3.2.1.3 Subtest 3: Multiplication

Table 53

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
% Pupils who had the item incorrect	23	43	49	41	64	70	70	62	69	71	84	81	74	77	70	60	63,0
No pupils who had the item incorrect	23	43	49	41	64	70	70	62	69	71	84	81	74	77	70	60	1 008

Item 1 involved no carrying. 23% of the pupils was unable to solve this item correctly. This indicates that these pupils do not know the multiplication tables well enough.

In items 2 and 3 two difficulties were present. The first one was the zero difficulty and the second one was carrying. 43% of the pupils solved item 2 incorrectly and 49% of the pupils could not solve item 3 correctly.

Item 4 differed from items 2 and 3. It did not have a zero difficulty, but it had the problem of successive carrying. 41% of the pupils were unable to solve item 4 correctly.

Items 5-8 involved multiplication by a multiple of 10. In these items the average percentage of incorrect solutions is 66,5%. This indicates that most of the pupils failed to add zero at the end of the products.

Items 9-12 required multiplication, addition and carrying. In these items the average percentage of incorrect solutions is 76,3%. This indicates that most of the pupils were unable to perform multiplication which was coupled with addition and carrying.

In items 13-15 multiplication involved addition, carrying and the zero difficulty. The average percentage of incorrect solutions is 73,7%. Once more, most of the pupils were unable to multiply correctly if the problem involved addition, carrying and the zero difficulty.

Item 16 involved multiplication by a multiple of 100. 60% of the pupils gave incorrect solutions to this problem. This shows that some of the pupils did not write two zeroes at the end of the product.

3.2.1.4 Subtest 4: Division

Table 54

Item	1	2	3	4	5	6	7	8	9
% Pupils who had the item incorrect	78	80	87	96	70	87	80	92	82
No pupils who had the item incorrect	78	80	87	96	70	87	80	92	82

10	11	12	13	14	15	16	17	18	19	20	Total
80	98	99	81	99	98	99	97	97	100	100	90,0
80	98	99	81	99	98	99	97	97	100	100	1 800

In item 1 the zero difficulty was not coupled with carrying. 78% of the pupils were unable to solve this item correctly. This shows that most of the pupils cannot do simple division.

Initems 2-4 the zero difficulty was coupled with carrying. The average percentage of incorrect solutions of these items is 87,7%. This indicates that most of the pupils were unable to overcome the zero difficulty coupled with carrying.

In items 5-20 pupils performed very badly. The average percentage of incorrect solutions of items 5-20 is 91,2%. Not a single pupil was able to solve items 19 and 20 correctly.

3.2.2 Summary: Test 6: Vulgar fractions

Number of pupils tested: 100

3.2.2.1 Subtest 1: Concept of vulgar fractions

Table 55

Item	1A	2A	3A	4A	5A	6A	7A	8A	9A
% Pupils who had the item incorrect	9	23	15	64	21	23	18	43	52
No pupils who had the item incorrect	9	23	15	64	21	23	18	43	52

Item	A 10	A 11	A 12	A 13	A 14	A 15	A 16	B 1	B 2	B 3	B 4	C 1	C 2	C 3	C 4	D 1	D 2	D 3	D 4
% Pupils who had the item incorrect	43	44	66	70	52	78	61	57	61	59	61	11	23	74	49	18	25	26	23
No pupils who had the item incorrect	43	44	66	70	52	78	61	57	61	59	61	11	23	74	49	18	25	26	23

Item	1E	E2	3E	4E	Total
% Pupils who had the item incorrect	12	12	13	13	38,1
No pupils who had the item incorrect	12	12	13	13	1 219

Items 1A-16A tested the pupils' understanding of the concept of equivalent fractions. Here the percentage of incorrect solutions range from 9% to 78%. On the average the percentage of incorrect solutions of items 1A-16A is 42,6%. This indicates that most of the pupils have not yet grasped the concept of equivalent fractions.

In items 1B-4B the pupils were to identify the fractions which were in the simplest form. In these items the percentage of incorrect solutions per item range from 57% to 61%. On the average the percentage of incorrect solutions of items 1B-4B is 59,5%. This shows that the majority of the pupils were unable to identify fractions which were in the simplest form.

In items 1C-4C the pupils were to rewrite the given fractions in the simplest form. The average percentage of incorrect solutions is 39,2%. This indicates that some of the pupils were unable to simplify a given fraction.

In items 1D-4D the pupils were required to rewrite the improper fractions as mixed numbers. The table shows that 77% of the pupils could give the correct answers as they appeared on the memorandum. On the average 23% of the pupils gave incorrect solutions to items 1D-4D. This shows that the majority of the pupils were able to change improper fractions to mixed numbers.

In items 1E-4E the pupils were required to rewrite mixed numbers as improper fractions. It is evident from the table that a mere 12,5% of the pupils on the average could not give the correct solutions to items 1E-4E. This means that most of the pupils, 87,5%, knew how to convert mixed numbers to improper fractions.

3.2.2.2 Subtest 2: Addition

Table 56

Item	1	2	3	4	5	6	7	8	9
% Pupils who had the item incorrect	15	9	20	13	65	75	43	74	20
No pupils who had the item incorrect	15	9	20	13	65	75	43	74	20

10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
23	19	27	69	72	65	68	17	37	19	57	24	34	31	37	73	73	68	84
23	19	27	69	72	65	68	17	37	19	57	24	34	31	37	73	73	68	84

29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
30	24	33	39	51	53	42	46	69	76	72	76	62	76	78	76	20	33	31
30	24	33	39	51	53	42	46	69	76	72	76	62	76	78	76	20	33	31

48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66
34	29	37	36	38	41	39	67	76	49	52	76	79	29	34	31	50	31	43
34	29	37	36	38	41	39	67	76	49	52	76	79	29	34	31	50	31	43

67	68	69	70	71	72	Total
44	45	46	60	57	73	47,4
44	45	46	60	57	73	3 414

In items 1-16 the pupils were to add two vulgar fractions which had the same denominator. The table shows that

42,3% of the pupils could not solve problems 1-16 correctly. Item 2 was incorrectly solved by 9% of the pupils while 75% of the pupils could not solve item 6 correctly. This may indicate that the pupils are more familiar with 10 as a denominator compared to 6 as a denominator as is clearly evidenced in the results of items 2 and 6.

In items 17-19 the pupils were required to add a vulgar fraction to a natural number. The average percentage of incorrect solutions of items 17-19 is 24,3%. On the whole, the majority of the pupils, 75,7%, performed well.

In item 20 the pupils were required to add a natural number to a mixed number. 57% of the pupils could not solve this item correctly. This indicates that the pupils had problems in adding a natural number to a mixed number.

In items 21-28 the pupils were required to add a vulgar fraction to a mixed number. The average percentage of incorrect solutions of items 21-28 is 53%. This shows that the majority of the pupils were unable to add a vulgar fraction to a mixed number.

In items 29-44 the pupils were required to add a mixed number to another mixed number. The average percentage of incorrect solutions is 56,4%. This shows clearly that the majority of the pupils were unable to add a mixed number to another mixed number.

In items 45-52 the pupils were required to add vulgar fractions with different denominators. The second denominator was a multiple of the first one. The average percentage of incorrect solutions of items 45-52 is 32,2%. This indicates that the majority of the pupils were able to add two vulgar fractions with different denominators together.

In items 53-56 the pupils were required to add a vulgar fraction to a mixed number. The denominator of the first fraction was a multiple of the denominator of the second fraction. The average percentage of incorrect solutions of items 53-56 is 55,8%. This indicates that the majority of the pupils were unable to add a vulgar fraction to a mixed number correctly.

In items 57-60 the pupils were required to add a mixed number to another mixed number. As in the previous items, namely, 53-56, the denominator of the second fraction was a multiple of the denominator of the first fraction. The average percentage of incorrect solutions of items 57-60 is 64%. This shows very well that the majority of the pupils were unable to add mixed numbers.

In items 61-72 the pupils were required to add one vulgar fraction to another. Both vulgar fractions were having different denominators. The average percentage of incorrect solutions of items 61-72 is 45,3%. On the whole the performance of some of the pupils in these items was not satisfactory.

3.2.3 Summary: Test 7: Vulgar fractions

Number of pupils tested: 100

3.2.3.1 Subtest 1: Subtraction

Table 57

Item	1	2	3	4	5	6	7	8	9
% Pupils who had the item incorrect	34	55	58	49	45	75	25	50	29
No pupils who had the item incorrect	34	55	58	49	45	75	25	50	29

10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
52	49	68	84	100	99	97	90	89	94	90	88	98	97	98	98	100	100
52	49	68	84	100	99	97	90	89	94	90	88	98	97	98	98	100	100

28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46
99	76	88	93	88	88	94	91	93	90	94	93	95	99	99	100	99	73	73
99	76	88	93	88	88	94	91	93	90	94	93	95	99	99	100	99	73	73

47	48	49	50	51	52	53	54	55	56	Total
74	93	95	95	95	99	92	92	100	100	83,4
74	93	95	95	95	99	92	92	100	100	4671

In items 1-8 the pupils were required to subtract vulgar fractions with the same denominators. In these items the average percentage of incorrect solutions is 48,9%. This shows that some of the pupils were unable to subtract vulgar fractions. Here the pupils should have performed better because there was no problem of the Lowest Common Multiple, since the denominators of both fractions were the same.

In items 9-12 the pupils were required to subtract a natural number from a mixed number. In these items the average percentage of incorrect solutions is 49,5%. This shows that almost half of the pupils were unable to subtract a natural number from a mixed number correctly.

In items 13-16 the pupils were required to subtract a vulgar fraction from a natural number. The average percentage of incorrect solutions of items 13-16 is 95%. This percentage is very high and it shows clearly that almost all the pupils were unable to subtract a vulgar fraction from a natural number. Only 5% of the problems were correctly solved. The pupils were unable to convert the natural number to an equivalent improper fraction and to look for the correct Lowest Common Multiple.

In items 17-24 the pupils were required to subtract mixed numbers. Both mixed numbers were having the same denominator. The average percentage of incorrect solutions of items 17-24 is 93%. This shows that almost all the pupils were unable to subtract a mixed number from another mixed number. It also indicates that the pupils failed to convert the mixed numbers to improper fractions.

In items 25-28 the pupils were required to subtract a mixed number from a natural number. The average percentage of incorrect solutions is 99,3%. This indicates clearly that almost all the pupils experienced serious difficulties when subtracting a mixed number from a natural number.

In items 29-36 the pupils were required to subtract a vulgar fraction from another vulgar fraction. The denominator of the first vulgar fraction was a multiple of the denominator of the second vulgar fraction. The average percentage of incorrect solutions of items 29-36 is 88,9%. This shows that most of the pupils were unable to calculate the Lowest Common Multiple. As a result they failed to solve these problems.

In items 37-40 the pupils were required to subtract a vulgar fraction from a mixed number. The denominator of the vulgar fraction was a multiple of the denominator of the mixed number. The average percentage of incorrect solutions of items 37-40 is 93%. This also indicates that almost all the pupils were unable to subtract a vulgar fraction from a mixed number.

In items 41-44 the pupils were required to subtract a mixed number from another mixed number. The denominator of the first mixed number was a multiple of the denominator of the second mixed number. The average percentage of incorrect solutions of items 41-44 is 99,3%. This high percentage of incorrect solutions indicates clearly that most of the pupils were unable to subtract a mixed number from another mixed number.

In items 45-52 the pupils were required to subtract a vulgar fraction from another vulgar fraction. The denominators of the two vulgar fractions were different. The average percentage of incorrect solutions of items 45-52 is 87,1%. This percentage indicates clearly that the majority of the pupils were unable to subtract one vulgar fraction from another.

In items 53-56 the pupils were required to subtract a mixed number from another mixed number. The denominators of the two mixed numbers were different. The average percentage of incorrect solutions of items 53-56 is 96%. This indicates that the majority of the pupils could not solve these problems.

3.2.4 Summary: Test 8: Vulgar fractions

Number of pupils tested: 100

3.2.4.1 Subtest 1: Multiplication

Table 58

Item	1	2	3	4	5	6	7	8	9
% Pupils who had the item incorrect	69	78	75	76	79	80	89	97	56
No pupils who had the item incorrect	69	78	75	76	79	80	89	97	56

10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
51	48	64	98	93	98	94	47	72	87	62	77	79	92	92	89	92	91	95
51	48	64	98	93	98	94	47	72	87	62	77	79	92	92	89	92	91	95

29	30	31	32	33	34	35	36	37	38	39	40	Total
92	89	93	83	94	94	93	95	92	98	99	97	83,5
92	89	93	83	94	94	93	95	92	98	99	97	3 341

In items 1-8 the pupils were required to multiply a vulgar fraction by a natural number. The natural number was a multiple of the denominator of the given vulgar fraction and in some cases the denominator of the vulgar fraction was a multiple of the natural number. The average percentage of incorrect solutions of items 1-8 is 80,4%. Almost all the pupils were unable to multiply a vulgar fraction by a natural number correctly.

In items 9-12 the pupils were supposed to multiply a vulgar fraction by a natural number. The natural number was a multiple of the numerator of the vulgar fraction. The average percentage of incorrect solutions of items 9-12 is 54,8%. It is evident from the average percentage of incorrect solutions that the majority of the pupils performed badly in these items.

In items 13-16 the pupils were required to multiply a natural number by a vulgar fraction. The denominator of the vulgar fraction and the given natural number were both multiples of 2. The average percentage of incorrect solutions of items 13-16 is 95,8%. From the average percentage of incorrect solutions it is clear that a mere 4,2% of the problems were correctly solved by the pupils. Almost all of the pupils were unable to multiply a natural number by a vulgar fraction correctly.

In items 17-22 the pupils were required to multiply a vulgar fraction by another vulgar fraction. The denominator of the first vulgar fraction was a multiple of the numerator of the second vulgar fraction. The average percentage of incorrect solutions of items 17-22 is 70,7%. This indicates that the majority of the pupils were unable to multiply vulgar fractions correctly.

In items 23-32 the pupils were required to multiply a vulgar fraction by an improper fraction. The denominator of the vulgar fraction was a multiple of the numerator of the improper fraction. The average percentage of incorrect solutions of items 23-32 is 90,8%. This indicates that almost all the pupils were unable to multiply a vulgar fraction by an improper fraction correctly.

In items 33-40 the pupils were required to multiply a mixed number by another mixed number. The average percentage of incorrect solutions of items 33-40 is 95,3%. This indicates that almost all the pupils were unable to multiply a mixed number by another mixed number correctly.

3.2.4.2 Subtest 2: Division

Table 59

Item	1	2	3	4	5	6	7	8	9
% Pupils who had the item incorrect	61	60	59	62	94	94	93	96	95
No pupils who had the item incorrect	61	60	59	62	94	94	93	96	95

Item	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
% Pupils who had the item incorrect	94	96	94	96	98	99	95	99	98	96	98	82	90	86	87	87	92	89	91
No pupils who had the item incorrect	94	96	94	96	98	99	95	99	98	96	98	82	90	86	87	87	92	89	91

Item	29	30	31	32	33	34	35	36	Total
% Pupils who had the item incorrect	89	91	98	90	100	100	99	99	90,3
No pupils who had the item incorrect	89	91	98	90	100	100	99	99	3 249

In items 1 and 3 the pupils were required to divide a vulgar fraction by a natural number. The average percentage of incorrect solutions of items 1 and 3 is 60%. This indicates that the majority of the pupils were unable to divide a vulgar fraction by a natural number correctly.

In items 2 and 4-8 the pupils were required to divide an improper fraction by a natural number. The numerator of the improper fraction was a multiple of the given natural number. The average percentage of incorrect solutions of items 2 and 4-8 is 83,2%. This indicates clearly that almost all the pupils were unable to divide an improper fraction by a natural number correctly.

In items 9-16 the pupils were required to divide a mixed number by a natural number. The numerator of the mixed number was a multiple of the natural number and in some cases the denominator of the mixed number was a multiple of the given natural number. The average percentage of incorrect solutions of items 9-16 is 95,6%. This shows that almost all the pupils were unable to divide a mixed number by a natural number correctly.

In items 17-20 the pupils were required to divide a natural number by a vulgar fraction. The average percentage of incorrect solutions is 97,8%. This indicates clearly that almost all the pupils were unable to divide a natural number by a vulgar fraction correctly.

In items 21-24 the pupils were required to divide a vulgar fraction by another vulgar fraction. The average percentage of incorrect solutions of items 21-24 is 86,3%. This indicates that the majority of the pupils were unable to solve these problems correctly.

In items 25-27 the pupils were required to divide a vulgar fraction by another vulgar fraction. The denominator of the first vulgar fraction was a multiple of the denominator of the second vulgar fraction. The average percentage of incorrect solutions of items 25-27 is 89,3%. This indicates that the majority of the pupils were unable to solve these problems correctly.

In items 28 and 29 the pupils were required to divide a vulgar fraction by another vulgar fraction. The numerators of the two vulgar fractions were the same. The average percentage of incorrect solutions of these items is 90%. This indicates that almost all the pupils were unable to solve items 28 and 29 correctly.

In items 30-32 the pupils were required to divide a vulgar fraction by another vulgar fraction. The numerator of one vulgar fraction was a multiple of the numerator of another vulgar fraction, and the denominator of the fraction was a multiple of the denominator of another vulgar fraction. The average percentage of incorrect solutions of these items is 93%. This indicates that almost all the pupils were unable to solve items 30-32 correctly.

In items 33-36 the pupils were required to divide a mixed number by another mixed number. The denominator of the first mixed number was the same as the denominator of the second mixed number. The average percentage of incorrect solutions of these items is 99,5%. This shows clearly that almost all the pupils were unable to divide a mixed number by another mixed number.

3.2.5 Summary: Test 9: Decimal fractions

Number of pupils tested: 100

3.2.5.1 Subtest 1: Concept of decimal fractions

Table 60

Item	1A	2A	3A	4A	5A	6A	7A	8A	9A
% Pupils who had the item incorrect	3	47	84	97	56	36	53	95	13
No pupils who had the item incorrect	3	47	84	97	56	36	53	95	13

10A	11A	12A	13A	14A	15A	16A	17A	18A	19A	20A	1B	2B	3B	4B
81	90	95	16	83	78	96	76	72	61	65	9	29	63	65
81	90	95	16	83	78	96	76	72	61	65	9	29	63	65

	5B	6B	7B	8B	Total
→	59	62	72	77	61,9
	59	62	72	77	1 733

In items 1A-20A the pupils were required to rewrite common fractions in decimal notation. Out of 100 pupils only 3 pupils were unable to solve item 1A but 97 pupils were unable to solve item 4A. The average percentage of incorrect solutions of items 1A-20A is 64,9%. This shows clearly that the majority of the pupils were unable to convert vulgar fractions and improper fractions to decimal fractions.

In items 1B-4B the pupils were required to rewrite decimal fractions as vulgar fractions. Nine pupils out of 100 pupils were unable to convert item 1B to a vulgar fraction while 65 pupils were unable to solve item 4B correctly. The average percentage of incorrect solutions of items 1B-4B is 41,5%. This shows that the majority of the pupils were able to convert decimal fractions to vulgar fractions.

In items 5B-8B the pupils were required to rewrite decimal fractions as mixed numbers. The average percentage of incorrect solutions of items 5B-8B is 67,5%. This indicates that the majority of the pupils were unable to convert decimal fractions to mixed numbers.

3.2.5.2 Subtest 2: Addition

Table 61

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
% Pupils who had the item incorrect	10	8	52	51	4	8	46	45	12	24	33	36	11	20	17	31	13	15	43	39	25,9
No pupils who had the item incorrect	10	8	52	51	4	8	46	45	12	24	33	36	11	20	17	31	13	15	43	39	518

In items 1-4 the pupils were required to add a decimal fraction of one decimal place to another similar decimal fraction. In items 1 and 2 the pupils performed well because only 10 out of 100 pupils failed to solve item 1 correctly and only 8 pupils could not solve item 2 correctly. This shows that the majority of the pupils were able to add correctly one decimal fraction of one decimal place to another similar one if the sum total is less than 1. The average percentage of incorrect solutions of items 1-4 is 30,3%. This shows that the majority of the pupils were able to add a decimal fraction of one decimal place to the same type of decimal fraction. The pupils only encountered some difficulties when the sum total of the two decimal fractions was greater than 1 as evidenced in the results of items 3 and 4 respectively where 52% and 51% of incorrect responses were found.

In items 5-8 the pupils were required to add a decimal fraction of one decimal place having the units digit to another number of the same kind. The average percentage of incorrect solutions of items 5-8 is 25,8%. The majority of the pupils performed well in these items. Pupils performed well in items 5 and 6 because there was no carrying involved. In items 7 and 8 the percentages of incorrect solutions were 46% and 45% respectively. This shows that the pupils were having difficulty with the problems where carrying in addition of decimal fractions was involved.

In items 9-12 the pupils were required to add a decimal fraction of two decimal places to another one of the same kind. The average percentage of incorrect solutions of these items is 26,3%. This shows that the majority of the pupils were able to add a decimal fraction of two decimal places to another similar one correctly.

In items 13-16 the pupils were required to add a decimal fraction of two decimal places, having the units digit, to another similar number. The average percentage of incorrect solutions of items 13-16 is 19,8%. This indicates that the majority of the pupils have mastered this kind of work.

In items 17-20 the pupils were required to add a decimal fraction of two decimal places, having the tens and units digits, to another similar decimal fraction. The average percentage of incorrect solutions of items 17-20 is 27,5%. It is evident that the majority of the pupils have mastered the addition of decimal fractions.

3.2.5.3 Subtest 3: Subtraction

Table 62

Item	1	2	3	4	5	6	7	8	9
% Pupils who had the item incorrect	8	38	35	18	14	11	24	36	16
No pupils who had the item incorrect	8	38	35	18	14	11	24	36	16

Item	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
% Pupils who had the item incorrect	35	47	89	4	28	49	39	46	56	61	54	88	89	85	89	89	70	88	85
No pupils who had the item incorrect	35	47	89	4	28	49	39	46	56	61	54	88	89	85	89	89	70	88	85

Item	Total
% Pupils who had the item incorrect	49,7
No pupils who had the item incorrect	1 391

In items 1-4 the pupils were required to subtract a decimal fraction of one decimal place, having the units digit, from another similar one. The lowest percentages of incorrect solutions is 8% for item 1 and 18% for item 4. The reason for this good performance is that these items did not involve carrying. In items 2 and 3 the performance of the pupils was not as food as in items 1 and 4 because these solutions involved the problem of carrying. The percentages of incorrect solutions of items 2 and 3 are 38% and 35% respectively. The average percentage of incorrect solutions of items 1-4 is 24,8%. This indicates that the majority of the pupils performed well in this aspect of work.

In items 5-12 the pupils were required to subtract a decimal fraction of two decimal places from another similar number. Pupils performed well in items 5 and 6 because these items did not involve carrying. The percentages of incorrect solutions of items 5 and 6 are 14% and 11% respectively. But 89% of the 100 pupils got item 12 incorrect because it involved carrying. The average percentage of incorrect solutions of items 5-12 is 34%. The performance of the majority of the pupils in items 5-12 was satisfactory.

In items 13-20 the pupils were required to subtract a decimal fraction of two decimal places, having the units digit, from another similar number. Pupils performed very well in item 13 as only 4% could not solve it correctly and performed badly in item 19 where 61% of the pupils could not solve it correctly. The reason for this great difference in performance is that item 13 did not involve carrying while item 19 did. The average percentage of incorrect solutions of items 13-20 is 42,1%. This indicates that the majority of the pupils performed well in these problems.

In items 21-24 the pupils were required to subtract a decimal fraction of two decimal places, having the units digit, from another decimal fraction of one decimal place also having the units digit. The average percentage of incorrect solutions of items 21-24 is 87,8%. This shows clearly that the majority of the pupils performed badly whenever carrying was involved. All the four items involved the difficulty of carrying.

In items 25-28 the pupils were required to subtract a decimal fraction of two decimal places, having the units digit, from a natural number. The average percentage of incorrect solutions of items 25-28 is 83%. This indicates clearly that the majority of the pupils encountered serious difficulties with successive carrying as evidenced by the high average percentage of incorrect solutions of items 25-28.

3.3 CONCLUSION

In this chapter the researcher has discussed the analysis and interpretation of the results of the standardized tests of Buphuthatswana.

In chapter 4 we are going to compare the results of the standardized tests of Bophuthatswana and Lebowa. J A T Wentzel conducted a research similar to the one conducted by the researcher in Lebowa. His work is entitled "Ondersoek na die prestasies in Wiskunde van die St 5 — leerlinge van Lebowa."

CHAPTER 4

COMPARISON OF THE RESULTS OF THE STANDARDIZED TESTS OF BOPHUTHATSWANA AND LEBOWA STANDARD 5 PUPILS

The main purpose of this chapter is to compare the results of the Bophuthatswana Standard 5 pupils with the results of the Lebowa Standard 5 pupils. This can assist towards the identification of problem areas in the teaching and learning of Mathematics in the primary schools. It would also be possible to determine the standard of work and whether there are similarities among the typical errors made by the pupils of these two states. This comparison could also provide guidelines for the direction in which further research should be undertaken.

In his work, "Ondersoek na die prestasies in Wiskunde van die St 5-leerlinge van Lebowa", J A T Wentzel gave the results of the "Scholastic Achievement Tests in Arithmetic - Standard 4" and the "Diagnostic Mathematical Tests" of Lebowa Standard 5 pupils. The above-mentioned standardized tests have been compiled by the Human Sciences Research Council. These results of Lebowa Standard 5 pupils will be compared with the results of Bophuthatswana Standard 5 pupils.

In this chapter a comparison will be drawn between the achievements of the Standard 5 pupils of the two states.

In comparing the results of the diagnostic tests attention will be given to a group of similar questions as well as the whole subtest. Tables indicate how pupils of each state performed and which typical errors were made throughout. Each table of incorrect solutions is followed by a discussion of the performance of the pupils.

4.1 COMPARISON OF THE RESULTS OF THE "SCHOLASTIC ACHIEVEMENT TESTS IN ARITHMETIC - STANDARD 4" OF THE STANDARD 5 PUPILS OF BOPHUTHATSWANA AND THE STANDARD 5 PUPILS OF LEBOWA

4.1.1 Analysis of the raw scores of the whole group of Bophuthatswana and Lebowa

Number of pupils tested: Bophuthatswana 495; Lebowa 1 047

4.1.1.1 Test 1

Table 63

	Bophuthatswana		Lebowa	
Raw scores out of 20	No of pupils	Percentage of pupils	No of pupils	Percentage of pupils
0	14	2,8	42	4,0
1	28	5,7	94	9,0
2	62 } 322	12,5	147 } 795	14,0
3	67	13,5 } ±65	183	17,4 } 75,9
4	71	14,3	187	17,8
5	80	16,2	142	13,5
6	43	8,7	110	10,5
7	50	10,1	66	6,3
8	28 } 159	5,7 } ±32,1	28 } 235	2,7 } 22,5
9	22	4,4	22	2,1
10	16	3,2	9	0,8
11	4	0,8	8	0,7
12	9	1,8	5	0,4
13	0 } 14	0,0 } ±2,8	3 } 17	0,3 } 1,6
14	0	0,0	1	0,1
15	1	0,2	0	0,0

Three hundred and twenty two out of the sample of 495 or ±65% of the pupils from Bophuthatswana solved 5 or less problems correctly. Of the 1 047 pupils of Lebowa tested 795 or 75,9% could only solve 5 or less problems correctly. Although both groups have achieved below expectation, the Bophuthatswana group performed slightly better than the Lebowa group in this test. It is also evident from the average percentages that the majority of the pupils from the two countries could solve only 25% or less of the items correctly.

±32,1% of the pupils tested in Bophuthatswana solved 6 to 10 items correctly compared with 22,5% of the Lebowa pupils. This indicates that more pupils from Bophuthatswana scored from 6 to 10 correct solutions. It also indicates that the pupils from Bophuthatswana performed slightly better than the pupils from Lebowa.

A mere ±2,8% of the pupils tested in Bophuthatswana obtained more than 50% in Test 1 while only 1,6% of the pupils tested in Lebowa scored more than 50% in Test 1. In other words, very few pupils from the two groups obtained satisfactory scores. The highest percentage of correct solutions of the Bophuthatswana group is 75% while that of the Lebowa group is 70%. 0,2% of the Bophuthatswana group scored 75% while 0,1% of the Lebowa group obtained 70%. Here again the pupils from Bophuthatswana performed slightly better than the pupils from Lebowa.

If grouped differently we find that ±97,1% of the pupils from Bophuthatswana scored from 0% to 50% correct solutions compared with 98,4% of the pupils from Lebowa. These percentages indicate clearly that the performance of almost all the pupils from the two states was poor. In each state more than 97% of the pupils failed to solve more than 50% of the items correctly.

The main purpose of Test 1 was to determine the pupils' skill in dealing with the four basic mathematical operations, for example, addition, subtraction, multiplication and division. The results of the pupils from the two states as reflected on Table 63 indicate that most of the pupils experienced serious difficulties in the four basic mathematical operations. This means that the standard of work of the pupils from Bophuthatswana and Lebowa as far as the four basic mathematical operations are concerned, is below expectation.

4.1.1.2 Test 2

The number of pupils tested: Bophuthatswana 495; Lebowa 1 047

Table 64

	Bophuthatswana				Lebowa			
Raw scores our of 20	No of pupils		Percentage of pupils		No of pupils		Percentage of pupils	
0	10		2,0		41		3,9	
1	31		6,3		123		11,7	
2	66		13,3		155		14,8	
3	66	343	13,3	69,3	184	847	17,5	± 80,7
4	99		20,0		190		18,1	
5	71		14,4		154		14,7	
6	71		14,4		86		8,2	
7	45		9,1		53		5,0	
8	21	150	4,2	30,3	38	192	3,6	± 18,2
9	9		1,8		11		1,0	
10	4		0,8		4		0,4	
11	0		0,0		5		0,5	
12	2	2	0,4	0,4	2	8	0,2	±0,8
13	0		0,0		1		0,1	

69,3% of the pupils from Bophuthatswana could solve 5 or less problems correctly while ±80,7% of the pupils tested in Lebowa also scored 5 or less out of 20 possible answers. Both groups performed below expectation. From the average percentages of the two groups it is clear that the majority of the pupils from the two states could solve 5 or less problems correctly. The average percentages also indicate that the pupils from Bophuthatswana performed slightly better than the pupils from Lebowa.

30,3% of the pupils tested in Bophuthatswana solved 6 to 10 items correctly compared with ±18,2% of the Lebowa pupils. This indicates that more pupils from Bophuthatswana obtained 6 to 10 correct solutions. It can also be deduced that the pupils from Bophuthatswana performed slightly better than the pupils from Lebowa.

Only 0,4% of the pupils tested in Bophuthatswana obtained above 50% in Test 2 while ±0,8% of the pupils tested in Lebowa also obtained above 50% in the same test. The highest percentage of correct solutions obtained in Bophuthatswana is 60% while that of Lebowa is 65%. It can be concluded that the pupils from Lebowa performed slightly better than the pupils from Bophuthatswana.

If grouped differently we find that 99,6% of the pupils from Bophuthatswana scored from 0% to 50% correct solutions compared with $\pm$98,9% of the pupils from Lebowa. These percentages indicate clearly that the performance of almost all the pupils from the two states was poor. In each state more than 98% of the pupils failed to solve more than 50% of the items correctly.

The purpose of Test 2 was to determine to what extent the pupils understood Mathematics and to what extent they could think logically about numbers. In the main, the content of the items in this test dealt with the following aspects:

i) Calculations on the number line.

ii) Place value of digits.

iii) The four basic mathematical operations with natural numbers.

iv) Vulgar and decimal fractions.

From Table 64 it can be deduced that the majority of the pupils from the two states experienced difficulties in the above-mentioned aspects of Mathematics. It is also evident from the table that the standard of work of the pupils from Bophuthatswana and Lebowa is very low.

4.1.1.3 Test 3

The number of pupils tested: Bophuthatswana 495; Lebowa 1 047

Table 65

	Bophuthatswana		Lebowa	
Raw scores out of 20	No of pupils	Percentage of pupils	No of pupils	Percentage of pupils
0	12	2,4	79	7,5
1	47	9,5	117	11,2
2	57	11,5	187	17,9
3	102 } 391	20,6 } 79,0	218 } 927	20,8 } ±88,5
4	104	21,1	198	18,9
5	69	13,9	128	12,2
6	54	10,9	66	6,3
7	29	5,9	30	2,9
8	9 } 99	1,8 } 20,0	14 } 117	1,3 } ± 11,2
9	2	0,4	3	0,3
10	5	1,0	4	0,4
11	2	0,4	1	0,1
12	2	0,4	1	0,1
13	0 } 5	0,0 } 1,0	0 } 2	0,0 } ± 0,2
14	0	0,0	0	0,0
15	1	0,2	0	0,0

79% of the pupils from Bophuthatswana solved 5 or less problems correctly. Of the 1 047 pupils tested in Lebowa ±88,5% of them could solve 5 or less problems correctly. Although both groups have performed badly, the pupils from Bophuthatswana performed slightly better than the pupils from Lebowa in Test 3. More than 75% of the pupils from the two groups could only solve 5 or less problems correctly. This indicates that the majority of the pupils from the two states could solve only 25% or less of the items correctly.

20% of the pupils tested in Bophuthatswana solved 6 to 10 items correctly compared with ±11,2% of the Lebowa pupils. This indicates that more pupils from Bophuthatswana could solve 6 to 10 problems correctly. It also means that the Bophuthatswana group performed slightly better than the Lebowa group.

Only 1% of the pupils tested in Bophuthatswana obtained more than 50% in Test 3, while a mere ±0,2% of the pupils tested in Lebowa also obtained more than 50%. In other words, a handful of pupils from the two states obtained satisfactory scores. The highest percentage of correct solutions of the pupils from Bophuthatswana is 75% while that of the pupils from Lebowa is 60%. 0,2% of the pupils from Bophuthatswana scored 75% and ±0,2% of the pupils from Lebowa scored 60%. This shows that the pupils from Bophuthatswana performed slightly better than the pupils from Lebowa.

If grouped differently we find that 99% of the pupils from Bophuthatswana scored from 0% to 50% correct solutions compared with ±99,7% of the pupils from Lebowa. It is evident from these high percentages that almost all the pupils from the two states performed badly.

The purpose of Test 3 was to determine the pupils' ability to solve mathematical word problems. Each item tested whether the pupils understood what was asked and whether they could carry out the correct calculations in order to find the solution. The calculations of these word problems were of an extremely elementary nature. It is clear from Table 65 that almost all the pupils from Bophuthatswana and Lebowa encountered serious difficulties in solving word problems.

4.1.2 Analysis of the percentile ranks of the whole group of Bophuthatswana and Lebowa

4.1.2.1 Test 1

The number of pupils tested: Bophuthatswana 495; Lebowa 1 047

Table 66

	Bophuthatswana		Lebowa	
Percentile ranks	No of pupils	Percentage	No of pupils	Percentage
0-5	241	48,7	655	62,6
6-10	80 } 321 } 365	16,2 } 64,9 } 73,8	141 } 796 } 907	13,5 } 76,1 } ±86,7
11-15	44	8,9	111	10,6
16-20	0	0,0	1	0,1
21-25	50	10,1	65	6,2
26-30	0	0,0	0	0,0
31-35	28	5,7	28	2,6
36-40	22	4,4	22	2,1
41-45	0	0,0	0	0,0
46-50	16 } 130	3,2 } 26,2	9 } 140	0,9 } ± 13,4
51-55	4	0,8	7	0,7
56-60	0	0,0	0	0,0
61-65	9	1,8	5	0,5
66-70	0	0,0	3	0,3
71-75	0	0,0	0	0,0
76-80	0	0,0	0	0,0
81-85	1	0,2	0	0,0

321 or 64,9%of the 495 pupils from Bophuthatswana fall within the range of 0-10 percentile ranks while 796 or ± 76,1% of the 1 047 pupils in Lebowa fall within the same range. 365 or 73,8% of the pupils from Bophuthatswana fall within the range of 0-15 percentile ranks. Of the 1 047 pupils tested in Lebowa 907 or ±86,7% of them also fall within the range of 0-15 percentile ranks. This indicates that the majority of the pupils from the two states obtained very low scores. It also means that the pupils from Bophuthatswana performed slightly better than the pupils from Lebowa.

Only 130 or 26,2% of the pupils from Bophuthatswana reached percentile ranks of 16 or higher compared with 140 or ±13,4% of the pupils from Lebowa. Just 1 or o,2% of the pupils from Bophuthatswana falls within the range of 81-85 percentile ranks while none of the pupils from Lebowa falls within the same range. It follows therefore that the pupils from Bophuthatswana again performed slightly better than the pupils from Lebowa.

From Table 66 it is clear that the majority, more than 73%, of the pupils from these two states fall within the range of 0-15 percentile ranks. It can also be deduced from the table that the Bophuthatswana group again performed slightly better than the Lebowa group.

4.1.2.2 Test 2

The number of pupils tested: Bophuthatswana 495; Lebowa 1 047

Table 67

	Bophuthatswana		Lebowa	
Percentile ranks	No of pupils	Percentage	No of pupils	Percentage
0-5	173	34,9	504	48,1
6-10	102 275 344	20,6 ±55,5 ±69,4	191 695 845	18,2 ± 66,3 ±80,7
11-15	69	13,9	150	14,4
16-20	71	14,3	85	8,1
21-25	45	9,1	55	5,2
26-30	20	4,0	38	3,6
31-35	0	0,0	0	0,0
36-40	9	1,8	11 202	1,0
41-45	4 151	0,8 ±30,4	4	0,4 ± 19,2
46-50	0	0,0	0	0,0
51-55	0	0,0	6	0,6
56-60	0	0,0	0	0,0
61-65	2	0,4	2	0,2
66-70	0	0,0	1	0,1

275 or ± 55,5% of the 495 pupils from Bophuthatswana fall within the range 0-10 percentile ranks while 695 or ±66,3% of the 1 047 pupils in Lebowa fall within the same range. 344 or ±69,4% of the pupils from Bophuthatswana fall within the range of 0-15 percentile ranks. Of the 1 047 pupils tested in Lebowa 845 or ±80,7% of them also fall within the range of 0-15 percentile ranks. Although the Bophuthatswana and Lebowa groups have performed badly, the Bophuthatswana group performed slightly better than the Lebowa group in this test.

Only 151 or ±30,4% of the Bophuthatswana group reached percentile ranks of 16 or higher compared with 202 or ±19,2% of the Lebowa group. Just 1 or ±0,1% of the pupils from Lebowa falls within the higher percentile ranks range of 66-70 while none of the pupils from Bophuthatswana falls within the same percentile ranks range. It follows therefore that the Bophuthatswana group again performed slightly better than the Lebowa group.

From Table 67 it is clear that the majority of the pupils from these two states fall within the range of 0-15 percentile ranks. More than 69% of the pupils from each group fall within the range of 0-15 percentile ranks. It follows from the table that the Bophuthatswana group again performed slightly better than the Lebowa group.

4.1.2.3 Test 3

The number of the pupils tested: Bophuthatswana 495; Lebowa 1 047

Table 68

	Bophuthatswana		Lebowa	
Percentile ranks	No of pupils	Percentage	No of pupils	Percentage
0-5	218 391 391	44,0 ±78,9 ±78,9	597 925 930	57,0 88,3 88,8
6-10	173	34,9	328	31,3
11-15	0	0,0	5	0,5
16-20	53	10,7	64	6,1
21-25	30	6,1	30	2,9
26-30	0	0,0	14	1,3
31-35	9	1,8	0	0,0
36-40	2	0,4	3	0,3
41-45	0	0,0	0	0,0
46-50	5	1,0 ±21,0	4	0,4
51-55	1 104	0,2	0 117	0,0 11,2
56-60	1	0,2	1	0,1
61-65	0	0,0	0	0,0
66-70	2	0,4	1	0,1
71-75	0	0,0	0	0,0
76-80	0	0,0	0	0,0
81-85	0	0,0	0	0,0
86-90	1	0,2	0	0,0

391 or ±78,9% of the 495 pupils from Bophuthatswana fall within the range of 0-10 percentile ranks while 925 or 88,3% of the 1 047 pupils in Lebowa fall within the same range of percentile ranks. 391 or ±78,9% of the pupils from Bophuthatswana fall within the range of 0-15 percentile ranks. Of the 1 047 pupils tested in Lebowa 930 or 88,8% of them also fall within the range of 0-15 percentile ranks. This indicates that the majority of the pupils from the two states obtained low scores. It also means that the Bophuthatswana group performed slightly better than the Lebowa group.

Only 104 or ±21% of the pupils from Bophuthatswana reached percentile ranks of 16 or higher compared with 117 or 11,2% of the pupils from Lebowa. Just 1 or ±0,2% of the pupils from Bophuthatswana falls within the higher percentile ranks range of 86-90 while none of the pupils from Lebowa falls within the same range of percentile ranks. It follows therefore that the Bophuthatswana group again performed slightly better than the Lebowa group.

From Table 68 it is evident that the majority of the pupils fall within the range of 0-15 percentile ranks. More than 78% of the pupils from each state fall within the range of 0-15 percentile ranks. It can also be deduced from the table that the Bophuthatswana group performed slightly better than the Lebowa group.

4.1.3 Analysis of the stanines of the whole group of Bophuthatswana and Lebowa

4.1.3.1 Test 1

The number of pupils tested: Bophuthatswana 495; Lebowa 1 047

Table 69

	Bophuthatswana		Lebowa	
Stanines	No of pupils	Percentage	No of pupils	Percentage
1	171	34,5	465 } 907	44,4
2	151 } 365	30,5 } ±73,7	330	31,5 } 86,6
3	43	8,7	112	10,7
4	100	20,2	115 } 140	11,0
5	20 } 129	4,0 } ±26,0	16	1,5 } 13,4
6	9	1,8	9	0,9
7	1 } 1	0,2 } ±0,2	0 } 0	0,0 } 0,0

365 out of a sample of 495 or $\pm 73{,}7\%$ of the pupils from Bophuthatswana obtained stanines 1-3 compared with 907 or 86,6% of the pupils in Lebowa. This means that the performance of $\pm 73{,}7\%$ of the pupils from Bophuthatswana was weak compared with 86,6% of the pupils from Lebowa. In other words, the performance of the majority of the pupils from Bophuthatswana and Lebowa was weak. Although both groups' performance can be regarded as weak, the Bophuthatswana group again performed slightly better than the Lebowa group.

129 or $\pm 26\%$ of the pupils from Bophuthatswana obtained stanines 4-6 while 140 or 13,4% of the pupils from Lebowa also obtained stanines 4-6. This means that the performance of $\pm 26\%$ of the pupils from Bophuthatswana was average compared with 13,4% of the pupils from Lebowa. The Bophuthatswana group again performed slightly better than the Lebowa group.

1 or ±0,2% of the pupils from Bophuthatswana obtained stanine 7, while no Lebowa pupil could score higher than stanine 6. Out of the groups of Bophuthatswana and Lebowa just one pupil's performance could be regarded as good. The Bophuthatswana group again performed slightly better than the Lebowa group.

To summarize, the majority of the pupils from Bophuthatswana and Lebowa obtained stanines 1-3. More than 73% of the pupils from each group obtained stanines 1-3. This means that the performance of more than 73% of all the pupils could be regarded as weak. It can also be deduced from the table that the pupils from Bophuthatswana again performed slightly better than the pupils from Lebowa.

4.1.3.2 Test 2

The number of pupils tested: Bophuthatswana 495; Lebowa 1 047

Table 70

	Bophuthatswana		Lebowa	
Stanines	No of pupils	Percentage	No of pupils	Percentage
1	108	21,8 ±83,8	319 929	30,5
2	167 415	33,7	375	35,8 88,7
3	140	28,3	235	22,4
4	74	14,9 ±16,1	104 118	9,9
5	4 80	0,8	10	1,0 11,3
6	2	0,4	4	0,4

415 out of a sample of 495 or $\pm$83,8% of the pupils from Bophuthatswana obtained stanines 1-3 compared with 929 or 88,7% of the pupils from Lebowa. This indicates that the performance of $\pm$83,8% of the pupils from Bophuthatswana was weak compared with 88,7% of the pupils from Lebowa. In other words, the performance of the majority of the pupils from Bophuthatswana and Lebowa was weak. The Bophuthatswana group again performed slightly better than the Lebowa group.

80 or $\pm$16,1% of the pupils from Bophuthatswana obtained stanines 4-6 while 118 or 11,3% of the pupils from Lebowa also obtained stanines 4-6. This means that the performance of $\pm$16,1% of the pupils from Bophuthatswana was average while the performance of 11,3% of the pupils from Lebowa was also average. The Bophuthatswana group again performed slightly better than the Lebowa group.

To summarize, the majority of the pupils from Bophuthatswana and Lebowa obtained stanines 1-3. More than 83% of the pupils from each state obtained stanines 1-3. This means that the performance of more than 83% of all the pupils can be regarded as weak. It is also noteworthy that none of the pupils from the two states obtained stanines 7-9. This means that none of the pupils' performance from the two states can be regarded as good.

Be that as it may, the Bophuthatswana group again performed slightly better than the Lebowa group.

4.1.3.3 <u>Test 3</u>

The number of pupils tested: Bophuthatswana 495; Lebowa 1 047

<u>Table 71</u>

	Bophuthatswana		Lebowa	
Stanines	No of pupils	Percentage	No of pupils	Percentage
1 2 3	116 275 53 } 444	23,4 55,6 10,7 } 89,7	383 542 67 } 992	36,6 51,8 6,3 } 94,7
4 5 6	41 7 2 } 50	8,3 1,4 0,4 } 10,1	49 5 1 } 55	4,7 0,5 0,1 } 5,3
7	1 } 1	0,2 } 0,2	0 } 0	0,0 } 0,0

444 out of a sample of 495 or 89,7% of the pupils from Bophuthatswana obtained stanines 1-3 compared with 992 or 94,7% of the 1 047 pupils from Lebowa. This means that the performance of 89,7% of the pupils from Bophuthatswana was weak compared with 94,7% of the pupils from Lebowa. In other words, the performance of the majority of the pupils from Bophuthatswana and Lebowa was weak. The Bophuthatswana group again performed slightly better than the Lebowa group.

50 or 10,1% of the pupils from Bophuthatswana obtained stanines 4-6 while 55 or 5,3% of the pupils from Lebowa also obtained stanines 4-6. This means that the performance of 10,1% of the pupils from Bophuthatswana was average while the performance of 5,3% of the pupils from Lebowa was also average. The pupils from Bophuthatswana again performed slightly better than the pupils from Lebowa.

1 or 0,2% of the pupils from Bophuthatswana obtained stanine 7 while no Lebowa pupil could score higher than stanine 6. Out of the groups of Bophuthatswana and Lebowa just one pupil's performance can be regarded as good. The Bophuthatswana group again performed slightly better than the Lebowa group.

To summarize, the majority of the pupils from Bophuthatswana and Lebowa obtained stanines 1-3. This means that the performance of more than 89% of all the pupils from each group can be regarded as weak. It can also be deduced from the table that the pupils from Bophuthatswana again performed slightly better than the pupils from Lebowa.

4.2 COMPARISON OF THE RESULTS OF THE "DIAGNOSTIC MATHEMATICAL TESTS" OF BOPHUTHATSWANA AND LEBOWA

4.2.1 Summary: Test 5: Basic operations ($<$100 000)

4.2.1.1 Subtest 1: Addition

The number of pupils tested: Bophuthatswana 100; Lebowa 156

Table 72

Item	1a	1b	1c	1d	2	3	4	5	6	7	8	9	Total
Bop: % Pupils who had the item incorrect	13	11	44	68	20	19	40	28	20	29	31	29	29,3
Leb: % Pupils who had the item incorrect	16,0	23,1	57,1	91,7	41,7	32,1	47,4	38,5	40,4	47,4	45,5	42,3	43,1
Bop: No pupils who had the item incorrect	13	11	44	68	20	19	40	28	20	29	31	29	352
Leb: No pupils who had the item incorrect	25	36	89	143	65	50	74	60	63	74	71	66	807

In items 1a-1d the pupils were required to add the natural numbers which were arranged horizontally. The average percentage of incorrect solutions of items 1a-1d in Bophuthatswana is 34% while of Lebowa is 47%. The pupils from Bophuthatswana performed better than the pupils from Lebowa. Item 1d whose numbers were written in words, was incorrectly solved by 68% of the pupils from Bophuthatswana while 91,7% of the pupils from Lebowa also solved it incorrectly. This indicates that the majority of the pupils from the two states performed badly in item 1d.

In items 2-9 the pupils were required to add numbers which were arranged in columns. The average percentage of incorrect solutions of items 2-9 of the pupils from Bophuthatswana is 27% while of the pupils from Lebowa is 41,9%. This means that the pupils from Bophuthatswana performed better than the pupils from Lebowa.

The average percentage of incorrect solutions of the pupils from Bophuthatswana of Subtest 1 as reflected on Table 72 is 29,3% while of the pupils from Lebowa as reflected on the same table is 43,1%. The average percentage of incorrect solutions of the pupils from Bophuthatswana is calculated as follows: $\frac{352}{12 \times 100} \times \frac{100}{1}$; and of the pupils from Lebowa is calculated as follows: $\frac{807}{12 \times 156} \times \frac{100}{1}$. In other words, the average percentage of incorrect solutions is calculated as follows: The total of incorrect solutions over the product of the number of items and the number of the pupils, all multiplied by hundred. This indicates that the pupils from Bophuthatswana performed better than the pupils from Lebowa in Subtest 1. It also shows that the performance of the majority of the pupils from the two states is satisfactory.

4.2.1.2 Subtest 2: Subtraction

The number of pupils tested: Bophuthatswana 100; Lebowa 156

Table 73

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Bop: % Pupils who had the item incorrect	5	18	18	16	36	73	26	71	37	49	37	36	44	47	49	41	37,7
Leb: % Pupils who had the item incorrect	7,1	23,1	24,4	16,0	68,0	81,4	56,4	71,2	74,4	85,3	66,0	70,5	68,0	77,6	82,7	87,2	59,9
Bop: No pupils who had the item incorrect	5	18	18	16	36	73	26	71	37	49	37	36	44	47	49	41	603
Leb: No pupils who had the item incorrect	11	36	38	25	106	127	88	111	116	133	103	110	106	121	129	136	1 496

Items 1-4 did not involve borrowing when subtracting. The average percentage of incorrect solutions of items 1-4 of the pupils from Bophuthatswana is 14,3% while of the pupils from Lebowa is 17,7%. In these items the pupils from the two states performed almost equally well. The pupils from Bophuthatswana performed slightly better than the pupils from Lebowa.

In items 5-16 borrowing was necessary. The average percentage of incorrect solutions of items 5-16 of the pupils from Bophuthatswana is 45,5% while of the pupils from Lebowa is 74,1%. In these items the pupils from Bophuthatswana performed better than the pupils from Lebowa.

The average percentage of incorrect solutions of the pupils from Bophuthatswana of Subtest 2 as reflected on Table 73 is 37,7% while of the pupils from Lebowa is 59,9%. The average percentage of incorrect solutions of the pupils from Bophuthatswana is calculated as follows: $\frac{603}{16 \times 100} \times \frac{100}{1}$; and of the pupils from Lebowa is calculated as follows: $\frac{1496}{16 \times 156} \times \frac{100}{1}$. In other words, the average percentage is calculated as follows: The total of incorrect solutions over the product of the number of items and the number of pupils, all multiplied by hundred. From these average percentages of incorrect solutions it is evident that the pupils from Bophuthatswana performed better than the pupils from Lebowa in Subtest 2. They also indicate that the performance of the majority of the pupils from Bophuthatswana is satisfactory while of the majority of the pupils from Lebowa is poor.

4.2.1.3 Subtest 3: Multiplication

The number of pupils tested: Bophuthatswana 100; Lebowa 156

Table 74

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Bop: % Pupils who had the item incorrect	23	43	49	41	64	70	70	62	69	71	84	81	74	77	70	60	63,0
Leb: % Pupils who had the item incorrect	37,2	55,1	55,8	59,6	70,5	80,1	83,3	70,5	75,0	78,9	85,3	85,3	84,0	84,6	85,3	78,9	73,1
Bop: No pupils who had the item incorrect	23	43	49	41	64	70	70	62	69	71	84	81	74	77	70	60	1 008
Leb: No pupils who had the item incorrect	58	86	87	93	110	125	130	110	117	123	133	133	131	132	133	123	1 824

In item 1 the pupils were required to multiply a natural number by another natural number. This item did not involve carrying. 23% of the pupils from Bophuthatswana solved this item incorrectly while 37,2% of the pupils from Lebowa also solved it incorrectly. This indicates that the pupils from Bophuthatswana performed better than the pupils from Lebowa.

In items 2 and 3 the problems of zero and carrying were involved. The average percentage of incorrect solutions of items 2 and 3 of the pupils from Bophuthatswana is 46% while of the pupils from Lebowa is 55,5%. This shows that the pupils from Bophuthatswana performed better than the pupils from Lebowa.

In item 4 the problem of successive carrying was involved. 41% of the pupils from Bophuthatswana solved this item incorrectly while 59,6% of the pupils from Lebowa also solved it incorrectly. This indicates that the pupils from Bophuthatswana performed better than the pupils from Lebowa.

In items 5-8 the pupils were required to multiply a natural number by a multiple of 10. The average percentage of incorrect solutions of items 5-8 of the pupils from Bophuthatswana is 66,5% while of the pupils from Lebowa is 76,1%. The majority of the pupils from the two states were unable to solve items 5-8 correctly. Pupils from Bophuthatswana performed better than the pupils from Lebowa.

In items 9-12 addition and carrying were involved. The average percentage of incorrect solutions of items 9-12 of the pupils from Bophuthatswana is 76,3% while of the pupils from Lebowa is 81,1%. The pupils from the two states performed badly for in each state more than 75% of the pupils solved items 9-12 incorrectly. The Bophuthatswana group performed better than the Lebowa group.

Items 13-15 involved addition, carrying and the zero difficulty. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 73,7% while of the pupils from Lebowa is 84,6%. This indicates that the majority of the pupils from each state performed badly. More than 73% of the pupils from each state solved items 13-15 incorrectly. The Bophuthatswana group performed better than the Lebowa group.

In item 16 the pupils were required to multiply a natural number by a multiple of hundred. 60% of the pupils from Bophuthatswana solved this item incorrectly while 78,9% of the pupils from Lebowa also solved it incorrectly. This indicates that the majority of the pupils from the two states were unable to solve item 16 correctly. The Bophuthatswana group performed better than the Lebowa group.

The average percentage of incorrect solutions of the pupils from Bophuthatswana of Subtest 3 as reflected on Table 74 is 63% while of the pupils from Lebowa is 73,1%. The average percentage of incorrect solutions of the pupils from Bophuthatswana is calculated as follows:

$\frac{1008}{16\text{x}100} \text{ X } \frac{100}{1}$; and of the pupils from Lebowa is calculated as follows: $\frac{1824}{16\text{x}156} \text{ X } \frac{100}{1}$. In other words, the average percentage is calculated as follows: The total of incorrect solutions over the product of the number of items and the number of pupils, all multiplied by hundred. From the average percentages of incorrect solutions it is clear that the performance of the majority of the pupils of the two states is poor. It is also evident that the pupils from Bophuthatswana performed better than the pupils from Lebowa.

4.2.1.4 Subtest 4: Division

The number of pupils tested: Bophuthatswana 100; Lebowa 156

Table 75

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
Bop: % Pupils who had the item incorrect	78	80	87	96	70	87	80	92	82	80	98	99	81	99	98	99	97	100	100	100	90,0
Leb: % Pupils who had the item incorrect	79,5	94,9	93,6	99,4	68,0	88,5	93,0	98,7	93,6	92,3	98,1	98,7	90,4	98,1	98,1	100	98,1	98,7	98,1	98,7	93,9
Bop: No pupils who had the item incorrect	78	80	87	96	70	87	80	92	82	80	98	99	81	99	98	99	97	100	100	100	1 800
Leb: No pupils who had the item incorrect	124	148	146	155	106	138	145	154	146	144	153	154	141	153	153	156	153	154	153	154	2 930

Item 1 involved the zero difficulty only. 78% of the pupils from Bophuthatswana were unable to solve this item correctly while 79,5% of the pupils from Lebowa were unable to solve it correctly. This indicates that the majority of the pupils from the two states were unable to solve item 1 correctly. More than 75% of the pupils from the two states solved item 1 incorrectly. The Bophuthatswana group performed better than the Lebowa group.

Items 2-4 involved the zero difficulty and carrying. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 87,7% while of the pupils of Lebowa is 97,3%. This indicates that almost all the pupils from the two states were unable to solve items 2-4 correctly. More than 87% of the pupils from each state solved items 2-4 incorrectly. The Bophuthatswana group performed better than the Lebowa group.

In items 5-8 the pupils were required to divide a natural number by a multiple of 10. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 82,3% while of the pupils of Lebowa is 87,1%. This shows that the majority of the pupils from the two states were unable to solve items 5-8 correctly. More than 82% of the pupils from each state solved items 5-8 incorrectly. The pupils from Bophuthatswana performed better than the pupils from Lebowa.

In items 9-20 the pupils were required to multiply a natural number by another natural number. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 94,2% while of the pupils from Lebowa is 96,9%. This indicates that almost all the pupils from the two states were unable to solve items 9-20 correctly. More than 94% of the pupils from each state solved items 9-20 incorrectly. The Bophuthatswana group performed better than the Lebowa group.

The average percentage of incorrect solutions of the pupils from Bophuthatswana of Subtest 4 as reflected on Table 75 is 90% while of the pupils from Lebowa is 93,9%. The average percentage of incorrect solutions of the pupils from Bophuthatswana is calculated as follows: $\frac{1800}{20x100} \text{ X } \frac{100}{1}$; and of the pupils from Lebowa is calculated as follows: $\frac{2930}{20x156} \text{ X } \frac{100}{1}$. In other words, the average percentage of incorrect solutions is calculated as follows: The total of incorrect solutions over the product of the number of items and the number of pupils, all multiplied by hundred. From the average percentages of the two groups it is clear that almost all the pupils from the two states were unable to solve items of Subtest 4 correctly. It is also evident that the pupils from Bophuthatswana performed better than the pupils from Lebowa.

4.2.2 Summary: Test 6: Vulgar fractions

4.2.2.1 Subtest 1: Concept of vulgar fractions

The number of pupils tested: Bophuthatswana 100; Lebowa 156

Table 76

Item	1A	2A	3A	4A	5A	6A
Bop: % Pupils who had the item incorrect	9	23	15	64	21	23
Leb: % Pupils who had the item incorrect	16,7	69,9	69,9	89,7	64,7	69,9
Bop: No pupils who had the item incorrect	9	23	15	64	21	23
Leb: No pupils who had the item incorrect	26	109	109	140	101	109

Item	7A	8A	9A	10A	11A	12A	13A	14A	15A	16A	1B	2B
Bop: % Pupils who had the item incorrect	18	43	52	43	44	66	70	52	78	61	57	61
Leb: % Pupils who had the item incorrect	65,4	65,4	64,7	68,0	73,7	87,2	85,3	80,8	82,7	66,0	49,4	68,0
Bop: No pupils who had the item incorrect	18	43	52	43	44	66	70	52	78	61	57	61
Leb: No pupils who had the item incorrect	102	102	101	106	115	136	133	126	129	103	77	106

Item	3B	4B	1C	2C	3C	4C	1D	2D	3D	4D	1E
Bop: % Pupils who had the item incorrect	59	61	11	23	74	49	18	25	26	23	12
Leb: % Pupils who had the item incorrect	69,2	61,5	74,4	81,4	96,2	88,5	51,3	56,4	59,6	54,5	54,5
Bop: No pupils who had the item incorrect	59	61	11	23	74	49	18	25	26	23	12
Leb: No pupils who had the item incorrect	108	96	116	127	150	138	80	88	93	85	85

Item	2E	3E	4E	Total
Bop: % Pupils who had the item incorrect	12	13	13	38,1
Leb: % Pupils who had the item incorrect	43,6	50,6	53,9	66,6
Bop: No pupils who had the item incorrect	12	13	13	1 219
Leb: No pupils who had the item incorrect	68	79	84	3 327

Items 1A-16A tested the pupils' understanding of equivalent fractions. The average percentage of incorrect solutions of items 1A-16A of the pupils from Bophuthatswana is 42,6% while of the pupils from Lebowa is 70,0%. This indicates that the pupils from Bophuthatswana performed better than the pupils from Lebowa. In other words, the pupils from Bophuthatswana have a better understanding of the concept of equivalent fractions than the pupils from Lebowa.

In items 1B-4B the pupils were required to identify the fractions which are in the simplest form. The average percentage of incorrect solutions of items 1B-4B of the pupils from Bophuthatswana is 59,5% while of the pupils from Lebowa is 62,0%. This indicates that the pupils from the two states performed almost equally poor. In these states the majority of the pupils were unable to identify fractions which were in the simplest form. The Bophuthatswana group performed better than the Lebowa group.

In items 1C-4C the pupils were to rewrite the given fractions in the simplest form. The average percentage of incorrect solutions of items 1C-4C of the pupils from Bophuthatswana is 39,2% while of the pupils from Lebowa is 85,1%. This indicates that the performance of the pupils from Bophuthatswana is satisfactory while the performance of the pupils from Lebowa is poor. It also means that the pupils from Bophuthatswana were more able than the pupils from Lebowa to simplify the given fractions.

In items 1D-4D the pupils were required to rewrite improper fractions as mixed numbers. The average percentage of incorrect solutions of items 1D-4D of the pupils from Bophuthatswana is 23,0% while of the pupils from Lebowa is 55,5%. This indicates that the performance of the pupils from Bophuthatswana is good while the performance of the pupils from Lebowa is poor. From the average percentages it is evident that the pupils from Bophuthatswana performed better than the pupils from Lebowa. It also means that the pupils from Bophuthatswana were more able to convert improper fractions to mixed numbers than the pupils from Lebowa.

In items 1E-4E the pupils were required to rewrite mixed numbers as improper fractions. The average percentage of incorrect solutions of items 1E-4E of the pupils from Bophuthatswana is 12,5% while of the pupils from Lebowa is 50,7%. This indicates that the performance of the pupils from Bophuthatswana is good while the performance of the pupils from Lebowa is poor. The Bophuthatswana group performed better than the Lebowa group. It also means that the pupils from Bophuthatswana were more able to convert mixed numbers to improper fractions than the pupils from Lebowa.

The average percentage of incorrect solutions of the pupils from Bophuthatswana of Subtest 1 as reflected on Table 76 is 38,1% while of the pupils from Lebowa is 66,6%. The average percentage of incorrect solutions of the pupils from Bophuthatswana is calculated as follows: $\frac{1219}{32x100}$ X $\frac{100}{1}$; and of the pupils from Lebowa is calculated as follows: $\frac{3327}{32x156}$ X $\frac{100}{1}$. In other words, the average percentage of incorrect solutions is calculated as follows: The total of incorrect solutions over the product of the number of items and the number of pupils, all multiplied by hundred. This indicates that the performance of the pupils from Bophuthatswana is satisfactory while of the pupils from Lebowa is poor. It also means that the Bophuthatswana group performed better than the Lebowa group.

4.2.2.2 Subtest 2: Addition

The number of pupils tested: Bophuthatswana 100; Lebowa 156.

Table 77

Item	1	2	3	4	5	6
Bop: % Pupils who had the item incorrect	15	9	20	13	65	75
Leb: % Pupils who had the item incorrect	44,9	42,3	45,5	43,6	80,8	94,2
Bop: No pupils who had the item incorrect	15	9	20	13	65	75
Leb: No pupils who had the item incorrect	70	66	71	68	126	147

7	8	9	10	11	12	13	14	15	16	17	18
43	74	20	23	19	27	69	72	65	68	17	37
81,4	93,6	84,0	84,6	85,9	86,5	97,4	96,2	96,8	96,2	87,8	90,4
43	74	20	23	19	27	69	72	65	68	17	37
127	146	131	132	134	135	152	150	151	150	137	141
19	20	21	22	23	24	25	26	27	28	29	30
19	57	24	34	31	37	73	73	68	84	30	24
90,4	82,7	76,9	76,9	73,1	79,5	93,6	94,9	94,9	90,4	76,9	74,4
19	57	24	34	31	37	73	73	68	84	30	24
141	129	120	120	114	124	146	148	148	141	120	116
31	32	33	34	35	36	37	38	39	40	41	42
33	39	51	53	42	46	69	76	72	76	62	76
75,6	78,2	91,7	87,8	91,0	88,5	96,8	93,6	94,2	94,9	94,2	97,4
33	39	51	53	42	46	69	76	72	76	62	76
118	122	143	137	142	138	151	146	147	148	147	152
43	44	45	46	47	48	49	50	51	52	53	54
78	76	20	33	31	34	29	37	36	38	41	39
98,1	97,4	77,6	73,1	75,0	76,3	84,0	84,0	78,9	82,7	76,9	85,9
78	76	20	33	31	34	29	37	36	38	31	39
153	152	121	114	117	119	131	131	123	129	120	134

55	56	57	58	59	60	61	62	63	64	65	66
67	76	49	52	76	79	29	34	31	50	31	43
94,2	96,2	85,9	86,5	96,8	96,2	79,5	79,5	83,3	90,4	82,7	89,1
67	76	49	52	76	79	29	34	31	50	31	43
147	150	134	135	151	150	124	124	130	141	129	139

67	68	69	70	71	72	Total
44	45	46	60	57	73	47,4
85,3	87,8	90,4	93,0	95,5	96,8	85,0
44	45	46	60	57	73	3 414
133	137	141	145	149	151	9 546

In items 1-16 the pupils were required to add two vulgar fractions which have the same denominator. The average percentage of incorrect solutions of items 1-16 of the pupils from Bophuthatswana is 42,3% while of the pupils from Lebowa is 78,4%. This shows that the pupils from Bophuthatswana performed better than the pupils from Lebowa. It also indicates that the pupils from Bophuthatswana were more able to add two vulgar fractions which have the same denominator than the pupils from Lebowa.

In items 17-19 the pupils were required to add a vulgar fraction to a natural number. The average percentage of incorrect solutions of items 17-19 of the pupils from Bophuthatswana is 24,3% while of the pupils from Lebowa is 89,5%. The pupils from Bophuthatswana again performed better than the pupils from Lebowa. This indicates that the pupils from Lebowa were experiencing more difficulties in the addition of vulgar fractions to natural numbers than the pupils from Bophuthatswana.

In item 20 the pupils were required to add a natural number to a mixed number. 57% of the pupils from Bophuthatswana could not solve this item correctly while 82,7% of the pupils from Lebowa were also unable to solve it correctly. This indicates that the majority of the pupils from the two states were unable to add a natural number to a mixed number. The Bophuthatswana group again performed better than the Lebowa group.

In items 21-28 the pupils were required to add a vulgar fraction to a mixed number. The average percentage of incorrect solutions of items 21-28 of the pupils from Bophuthatswana is 53% while of the pupils from Lebowa is 85%. In both states more than 50% of the pupils were unable to add vulgar fractions to mixed numbers correctly. The Bophuthatswana group again performed better than the Lebowa group.

In items 29-44 the pupils were required to add a mixed number to another mixed number. The average percentage of incorrect solutions of items 29-44 of the pupils from

Bophuthatswana is 56,4% while of the pupils from Lebowa is 89,4%. In the two states more than 56% of the pupils were unable to add a mixed number to another mixed number correctly. The pupils from Bophuthatswana again performed better than the pupils from Lebowa.

In items 45-52 the pupils were required to add vulgar fractions with different denominators. The average percentage of incorrect solutions of items 45-52 of the pupils from Bophuthatswana is 32,2% compared with 79% of the pupils from Lebowa. This indicates that the pupils from Bophuthatswana were more able to add vulgar fractions with different denominators together than the pupils from Lebowa. The Bophuthatswana group again performed better than the Lebowa group.

In items 53-56 the pupils were required to add a vulgar fraction to a mixed number. The average percentage of incorrect solutions of items 53-56 of the pupils from Bophuthatswana is 55,8% compared with 88,3% of the pupils from Lebowa. More than 55% of the pupils from the two states were unable to add a vulgar fraction to a mixed number correctly. Although the performance of the pupils from the two states is poor, the Bophuthatswana group performed better than the Lebowa group.

In items 57-60 the pupils were required to add a mixed number to another mixed number. The average percentage of incorrect solutions of items 57-60 of the pupils from Bophuthatswana is 64% compared with 91,4% of the pupils from Lebowa. This indicates that more than 60% of the pupils from the two states were unable to add a mixed number to another mixed number correctly.

In items 61-72 the pupils were required to add one vulgar fraction to another. The average percentage of incorrect solutions of items 61-72 of the pupils from Bophuthatswana is 45,3% compared with 87,8% of the pupils from Lebowa. This indicates that the pupils from Bophuthatswana were more able to add two vulgar fractions together than the pupils from Lebowa.

The average percentage of incorrect solutions of Subtest 2 of the pupils from Bophuthatswana as reflected on Table 77 is 47,4% compared with 85,0% of the pupils from Lebowa. The average percentage of incorrect solutions of the pupils from Bophuthatswana is calculated as follows: $\frac{3414}{72\text{x}100} \text{ X } \frac{100}{1}$; and of the pupils from Lebowa is calculated as follows: $\frac{9546}{72\text{x}156} \text{ X } \frac{100}{1}$. In other words, the average percentage of incorrect solutions is calculated as follows: The total of incorrect solutions over the product of the number of items and the number of pupils, all multiplied by hundred. In this subtest the performance of the pupils from Bophuthatswana is satisfactory while of the pupils from Lebowa is poor. The Bophuthatswana group again performed better than the Lebowa group in Subtest 2.

4.2.3 Summary: Test 7: Vulgar fractions

4.2.3.1 Subtest 1: Subtraction

The number of pupils tested: Bophuthatswana 100; Lebowa 145

Table 78

Item	1	2	3	4	5	6
Bop: % Pupils who had the item incorrect	34	55	58	49	45	75
Leb: % Pupils who had the item incorrect	16,6	22,8	20,7	21,4	58,6	67,6
Bop: No pupils who had the item incorrect	34	55	58	49	45	75
Leb: No pupils who had the item incorrect	24	33	30	31	85	98

7	8	9	10	11	12	13	14	15	16	17	18
25	50	29	52	49	64	84	100	99	97	90	89
64,8	62,8	59,3	64,1	62,1	64,8	62,1	64,8	66,9	57,2	57,2	71,7
25	50	29	52	49	64	84	100	99	97	90	89
94	91	86	93	90	94	90	94	97	83	83	104
19	20	21	22	23	24	25	26	27	28	29	30
94	90	88	98	97	98	98	100	100	99	76	88
69,7	66,2	72,4	77,9	75,9	75,2	70,3	70,3	69,0	64,1	42,1	40,7
94	90	88	98	97	98	98	100	100	99	76	88
101	96	105	113	110	109	102	102	100	93	61	59

31	32	33	34	35	36	37	38	39	40	41	42
93	88	88	94	91	93	90	94	93	95	99	99
→ 41,4	45,5	65,5	66,2	66,9	64,8	57,2	53,8	53,8	55,9	71,7	75,9 →
93	88	88	94	91	93	90	94	93	95	99	99
60	66	95	96	97	94	83	78	78	81	104	110
43	44	45	46	47	48	49	50	51	52	53	54
100	99	73	73	74	93	95	95	95	99	92	92
→ 69,0	74,5	44,1	43,5	49,7	55,9	55,2	61,4	69,0	85,2	57,9	56,6 →
100	99	73	73	74	93	95	95	95	99	92	92
100	108	64	63	72	81	80	89	100	125	84	82

55	56	Total
100	100	83,4
→ 69,7	76,6	59,8
100	100	4 671
101	111	4 856

In items 1-8 the pupils were required to subtract a vulgar fraction from another one with the same denominator. The average percentage of incorrect solutions of items 1-8 of the pupils from Bophuthatswana is 48,9% compared with 41,9% of the pupils from Lebowa. This indicates that the pupils from Lebowa performed slightly better than the pupils from Bophuthatswana. It also indicates that more than 50% of the pupils from the two states were able to subtract vulgar fractions with the same denominators.

In items 9-12 the pupils were required to subtract natural numbers from mixed numbers. The average percentage of incorrect solutions of items 9-12 of the pupils from Bophuthatswana is 49,5% compared with 62,6% of the pupils from Lebowa. This means that the pupils from Bophuthatswana performed slightly better than the pupils from Lebowa. It also indicates that the Bophuthatswana group were more able to subtract natural numbers from mixed numbers than the Lebowa group.

In items 13-16 the pupils were required to subtract vulgar fractions from natural numbers. The average percentage of incorrect solutions of items 13-16 of the pupils from Bophuthatswana is 95% compared with 62,8% of the pupils from Lebowa. This shows that the Lebowa group performed better than the Bophuthatswana group. It also indicates that more than 62% of the pupils from the two states were unable to subtract vulgar fractions from natural numbers correctly.

In items 17-24 the pupils were required to subtract a mixed number from another one. The average percentage of incorrect solutions of items 17-24 of the pupils from Bophuthatswana is 93% compared with 70,8% of the pupils from Lebowa. In these items the Lebowa group performed slightly better than the Bophuthatswana group. More than 70% of the pupils from the two states were unable to subtract a mixed number from another one correctly.

In items 25-28 the pupils were required to subtract a mixed number from a natural number. The average percentage of incorrect solutions of items 25-28 of the pupils from Bophuthatswana is 99,3% compared with 68,4% of the pupils from Lebowa. In these items the Lebowa group performed better than the Bophuthatswana group. It is also worth noting that in the two states more than 68% of the pupils were unable to subtract a mixed number from a natural number correctly.

In items 29-36 the pupils were required to subtract a vulgar fraction from another one. The average percentage of incorrect solutions of items 29-36 of the pupils from Bophuthatswana is 88,9% compared with 54,1% of the pupils from Lebowa. Again the pupils from Lebowa performed better than the pupils from Bophuthatswana. In both states more than 54% of the pupils were unable to subtract a vulgar fraction from another one correctly.

In items 37-40 the pupils were required to subtract a vulgar fraction from a mixed number. The average percentage of incorrect solutions of items 37-40 of the pupils from Bophuthatswana is 93% compared with 55,2% of the pupils from Lebowa. This indicates that the pupils from Lebowa again performed better than the pupils from Bophuthatswana.

It also means that in the two states more than 55% of the pupils were unable to subtract a vulgar fraction from a mixed number correctly.

In items 41-44 the pupils were required to subtract a mixed number from another one. The denominator of the first mixed number was a multiple of the denominator of the second mixed number. In these items the average percentage of incorrect solutions of the pupils from Bophuthatswana is 99,3% compared with 72,8% of the pupils from Lebowa. This shows that the Lebowa group again performed better than the Bophuthatswana group. It also means that in both states more than 72% of the pupils had problems in subtracting a mixed number from another one.

In items 45-52 the pupils were required to subtract vulgar fractions with different denominators. The average percentage of incorrect solutions of items 45-52 of the pupils from Bophuthatswana is 87,1% compared with 58% of the pupils from Lebowa. This indicates that the pupils from Lebowa again performed better than the pupils from Bophuthatswana. It also means that more than 57% of the pupils from the two states experienced problems in subtracting vulgar fractions with different denominators.

In items 53-56 the pupils were required to subtract mixed numbers with different denominators. The average percentage of incorrect solutions of items 53-56 of the pupils from Bophuthatswana is 96% compared with 65,2% of the pupils from Lebowa. In these items the Lebowa group again performed better than the Bophuthatswana group. It is also evident that more than 65% of the pupils from the two states experienced problems in subtracting mixed numbers with different denominators.

The average percentage of incorrect solutions of Subtest 1 of the pupils from Bophuthatswana as reflected on Table 78 is 83,4% compared with 59,8% of the pupils from Lebowa. The average percentage of incorrect solutions of the pupils from Bophuthatswana is calculated as follows: $\frac{4671}{56x100} \text{ X } \frac{100}{1}$; and of the pupils from Lebowa is calculated as follows: $\frac{4856}{56x145} \text{ X } \frac{100}{1}$. In other words, the average percentage of incorrect solutions is calculated as follows: The total of incorrect solutions over the product of the number of items and the number of pupils, all multiplied by hundred. In this subtest the Lebowa group performed better than the Bophuthatswana group. It also indicates that the performance of the majority of the pupils from the two states is poor.

4.2.4 Summary: Test 8: Vulgar fractions

4.2.4.1 Subtest 1: Multiplication

The number of pupils tested: Bophuthatswana 100; Lebowa 145

Table 79

Item	1	2	3	4	5	6
Bop: % Pupils who had the item incorrect	69	78	75	76	79	80
Leb: % Pupils who had the item incorrect	44,1	42,8	42,8	43,5	35,9	33,1
Bop: No pupils who had the item incorrect	69	78	75	76	79	80
Leb: No pupils who had the item incorrect	64	62	62	63	52	48

7	8	9	10	11	12	13	14	15	16	17	18
89	97	56	51	48	64	98	93	98	94	47	72
43,5	43,5	49,0	37,2	43,5	51,7	55,2	65,5	66,9	73,1	61,4	44,1
89	97	56	51	48	64	98	93	98	94	47	72
63	63	71	54	63	75	80	95	97	106	89	64

19	20	21	22	23	24	25	26	27	28	29	30
87	62	77	79	92	92	89	92	91	95	92	89
51,7	58,6	28,3	50,3	57,2	56,6	60,0	64,1	68,3	71,7	62,8	52,4
87	62	77	79	92	92	89	92	91	95	92	89
75	85	41	73	83	82	87	93	99	104	91	76

31	32	33	34	35	36	37	38	39	40	Total
93	83	94	94	93	95	92	98	99	97	83,5
70,0	53,1	53,1	65,5	62,8	64,8	50,3	61,4	67,6	61,4	54,8
93	83	94	94	93	95	92	98	99	97	3 341
100	77	77	95	91	94	73	89	98	89	3 181

In items 1-8 the pupils were required to multiply a vulgar fraction by a natural number. The natural number was a multiple of the denominator of the given vulgar fraction. The average percentage of incorrect solutions of items 1-8 of the pupils from Bophuthatswana is 80,4% compared with 41,2% of the pupils from Lebowa. This shows clearly that the Lebowa group performed far much better than the Bophuthatswana group. It also indicates that the pupils from Lebowa were more able to multiply a vulgar fraction by a natural number than the pupils from Bophuthatswana.

In items 9-12 the pupils were supposed to multiply a vulgar fraction by a natural number. The natural number was a multiple of the numerator of the vulgar fraction. In these items the average percentage of incorrect solutions of the pupils from Bophuthatswana is 54,8% compared with 45,4% of the pupils from Lebowa. This indicates that the pupils from Lebowa again performed slightly better than the pupils from Bophuthatswana. It also means that the Lebowa group was more able to multiply a vulgar fraction by a natural number than the Bophuthatswana group.

In items 13-16 the pupils were required to multiply a natural number by a vulgar fraction. The denominator of the vulgar fraction and the natural number were both multiples of 2. The average percentage of incorrect solutions of items 13-16 of the pupils from Bophuthatswana is 95,8%

compared with 65,2% of the pupils from Lebowa. In these items the Lebowa group again performed better than the Bophuthatswana group. It is also noteworthy that in the two states more than 65% of the pupils had problems in multiplying a natural number by a vulgar fraction.

In items 17-32 the pupils were required to multiply a vulgar fraction by another vulgar fraction and in some cases vulgar fractions were to be multiplied by improper fractions. The average percentage of incorrect solutions of items 17-32 of the pupils from Bophuthatswana is 83,3% compared with 56,9% of the pupils from Lebowa. This indicates that the Lebowa group again performed better than the Bophuthatswana group. It is also evident from the average percentages of incorrect solutions that in the two states more than 56% of the pupils experienced problems in multiplying a vulgar fraction by another vulgar fraction and in some cases vulgar fractions by improper fractions.

In items 33-40 the pupils were required to multiply a mixed number by another mixed number. The average percentage of incorrect solutions of items 33-40 of the pupils from Bophuthatswana is 95,3% compared with 60,9% of the pupils from Lebowa. It is quite evident from these average percentages of incorrect solutions that the Lebowa group again performed better than the Bophuthatswana group. In the two states more than 60% of the pupils experienced problems in multiplying a mixed number by another mixed number.

The average percentage of incorrect solutions of Subtest 1 of the pupils from Bophuthatswana as reflected on Table 79 is 83,5% compared with 54,8% of the pupils from Lebowa. The average percentage of incorrect solutions of the pupils from Bophuthatswana is calculated as follows: $\frac{3341}{40x100} \times \frac{100}{1}$; and of the pupils from Lebowa is calculated as follows: $\frac{3181}{40x145} \times \frac{100}{1}$. In other words, the average percentage of incorrect solutions is calculated as follows: The total number of incorrect solutions over the product of the number of items and the number of pupils, all multiplied by hundred. It follows from the average percentages of incorrect solutions that the Lebowa group again performed better than the Bophuthatswana group. It is also evident that the performance of the majority of the pupils from the two states is poor.

4.2.4.2 Subtest 2: Division

The number of pupils tested: Bophuthatswana 100; Lebowa 145

Table 80

Item	1	2	3	4	5	6
Bop: % Pupils who had the item incorrect	61	60	59	62	94	94
Leb: % Pupils who had the item incorrect	49,0	44,1	45,5	46,2	49,0	52,4
Bop: No pupils who had the item incorrect	61	60	59	62	94	94
Leb: No pupils who had the item incorrect	71	64	66	67	71	76

Item	7	8	9	10	11	12	13	14	15	16	17	18
Bop: % Pupils who had the item incorrect	93	96	95	94	96	94	96	98	99	95	99	98
Leb: % Pupils who had the item incorrect	48,3	49,7	57,9	49,7	62,1	54,5	55,9	56,6	62,8	55,2	69,7	73,1
Bop: No pupils who had the item incorrect	93	96	95	94	96	94	96	98	99	95	99	98
Leb: No pupils who had the item incorrect	70	72	84	72	90	79	81	82	91	80	101	106

Item	19	20	21	22	23	24	25	26	27	28	29	30
Bop: % Pupils who had the item incorrect	96	98	82	90	86	87	87	92	89	91	89	91
Leb: % Pupils who had the item incorrect	68,3	60,0	40,7	58,6	46,2	51,0	54,5	54,5	48,3	50,3	47,6	64,1
Bop: No pupils who had the item incorrect	96	98	82	90	86	87	87	92	89	91	89	91
Leb: No pupils who had the item incorrect	99	87	59	85	67	74	79	79	70	73	69	93

Item	31	32	33	34	35	36	Total
Bop: % Pupils who had the item incorrect	98	90	100	100	99	99	90,3
Leb: % Pupils who had the item incorrect	73,1	47,6	71,7	68,3	61,4	55,9	55,7
Bop: No pupils who had the item incorrect	98	90	100	100	99	99	3 249
Leb: No pupils who had the item incorrect	106	69	104	99	89	81	2 905

In items 1-8 the pupils were required to divide a vulgar fraction by a natural number. In some instances the pupils were to divide an improper fraction by a natural number. The average percentage of incorrect solutions of items 1-8 of the pupils from Bophuthatswana is 77,4% compared with 48% of the pupils from Lebowa. This indicates that the Lebowa group again performed better than the Bophuthatswana group. It also means that the pupils from Lebowa were more able to divide a vulgar fraction or improper fraction by a natural number than the pupils from Bophuthatswana.

In items 9-16 the pupils were required to divide a mixed number by a natural number. The average percentage of incorrect solutions of items 9-16 of the pupils from Bophuthatswana is 95,9% compared with 56,8% of the pupils from Lebowa. It is evident from the average percentages that the Lebowa group again performed better than the Bophuthatswana group. It also means that the pupils from Lebowa were more able to divide a mixed number by a natural number than the pupils from Bophuthatswana.

In items 17-20 the pupils were required to divide a natural number by a vulgar fraction. The average percentage of incorrect solutions of items 17-20 of the pupils from Bophuthatswana is 97,8% compared with 67,8% of the pupils from Lebowa. This shows clearly that the Lebowa group again performed better than the Bophuthatswana group. In the two states more than 67% of the pupils experienced problems in dividing a natural number by a vulgar fraction.

In items 21-32 the pupils were required to divide a vulgar fraction by another vulgar fraction. The average percentage of incorrect solutions of items 21-32 of the pupils from Bophuthatswana is 89,3% compared with 53% of the pupils from Lebowa. It follows from the average percentages that the Lebowa group again performed far much better than the Bophuthatswana group. In the two states more than 52% of the pupils had problems in dividing a vulgar fraction by another vulgar fraction.

In items 33-36 the pupils were required to divide a mixed number by another mixed number. The average percentage of incorrect solutions of items 33-36 of the pupils from Bophuthatswana is 99,5% compared with 64,3% of the pupils from Lebowa. It is quite clear from the average percentages of incorrect solutions that the pupils from Lebowa again performed better than the pupils from Bophuthatswana. It also means that in the two states more than 64% of the pupils experienced problems in dividing a mixed number by another mixed number.

The average percentage of incorrect solutions of Subtest 2 of the pupils from Bophuthatswana as reflected on Table 80 is 90,3% compared with 55,7% of the pupils from Lebowa. The average percentage of incorrect solutions of the pupils from Bophuthatswana is calculated as follows: $\frac{3249}{36 \times 100} \times \frac{100}{1}$; and of the pupils from Lebowa is calculated as follows: $\frac{2905}{36 \times 145} \times \frac{100}{1}$. In other words, the average percentage of incorrect solutions is calculated as follows: The total number of incorrect solutions over the product of the number of items and the number of pupils, all multiplied by hundred. In this subtest the Lebowa group again performed better than the Bophuthatswana group. It also follows from the average percentages of incorrect solutions that the majority of the pupils from the two states performed badly.

4.2.5 Summary: Test 9: Decimal fractions

4.2.5.1 Subtest 1: Concept of decimal fractions

The number of pupils tested: Bophuthatswana 100; Lebowa 144

Table 81

Item	1A	2A	3A	4A	5A	6A
Bop: % Pupils who had the item incorrect	3	47	84	97	56	36
Leb: % Pupils who had the item incorrect	10,4	67,4	81,9	89,0	38,2	16,7
Bop: No pupils who had the item incorrect	3	47	84	97	56	36
Leb: No pupils who had the item incorrect	15	97	118	128	55	24

7A	8A	9A	10A	11A	12A	13A	14A	15A	16A	17A	18A
53	95	13	81	90	95	16	83	78	96	76	72
76,4	84,7	56,9	84,7	92,4	93,8	70,1	86,8	91,7	94,4	61,1	87,5
53	95	13	81	90	95	16	83	78	96	76	72
110	122	82	122	133	135	101	125	132	136	88	126

19A	20A	1B	2B	3B	4B	5B	6B	7B	8B	Total
61	65	9	29	63	65	59	62	72	77	61,9
84,0	91,7	22,9	20,1	41,0	44,4	86,8	84,0	84,7	84,7	68,9
61	65	9	29	63	65	59	62	72	77	1 733
121	132	33	29	59	64	125	121	122	122	2 777

In items 1A-20A the pupils were required to rewrite common fractions in decimal notation. The average percentage of incorrect solutions of items 1A-20A of the pupils from Bophuthatswana is 64,9% compared with 73% of the pupils from Lebowa. This indicates that the pupils from Bophuthatswana performed slightly better than the pupils from Lebowa. It also means that more than 64% of the pupils from the two states experienced problems in converting common fractions to decimal fractions.

In items 1B-4B the pupils were required to rewrite decimal fractions as common fractions. The average percentage of incorrect solutions of items 1B-4B of the pupils from Bophuthatswana is 41,5% compared with 32,1% of the pupils from Lebowa. From these average percentages it is clear that the pupils from Lebowa performed slightly better than the pupils from Bophuthatswana. It is worth noting that the majority of the pupils from the two states were able to convert decimal fractions to common fractions.

In items 5B-8B the pupils were required to rewrite decimal fractions as mixed numbers. The average percentage of incorrect solutions of items 5B-8B of the pupils from Bophuthatswana is 67,5% compared with 85,1% of the pupils from Lebowa. It follows from the average percentage of incorrect solutions that the Bophuthatswana group performed slightly better than the Lebowa group. It also means that more than 67% of the pupils from the two states had problems in converting decimal fractions to mixed numbers.

The average percentage of incorrect solutions of Subtest 1 of the pupils from Bophuthatswana as reflected on Table 81 is 61,9% compared with 68,9% of the pupils from Lebowa. The average percentage of incorrect solutions of the pupils from Bophuthatswana is calculated as follows: $\frac{1733}{28 \times 100} \times \frac{100}{1}$; and of the pupils from Lebowa is calculated as follows: $\frac{2777}{28 \times 144} \times \frac{100}{1}$. In other words, the average percentage of incorrect solutions is calculated as follows: The total number of incorrect solutions over the product of the number of items and the number of pupils, all multiplied by hundred. In this subtest the Bophuthatswana group performed slightly better than the Lebowa group.

It also follows from the average percentages of incorrect solutions that the majority of the pupils from the two states performed badly.

4.2.5.2 Subtest 2 : Addition

The number of pupils tested: Bophuthatswana 100; Lebowa 144

Table 82

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
Bop: % Pupils who had the item incorrect	10	8	52	51	4	8	46	45	12	24	33	36	11	20	17	31	13	15	43	39	25,9
Leb: % Pupils who had the item incorrect	3,5	1,4	46,5	45,1	3,5	4,2	29,2	29,2	8,3	21,5	35,4	38,9	6,3	31,3	31,9	37,5	19,4	20,8	38,2	34,0	24,3
Bop: No pupils who had the item incorrect	10	8	52	51	4	8	46	45	12	24	33	36	11	20	17	31	13	15	43	39	518
Leb: No pupils who had the item incorrect	5	2	67	65	5	6	42	42	12	31	51	56	9	45	46	54	28	30	55	49	700

In items 1-4 the pupils were required to add decimal fractions of one decimal place together. The average percentage of incorrect solutions of items 1-4 of the Bophuthatswana group is 30,3% compared with 24,1% of the Lebowa group. This indicates that the pupils from Lebowa performed slightly better than the pupils from Bophuthatswana. It also means that in the two states the majority of the pupils were able to add decimal fractions of one decimal place together correctly.

In items 5-8 the pupils were required to add decimal fractions of one decimal place, having the units digit together. The average percentage of incorrect solutions of items 5-8 of the Bophuthatswana group is 25,8% compared with 16,5% of the Lebowa group. It follows from the average percentages of incorrect solutions that the pupils from Lebowa again performed slightly better than the pupils from Bophuthatswana. From the low average percentages of incorrect solutions of the pupils from the two states it is clear that the majority of the pupils performed well.

In items 9-12 the pupils were required to add decimal fractions of two decimal places together. The average percentage of incorrect solutions of items 9-12 of the Bophuthatswana group is 26,3% compared with 26% of the Lebowa group. It is quite evident from these average percentages of incorrect solutions that the pupils from Lebowa again performed slightly better than the pupils from

Bophuthatswana. They also indicate that the majority of the pupils from the two states were able to add decimal fractions of two decimal places together correctly.

In items 13-16 the pupils were required to add a decimal fraction of two decimal places, having the units digit, to another similar number. The average percentage of incorrect solutions of items 13-16 of the pupils from Bophuthatswana is 19,8% compared with 26,8% of the pupils from Lebowa. This indicates that the pupils from Bophuthatswana performed slightly better than the pupils from Lebowa. It also means that the majority of the pupils from the two states were able to add a decimal fraction of two decimal places, having the units digit, to another similar number correctly.

In items 17-20 the pupils were required to add a decimal fraction of two decimal places, having the tens and units digits, to another similar number. The average percentage of incorrect solutions of items 17-20 of the pupils from Bophuthatswana is 27,5% compared with 28,1% of the pupils from Lebowa. This indicates that the Bophuthatswana group again performed slightly better than the Lebowa group. It also means that the majority of the pupils from the two states were able to add a decimal fraction of two decimal places, having the tens and units digits, to another similar number correctly.

The average percentage of incorrect solutions of Subtest 2 of the pupils from Bophuthatswana as reflected on Table 82 is 25,9% compared with 24,3% of the pupils from Lebowa. The average percentage of incorrect solutions of the Bophuthatswana group is calculated as follows: $\frac{518}{20x100}$ X $\frac{100}{1}$; and of the Lebowa group is calculated as follows: $\frac{700}{20x144}$ X $\frac{100}{1}$. In other words, the average percentage of incorrect solutions is calculated as follows: The total number of incorrect solutions over the product of the number of items and the number of pupils, all multiplied by hundred. It follows from the average percentages of incorrect solutions that the Lebowa group performed slightly better than the Bophuthatswana group. It also means that the performance of the majority of the pupils from the two states is good.

4.2.5.3 Subtest 3: Subtraction

The number of pupils tested: Bophuthatswana 100; Lebowa 144

Table 83

Item	1	2	3	4	5	6
Bop: % Pupils who had the item incorrect	8	38	35	18	14	11
Leb: % Pupils who had the item incorrect	6,3	27,1	26,4	22,9	6,9	8,3
Bop: No pupils who had the item incorrect	8	38	35	18	14	11
Leb: No pupils who had the item incorrect	9	39	38	33	10	12

Item	7	8	9	10	11	12	13	14	15	16	17	18
Bop: % Pupils who had the item incorrect	24	36	16	35	47	89	4	28	49	39	46	56
Leb: % Pupils who had the item incorrect	29,2	29,2	13,2	34,0	40,3	70,8	13,9	29,9	47,9	45,8	43,8	51,4
Bop: No pupils who had the item incorrect	24	36	16	35	47	89	4	28	49	39	46	56
Leb: No pupils who had the item incorrect	42	42	19	49	58	102	20	43	69	66	63	74

Item	19	20	21	22	23	24	25	26	27	28	Total
Bop: % Pupils who had the item incorrect	61	54	88	89	85	89	89	70	88	85	49,7
Leb: % Pupils who had the item incorrect	63,2	63,2	74,3	78,5	72,9	73,6	76,4	62,5	65,3	70,8	44,6
Bop: No pupils who had the item incorrect	61	54	88	89	85	89	89	70	88	85	1 391
Leb: No pupils who had the item incorrect	91	91	107	113	105	106	110	90	94	102	1 797

In items 1-4 the pupils were required to subtract a decimal fraction of one decimal place, having the units digit, from another similar number. The average percentage of incorrect solutions of items 1-4 of the pupils from Bophuthatswana is 24,8% compared with 20,7% of the pupils from Lebowa. This indicates that the Lebowa group performed slightly better than the Bophuthatswana group. It also means that the majority of the pupils from the two states were able to subtract a decimal fraction, having the units digit, from another similar number correctly.

In items 5-12 the pupils were required to subtract a decimal fraction of two decimal places from another similar number. The average percentage of incorrect solutions of items 5-12 of the pupils from Bophuthatswana is 34% compared with 29% of the pupils from Lebowa. From the average percentages of incorrect solutions it follows that the Lebowa group again performed slightly better than the Bophuthatswana group. It can also be deduced from the average percentages of incorrect solutions that the majority of the pupils of the two states were able to subtract a decimal fraction of two decimal places from another similar number correctly.

In items 13-20 the pupils were required to subtract a decimal fraction of two decimal places, having the units digit, from another similar number. The average percentage of incorrect solutions of items 13-20 of the pupils from Bophuthatswana is 42,1% compared with 44,9% of the pupils from Lebowa. This indicates that the pupils from Bophuthatswana performed slightly better than the pupils from Lebowa. It also means that the majority of the pupils from the two states were able to subtract a decimal fraction of two decimal places, having the units digit, from another similar number.

In items 21-24 the pupils were required to subtract a decimal fraction of two decimal places, having the units digit, from another decimal fraction of one decimal place also having the units digit. The average percentage of incorrect solutions of items 21-24 of the pupils from Bophuthatswana is 87,8% compared with 74,8% of the pupils from Lebowa. In these items the Lebowa group performed slightly better than the Bophuthatswana group. The average percentages of incorrect solutions indicate that in the two states more than 74% of the pupils encountered serious difficulties in subtracting a decimal fraction of two decimal places, having the units digit, from another decimal fraction of one decimal place also having the units digit.

In items 25-28 the pupils were required to subtract a decimal fraction of two decimal places, having the units digit, from a natural number. The average percentage of incorrect solutions of items 25-28 of the pupils from Bophuthatswana is 83% compared with 68,8% of the pupils from Lebowa. From these average percentages of incorrect solutions it follows that the Lebowa group again performed slightly better than the Bophuthatswana group. It can also be deduced from the average percentages of incorrect solutions that in the two states more than 68% of the pupils experienced problems in subtracting a decimal fraction of two decimal places, having the units digit, from a natural number.

The average percentage of incorrect solutions of Subtest 2 of the pupils from Bophuthatswana as reflected on Table 83 is 49,7% compared with 44,6% of the pupils from Lebowa. The average percentage of incorrect solutions of the Bophuthatswana group is calculated as follows: $\frac{1391}{28x100} \times \frac{100}{1}$; and of the Lebowa group is calculated as follows: $\frac{1797}{28x144} \times \frac{100}{1}$. In other words, the average percentage of incorrect solutions is calculated as follows: The total number of incorrect solutions over the product of the number of items and the number of pupils, all multiplied by hundred. It follows from the average percentages of incorrect solutions that the Lebowa group performed slightly better than the Bophuthatswana group. It also means that the performance of the majority of the pupils from the two states is satisfactory.

4.3 CONCLUSION

In this chapter the researcher has compared the results of the standardized tests of Bophuthatswana and Lebowa. It has been discovered that the Bophuthatswana group performed slightly better than the Lebowa group in the "Scholastic Achievement Tests in Arithmetic - Standard 4". In the case of the "Diagnostic Mathematical Tests" the Bophuthatswana group performed slightly better than the

Lebowa group in Tests 5 and 6 while the Lebowa group performed slightly better than the Bophuthatswana group in Tests 7,8 and 9.

In chapter 5 we are going to discuss the general findings, possible reasons which might have influenced the pupils' performance and recommendations.

CHAPTER 5

GENERAL FINDINGS, POSSIBLE REASONS WHICH MIGHT HAVE INFLUENCED THE PUPILS' PERFORMANCE AND RECOMMENDATIONS

5.1 SUMMARY OF FINDINGS

5.1.1 Results of the pupils from Bophuthatswana

5.1.1.1 Results of the "Scholastic Achievement Tests in Arithmetic - Standard 4"

It was impossible to test all the Standard 5 pupils in the primary and middle schools of Bophuthatswana. As a result a sample was used which consisted of schools from rural as well as urban areas, big and small schools. The selected schools came from three inspection circuits, namely, Ga-Rankuwa, Mankwe and Tlhabane. In this way the sample could be regarded as representative.

The schools were chosen as follows:

i) Four small schools (enrolment 1-399)
Six big schools (enrolment 400+)

ii) Four schools from urban areas: Ga-Rankuwa.
Four schools from rural areas: Mankwe.

One school from urban area: Tlhabane.

One school from rural area: Tlhabane.

Ten schools were tested in Bophuthatswana. As seen in Chapter 3, the results of the ten schools do not differ significantly from each other. In general, the performance of the pupils from Bophuthatswana is below expectation.

The poor performance of the pupils from Bophuthatswana is not area-bound. The three inspection circuits performed almost equally poor. This means that schools from urban areas performed almost equally poor as the schools from rural areas. Small schools also performed almost equally poor as big schools.

The school which performed better than the other schools in Bophuthatswana is school C, see Tables 7, 8 and 9. School J, see Tables 28,29 and 30, performed worse than the other schools in Bophuthatswana. It is of interest to note that school J is from an urban area.

5.1.1.2 <u>Results of the "Diagnostic Mathematical Tests"</u>

Five schools wrote the "Diagnostic Mathematical Tests" in Bophuthatswana. Two of the schools were from rural areas while three of them were from urban areas. Two schools are in Ga-Rankuwa inspection circuit, one is in Mankwe inspection circuit and two are in Tlhabane inspection circuit. Each school wrote a different test. The "Diagnostic Mathematical Tests" consisted of the following five tests, Test 5, Test 6, Test 7, Test 8 and Test 9.

One hundred pupils in school B wrote Test 5. The average percentage of incorrect solutions of Test 5: Subtest 1 as reflected on Table 51 is 29,3%. The majority of the pupils performed well in this subtest.

The average percentage of incorrect solutions of Test 5: Subtest 2 as reflected on Table 52 is 37,7%. Again the majority of the pupils performed well.

The average percentage of inocrrect solutions of Test 5: Subtest 3 as reflected on Table 53 is 63%. The majority of the pupils performed below expectation.

The average percentage of incorrect solutions of Test 5: Subtest 4 as reflected on Table 54 is 90%. Almost all the pupils performed below expectation.

One hundred pupils in school C wrote Test 6. The average percentage of incorrect solutions of Test 6: Subtest 1 as reflected on Table 55 is 38,1%. The majority of the pupils performed well.

The average percentage of incorrect solutions of Test 6: Subtest 2 as reflected on Table 56 is 47,4%. The performance of the majority of the pupils is satisfactory.

One hundred pupils in school J wrote Test 7. The average percentage of incorrect solutions of Test 7: Subtest 1 as reflected on Table 57 is 83,4%. Almost all the pupils performed below expectation.

One hundred pupils in school G wrote Test 8. The average percentage of incorrect solutions of Test 8: Subtest 1 as reflected on Table 58 is 83,5%. Again, almost all the pupils performed below expectation.

The average percentage of incorrect solutions of Test 8: Subtest 2 as reflected on Table 59 is 90,3%. Again, almost all the pupils performed below expectation.

One hundred pupils in school I wrote Test 9. The average percentage of incorrect solutions of Test 9: Subtest 1 as reflected on Table 60 is 61,9%. The majority of the pupils performed below expectation.

The average percentage of incorrect solutions of Test 9: Subtest 2 as reflected on Table 61 is 25,9%. Most of the pupils performed well.

The average percentage of incorrect solutions of Test 9: Subtest 3 as reflected on Table 62 is 49,7%. Slightly more than half of the pupils performed well.

5.1.2 Results of the pupils from Lebowa

5.1.2.1 Results of the "Scholastic Achievement Tests in Arithmetic - Standard 4"

A sample of the Lebowa schools was composed as follows:

Two schools from rural areas: Bochum
Two schools from rural areas: Sekhukhune
One school from rural area: Bosbokrand
Two schools from urban areas: Mahwelereng
Two schools from urban areas: Polokwane

Nine schools were tested in Lebowa. As seen in chapter 4 the results of the nine schools differ slightly. The performance of the pupils is also below expectation.

The poor performance of the pupils from Lebowa is not confined to a particular inspection circuit. The five inspection circuits performed below expectation. In other words, schools from rural areas performed almost equally poor as the schools from urban areas.

5.1.2.2 Results of the "Diagnostic Mathematical Tests"

Four schools wrote the "Diagnostic Mathematical Tests" in Lebowa. Each school wrote a different test except one school which wrote two tests.

156 pupils wrote Test 5. The average percentage of incorrect solutions of Test 5: Subtest 1 as reflected on Table 72 is 43,1%. Slightly more than half of the pupils performed well.

The average percentage of incorrect solutions of Test 5: Subtest 2 as reflected on Table 73 is 59,9%. Slightly more than half of the pupils performed below expectation.

The average percentage of incorrect solutions of Test 5: Subtest 3 as reflected on Table 74 is 73,1%. The majority of the pupils performed below expectation.

The average percentage of incorrect solutions of Test 5: Subtest 4 as reflected on Table 75 is 93,9%. Almost all the pupils performed below expectation.

156 pupils wrote Test 6. The average percentage of incorrect solutions of Test 6: Subtest 1 as reflected on

Table 76 is 66,6%. The majority of the pupils performed below expectation.

The average percentage of incorrect solutions of Test 6: Subtest 2 as reflected on Table 77 is 85%. Almost all the pupils performed below expectation.

145 pupils wrote Test 7. The average percentage of incorrect solutions of Test 7: Subtest 1 as reflected on Table 78 is 59,8%. Slightly more than half of the pupils performed below expectation.

145 pupils wrote Test 8. The average percentage of incorrect solutions of Test 8: Subtest 1 as reflected on Table 79 is 54,8%. Again, slightly more than half of the pupils performed below expectation.

The average percentage of incorrect solutions of Test 8: Subtest 2 as reflected on Table 80 is 55,7%. Once more, slightly more than half of the pupils performed below expectation.

144 pupils wrote Test 9. The average percentage of incorrect solutions of Test 9: Subtest 1 as reflected on Table 81 is 68,9%. The majority of the pupils performed below expectation.

The average percentage of incorrect solutions of Test 9: Subtest 2 as reflected on Table 82 is 24,3%. The majority of the pupils performed well.

The average percentage of incorrect solutions of Test 9: Subtest 3 as reflected on Table 83 is 44,6%. Slightly more than half of the pupils performed well.

5.1.3 A comparative review of the results of the pupils from Bophuthatswana and Lebowa

5.1.3.1 Results of the "Scholastic Achievement Tests in Arithmetic - Standard 4"

The "Scholastic Achievement Tests in Arithmetic - Standard 4" of the Human Sciences Research Council was applied to determine the general standard of performance in Mathematics of the pupils. The tests were given to 495 Bophuthatswana Standard 5 pupils and to 1 047 Lebowa Standard 5 pupils.

In Bophuthatswana pupils of ten schools and in Lebowa pupils of nine schools were tested. The schools from Bophuthatswana were chosen from three inspection circuits while in Lebowa the schools were chosen from five inspection circuits.

The results of the majority of the pupils from the two states in Test 1 as reflected on Table 63 are below expectation. Test 1 was to determine the pupils' skill in dealing with the four basic mathematical operations. The results of the pupils from Bophuthatswana and Lebowa indicate that most of the pupils experienced problems in the four basic mathematical operations.

The results of the majority of the pupils from the two states in Test 2 as reflected on Table 64 are also below expectation. Test 2 was to determine to what extent the pupils understood Mathematics and to what extent they could think logically about numbers. The results of the pupils from Bophuthatswana and Lebowa indicate that the majority of the pupils experienced problems in Test 2.

The results of the majority of the pupils from the two states in Test 3 as reflected on Table 65 are also below expectation. Test 3 was to determine the pupils' ability to solve mathematical word problems. The results of Test 3 of the pupils from Bophuthatswana and Lebowa indicate that the majority of the pupils encountered serious difficulties in solving word problems.

5.1.3.2 Results of the "Diagnostic Mathematical Tests"

The "Diagnostic Mathematical Tests" of the Human Sciences

Research Council were used to determine the problem areas. The aim was to determine as to whether the weakness was general or whether it was in specific aspects. The tests were written by 500 Bophuthatswana Standard 5 pupils and by 590 Lebowa Standard 5 pupils.

To identify certain problem areas, the following diagnostic tests of the Human Sciences Research Council were applied:

i) Test 5: Basic operations ($<$100 000)
Subtest 1: Addition
Subtest 2: Subtraction
Subtest 3: Multiplication
Subtest 4: Division

ii) Test 6: Vulgar fractions
Subtest 1: Concept of vulgar fractions
Subtest 2: Addition

iii) Test 7: Vulgar fractions
Subtest 1: Subtraction

iv) Test 8: Vulgar fractions
Subtest 1: Multiplication
Subtest 2: Division

v) Test 9: Decimal fractions

Subtest 1: Concept of decimal fractions

Subtest 2: Addition

Subtest 3: Subtraction

The results of the majority of the pupils from Bophuthatswana and Lebowa in Test 5: Subtest 1 as reflected on Table 72 are satisfactory. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 29,3% compared with 43,1% of the pupils from Lebowa. Bophuthatswana group performed slightly better than the Lebowa group.

The results of the majority of the pupils from Bophuthatswana in Test 5: Subtest 2 as reflected on Table 73 are satisfactory while the results of the majority of the pupils from Lebowa are poor. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 37,7% compared with 59,9% of the pupils from Lebowa. Again, the results of the pupils from Bophuthatswana are slightly better than those of the pupils from Lebowa.

The results of the majority of the pupils from the two states in Test 5: Subtest 3 as reflected on Table 74 are poor. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 63% compared with 73,1% of the pupils from Lebowa. The Bophuthatswana group again performed slightly better than the Lebowa group.

The results of the majority of the pupils from the two states in Test 5: Subtest 4 as reflected on Table 75 are poor. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 90% compared with 93,9% of the pupils from Lebowa. The Bophuthatswana group again performed slightly better than the Lebowa group.

The results of the majority of the pupils from Bophuthatswana in Test 6: Subtest 1 as reflected on Table 76 are satisfactory while the results of the majority of the pupils from Lebowa are poor. The average percentage of incorrect solutions of the Bophuthatswana group is 38,1% compared with 66,6% of the Lebowa group. The Bophuthatswana group again performed slightly better than the Lebowa group.

The results of the majority of the pupils from Bophuthatswana in Test 6: Subtest 2 as reflected on Table 77 are satisfactory while the results of the majority of the pupils from Lebowa are poor. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 47,4% compared with 85% of the pupils from Lebowa. The Bophuthatswana group again performed better than the Lebowa group.

The results of the majority of the pupils from the two states in Test 7: Subtest 1 as reflected on Table 78 are poor. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 83,4% compared with 59,8% of the pupils from Lebowa. The Lebowa group performed better than the Bophuthatswana group.

The results of the majority of the pupils from the two states in Test 8: Subtest 1 as reflected on Table 79 are poor. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 83,5% compared with 54,8% of the pupils from Lebowa. The Lebowa group again performed better than the Bophuthatswana group.

The results of the majority of the pupils from the two states in Test 8: Subtest 2 as reflected on Table 80 are poor. The average percentage of incorrect solutions of the Bophuthatswana group is 90,3% compared with 55,7% of the Lebowa group. The Lebowa group again performed better than the Bophuthatswana group.

The results of the majority of the pupils from the two states in Test 9: Subtest 1 as reflected on Table 81 are poor. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 61,9% compared with 68,9% of the pupils from Lebowa. The Bophuthatswana group performed slightly better than the Lebowa group.

The results of the majority of the pupils from the two states in Test 9: Subtest 2 as reflected on Table 82 are good. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 25,9% compared with 24,3% of the pupils from Lebowa. The Lebowa group performed slightly better than the Bophuthatswana group.

The results of the majority of the pupils from the two states in Test 9: Subtest 3 as reflected on Table 83 are satisfactory. The average percentage of incorrect solutions of the pupils from Bophuthatswana is 49,7% compared with 44,6% of the pupils from Lebowa. The Lebowa group again performed slightly better than the Bophuthatswana group.

The difference in the performance of the pupils from the two states may indicate that certain aspects of Mathematics are taught better in one state than in the other state.

The pupils from Bophuthatswana performed slightly better than the pupils from Lebowa in Test 5. This may indicate that the basic operations with respect to natural numbers are taught better in the Bophuthatswana schools than in the Lebowa schools.

In Test 6 the pupils from Bophuthatswana again performed slightly better than the pupils from Lebowa. This may indicate that the concept of and addition of vulgar fractions are taught better in the schools of Bophuthatswana than in the schools of Lebowa.

The results of Test 7 indicates that the Lebowa group performed slightly better than the Bophuthatswana group. The difference in performance of the two groups may be due to the fact that the subtraction of vulgar fractions was taught better in the Lebowa schools than in the Bophuthatswana schools.

The Lebowa group again performed slightly better than the Bophuthatswana group in Test 8. This may be due to the fact that the multiplication and division of vulgar fractions are taught better in the Lebowa schools than in the Bophuthatswana schools.

The pupils from Lebowa again performed slightly better than the pupils from Bophuthatswana in Test 9. This may indicate that decimal fractions are taught better in the schools of Lebowa than in the schools of Bophuthatswana.

5.1.4 Typical errors committed by the pupils from Bophuthatswana (cf Chapter 3)

5.1.4.1 Natural numbers

The following mistakes were made by the pupils when dealing with natural numbers:

i) 68% of the pupils was unable to add numbers which were written in words.

ii) 29,3% of the pupils was unable to add, 37,7% to subtract, 63% to multiply and 90% to divide natural numbers correctly if the problems involved carrying and the zero difficulty.

5.1.4.2 Vulgar fractions

The pupils committed the following errors when dealing with vulgar fractions:

i) 24,3% of the pupils failed to add a natural number to a mixed number.

ii) 56,4% of the pupils failed to add mixed numbers together.

iii) 53% of the pupils failed to add a vulgar fraction to a mixed number.

iv) 95% of the pupils failed to subtract a vulgar fraction from a natural number.

v) 93,1% of the pupils was unable to subtract mixed numbers. Both mixed numbers were having the same denominator.

vi) 49,5% of the pupils was unable to subtract a natural number from a mixed number.

vii) 99,3% of the pupils was unable to subtract a mixed number from a natural number.

viii) 42,6% of the pupils did not understand the equivalent fractions.

ix) 48,9% of the pupils was unable to subtract vulgar fractions with the same denominator.

X) Most pupils were unable to determine the correct Lowest Common Multiple.

xi) Most pupils failed to give the answers in the simplest form.

xii) Most pupils failed to give the reciprocals of mixed numbers.

5.1.4.3 Decimal fractions

The following mistakes were made by the pupils when dealing with decimal fractions:

i) 67,5% of the pupils was unable to convert decimal fractions to mixed numbers.

ii) 64,9% of the pupils failed to convert common fractions to decimal fractions.

iii) 41,5% of the pupils failed to convert decimal fractions to common fractions.

iv) 24,8% of the pupils failed to subtract a decimal fraction of one decimal place, having the units digit, from another similar number.

v) 42,1% of the pupils was unable to subtract a decimal fraction of two decimal places, having the units digit, from another similar number.

vi) 83% of the pupils failed to subtract a decimal fraction of two decimal places, having the units digit, from a natural number.

vii) 87,8% of the pupils failed to subtract a decimal fraction of two decimal places, having the units digit, from another decimal fraction of one decimal place also having the units digit.

5.2 POSSIBLE FACTORS WHICH MIGHT HAVE INFLUENCED THE PUPILS' PERFORMANCE

5.2.1 Possible factors which have been mentioned in the completed investigations

Owing to the unsatisfactory results of the pupils it is necessary to look into the possible factors which might have contributed to the poor performance. The following completed researches have identified the possible reasons that influence the pupils' performance in Mathematics:

i) Van den Berg, D J (1978, pp5-20): A pedagogical study of the Blackman's mathematical ability.

ii) Groenewald, F P (1976, pp.15-20): Aspects in the traditional world of culture of the Black child which hamper the actualization of his intelligence: a cultural-educational exploratory study.

iii) Wentzel, J A T (1982, pp.43-64): Ondersoek na die prestasies in Wiskunde van die St 5 leerlinge van Lebowa.

iv) Wilkinson, A C (1981, pp.100-118): An analysis of the problems experienced by pupils in Mathematics of Standard 5 level in the developing states in the South African context.

5.2.1.1 Domestic environment of the black child

5.2.1.1.1 The domestic milieu of the black child is uncomplicated. The traditional hut consists of one room. This room often serves as kitchen, sitting-room and bedroom. There is lack of household implements and personal possessions. There are also no books or magazines to read.

5.2.1.1.2 The traditional educational aim of the black child is to introduce him to the habits and customs of the tribe into which he is born. Individual thought and actions are tolerated only if they express the group's norms and views. Obedience to the tribal norms is of paramount importance.

5.2.1.1.3 The traditional culture of the black people offer much less chance of individual development. Personal opinions are only accepted if they do not clash with the interest of the group.

5.2.1.1.4 Since the traditional black culture has no written language, all the tribe's history and traditions had to be memorized and imparted to the next generation orally. As a result memorization played an important role in the learning activities of the black child.

5.2.1.1.5 The slavish obedience expected from the black child may dampen his initiative, originality and creativeness to a great extent.

5.2.1.1.6 Once the black baby or child has been weaned, the black mother does not give the child much of her time. School-going children expect no help from their parents who are often illiterates.

5.2.1.2 Development of the black child's intellectual potential

In his indepth study of the traditional cultural milieu of the black child, Groenewald points out that a child forms his impressions of and attitudes towards his environment from the age of three onwards. Research, as quoted by Van den Berg, shows that the crucial age for the development of a child's intellectual capabilities is between the ages of 10 months and 18 months and that the child's future competence is determined by the mother's actions. These first impressions and attitudes form the basis for his further intellectual development. Where the home environment is educationally poor and inadequate, the intellectual development of the child will be retarded.

5.2.1.3 Mathematics in everyday life of the black people

Trade has through the ages stimulated the development of Mathematics among various civilizations. Once the economic activity among a group of people becomes pronounced and trade with other nations develops, so too the need for more sophisticated ways of counting and computing is created. Contact with other nations means the transmission of knowledge and especially ways of conducting trade.

Van den Berg (1978, pp.15-20) discusses a number of interesting examples of mathematical ability among black people. He mentions that the black people have a remarkable flair for producing beautiful and intricate geometrical patterns as seen in their construction of houses. The walls of their houses are painted with intricate colourful geometrical patterns. The women take great pride and pleasure in weaving similar intricate geometrical patterns with coloured beads. The construction of houses and the weaving of geometrical patterns with beads are not only indications of mathematical concepts, but also how those concepts are put into practice.

5.2.1.4 Formation of mathematical concepts

Jean Piaget broke away from theorising about the way in which a child learnt Mathematics. He attempted to find an answer to this question by careful experiments with children. He and his students identified what they thought were the basic mathematical concepts and set about devising a number of ingenious experiments with which they succeeded in throwing light upon the stages of mathematical insight gained by a child during maturation. Piaget shed light on the matter of how long it takes to develop mathematical insight and how much it depends on the opportunity to manipulate material. He also underlined the danger of imparting mathematical knowledge to children who do not have adequate and applicable experience, for it actually impedes their development.

Piaget (Auleta, 1969, p.279) believed that children acquire certain basic mathematical concepts through experiences in many different situations and that they need more and more experience in order to be able to make the necessary generalizations. Further, he mentioned the danger of teaching Mathematics before the following basic concepts have been understood:

i) The concept of number (Fielker, 1976, p.13).

ii) The concept of space (Keats, 1978, p.28).

iii) The concept of time (Copeland, 1974, pp.247-269).

iv) The concepts of length and measurement (Keats, 1978, p.29).

v) Concepts associated with area and volume (Piaget, 1957, p.16).

vi) The concept of substance (Piaget, 1957, p.16).

vii) The concept of weight (Hyde, 1970, pp.82-101).

From his experiments Piaget was able to indicate three main stages of cognitive development in the growing child, namely, a non-operational stage, a pre-operational stage and an operational stage (Berlyne, 1965, pp.175-190).

i) The non-operational stage is from birth to approximately two years of age and is overwhelmingly sensori-motor in character. At this stage the child cannot foresee the outcome of his actions.

ii) The pre-operational stage extends in age from approximately two years to approximately seven years. At this stage the child comes to conclusions based on direct observation. His thinking is dominated by his observation.

iii) The third main stage is the operational stage which starts at about the age of seven years. During this stage insight into reversibility, equivalence and conservation break through. This last stage is characterized by the child being able to think hypothetically and deductively. He can think abstractly so that he can solve problems solely by means of language and symbols.

5.2.1.5 Medium of instruction

Researchers (Wentzel, 1982, pp.51-55 and Wilkinson, 1981, pp.111-113) attribute poor performance of pupils in Mathematics to the fact that a foreign language is used as medium of instruction. Pupils who have to learn through the medium of a foreign language may encounter tremendous problems.

In Bophuthatswana and Lebowa the changeover to English as medium of instruction takes place at the beginning of Standard 3. At the stage when the pupils change over to English as a medium of instruction, they may not yet be able to express themselves fluently. As a result they may be unable to understand some of the concepts. The children may then resort to memorization without understanding.

Some times the pupils in a Mathematics classroom may be passive. This may be due to the lack of the command of English. Pupils may be unable to ask and answer questions because they may not be able to express themselves clearly in English. Even in the examinations pupils may perform badly because they may not understand some of the questions asked in English.

The changeover from the vernacular to English as medium of instruction has received lots of attention from educationists. There is no general agreement about the best time to change from one medium to another. In practice we find that the changeover usually occurs during one of the following periods:

5.2.1.5.1 <u>During the first year of the school career</u>

If the child begins his formal school career in a foreign language, then there is no need to change to another medium of instruction at a later stage. Before a child can use a language as a thinking medium, he must first be fluent in it. Therefore the child must be first fluent in a foreign language before it can be used as a medium of instruction. The child will experience serious problems if a foreign language, which he does not understand, is used as a medium of instruction during his first year of his schooling.

5.2.1.5.2 <u>During the primary school phase</u>

In Bophuthatswana and Lebowa the changeover to English as a medium of instruction takes place at the beginning of Standard 3.

Pupils are taught English as a subject in Sub-Standard A or Grade 1, as it is called in Bophuthatswana. The pupils may find this changeover easier because at that stage they have already studied English for four years. With that they will have a certain command of the language as concept formation through a foreign language can only take place after the child has a command of that language.

It is an accepted psychological fact that any person learns a foreign language easily before he reaches the stage of puberty (Stern, 1974, p.21). The stage of puberty starts when the child is about 12 years of age. Pupils in Bophuthatswana and Lebowa complete Standard 2 before they reach puberty. Thus the changeover at this stage is psychologically sound.

5.2.1.5.3 During the secondary school phase

In our black schools many children leave school after completing primary education. A large number of the pupils leave school at this stage and to change over to a foreign medium of instruction may not encourage the pupils to continue with secondary school education. Pupils who leave school at the beginning of the secondary school phase may find it difficult to adapt themselves in a work situation because they may be unable to speak the official languages. Furthermore, it would be psychologically unsound because the pupils will already be in the stage of puberty.

5.3 TEACHERS' QUALIFICATIONS IN MATHEMATICS

A survey was made by the researcher to determine with the help of a questionnaire the qualifications of the Standard 5 Mathematics teachers. The following table reflects the qualifications in Mathematics of the teachers in the schools taken as a sample of Bophuthatswana.

Table 84

School	No of Maths teachers at a school	Qualifications in maths or Arithmetic	Teaching expe-rience	Teacher's attitude to Maths
School A	Teacher 1	JC	13 Yrs	Positive
School A	Teacher 2	JC	9 Yrs	Positive
School A	Teacher 3	JC	7 Yrs	Positive
School A	Teacher 4	Std 6	21 Yrs	Positive
School A	Teacher 5	JC	8 Yrs	Positive
School A	Teacher 6	JC	8 Yrs	Positive
School A	Teacher 7	JC	12 Yrs	Positive
School B	Teacher 1	Std 10	3 Yrs	Positive
School B	Teacher 2	Std 10	6 Yrs	Positive
School B	Teacher 3	JC	12 Yrs	Positive
School C	Teacher 1	JC	13 Yrs	Positive
School D	Teacher 1	JC	12 Yrs	Positive
School E	Teacher 1	JC	5 Yrs	Positive
School F	Teacher 1	JC	22 Yrs	Positive
School F	Teacher 2	JC	5 Yrs	Positive
School G	Teacher 1	JC	2 Yrs	Positive
School H	Teacher 1	JC	25 Yrs	Positive
School I	Teacher 1	JC	8 Yrs	Positive
School J	Teacher 1	JC	13 Yrs	Positive
School J	Teacher 2	JC	6 Yrs	Positive
School J	Teacher 3	JC	5 Yrs	Positive

From Table 84 it is clear that almost all the Mathematics teachers in the sample schools have Standard 8 as their highest qualification in Mathematics or Arithmetic. Only

2 out of the 21 teachers have Standard 10 as their qualification in Mathematics and one teacher has only Standard 6 as his qualification in Arithmetic.

From Table 84 it can be deduced that there is a shortage of suitably qualified teachers of Mathematics in the primary and middle schools of Bophuthatswana. A suitably qualified primary school teacher is the one who have at least passed Standard 10 Mathematics. This implies that great demands will have to be made on the Education Department as regards the initial training as well as in-service training of primary school Mathematics teachers.

The years of teaching experience of Mathematics teachers range from 2 to 25 years. There was no marked difference in the performance of the pupils being taught by the experienced or inexperienced teachers.

All Mathematics teachers from the sample schools indicated that they like teaching Mathematics. A teacher who loves his subject is able to inspire his pupils. The results of the questionnaire on Table 84 indicate that the love for the subject must be coupled with a thorough knowledge of the subject to produce the desired results.

5.4 RECOMMENDATIONS

In the light of the research done in this study as well as the findings of other related studies, the following recommendations are made:

5.4.1 Teaching methods should be in line with the developmental stages of the child (cf Chapter 2)

Most of the pupils may spend their time in the primary schools computing without understanding what they were doing because insufficient attention has been paid to the necessary basic mathematical concepts. They proceed to Algebra in the middle and high schools without the necessary conceptual development. The pupils may have built up a lot of misunderstandings at the primary school level. Piaget's investigation of children's conceptual development refers to the cognitive development that is necessary before the children can be expected to compute with understanding. With an understanding of the cognitive developmental stages a teacher can make appropriate decisions about the kind of provisions that might match the cognitive capacities of the pupils of a particular level of development. Such an understanding should enable a teacher to ask questions which will help him to ascertain the pupils' conceptual development (Tamburini, 1975, p.6).

Piaget's work has some implications for the teaching and learning of Mathematics. In keeping with Piaget's theory of intellectual development, it is strongly recommended that the methods of teaching should be appropriate to the level of cognitive development of the pupil, that is, the pupil must be ready for the new material which is to be presented to him.

In the case of Mathematics Piaget maintains that the pupil must first acquire certain basic mathematical concepts before he can tackle Mathematics successfully. This could perhaps be tested by using a battery of tests similar to those which were used by Piaget.

The cognitive organization which the pupil must possess in order to cope with Mathematics are regarded as integrated complex operations which the pupil can perform mentally. But before this can happen, the pupil must first carry out operations on concrete materials - arranging in groups, that is classifying, placing in one-to-one correspondence, arranging in series, counting and measuring quantities (Piaget, 1964, pp.1-146). In keeping with Piaget's theory, it is strongly recommended that the activity methods should be applied in the primary schools.

Active participation by the pupils in the primary schools means that the pupils should learn by doing, or by finding their own solutions to problems. In many school subjects self-discovery involves finding information from books or other printed materials, but in Mathematics, it is strongly recommended that the emphasis should be on doing, observing, comparing, reasoning and calculating.

5.4.2 The audio-visual centre

It is strongly recommended that each inspection circuit should have its own audio-visual centre. Regardless of the size of the inspection circuit, there must be a central place where teachers can get assistance, materials and equipments.

This audio-visual centre must be the place where instructional materials are housed and from where these audio-visual aids can be distributed to the schools. These centres should have interesting displays of new audio-visual aids, and there should be easy access to source indexes and there should be a preview or prelistening room. The audio-visual centre should also have facilities for teachers to produce their own inexpensive teaching aids such as charts, graphs and handmade lantern slides. At each centre there should be an audio-visual aids director and other staff members who should be trained in curriculum problems at the different levels and they should be available to assist teachers in the selection of materials and methods of using different aids. The audio-visual centre should be a service centre and should provide help to teachers so that they may teach more effectively (Richmond, 1969, p.172).

According to De Kieffer and Cochran (1962, p.233) the major functions of the audio-visual centre are the following:

i) To assist teachers with the selection and evaluation of audio-visual aids.

ii) To catalogue all audio-visual aids and equipments and issue lists of available materials.

iii) To purchase, produce, inspect, repair and maintain audio-visual aids and equipments.

iv) To distribute audio-visual aids and equipments by a delivery van.

v) To conduct in-service programmes on the improved use of audio-visual aids.

5.4.3 Mathematics teachers and their qualifications

Most of our primary school teachers are underqualified in

Mathematics. An attempt should be made to improve their qualifications so that most of them must at least pass Standard 10.

Another possibility is to introduce subject teaching in the primary schools as in middle and high schools. It is hardly possible that a teacher will be able to teach all the subjects offered in the primary school equally well. By introducing subject teaching it can be assured that all teachers will be well qualified in the subjects they have to teach. The limiting of the number of subjects a teacher has to offer may enable him to improve his subject knowledge as well as the methods of teaching those specific subjects. Through subject teaching teachers who are suitably qualified to teach Mathematics and who also love teaching Mathematics, could be made responsible for teaching this subject. This may help to raise the standard of Mathematics in our primary schools. Consequently subject teaching in Mathematics at the primary school level is strongly recommended.

5.4.3.1 Initial training of teachers

Colleges of education in preparing future primary school teachers should also offer less subjects to their students. In other words, specialization must be introduced during the initial training of primary school teachers. This will give students a chance to specialize in subjects which they like and in which they have performed well at Standard 10 level. By reducing the number of subjects offered at colleges of education in the Primary Teachers' Diploma, an opportunity will be created for more intensive study of the subject content as well as extensive teaching experience. Primary school teachers may then be better equiped for the offering of Mathematics at primary school level.

Lecturers of Mathematics at our colleges of education should acquaint their students with games in Mathematics, mathematics learning centres and mathematics laboratories which may make their teaching more interesting and effective. Each teacher should incorporate this approach into his teaching. This may lead to self-activity and better understanding of the subject content.

Mathematical games are specifically designed to teach certain topics or reinforce certain concepts. It is possible to modify the rules of the game to achieve this aim (Fielker, 1971, p.11).

Mathematics learning centres can be used for reinforcement of a skill or concept and to provide additional practice of a skill. Mathematics learning centres can also enrich, drill, enlighten, relieve boredom and reduce disciplinary problems. These laboratories may also be useful in assisting the teacher when working with large groups, small groups or individuals. A learning centre is both a teaching tool and a teaching technique (Manning, 1980, p.703).

A mathematics learning centre can be located in any corner or area in a classroom. For example, on top of a table, under a table, unused wall space, bulletin-boards, cardboard boxes, shelves or in a file cabinet (Manning, 1980, p.703).

Pupils in the mathematics laboratory may spend a lot of time with paper-and-pencil kind Mathematics' skill building. Diagnostic tests followed by specific skill development and retests are an integral part of mathematics laboratory activities. The laboratory also contains ongoing activities in an attempt at creating a mathematical environment and an awareness of Mathematics in the real world (Schussheim, 1980, p.637).

In mathematics laboratory lessons, the emphasis is on independent discovery. The pupil explore situations and look for patterns, for applications, for relationships, and for important ideas. As they do all these activities, they record the results and write conclusions. In the process pupils may learn to compute better and they should find illustrations of mathematical ideas. For many pupils it will be a new experience that gives a new outlook on school life and may give them new confidence in their ability to learn (Johnson and Rising, 1967, pp.469-470).

As has been indicated in the beginning of this paragraph, the mastery of the fore-mentioned approaches will help to make the teaching of Mathematics more interesting and enjoyable; and may help to increase the love for Mathematics among the pupils. Consequently pupils' performance in Mathematics may improve considerably.

5.4.3.2 In-service training of teachers

Underqualified serving teachers can be helped by means of crush courses and subject advisors. Teachers should meet at regular intervals to attend Mathematics crush courses. At these courses the stress should fall on content and method. At each meeting certain portions of the syllabus as well as the relevant methods should be treated.

Teachers of different standards should be taken separately. In this way the work of a particular standard will be treated thoroughly. If teachers are not grouped according to the different standards, then tuition will be of a general nature. Teachers of different standards may have different problems which will need to be solved separately. Crush courses are strongly recommended to equip serving teachers with the content and methods applied in Mathematics in different standards.

It is also recommended that each inspection circuit should have a subject adviser of Mathematics apart from the inspectors. These subject advisers should always be among the teachers of their specific circuits. The subject adviser should work independently of the inspectors. He does no inspection. His main duty should be to help the teachers in the classrooms and to organize courses. If necessary, he should stay for some days at a particular school. He should advice, give tuition and demonstrate in the classroom how to teach specific topics. He should not send formal inspection reports to the Department of Education as this will result in the teachers not being free and open to him. The subject adviser should work closely with the principals of schools. The principal must also be free and willing to contact the subject adviser regularly (Wentzel, 1982, pp.68-69).

Teachers must also attend courses organized by the In-Service Training Centre. At these courses they should be encouraged to improve their qualifications. Special attention must be given to teachers who have not yet passed

Standard 10. They should be encouraged to improve their subject knowledge and methods of the subjects they are offering at their schools. Subject knowledge may even be more important in the primary schools than in the middle and high schools because these teachers have to lay the correct mathematical foundation.

To impart the basics of Mathematics to the pupils successfully, the teacher must be well-grounded in them. If not, then he is bound to follow slavishly the textbook. A teacher who does not understand the basics of Mathematics will fail to adapt the explanations offered in the textbook to the needs of his class.

A recommendation with regard to the lecturers of Mathematics at the colleges of education is that they should also attend crush courses. At such a course the whole field of Mathematics teaching should be covered. The duration of such a course may be a month. Aspects which should receive attention at such courses are, the aim of teaching Mathematics, the content of the syllabi and methods. Each aspect should be treated thoroughly. The integrated approach should be followed. The method and the content should not be treated separately. Due to the fact that pupils use different methods in learning, teachers should be taught different methods of teaching each aspect. These different methods may help the pupils to understand the work better.

5.4.4 Mathematics textbooks

It has been pointed out that innovation in respect of Mathematics textbooks composition should take into consideration the black child's numeration system. In his counting the black child makes use of the expanded notation (Van den Berg, 1978, p.10). This implies that he has a better understanding of the place value but lacks the understanding of the concept of cardinal number. The exercises in the Mathematics textbooks of primary schools must be designed in such a way that they will help to develop the black child's understanding of the concept of cardinal number.

It is also important that textbook writers should take the language problems into consideration when compiling textbooks for the primary school black pupils. Particular attention must be paid to the choice of words, sentence structure as well as the choice and grading of examples. Language specialists should be consulted in order to make sure that the language used in the textbook is suitable for a particular class. Writers of textbooks should also pay special attention to the use as well as the explanation of mathematical terms. In this way the effect of a foreign medium of instruction on the performance of the primary school pupils can be minimized (Wilkinson, 1981, p.137).

5.4.5 School furniture

An attempt should be made to provide the necessary school furniture. In many of the classrooms there is a shortage of relevant school furniture. This makes it difficult to create a learning situation. In many classrooms we only find a table, a chair and desks. Classrooms should have space for demonstrations and games, cupboards and filing cabinets for keeping teaching aids. Most classrooms do not have enough space for the use of audio-visual aids and for demonstrations. If one keeps in mind that mathematical games, a mathematics learning centre and a mathematics laboratory must be introduced, then it becomes clear that more furniture and space is needed.

5.4.6 Medium of instruction

In Bophuthatswana the medium of instruction is English from the beginning of Standard 3 (Lekhela, 1978, p.40). Special attention should be given to the teaching of English as from Grade 1. It is strongly recommended that half of the periods for the day should be devoted to the teaching of English from Grade 1 to Standard 2. This will help the pupils to have a reasonable command of

English when they reach Standard 3. Language and thought are interdependent and an inadequate knowledge of the language hampers the acquisition of concepts (Wentzel, 1982, p.71).

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