A DISCURSIVE ANALYSIS OF THE USE OF MATHEMATICAL VOCABULARY IN A GRADE 9 MATHEMATICS CLASSROOM

by

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DEDICATION

This dissertation is dedicated to my MOTHER, Martha Redah Sihlangu and FATHER, Stefaans Maphoyisa Sihlangu for always encouraging me to study further. You always made sure that I prioritise attending school, otherwise I would not be here today.

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ABSTRACT

A classroom in which learners are afforded opportunities to engage in meaningful mathematical discourse (Sfard, 2008) is desirable for the effective teaching and learning of mathematics. However, engagement in mathematical discourse requires learners to use appropriate mathematical vocabulary to think, learn, communicate and master mathematics (Monroe & Orme, 2002). Hence, I have undertaken this study to explore how mathematical vocabulary is used during mathematical classroom discourse using the lens of the commognitive framework. I chose a qualitative approach as an umbrella for the methodology with ethnography as the research design whereby participant observation, structured interviews and documents were used to collect data. One Grade 9 mathematics classroom with 25 learners and their mathematics teacher were purposefully selected as participants in the study.

During data analysis, I looked at Sfard's (2008) constructs of the commognitive theory to analyse the data and identify the mathematics vocabulary used in the discourse. This was followed by the use of realisation trees that I constructed for the teacher and learners' discourse, which I used to identify learners thinking as either being explorative or ritualistic. Results indicate that both the teacher and learners use mathematical vocabulary objectively with positive whole numbers to produce endorsed narrative regulated by explorative routines. However, with algebraic terms both positive and negative, the teacher and learners' discourse is mostly disobjectified, and produces narratives that are not endorsed and are regulated by ritualistic routines. It also became evident that the mathematical vocabulary that the teacher and learners use in the classroom discourse includes words that are mathematical in nature and colloquial words that learners use for mathematical meaning.

Furthermore, learners' responses to the given mathematics questions which they are solving are mostly correct, hence it can be argued that the learners' narratives were endorsed. However, their realisation trees indicates that learners were not working with mathematical objects in their own right (Sfard, 2008) and hence their narratives were not endorsed. I have recommended in this study, that teachers need to be cautious when operating with entities and not separate operations from their mathematical terms. Furthermore, the department of basic education, during workshops should encourage educators to always request reasons from learners substantiating their answers to questions in order to enhance their explorative thinking.

DECLARATION

I SIPHIWE PAT SIHLANGU declare that the research report presented here is my own work and, where I have borrowed ideas from specific individuals, it is indicated in the acknowledgements, texts and references. I am submitting this dissertation in fulfilment of the requirement for the degree Master of Education at the University of Limpopo. I also declare that I have not submitted this dissertation before at any institution for any degree or examination.

30 September 2022

SP Sihlangu (Mr.) Date

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CHAPTER 1: INTRODUCTION

This chapter captures an outline I give as an orientation to this study for the reader by firstly narrating my encounter in a mathematics classroom that prompted me to carry out this study. I do this by providing a reflective story about the background that led to the discovery of the problem and locating the problem in literature. I secondly outline the purpose of the study and research questions that guided the study. Lastly, I give an overview of the research by providing a short description of how I have structured this dissertation.

1.1 AN ENCOUNTER FOR LANGUAGE USE IN THE MATHEMATICS CLASSROOM

In 2017, I was co-opted as one of the teachers to serve in the school management team (SMT) and appointed as an acting departmental head (DH) for mathematics and sciences. The mathematics pass rate in Grade 12 had been below 30% since my arrival at the school in 2014 and I always asked mathematics teachers about how they are teaching mathematics to learners in the classroom. Fortunate enough, as a DH one of my responsibilities is to conduct classroom visits for the purpose of monitoring and support. Immediately, I saw an opportunity for me to visit mathematics classrooms and observe how the subject is being taught. One of the reasons teachers were giving for poor performance in mathematics was that learners do not have basic mathematics knowledge from grades 8 and 9 but, most importantly, skills from Grade 9 mathematics. It then became my priority to begin my observation in Grade 9 mathematics classes as this grade was viewed as an exit point in the General Education and Training (GET) phase, which provides the foundation for Further Education and Training (FET) phase mathematics skills.

At the school, we had three Grade 9 classes, of which two were taught by a teacher who I will name X in this section. Teacher X has a Bachelor of Education degree and majored in mathematics and physical sciences. Teacher X has been employed since 2011 and had 8 years of teaching experience when I first envisaged conducting this study. She also had 5 years' experience of teaching Grade 9 mathematics and her mathematics classroom happened to be the first class that I observed during classroom visits. During my classroom visit, the teacher was teaching about the division of a binomial by a monomial. As they were busy with their lesson, one question required learners to divide two x-squared ($2x^2$) by x, a monomial by a monomial ($\frac{2x^2}{x}$); and the learners responded by saying that "x will cancel with the squared and they will be left with two x" ($\frac{2\mathrm{x}^{\cancel{2}}}{\cancel{\mathrm{x}}} = 2\mathrm{x}$). The teacher also demonstrated the 'cancelation' on the board and the answer was correct that way. It became interesting to me and I longed to be part of the lesson in order to understand how x cancels with a squared ($\square^2$) and still gives us the correct answer, $2x$, for the given fraction.

From that observation, I needed a mathematical explanation and I quickly asked a learner I was sitting close to for an explanation on how x will cancel with the squared to give two x. The learner, who I will call 'P' immediately responded 'the teacher told us that when we are dividing like this (*pointing at the expression*) the x will cancel with the other x on top since on top (*referring to the numerator*) we have two x's and below (*referring to the denominator*) we have one x; so, one x will cancel with one of the two x's on top so that we remain with one x on top and we will be left with two x'. P's use of words when talking about mathematical procedures prompted me to question further in order to understand his mathematical understanding in terms of the word 'cancel' as used in mathematics and how he uses 'this' and 'on top' when referring to a fraction and numerator respectively. Luckily, a learner sitting next to P, who I will call 'Q', became interested in the discussion and said 'no sir, the x in the denominator will divide the x–squared and we will be left with only two x'. Again, here I was not convinced that Q understands what she is talking about mathematically; the mathematical understanding of Q's explanation when saying 'x

divides x–squared'. However, P became intimidated by Q's response and substantiated his answer by saying 'Ah Sir, cancel each other and divide each other mean the same thing so it doesn't matter which one you say and it is way easier for me say they cancel each other, it is way much easy and understandable than saying they divide each other'. At this stage, my thinking was how is 'cancel each other' more understandable than 'divide each other' in mathematics but I reserved my question as the period was over. My interaction with P and Q motivated me to explore more about the language that learners use when communicating mathematics in the classroom, as well as how they use the language to create meaning in mathematics. I wanted to understand how P thinks about cancelling x and squared from a mathematical point of view, what meaning did he attach to the word 'cancel'. As a mathematics teacher myself I always advocate for the use of relevant words that have a mathematical meaning in whatever we do in mathematics.

Immediately after the lesson I sat with the teacher to give reflections on my observation and I asked the same question, namely how did the x and squared cancel each other? to which the teacher replied that 'we are used to say they cancel each other every time we are dividing a variable that will lead to a denominator of 1 and learners are used to say that as the short cut of the procedure, and that is how I was also taught and what matters is learners to get correct answers'. From this reflection, I wanted to change the perception that mathematical language is not that important when dealing with mathematical procedures but what matters is the learners being able to obtain correct answers. The only way to do this was to explore how learners use mathematics vocabulary during teaching and learning to communicate their meaning in the mathematics classroom. Conveniently, I chose to undertake the research using Grade 9 learners and their teacher because I discovered this in their classroom. Similarly, I had to use my school as the research site because, being the DH, I had access to the learners and their teacher, especially during class visits.

1.2 BACKGROUND

Discourse in linguistics is defined as a unit of language that is longer that a sentence characterised by the way in which a conversation flows (Saidovna, 2021). Similarly, discourse can also be referred to as the use of language (spoken or written) that is conceived as a social practice (Fairclough, 2013). Within the context of teaching and learning, discourse is referred to as the way in which language is used socially (Fairclough, 2013) to get a clear understanding. According to Dipper et al., (2021) discourse occurs when speakers use language to communicate information and are able to convey links between such pieces of information. This is also supported by Sfard (2008) who argues that communication can happen between two people or one person communicating with themselves verbally or non-verbally.

Classroom discourse, on the other hand, is defined as all the different forms of discourse; linguistic or non-linguistic, that take place in the classroom (Sfard, 2008; Philips, 2021). Linguistic elements include the use of language and interactions between the teacher and learners, while non-linguistic elements include paralinguistic gestures, prosody and silence (Sfard, 2008, 2009; Otheguy et al., 2015; Morgan & Sellner, 2017). If the discourse occurs when engaging on a mathematical task in the classroom, then it is regarded as 'mathematics classroom discourse' (Chapin et al., 2009).

Mathematics classroom discourse refers to learners' whole-class or small-group discussion whereby they discuss mathematics in a way that their understanding of concepts is revealed (Chapin et al., 2009; Shilo & Kramarski, 2019). Mathematics classroom discourse involves the use of mathematics language by mathematics experts and learners who are doing mathematics in the classroom communicating about mathematics (Moschkovich, 2003, 2013). This communication, which can be textual or verbal, includes, among others, proofs, definitions, problem-solving approaches and attitudes as well as behaviours that have to do with mathematics in the classroom (Morgan & Sellner, 2017). Mathematical discourse has three components namely; the mathematical register, the symbolic language and the mathematicians' informal jargon (Mohamed et al., 2020).

The mathematical register is where mathematical reasoning and facts are communicated, while the symbolic language of mathematics refers to expressions that are symbolic in nature and statements that are used in calculations and when presenting results (Mohamed et al., 2020). Also, mathematicians' informal jargon refers to expressions such as 'conceptual proof' (Mohamed et al., 2020). Discursive classrooms engage learners in mathematical discourse that enables learners to provoke each other's understanding about mathematical concepts and also justify their arguments in mathematics (Legesse et al., 2020). However, engagement in such discourse requires the use of appropriate mathematical vocabulary.

Mathematical vocabulary refers to the textual and verbal words that outline mathematical concepts and procedures that learners use when engaging in mathematical activity and discourse (National Council for Teachers of Mathematics (NCTM), 2000; Riccomini et al., 2015). Adoniou and Qing (2014) asserts that mathematical language has a crucial role to play in learners' mathematical understanding and learning. However, teachers seem not to be willing to engage in the recent trends of effectively teaching mathematics by improving their use of mathematics vocabulary in the classroom (Maluleke, 2019). This is also supported by Song (2022) who laments that the majority of learners are never given a chance to learn mathematical vocabulary in class. This lamentation was supported by Ogle et al., (2015), who argues that learners' inadequate mathematical vocabulary results from it not being explicitly taught during instruction practices.

Learners' inadequate mathematical vocabulary is mostly reported after learners have engaged in written assessments. For example, the South African Mathematics Annual National Assessments (ANA) reported that learners in Grade 9 performed poorly because of a lack of mathematical vocabulary (DBE ANA Report, 2014). Similarly, Sibanda (2017) also reported that learners experienced linguistic difficulties when solving problems in the 2013 ANA mathematics paper. The difficulty was attributed to the many questions asked on ANA mathematics paper using unfamiliar language and, as a result, learners could not get the answers correct.

Very little is known about learners' use of mathematical vocabulary in the classroom, in particular, the kind of vocabulary they use and how they use it when engaging in mathematical classroom discourse. As a result, using Sfard's (2008) commognitive theory of discursive analysis, the study reported on here was set out to explore how teachers and learners use mathematical vocabulary as they engage in mathematical discourse during teaching and learning. This was the case because I wanted to bring to the fore knowledge on how mathematical vocabulary is used in the classroom, as well as how the vocabulary used influences learners' mathematical thinking and communication. The study made use of discourse analysis as it shows the significant role that the use of language plays in metacognition and cognition for mathematics learning (Alzahrani, 2017). Amin et al., (2015) defines metacognition as a second level cognition, thinking about thinking which is the ability of one to self-reflect about an ongoing cognitive process. On the other hand, Alzahrani (2017) defines cognition as the thinking process of one's knowledge. The significant role that language plays in both cognition and metacognition can be explored through an analysis on how different words are used differently by learners, which impacts on meta-discursive rules (Kim et al., 2017; Sfard, 2008). This analysis afforded me an opportunity to consider a broader picture of thinking and learning in the mathematics classroom. Furthermore, the participation of learners in discourse offers opportunities that connect theory and practice in an authentic manner (Kim et al., 2017; Sfard, 2008).

1.3 PROBLEM STATEMENT

A classroom in which learners are afforded opportunities to engage in meaningful mathematical discourse (Sfard, 2008) is desirable in order for the teaching and learning of mathematics to be effective. These classrooms are dominated by discourses that are mathematical in nature and help learners to express their understanding, as well as how their mathematical tasks are interpreted (Siyepu & Ralarala, 2014). Furthermore, engaging learners in a mathematical discourse enables them to challenge one another about their mathematics understanding and to provide justification in their mathematical arguments (Walshaw & Anthony, 2008). Engagement in mathematical discourse requires learners to use appropriate mathematical vocabulary, which is the language of mathematics that helps learners to think, learn, communicate and master mathematics (Monroe & Orme, 2002). Mathematical vocabulary is one of the important aspects of developing learners' proficiency in mathematics (Riccomini et al., 2015). Therefore, to support learners' growth in mathematical knowledge, attention should be placed on how mathematical vocabulary is used to give learners the necessary skills that they need in order to think, talk and learn about mathematical concepts (Chard, 2003).

Studies in mathematics education research reported that learners' inadequate mathematical vocabulary results from it not being taught explicitly during instruction practices (Amen, 2006; Fletcher & Santoli, 2003; Rubenstein & Thompson, 2002; Maluleke, 2019). An inadequate mathematical vocabulary leads to many learners having difficulties in reading, writing and understanding mathematical content (Harmon et al, 2005). As a result, learners are unable to recognise and recall mathematical terminologies, which leads to failure to attempt and solve mathematical problems and tasks (Larson, 2007). Similarly, in South Africa it was also reported by the ANA that learners in Grade 9 were unfamiliar with mathematical terminology and properties and they often use them incorrectly (ANA Report, 2014).

There is a paucity of research focusing on how mathematical vocabulary is used during teaching and learning by both teachers and learners as they engage in classroom discourse. As a result, there is a gap in the creation of knowledge on how mathematical vocabulary is used during teaching and learning as the teacher and learners engage in mathematical discourse. Therefore, using Sfard's (2008) commognition theory of discursive analysis, the study reported on here was set out to explore how teachers and learners use mathematical vocabulary as they engage in mathematical discourse during teaching and learning. The use of commognitive framework to discursively analyse learners and teacher's mathematical discourse can bring to the fore knowledge on how mathematical vocabulary is used, and how it influences learners' mathematical thinking and communication in the classroom.

1.4 PURPOSE OF THE STUDY

The purpose of the study was to explore how mathematical vocabulary is used during mathematical classroom discourse using the lens of commognitive theory.

1.5 RESEARCH QUESTIONS

The guiding research question was:

How do teachers and learners use mathematics vocabulary during mathematics classroom discourse?

The sub-research questions were:

- What mathematics vocabulary do teachers and learners use during mathematics classroom discourse?
- In what way(s) does the used mathematical vocabulary influence learners' mathematical thinking and communication?

1.6 OVERVIEW OF THE DISSERTATION

This dissertation has been divided into **eight chapters** and in each chapter the reader is provided with an introduction to the chapter followed by the main contents of the chapter, while each chapter is concluded with a closing summary.

In **Chapter 1**, I introduced to the reader background that led to the identification of the research problem. This was followed by locating the research problem in literature, as well as by motivating for the significance of the study. Lastly, I highlight the purpose of the study and research question that guided the research, as well as providing an overview of the dissertation.

In **Chapter 2**, I look at philosophical world views in order to ensure that the reader understands the choice of the theoretical perspective that guided this study. Additionally, I further elaborate on how the theoretical perspective I chose fitted well with this study and how the framework helped me as a researcher to provide solutions to the research questions.

In **Chapter 3**, I elaborate on what classroom discourse is and how it gave rise to mathematics classroom discourse, with reference to my study. This is followed by an analysis of other studies on discourse analysis of mathematical classroom discourse. Here I argue about the limitations and significance of discourse analysis in my study.

In **Chapter 4**, I locate mathematical vocabulary in the study and how it can be identified in a mathematics classroom discourse. I further provide an analysis of other studies on mathematical vocabulary, thereby indicating their limitations and pointing out the significance of how I explored mathematical vocabulary in my study.

Chapter 5 is the centre of the research. Here I explain to the reader the process of the research methodology, the research approach and the design used in this research with motivation for their selection. I also argue and defend the sampling procedure used to engage participants and the data gathering techniques. In this chapter, I also deal with ethical considerations, quality criteria and how data was collected and analysed.

In **chapters 6 and 7**, I present an analysis of data collected using Sfard's (2008) discourse constructs as subheadings for each excerpt analysed. During the analysis I also discuss interpretation of the analysis and provide a summary of the analysis and interpretation. Typically, data analysis is presented in one chapter, however in this study I chose to present data analysis using two chapters for the purpose of coherently presenting the analysis in terms of the topics which activities focused on during data collection. In Chapter 6 I focus only on the activities that were on algebraic expressions and equations, while in Chapter 7, I focus on activities that were based on geometry of straight lines.

I lastly present **Chapter 8**, in which I deal with the conclusions based on the data analysis and interpretation, as well as study limitations and recommendations for future research.

1.7 SUMMARY

In this chapter, I start by defining what discourse is and how it has come to be called classroom discourse. Additionally, I clarified to the reader, that the research will only focus on classroom discourse that is mathematical in nature. I then illustrated how the problem was identified during classroom mathematics discourse before I discussed the significance of carrying out this research to try and understand the problem. This chapter also looked at the purpose of the study as well as the research questions that guided this study. I closed this chapter by providing the reader with an overview of what to expect in the dissertation.

CHAPTER 2: THEORETICAL PERSPECTIVES AND FRAMEWORK

2.1 INTRODUCTION

I begin the chapter by giving a distinction between philosophical world views that guided this research. In particular, I differentiate between positivist and constructivist philosophical world views in order to substantiate reasons for choosing the epistemology and ontology that this study followed. Furthermore, I argue why I chose discourse as the central phenomenon of the study by linking it to the philosophical world view I used in the study. In this way, I defend the choice of the theoretical framework chosen as the lens through which I viewed the study using the epistemology and ontology. I conclude this chapter with a summary detailing critical concepts that builds this research.

2.2 PHILOSOPHICAL WORLD VIEWS

There are a number of prominent world views or beliefs that a researcher can bring to a study to serve as a stepping stone in guiding the study throughout the research process. In this chapter, I highlight four widely discussed world views in the literature, namely post-positivism (Smith, 1983; Phillips & Burbules, 2000), constructivism (Berger & Luekmann, 1967; Lincoln & Guba, 1985), transformative (Fay, 1987; Heron & Reason, 1997; Kemmis & Wilkinson, 1998; Mertens, 2010) and pragmatism (Patton, 1990; Tashakkori & Teddlie, 2010). Furthermore, I also defend my decision for using constructivism as world view that guided this study. Correspondingly, I use the epistemology and ontology to defend the framework I embraced in this study, which serves as a lens through which I view the study.

The assumptions about post-positivism represents the traditional form of research that is in line with procedures for quantitative research and are not suitable for qualitative research (Creswell, 2014). Furthermore, this world view reflects on the need for researchers to make an assessment and further identify the causes that can influence outcomes. Similarly, their intention is to reduce ideas into a discreet and small set to test and develop numeric measures of observations (Smith, 1983; Phillips & Burbules, 2000).

Phillips and Burbules (2000), outlined key assumptions about post-positivists, which addresses what reality is, what knowledge is and, finally, how knowledge is constructed. In terms of the stance on reality they contemplated that one needs to be impartial when judging a claim in order for such judgement to be sufficiently reliable and competent for others to believe it. Regarding knowledge, these authors stated that it is the truth that one looks for through research claims, refining the claim until it makes sense and, if not, then it can be overlooked for another one making sense. Finally, in terms of how knowledge is constructed, they argued that knowledge can be shaped by collecting evidence of facts through research in order to develop relevant statements to back up your claim.

Based on the assumptions above, it is quite evident that post-positivism follows the route of quantitative research because researchers using this world view do not need to prove a hypothesis but rejecting it is deemed as a failure. The aim of the world view is to test theory by collecting information using instruments completed by participants or based on researcher's recorded observations. Similarly, researchers who follow this epistemology advance the relationship between variables in terms of hypothesis or questions, maintaining the standard of validity and reliability (Phillips & Burbules, 2000).

Most researchers view constructivist world view as an approach that follows in the footsteps of qualitative research (Berger & Luekmann, 1967; Lincoln & Guba, 1985). They believe that one seeks an understanding of the world in which they work and live in order to develop meanings that are subjective about their experiences, which can be directed towards particular objects or things (Crotty, 1998; Mertens, 2010; Lincoln et al., 2011). Furthermore, researchers look for the convolution of participant's views rather than to narrow their meanings into a few ideas. Research questions that follow this world view are broad and generalised in order for participants to construct meanings about a situation, in most cases conceived in discussions or interactions with others (Creswell, 2014).

Key assumptions are identified by Crotty (1998) with reference to the constructivist world view. He argues that one engages with the world around one and interprets it in order to construct meaning; we do this in order for the world to make sense with the culture infused in us. Lastly, for meaning to be generated one must interact with others around them.

I can confidently argue that constructivism has to do with qualitative research in a sense that it deals with individuals interpreting the world they live in through engagement. For the researcher to understand one's culture or context they live in, research should be done by physically being present in the context and the researcher should gather information personally. Interpretation of collected information in this case will be based on the inquirer's personal experience as well as their background.

Researchers who hold the transformative approach to be valid feel represented by the post-positivism and constructivism with an assumption that a research query must be intertwined with a political agenda and politics with the purpose of confronting social oppression at the levels at which it occurs (Mertens, 2010). According to Creswell (2014), research that follows this world view has an action agenda with the purpose of bringing about reform that may change the researcher's life, participant's lives or their working institutions. It is further argued that the research gives participants a voice to raise their consciousness or advance their agenda for change in order for their lives to improve (Fay, 1987; Heron & Reason, 1997; Kemmis & Wilkinson, 1998).

Mertens (2010) gives a generalisation of the key features of the transformative world view, which addresses the study of lives, inequalities and the theory of beliefs. In the study of lives, the focus is on the experiences faced by traditional, marginalised and diverse groups, looking at the link between politics and diversity that result. Theory of beliefs talks about the existence of programmes that work, oppression and domination of power relationships.

The last philosophical view is pragmatism, which emerges out of situations, actions and consequences (Patton, 1990). A researcher in this world view places greater emphasis on the research problem and makes use of all available approaches to ensure that the problem is understood (Rossman & Wilson, 1985). The world view advocates for a mixed methods research approach and, most importantly, attention should be focused on research problems that arise in the social sciences and derive knowledge about the problem using a pluralistic approach (Tashakkori & Teddlie, 2010)

In order to provide his view about the philosophical point of departure for pragmatism, Creswell (2014) looked at how pragmatists view commitment, how pragmatism's research is based as well as how it is conducted. In respect of commitment, there is a freedom of choice as there are no commitments to philosophical systems and reality. They also do not see the world as a complete unity; the truth is always what works for us at that moment. Additionally, their research is based at a certain context and can have intended consequences based on the what and how of the research. Lastly, their belief is based on their outside world that came in their mind or outside and, hence, they often employ mixed methods and a variety of data collection techniques (Cherryholmes, 1992).

In contrast, a post-positivist researcher begins with a theory and provides a hypothesis with the intention to test the theory. This researcher then collects data that will be analysed to check whether the theory is supported or not, which will then require new amendments in preparation for retest, when necessary, and, if not necessary, reach a conclusion (Creswell, 2014). In comparison with post-positivism, a constructivist researcher begins with an intention to understand, make interpretations for meaning and check whether they make sense about the world in order to generate or develop a theory or pattern about such meaning. This is achieved by asking broader and more general questions that will require participants to construct meaning of a situation through engagement or interactions with others, with the researcher's role being to provide questions that are more open-ended and carefully give an ear to the participants about what they do in their lives (Lincoln et al., 2011).

In transformativism, the researcher begins with specific issues that need to be addressed, which focus on social issues that are important (e.g. oppression, donation, empowerment etc) and identifies one of them as the study's focal point (Mertens, 2010). This is followed by a collaborative procedure that would not marginalise participants taking part in the research. Furthermore, the participants will assist with designing the questions, data collection, providing information for analysis, thereby reaping the research rewards. This kind of reform can transform participant's lives as well as the institutions in which they work or live. (Creswell, 2014). Lastly, pragmatist researchers start by looking at the required processes needed for them to carry out the research on the bases of the intended consequence so that they can establish a purpose for the study as well its goal, together with the mixed methods (Creswell, 2014).

The constructivism orientation underpins this study as it focuses on participants in their real-life settings and how they construct their meaning in their social contexts. In support of this, my study was conducted in the natural setting of a Grade 9 mathematics classroom. Similarly, constructivism focuses on how participants construct meaning by an interaction or engagement with others to seek an understanding of their environment. Hence, my study focuses on how learners and teachers use mathematics vocabulary in classroom discourse. It is quite evident that my study follows the path of a constructivism world view.

Furthermore, I describe discourse (which constructivists call engagements or interactions) in this study from three perspectives, namely the linguistic, non-linguistic and interdisciplinary perspectives. I further substantiate reasons for choosing one perspective as the point of departure. From the linguistic perspective, discourse is viewed as a unit of language that is longer than a single sentence. It can also be thought of as the manner in which a conversation flow (Lopez, 1999). From a non-linguistic perspective, discourse is referred to an understanding that contains basic elements that are called statements; they are also functional semiotics, rather than structural unities such as prepositions, acts of speech and utterances (Lopez, 1999). The statements depend upon ‘the conditions in which they emerge and exist within a field of discourse; the meaning of a statement is reliant on the succession of statements that precede and follow it’, as outlined by Gutting (1994). Lastly, interdisciplinary perspective views discourse as a concept that is three-dimensional and contains practice of discourse, text and sociocultural practice that results in a conversation that can either be institutional or casual. Furthermore, when talk is initiated, discourse is used so that there can be an exchange of information as the intended purpose of having a conversation is to exchange information for communication, which is the ultimate purpose of conversation (Schegloff et al., 1977). In this study, discourse was looked at in terms of linguistic and non-linguistic perspective, that is, the study focuses on discourse that occurs in a mathematics classroom that is about mathematics and nothing more. The mathematics discourse in this case can be in a form of written or spoken language such as speech or gestures.

2.3 THEORETICAL FRAMEWORK

My study draws on constructivism orientation which requires a framework that focuses on discourse analyses as a framework. I, therefore, selected Sfard's (2008) commognitive framework as a lens of conducting this research as well as a method of discourse analysis. In this framework, Sfard (2008) defines thinking as one's interpersonal communication that occurs verbally or non-verbally. She further argues that thinking happens in the learner's mind and can be expressed through communication, which can be referred to as 'commognition'.

Learning in a commognitive approach is witnessed when one becomes an active participant in a discursive community. Communication is perceived as an important expression of one's thinking, known as discourse, which occurs when a person is communicating with one's self or with others, sharing ideas. Sfard (2008) explains that there are different forms of communication that can usher people together or exclude others and, if that is the case, they are called discourses. It is only when these discourses feature mathematical words that they can count as mathematics discourse (Kim et al., 2017).

In developing this theory, Sfard (2008) claims that a study that is guided by an interpretive framework needs to cope with some issues to show that the framework has achieved its goal. The theory talks about focusing on the object of learning, the process and the outcome; this means that there must be a change that must occur to show that learning has taken place. Also, both the teacher and the learners need to be working towards that change and, finally, an evaluation must be conducted to ascertain whether the expectations have been met (Sfard, 2008).

Learners' difficulties in learning mathematics concept is of great concern in education and this has resulted in the emerging shift from acquisitionist to participationist metaphor. Kim et al., (2017) defines acquisition metaphor as being useful to provide a description about the aspect of teaching and learning where learners are assumed to be acquiring knowledge from the teacher. They further define participation metaphor as one that focuses shifts to the evolving relationships between an individual learner and other learners or their teacher as well as meta discursive rules (Kim et al., 2017). The emerging shift from acquisitionist to participationist metaphor was because of the ability of the participationist metaphor to offer a variety of solutions that explain how we develop as well as what it is that develops (Sfard, 2008). In a commognitive approach, there is a form of discourse that is distinctly characterised by four constructs that must be considered in a mathematics classroom discourse and those constructs comprise of words use, visual mediators, endorsed narratives and routines (Kim et al, 2017; Sfard, 2008).

2.3.1 Four Constructs of the Commognitive theory

There are four features that characterise mathematics as a discourse (Sfard, 2008), namely word use, visual mediators, routines and endorsed narratives. In order to provide descriptions of mathematical objects in a mathematical discourse, we make use of word use; in order for us to express mathematical objects we use routines; in order to signify mathematical objects, we make use of symbols; and for mathematical objects in which mathematicians as a community have come to an agreement, we use endorsed narratives to characterise them (Caspi & Sfard, 2012; Mudaly & Mpofy, 2019; Nachlieli & Tabach, 2012). Below I give a detailed explanation of these features in a more descriptive manner.

2.3.1.1 Word use

Sfard (2008) argues that word use focuses on the usage and process of mathematical words and, more specifically, inquires how learners use the synonyms and antonyms of those words to express meaning. Similarly, Ripardo (2017) adds that words allow us an opportunity to talk about the concerned object at that moment. That is, the categories of grammar for a given name that refers to an object shows the nature of the mathematical knowledge. With that being said, the noun ‘circle’ identifies knowledge and, at the same time, differentiates a circle from other objects; that is, the words ‘circle and ‘function’ cannot be used to refer to a similar object in a mathematical discourse (Ripardo, 2017). However, the words ‘multiply’ and ‘times’ refer to the same operational object in a mathematics discourse. In my study, word use refers to the vocabulary that is used by both the teacher and learners in the mathematics classroom to communicate about mathematical objects, processes or procedures. The mathematics language share words with the English language, however, the manner in which the same words are used as well as what the words mean in the two disciplines is different. For that, this research ONLY explores how words (mathematical or everyday English) are used for ‘mathematical meaning’ by both the teacher and learners during teaching and learning to influence learners’ mathematical thinking.

In a mathematical discourse, the use of words (in my case mathematical vocabulary) can be identified using four phases. That is, mathematical vocabulary can be used passively, in a routine driven way, in a phase driven way and also in an objective way (Sfard, 2008). The passive use of mathematical vocabulary refers to learners’ first encounter with a word or phrase, while routine driven talks about learners’ use of the word in the context of mathematical routines (Roberts, 2016; Sfard, 2008). In terms of phase driven usage, the learners have confidence in using the mathematical vocabulary. Mathematical vocabulary is used as part of constant phrases and in an objectified way, which means learners will now be able to use mathematical vocabulary as nouns (Roberts, 2016; Sfard, 2008).

In order to argue how mathematical vocabulary is used during mathematics classroom discourse by teachers and learners, I refer to Sfard’s (2008) and Morgan’s (2006) perspective of language in mathematics, which they used in a mathematics discourse. The use of language performs a specific function and it is not neutral in mathematics. As such, during the analyses I have identified the mathematical vocabulary used during classroom discourse. This was followed by looking at how the mathematical vocabulary is used by looking at the linguistic features and the corresponding meaning of the features (**Table 1**) in the context so that I could categorise learners' and the teacher's word use (Morgan, 2006; Sfard, 2008).

Table 1: Linguistic features and their associated meanings

Linguistic feature	Meaning in mathematical discourse
Nouns	Mathematical vs. colloquial words
Verb processes	Keywords with relational vs. material processes
Modal auxiliary verbs	Verbs with high modality (e.g. 'must') indicate obedience to authority
Subject and object	Keyword as subject or object vs. person as subject or object
Voice	Passive vs. active voice
Adverbs	Adverbs of place for spatial arrangement, adverbs of time for sequential action
Articles	Keywords prefaced with article
Pronouns	Reference pronouns for keywords

Source: Roberts, A., & Le Roux, K. (2019).

2.3.1.2 Visual mediators

Visual mediators can be described as the discursive cues such as symbolic artifacts, which includes tables, graphs, numbers, expressions and equations, that are made to communicate their relationships with and the operations they have with respect to mathematical objects (Roberts, 2016). Additionally, Sfard (2008) explains that visual mediators showcase how words used can be represented using symbolic mediators (i.e., tables, graphs and algebraic expressions) to create a medium for making meaning. Symbolic mediators are the different means that learners who are involved in discourses use to identify objects they talk about and to be able to coordinate their communication (Sfard, 2008). Discourses that are mathematical in nature are characterised by symbolic artefacts that are specifically created for a certain form of communication, which can include, among the others, mathematical symbols and rules (Sfard, 2008). Ripardo (2017) adds that visual mediators can be referred to as objects that are used for the purpose of communication in a mathematics discourse and that such objects (symbols) can help us to set up an organised mathematical discourse during interaction. In my study, I look at word use and the visual mediators used by learners and their teacher during mathematical discourse, and how they influence learners' mathematical thinking and communication.

2.3.1.3 Endorsed narratives

Word use and visual mediators are used to construct narratives, which can be any form of text that is spoken or can be written, used as 'description of objects, of relations between objects, or processes with or by objects' (Sfard, 2008). According to Roberts (2016), the word use of learners as well as their intended visual mediators during interaction can be used for construction of narratives but researchers can also construct narratives by interpreting what they see in the learners' discourse. Researchers are able to construct narratives by interpreting the manner in which learners describe and justify their procedures with reference to their use of words and their corresponding visual mediators during classroom discourse (Roberts, 2016). In mathematics discourse, Sfard (2008) refers to mathematics theories that include rules and formulae as endorsed narratives and further argues that endorsed narratives can appear in learners' responses, while not being articulated explicitly.

Narratives, according to Sfard (2008), are a description of what is done with mathematical objects, which will be called 'endorsed narratives' when indicating that such a narrative is true. Thoma (2018) supports this and argues that a narrative is called an endorsed narrative if it is derived from rules that are accepted, generally emanating from other endorsed narratives such as theorems and definitions. Furthermore, Ripardo (2017) agrees with Sfard (2008) and Thoma (2018) by adding that narratives that are endorsed are sequentially organised verbal expressions, which can also be texts that describe objects and the relationship that exists between the objects. In addition, the processes that are involved during the construction of such narratives, which the individuals who are involve in the discourse can either approve or disapprove. In this research, I looked at word use and visual mediators from the teacher's and learners' interactions during classroom discourse to construct narratives and then argue whether the word used with its corresponding visual mediator operated in an objectified (or disobjectified) way.

2.3.1.4 Routines

A routine, according to Sfard (2008), can be referred to as a procedure, if not a practice, that involves justifying, endorsing (or not endorsing) or the generalising of narratives in mathematics and, hence, is rule based. Routines are procedures that are applied when solving mathematical problems and can be easily perceived as being a result of the repeated steps that are displayed prior to reaching a solution (Sfard, 2008). Tabach and Nachlieli (2011) argue that routines are very helpful when learning a new discourse because the ability of an individual to act in a new situation depends solely on recalling the past experience of oneself or of others. According to Sfard (2008), routines can be looked at as patterns that are demonstrated by specific learners when engaging in a mathematical activity. She further argues that researchers should know when and how to identify patterns when provided with the opportunity to do so. This can be observed by determining how learners attempt to solve a mathematical problem when provided with one (Sfard, 2008).

Sfard (2008) categorises the rules that are used to define routines as follows: (1) object-level; and (2) metalevel. In terms of object level rules, she argues that they represent regularities based on the behaviour of the objects that are in a discourse. With regard to metalevel rules, she explains that they reflect the structure and frequent nature of the actions of the discussants by providing a definition based on the patterns observed in their activities (Sfard, 2008). When we look at a specific set of metalevel rules, we have one that is only concerned with the ‘how’ part of the routine, while the other will be concerned with the ‘when’ part of the routine; however, both of these sets are able to either determine or coerce the course of action (Roberts, 2016; Sfard, 2008). Sfard also suggests that, when teaching mathematics in the classroom, we mostly focus only on the ‘how’ part of the routine, which can result in learners’ mathematical development being constrained. Furthermore, we need to understand that routines are actions that recur in the discourse and those involved need to mobilise words and their intended visual mediators so that their narrative can be structured as per the needs of their discourse (Roberts, 2016).

The relationship between narratives and routines can be described as ‘noticing mathematical regularities whether when one is watching the use of mathematical words and mediators or following the process of creating and substantiating narratives about numbers or geometrical shapes’ (Sfard, 2008). Routines, in this case, are the regularities which Sfard also refers to as ‘the anatomy of mathematizing’ (Sfard, 2008).

In a mathematical discourse, the intended aim of a routine is to ensure that the narratives that are produced about mathematical objects are either categorised as being explorative, deeds or ritualistic (Roberts, 2016; Sfard, 2008, 2016). With regard to explorative routines, Sfard (2008) argues that is distinguished by talk about mathematical objects in an objectified way, which is characterised by endorsed narratives about mathematical objects. According to Roberts (2016), in a given equation it would mean that learners use words and phrases to signify algebraic terms and numbers as mathematical objects in one line of an equation. That is, the features of objectified discourse are characterised by the discourse that talks about the equivalence relationship between the left-hand and the right-hand side of an equation, as well as the discourse that talks about one equation that as a signifier that realises equivalent narrative and, hence, horizontal equivalence and vertical equivalence respectively (Roberts, 2016, Sfard, 2016).

Deeds, on the other hand, are described as routines with the primary aim of transforming the physical appearance of an object, rather than producing narratives (Sfard, 2016). Lasty, rituals are routines in which, in many discursive cases, the individuals involved in the discourse are not giving much attention to the closing of the routine or narrative construction or rather transforming the physical appearance the object (Ripardo, 2017; Sfard, 2016). Similarly, in a ritualistic routine, learners use words in a phase driven way (Sfard, 2008, 2016) in a sense that learners talk about mathematical objects as disobjectified entities. This happens when learners talk about mathematical objects by separating algebraic terms and integers into parts and, when that happens, we render them as digits, letters or operatory signs, which can be referred to as the spatial arrangement of an equation (Roberts, 2016, Sfard, 2016).

Additionally, the traits of rituals are governed by strict rules that are developed by a person who has authority in the discourse and this limits the intentions of the discourse to only justify how to do something and not when to do so or why it works (Sfard, 2008, 2016; Roberts, 2016). Similarly, rituals are regarded as an ‘acceptable interim phase’ in the process of learning because learners first imitate others and this imitation causes learners to gradually develop an understanding about the ‘how’ and the ‘when’ (Sfard, 2008, 2016). Roberts (2016) describes this imitation as being responsible in the discourse for the transition from rituals to an explorative way of thinking. According to Ripardo (2017), among the three mathematical discourse routines it is only rituals that do not count as a whole performance in the sense of being an object that is produced, while deeds and explorations place greater emphasis on the closing conditions. However, exploration routines contain closing performances that result in constructing endorsable narratives, while the closing condition is, indeed, the transformation of the physical object as a primary concern (Sfard, 2008, 2016; Roberts, 2016; Ripardo, 2017). With that being said, Ripardo (2017) argues that researchers should be aware that one cannot endorse a narrative without the consideration of the routine that has produced such a narrative. In this research I use routines that the teacher and learners showcase during classroom discourse in order to argue whether such routines produce endorsed narratives or not; that is, whether such narratives signify explorative routines or ritualistic routines. This will, in turn, help me to understand learners' mathematical thinking and communication during the classroom discourse.

While my argument for using learners and the teacher's routine to construct their narrative is supported by Ripardo (2017), however, it will not be acceptable for one to talk about the process (routine) and not mention its product (narrative) or to talk about the product without mentioning the process (Ripardo, 2017). Exploratory routines can be classified by looking at the relationship between the routine and its corresponding produced narrative as either being constructed, substantiated or recalled (Sfard, 2008, 2016). The purpose of construction in this case, is to create new endorsed narratives through direct realisation by either discursive discovery, observation or reflection. On the other hand, the purpose of substantiation, is to decide on whether to endorse previously created narratives by convincing a mathematician and lastly, recall's purpose is to call upon narratives that were endorsed in the past (Daher, 2020; Ho et al., 2019; Ngin, 2018; Sfard, 2008, 2016). This can be done immediately and if not immediately, may need to be reconstructed which may reveal a great deal about how the narratives were memorised, constructed and substantiated (Ripardo, 2017).

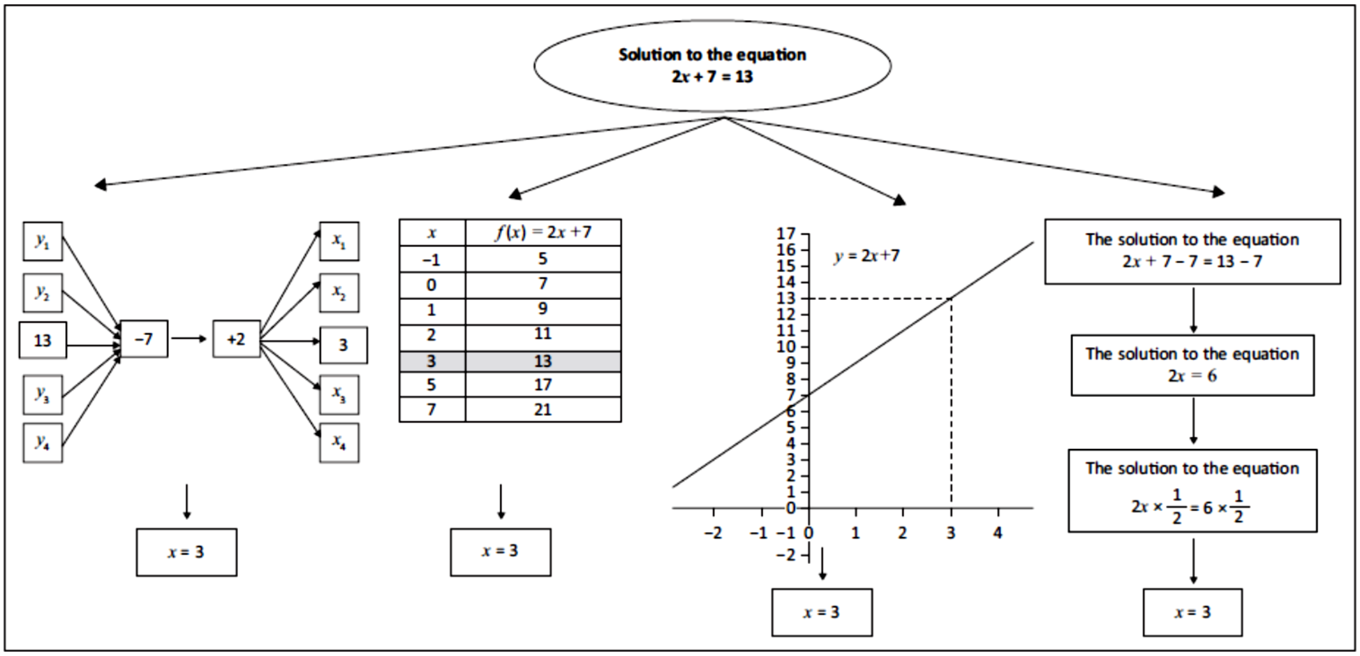
2.3.2 Realisation trees for objectification

There is a growing debate in mathematics education research about what mathematical objects are and what their roles are in a mathematics discourse (Sfard, 2008; Radford, 2008). According to Radford (2008), "mathematical objects are patterns of reflexive activity that are incrusted into the ever-changing world of social practice mediated by artefacts" (p. 222). On the other hand, Sfard (2008) talks about mathematical objects as the talk about discursive objects that are produced by a discourse and made up of various realisations. Similarly, they can also be referred to as the talk about mathematical signifiers of objectification (Roberts, 2016; Sfard, 2008). I see Sfard's explanation of what mathematical objects are as relevant to this study because she talks about mathematical objects being involved in a discourse which, in my case, the focus is on how learners and their teacher use mathematical vocabulary to describe mathematical objects in a mathematical discourse when solving mathematical problems for meaning making.

In terms of Sfard's (2008) theory, mathematics classroom discourse can be used to construct objects such as variables, numbers and functions in order to represent the picture or structure that comes out of a discourse. In order to do this, she made use of a realisation tree and demonstrated what the answer to the linear equation $2x + 7 = 13$ would look like (**Table 2**). The construction of a learner's realisation tree is deemed by Sfard (2008) to be a personal construct simply because it is a representation of that learner's discourse. The purpose of constructing this tree from learners' discourse is to obtain a visual representation of the learners' constructed discursive object. A representation of the learners' constructed discursive objects helps researchers to identify the routines of learners as either explorative or ritualised. I use the realisation tree in **Table 2** below to demonstrate how Roberts (2016) used it to determine learners' routines as explorative or ritualistic, which I will also employ in chapters 6 and 7 when analysing the teacher and learners' discourse.

The use of realisation trees to identify the learners' routines as explorative or ritualistic will help to decide whether learners' discourse is objectified and whether the learner acts with mathematical objects (Roberts, 2016). To do this, one needs to consider whether the learner's constructed endorsed (or not) narrative justifies the relationship between a realisation and its constructing signifier (Ripardo, 2017; Roberts, 2016; Sfard, 2008). Similarly, in chapters 6 and 7, I will show whether the teacher's and learners' discourse in the classroom is objectified or not and whether they constitute endorsed narratives or not.

Table 2: Realisation tree for the linear equation $2x + 7 = 13$



Source: Roberts, A. (2016).

The realisation tree above has four branches that can be used to solve one given problem, yet use different methods to arrive at the same solution. In order to explain how Sfard (2008) uses the realisation tree to identify explorative and ritualistic routine as well as narratives constructed being endorsed or not, I first explain how the realisation tree works, which will provide basis for how it is used in this study. The realisation tree has four branches that represent different methods of solving one mathematical problem, yet arriving at the same solution. A branch in this case begins with learners being given a signifier (algebraic equation) and realising its significance, which will result in the production of a written or spoken response, which is called a realisation. This realisation mediates meaning between one entity and another, which leads to the final mediation of the realisation, which is called the solution to the signifier (Sfard, 2008). For the purpose of this study, I report on one branch of the realisation tree because my study only focuses on the algebraic branch of the realisation tree to represent learners' solutions.

The algebraic branch of the realisation tree has a signifier as well as its realisation and, in the case of **Table 2** above, the signifier is the equation to be solved ($2x + 7 = 13$), while the realisation is the learners first encounter with solving the equation (application of additive inverse) and these two together are called signifier-realisation pair, which is also called a node (Sfard, 2008). Mediation for meaning between one entity and another leads to another realisation that leads to the final solution of $x = 3$, whereas the other realisation for mediating meaning is the application of multiplication inverse operation to reach the final solution (Roberts, 2016; Sfard, 2008). However, learners can have different realisations even though their signifiers are the same and Sfard (2008) uses the case of $-3x$ as a mathematical object as a result of learner's realisation and operation of the object in a mathematically endorsed way. However, one can talk about '-' and '$3x$' as separate mediators which signify different operations '$-$' and '$+$' for the '$3x$', which are not endorsable (Sfard, 2008).

2.4 SUMMARY

I started this chapter by providing the reader with a distinction between four philosophical world views that build this research. I also motivate for the choice of discourse that this study talks about, which led to the choice of the theoretical framework that guided this study. Similarly, I discussed tenets of the commognitive framework that I used as a lens through which I viewed this study and explained how each tenant will be used during data analysis in my study in order to answer the research questions. I further explained the purpose of realisation trees within the commognitive framework and how they are used to categorise learners' discourse about mathematical objects as objectified.

CHAPTER 3: DISCOURSE

3.1 INTRODUCTION

Since this study is entrenched in mathematical classroom discourse, I begin this chapter by clarifying the type of discourse the study focused on. I do this by giving the difference between Discourse with a ‘big D’ and discourse with a ‘little d’ (Gee, 1999). Thereafter, I explain the perspective of classroom discourse that I adopted in this study, which, in turn, gave rise to the perspective of mathematical classroom discourse. Furthermore, I present an analysis of mathematics education studies that focused on mathematical classroom discourse. Throughout this analysis, I continually reflect on the significance of the study in terms of its contribution to the literature. I end this chapter by giving a summary of the arguments I raised in this chapter.

3.2 CLASSROOM DISCOURSE

There is a distinction between two types of discourse outlined by Gee (1999, 2008), namely ‘Discourse’ with a ‘big D’ and ‘discourse’ with a ‘little d’. The latter refers to the language that is in use over time (Gee, 1999) and the features of language that enact activities and identities (Shepard-Carey, 2020). Discourse with a ‘small d’ has to do with the art of language and not the elements in the language that people use to demonstrate how they think about the situation around them. The perspective of discourse with a “little d” is not adopted in this study because it only focuses on the art of language that is used and not on how the language is used for understanding.

Discourse with ‘big D’ involves a socially accepted way in which associations use language, think, value, act and interact with the relevant people that are in the right places at the right time. It can be used as a way of identifying one as part of a group that is socially meaningful (Gee, 1999, 2008). This notion is also supported by Shepard-Carey (2020), who argues that discourse with ‘big D’ involves the different ways that discourse can help people construct how they look at the world; make meanings of texts they use; and allow them to behave differently based on how they do things, think and how they belong to the world. Therefore, in this study, discourse with ‘big D’ is adopted and taken as the point of departure because it involves how language is used in the classroom in order to give learners meaning for understanding. In this study, Discourse includes a set of social conventions in the form of mathematical vocabulary that learners and their teacher use to communicate with each other in the classroom (Gee, 2008)

Classroom discourse is not foreign in mathematics education research. For instance, White (2003) conducted a study on the importance of classroom discourse and its influence on learners’ mathematical thinking; Moschkovich (2003) examined descriptions of mathematical discourse and mathematical discourse practice Moschkovich (2007); Tsiu (2008) spoke about approaches to and perspectives of classroom discourse; Riesbeck (2009) described and analysed ways in which discourse can help to advance knowledge based on the teaching of mathematics in schools; and Johnson et al. (2013) explored tools for analysing mathematics classroom discourse, to name a few. However, in this study I use Tsiu’s (2008) perspective to describe what classroom discourse is, which is defined as all forms of discourse (interactions) that happen in the classroom. She further indicates that these interactions, by their nature, can be linguistic or non-linguistic (Tsui, 2008) and that they can occur as one interacts with the self or with other individuals (Sfard, 2009).

Self-interactions occur when one communicates by thinking, whereas interactions with other individuals occur when different thinking is shared through verbal communication between peers (Sfard, 2009). When these interactions happen in the classroom, they are regarded as classroom discourse (Tsiu, 2008), which provides opportunities to examine language usage as well as a variety of ways in which knowledge is constructed and displayed during interactions in the classroom Jocuns (2012). Hence, I carried out this study to discursively analyse the use of mathematical vocabulary during ‘classroom discourse’ in order to explore how the used vocabulary influences mathematical thinking and communication.

3.3 MATHEMATICAL CLASSROOM DISCOURSE

Mathematics discourse can be conceptualised as a form of discourse (Sfard, 2008) defined as the way in which people communicate, it is unique in its nature and makes use of vocabulary, a means of visuals and a routine way in which we do things that can result in a set of endorsable narratives (Sfard, 2012). Moschkovich (2003) talks about mathematics discourse as not only including interactions, how people communicate, act, believe, read and write but that it also includes values, beliefs and points of view that are mathematical in nature. In this research, the focus was on interactions that happened in the classroom during mathematics teaching and learning (classroom discourse) as a result of learners and their teacher being engaged in a mathematical task during teaching and learning.

In a discursive mathematics classroom, participation is viewed as a situation where learners and their teacher talk and act in a way that mathematically competent people talk and act when they talk about mathematics (Moschkovich, 2003). Correspondingly, when these talks are as a result of a mathematical activity that results in mathematical thinking in the classroom, then it is regarded as mathematics classroom discourse (Moschkovich, 2003). The focus of this study was on the interaction that take place in a mathematics classroom, which could be between the teacher and learners or between a learner and learner that are about mathematics, excluding any other talk. Furthermore, mathematics classroom discourse in this study can be looked at as interactions (teacher-learner or learner-learner) that take place in the classroom when learners are engaged in mathematical activities during the teaching and learning of mathematics.

3.4 RESEARCH ON DISCOURSE ANALYSIS IN MATHEMATICS

Discourse analyses in mathematics education research has increasingly dominated the discipline, where researchers use it to analyse the nature of classroom discourse in the classroom (Cooper et al., 2018; Knuth & Peressini, 2001; O'Halloran, 1998; Reinholz, & Shah 2018) and its ability to enhance learners' mathematical understanding (or thinking) (Gcasamba, 2014; Kersaint, 2015; Le Roux, 2008; Roberts, 2016). Furthermore, some researchers use discourse analysis to examine classroom discourse as an intervention to improve a teacher's classroom practice (Johansson & Kilhamn, 2022; Mbhiza, 2021; Zayyadi et al., 2020) and the role played by teachers in the initiation of classroom discourse that results in mathematical understanding (Legesse, 2020; Nathan & Knuth, 2003). In developed countries such as Spain, a commognitive approach was used as a method for discourse analysis using case study research to investigate teacher's pedagogical discourse when teaching the concept of derivative in a secondary school (Gallego-Sánchez et al., 2022). Also in Sweden, a case study was carried out to investigate a teacher's discourse in order to explore opportunities for or obstacles to learning algebra from a commognitive perspective (Johansson & Kilhamn, 2022).

Critical discourse analysis in Australia was used to analyse classroom interactions by examining the extent to which student agency is promoted and evident in a mathematics classroom (Thornton & Reynolds, 2006). A study conducted in Canada looked at the explanations of students in a discourse during mathematics teaching and learning (Esmonde, 2009). Similarly, discourse analysis studies were carried out in developing countries such as Nigeria (Hardman et al., 2008), Kenya (Pontefract & Hardman, 2005) and South Africa (Aineamani, 2018; Essack, 2016; Roberts, 2016), to name a few, which I highlight in the next paragraph.

A study was conducted in Nigeria to investigate teacher-learner classroom interaction and discourse practice and the central role it plays in improving the quality of teaching and learning (Hardman et al., 2008). Additionally, in Kenya the role of classroom discourse in supporting learners' learning was studied (Pontefract & Hardman, 2005). Meanwhile in South Africa, discourse analysis was used to understand learners' mathematical reasoning when communicating in a multilingual classroom (Aineamani, 2018), while in another study, the mathematical discourse of learners on functions was explored from a commognitive perspective (Essack, 2016). Additionally, the commognitive framework was used to analyse learners use of words, their gestures, narratives and routines, as well as how they linked up to construct a picture of their perceived mathematical object (Roberts, 2016; Sfard, 2009).

Furthermore, the discourse analyses studies conducted in South Africa mainly involved Grade 10 learners in order to explore how their teacher approaches the teaching of functions in algebra (Mbhiza, 2021) and a description about the use of an analytical framework that documented mathematical discourse in order to interpret how the teaching of mathematics differs in teachers' classrooms (Adler & Ronda, 2015). Additionally, Grade 11 learners were involved in discourse analysis research to explore their routines on functions (Essack, 2016), to explore how they communicate their mathematical thinking (Aineamani, 2018) and to examine their selected episodes of discourse shifts (Tyler, 2016). Furthermore, discourse analysis studies involving learners in grades 8 and 9 were conducted to explore how they were thinking about linear equations (Roberts, 2016; Roberts & Le Roux, 2019).

However, most of the studies discussed in the previous paragraphs were not conducted in a natural setting of a Grade 9 mathematics classroom and neither did they explore the use of mathematical vocabulary, as was the case in my study. Only Roberts (2016) and Roberts and Le Roux (2019) conducted studies involving grades 8 and 9 learners, which is similar to my study that only focussed on the level of Grade

9 learners. Furthermore, their studies explored learners' mathematical thinking through discourse analysis, while in my study the intention was to look at the mathematical vocabulary used, and explore how the used mathematical vocabulary influences learners' mathematical thinking. Additionally, it becomes apparent that I provide a deeper analysis of Roberts's (2016) study in order to compare and contrast our studies, particularly looking at how the commognitive theory as a method of discourse analysis was implemented in the study, which I will discuss later.

From the studies highlighted above, it is evident that different approaches or perspectives are used to conduct discourse analysis when looking at classroom interactions from international research and also from research in Africa. However, it is also evident that the commognitive framework was used as a method of discourse analyses in both developed (e.g. Sweden) and developing countries, such as South Africa. Similarly, in this study I saw it significant to use Sfard's (2008) commognitive perspective as the lens through which I navigate through the study because commognitive framework has proven to work very well as both a framework and a method of discourse analysis through which classroom discourse can be analysed (Sfard, 2008). Furthermore, in their studies, Aineamani (2018), Essack (2016) and Roberts (2016) used the same framework to explore mathematics classroom discourse and the impact it has on the teaching and learning of mathematics. This is the reason I decided to use the commognitive framework in my study to explore the use of mathematical vocabulary in a discursive mathematics classroom through discourse analysis.

To collect data in these commognitive studies, the researchers made use of audio and video recordings of the classroom (Gallego-Sánchez et al., 2022), classroom observations and document analysis (Aineamani, 2018) and in-depth interviews (Essack, 2016; Roberts, 2016), which Sfard (2008) also supported in her framework as techniques for data collection. Similarly, the choice of using participant observation and documents in my research, in addition to informed interviews, is substantiated by the literature. The arguments I make here substantiate the use of a commognitive perspective as a discourse analysis method to study mathematics classroom discourse in this research.

Esmonde (2009) made use of vignettes to discursively analyse explanations of the students' tasks in order to highlight how their talk can be used as social action and how the different categories of talk can support their learning. Roberts (2016), on the other hand, focused on how learners think about linear equations in grades 8 and 9. Similarly, Gallego-Sánchez et al. (2022) investigated discourse when the concept of a derivative was introduced and sought to identify the property of the discourse during analysis. They looked at how words were used and at visual mediators in order to infer and classify pedagogical routines. Thornton and Reynolds (2006) used critical discourse analysis to analyse the conversation patterns and content in a mathematics classroom. Even though discourse analysis is common in the studies discussed here, their research focus was different, and for this I make an assumption that the purpose of discourse analyses in mathematics research is to analyse mathematical interactions or conversation for a specific purpose. This is also the case in my research, discourse analysis was used to analyse the use of the mathematical vocabulary used by teachers and learners during mathematics classroom discourse.

Furthermore, I used a commognitive perspective to discursively explore how the mathematical vocabulary used during mathematics classroom discourse influences learners' mathematical thinking and communication. This also was indicated in Esmonde (2009) when looking at how forms of talk can support learners, which is also what Roberts (2016) used to look for learners' thinking. However, some researchers use discourse analysis to look at students' conversation pattern in mathematics classroom (Thornton & Reynolds, 2006) and still decide not to use the commognitive framework. This could happen because the study does not focus on learners' thinking, as was the case with Thornton and Reynolds (2006), since the focus was on socio-mathematical norms for the explanations.

Roberts's (2016) study was motivated by the poor performance of learners in South African schools, mainly in school mathematics and, as such, she carried out a small-scale study where she interviewed grades 8 and 9 learners in order to explore how they think about linear equations. The concept of ritualised and explorative discourse, in particular, from Sfard's (2008) commognitive framework was used to analyse the data. During data analysis, she looked at how the framework's four constructs (word use, gestures, routines and narratives) of learners linked up to construct a picture of their perceived mathematical object (Sfard, 2009). This was also the case in my study, I looked at Grade 9 learners' mathematical thinking, while Roberts looked at both grades 8 and 9 learners' thinking on a specific topic in mathematics. Similarly, my study also used the commognitive framework as a method of data analysis, using its four constructs. However, Roberts focused on how explorative and ritualised discourse builds learners perceived mathematical objects, while in my case, I look at how mathematical vocabulary was used to influence learners' mathematical thinking.

To investigate learners' thinking, the commognitive framework advocates for the researcher to look at learners' discourse as they solve the equation, to look at what are they saying and doing. This was achieved by developing a description of whether the discourse of learners was explorative, ritualised or is transitioning from being a ritual to becoming an exploration using Sfard's (2008) constructs of discourse. Similarly, in my study I also looked at learners' 'mathematical' thinking, which I achieved by looking at what they were saying and doing as they engaged in mathematical activities during teaching and learning. However, I achieved this by looking specifically at learner's use of mathematical vocabulary using the constructs of Sfard's (2008) commognitive framework. I therefore, argue that my work follows the same route as Roberts's (2016) work in terms of exploring or investigating learners' thinking, which, in my case is mathematical thinking. Also, during analysis we both used the four constructs from the commognitive framework, however, I looked at mathematical vocabulary while Roberts looked at ritualised, explorative or a transition between the two discourses.

Data analysis in Roberts's research comprised of two levels. Level 1 data analysis contained transcribed video interviews that were operationalised by Sfard's (2008) theory to identify the tools of the discourse. This was followed by a level 2 analyses, wherein Roberts used these tools of discourse to determine the nature of learners' discourse. In my study, data analysis followed a different route. I first looked at transcribed participant observation videos and documents and used Sfard's (2008) theory to firstly identify the mathematics vocabulary that the learners and the teacher used in the classroom and how they used visual mediators to make meaning of the used vocabulary (word use and visual mediators). This was followed by using their discourse to look at the routines and their endorsed narrative as a result of the word use and visual mediators. Lastly transcribed interviews were analysed for a specific reason, i.e., to give learners an opportunity to clarify their use of mathematical vocabulary that, in the classroom, I found to be worth exploring further.

The commognitive framework used in the study indicated the findings of learners' thinking in a ritualistic manner and that we cannot use learner scores on written assessment tasks to measure their understanding. However, what remains to be proven in my study is how the use of mathematical vocabulary influences learners' mathematical thinking. Also in my case, I used learners' written responses not to determine learners' mathematical understanding, but to determine the mathematical understanding by looking at how learners use mathematical vocabulary to account for the answers they provide.

Studies that have used commognitive theory in order to explore learners' mathematical thinking do exist in mathematics education research and they have made use of realisation trees (Roberts, 2016; Roberts & Le Roux, 2019), Discourse profile of a hyperbola (Mpofu & Pournara, 2018), discourse profile of a hyperbola and exponential function (Mpofu & Mudaly, 2020) and realisation tree assessment tools (Mudaly & Mpofu, 2019) to study learners' mathematical discourse. I also found it relevant that, in my study, I make use of realisation trees in order to further explore the teacher's and learners' discourse as a way of identifying aspects of their discourse which could help me to decide whether their thinking is explorative or ritualistic (Sfard, 2008).

3.5 SUMMARY

In this chapter I have captured a discussion about the origin of discourse as well as the stance I take in the study concerning the type of discourse that I focus on in this study. Furthermore, I gave details to the reader about what classroom discourse is as well as how it came about to be called mathematics classroom discourse in my study. I also compared and contrasted studies that used discourse analyses in mathematics education to study classroom discourse as well as commognitive framework. I presented this by a synthetic and integrated discussion of the studies, looking firstly at developed countries before heading on to talk about developing countries.

CHAPTER 4: VOCABULARY

4.1 INTRODUCTION

In chapter 3, I discussed what classroom discourse is and its identifying traits. I further elaborated on how classroom discourse come to be called mathematics classroom discourse. This was followed by a critical analysis of the literature on studies using discourse analysis, which detailed how researchers were analysing learners' interaction in the mathematics classroom. Therefore, I start this chapter by first discussing what mathematical vocabulary is in a mathematical classroom discourse in order to shape what this study seeks to address in respect of mathematical discourse, and what mathematical vocabulary looks like in a mathematics classroom discourse. Thereafter, I present an analysis of studies focusing on mathematical vocabulary, raising arguments about their limitations and, in turn, highlighting their significance in terms of this study. Finally, I present an argument for the need to explore how mathematical vocabulary is used in the mathematics classroom during teaching and learning of mathematics

4.2 MATHEMATICAL VOCABULARY

Mathematics classroom discourse has been clarified in respect of what it means in this research. Here I explicate what mathematical vocabulary is and how it is constituted in mathematics classroom discourse. A classroom that is dominated by mathematics discourse is characterised by the use of a mathematics register (Moschkovich, 2003), defined as package of meanings that are necessary for a certain function of language, which includes not only words or terms but structures that express the meanings of the words (Halliday, 1978). Moschkovich (2003) indicates that some of the words that are found in the mathematics register which are used during classroom discourse are mathematical in nature, while others have meaning that is different as used in our everyday life English. As such, I argue that the teaching and learning of mathematics during classroom discourse is a combination of two set of languages used during mathematics discourse. That is, when learners are using words or terms to express mathematical meanings during mathematical classroom discourse, they will be referred to as mathematical vocabulary regardless of them being mathematical in nature or having a meaning of everyday life English. I view mathematics vocabulary as the heart of this research, which is why it was imperative that I take a stance on what mathematical vocabulary is in terms of the study.

One of the questions I intend to answer in this study requires me to explore ways in which mathematical vocabulary is used to influence learners' mathematical thinking and communication. It is, however, imperative to note that Cuevas (1984) elaborated on the confusion that is brought by the use of mathematics language, such as symbols, signs, rules and formulae, among learners because they struggle to understand their meaning. This was also found by Roberts (2016) when discussing how a learner named Emily justifies how she divide by the coefficient of 'x' when she describes x as a common factor, which created a confusion that resulted in an error. However, it remains the responsibility of mathematics teachers to help learners to make sense of the mathematics language by using activities that investigate the relationship between meaning and sign and then improving on the existence of the relationship (Siyepu & Ralarala, 2014; Oers, 1997). Roberts (2016) calls such an intervention, where by the teacher help learners to making sense of the mathematical language, 'scaffolding', which helps learners to improve on the level of their correctability when they are engaged in their errors or confusions.

Similarly, this study looked at how teachers and learners use mathematical vocabulary during mathematical classroom discourse and at how the use of such vocabulary influences learners' mathematical thinking and communication. However, it remains to be proven in my study whether or not learners are able to create and demonstrate meaning about the mathematical vocabulary they use during mathematics classroom discourse and how such vocabulary, through communication, stimulates their mathematical thinking (White, 2003).

Mathematical discourse is reported by Shepherd (1973) to comprise of two languages, namely technical vocabulary and specialised symbols, which are necessary for teachers and learners to effectively communicate in mathematics. Within these languages, communication is divided in to different levels required for its success. Communication starts with letters that transform into words, then in to sentences that becomes paragraphs, which create discourse (Kid et al, 1993). However, Powell and Nelson (2017) categorise mathematical vocabulary according to different types of vocabulary that are found in mathematics and are described as: (1) technical vocabulary, which describes a word that has only one meaning in mathematics; (2) sub-technical vocabulary, which describes a word that has multiple meanings where one meaning is specified in mathematics; (3) symbolic vocabulary, described as words that explain numerals and symbols; and (4) general vocabulary, which can be described as the everyday English language that learners encounter in mathematics (Peng & Lin, 2019). Interestingly, in this study, mathematics vocabulary refers to all categories that are explained by Powell and Nelson (2017), Peng and Lin (2019) and Shepherd (1973), as long as during the classroom discourse the different categories are used for mathematical meaning that leads to mathematical understanding.

4.3 RESEARCH ON MATHEMATICS VOCABULARY

Quite a number of studies have been done on mathematical vocabulary in the mathematics classroom and most have focused on mathematical vocabulary considerations (Monroe & Panchyshyn, 1995), mathematical vocabulary knowledge (Powell & Nelson, 2017; Unal, 2019), mathematical vocabulary instruction (Larson, 2007; Monroe & Pendergrass, 1997; Moschkovich, 2007; Pierce & Fontaine, 2009; Wanjiru, 2015), comprehension of mathematics vocabulary (Kidd et al., 1993;

Monroe & Orme , 2002), teaching and learning of mathematics vocabulary (Harmon et al., 2005; Nelson & Carter, 2022; Riccomini et al., 2015), incorporation of vocabulary into mathematics (Fletcher & Santoli, 2003; Gharet, 2007) and mathematics vocabulary ability and performance (Bulos, 2021; Peng & Lin, 2019), to name a few. However, I did not find evidence of studies that focused on how mathematical vocabulary is used during the teaching and learning of mathematics, which is a gap in knowledge creation. Furthermore, the studies quoted above does not focus on finding out the type of mathematical vocabulary that learners and teachers use during teaching and learning, which is also a gap in knowledge creation. In some cases, learners were provided with a vocabulary assessment wherein they were required do a task to demonstrate their knowledge of mathematics vocabulary (Powell & Nelson, 2017), while in other cases a vocabulary test was given to measure performance (Bulos, 2021; Kidd et al., 1993). In my case, however, I observed a classroom where teaching and learning was taking place in order to identify mathematical vocabulary that is used and also explore how it is used during the classroom discourse to influence learners' mathematical thinking and communication.

Learners in a mathematics classroom must be provided with an opportunity to write, read and discuss, which can be achieved when there is use of natural language in the mathematics classroom (Kidd et al.,1993). However, in order for them to be able to write, read and discuss in the mathematics classroom, they need to know and understand mathematics vocabulary. Correspondingly, mathematical understanding can be built through learners' engagements during classroom discourse when making use of mathematical vocabulary, which, in turn, develops their ability to do mathematics (Gharet, 2007). However, there is a paucity of research that looks at how mathematical vocabulary is used during classroom discourse, which renders it difficult for one to assume that learners are, indeed, using mathematics vocabulary in the classroom and whether or not such vocabulary is used to build their mathematical understanding. Hence, this study was conducted in a natural setting of a mathematics classroom where there are engagements in order to identify the mathematics vocabulary used, as well as how the mathematical vocabulary used influences mathematical thinking.

Mathematical vocabulary instruction has dominated mathematics education research, where researchers mostly looked at comparing how two vocabulary instruction models, namely the integrated graphic (IG) model and the definition-only (DO) model, have an impact, looking at Grade 4 learners' use of mathematical vocabulary (Monroe & Pendergrass, 1997). With respect to the IG model, they made use of definitions that were modified using a Frayer discussion, while on the other hand, the DO model only required learners to provide answers after an oral review of the definition of terms. Findings from this study indicated significantly more mathematics concepts being registered by the group that made use of the integrated concept definition Frayer model and this deemed IG as an effective method to use to teach mathematics vocabulary.

Wanjiru (2015) also conducted a similar study with the intention of establishing how learners' achievement in secondary school is affected by mathematical vocabulary using a quasi-experimental design in which various forms of tests were given to learners before and after instruction. The learners who were used as the experimental group were taught mathematics vocabulary using IG that was based on the Frayer model but integrated with an information and communication technology (ICT) approach to instruction. The teaching of mathematical vocabulary to the focus group was undertaken through DO, as was the case in the Monroe and Pendergrass (1997) study. Their (Monroe & Pendergrass, 1997) findings indicated that the mean difference in performance was significant between the two groups. The finding from the studies discussed here is an indication that mathematical vocabulary instruction needs to be looked at from a different angle. That is, the tests or quizzes given to the participants are not enough for one to conclude how mathematical vocabulary has a great effect on teaching and learning. Hence, the study I conducted required me, as the researcher, to be part of the class as an observer in order to identify the vocabulary used and how vocabulary is used during teaching and learning to influence learners mathematical thinking.

Larson (2007) conducted an action research inquiry with Grade 6 mathematics learners in order to explore the role that mathematics vocabulary plays in learners' mathematical understanding and learning. Data collection was done using quizzes and activities that were based on mathematics vocabulary. Findings indicate limited exposure of learners to mathematics vocabulary consistently, which leaves them with an unfavourable impression of mathematics. As a result, Larson decided that, in his everyday teaching, mathematics vocabulary should form part of the teaching of mathematics and he noticed that not only did their confidence and attitude improve but also their test scores began to improve as they started to understand mathematical language. The study discussed here looked at the incorporated mathematical vocabulary knowledge of learners based on tests or quizzes given to learners and how learners responded to them. However, it remains to be proven how learners use the incorporated vocabulary during teaching and learning and how the use of the incorporated vocabulary influences their mathematical thinking and communication.

A study that focused on the impact of mathematical vocabulary teaching in the classroom on mathematical proficiency was presented by Riccomini et al. (2015). The study looked at strategies that can be used to infuse mathematical vocabulary into mathematics learning in the classroom. In my study, the focus was not on incorporating mathematics vocabulary into the classroom but on identifying the mathematical vocabulary that is used in the classroom and also to explore how the vocabulary used influences mathematical thinking. Additionally, Nelson and Carter (2022) examined the idea of whether it is important or not in mathematics education to not allow learners to use informal, everyday language and advocated for the use of formal technical vocabulary. This was achieved by making use of observations in order to perceive how formal and informal language is used, which was displayed on transcriptions of dimensions. During data analysis, they focused on the complexities of establishing meaning for non-core and core vocabulary, as well as how learners made use of words to create meaning or construct a new one. The results indicated the importance of using informal vocabulary (language) in a productive way so that one can explore concepts that are represented through technical vocabulary. Similarly, in my study I refer to both languages as mathematical vocabulary as long as they are used for mathematical meaning to account for learners' mathematical procedures. Also, in my study I use transcriptions obtained from observations of classroom interaction to identify the mathematical vocabulary used. However, my study remains significant in that it aims to establish how the mathematical vocabulary used influences mathematical thinking and communication, while in their study, Nelson and Carter (2022), the focus was on whether or not to stop using informal language in mathematics and prioritise technical vocabulary.

It has been reported that mathematics vocabulary is not being taught in schools and, if learners do not have access to a good mathematics textbook, then they will have nowhere to read mathematics vocabulary (Fletcher & Santoli, 2003). In their study, Fletcher and Santoli (2003) found that proper use of mathematics vocabulary frequently assists learners to better comprehend concepts in mathematics. They alluded to this after having explored mathematical vocabulary and how it can be incorporated into the mathematics classroom. This was also supported in Gharet's (2007) research, which aimed to establish whether infusing vocabulary into mathematics could improve the comprehension of learners' mathematical concepts. Various methods were used to introduce mathematical vocabulary into the school mathematics curriculum in an urban school and data was collected using vocabulary and mathematics tests, with quizzes on weekly bases prior to and after incorporation of the vocabulary. The results supported the notion that the introduction of mathematical vocabulary as part of the curriculum to be taught increases how learners comprehend concepts in mathematics. This was observed when the students' verbal use of these words and their test scores increased.

It is imperative that the use of mathematical vocabulary has an impact on learners' development of mathematical understanding (concepts), as the results from Gharet (2007) and Fletcher & Santoli (2003) indicate. However, it is also significant that I explore how mathematical vocabulary is used during teaching and learning without tampering with the school curriculum and checking how the vocabulary used influences learners thinking. In this research, the mathematical vocabulary used will be independent of any influence from the research and will only depend on learners' and educators' mathematical knowledge.

In their article titled *Mathematical Vocabulary Considerations*, Monroe and Panchyshyn (1995) classified the kind of mathematical vocabulary that teachers must teach in order to help learners to develop their mathematical concepts into technical, sub-technical, general and symbolic. However, it is not clear how this vocabulary should be taught and used in the mathematical classroom to develop the learners' mathematical concepts. In their first-grade study, Powell and Nelson (2017) composed a test that contained mathematics vocabulary generated from four topics, namely algebra and operations; operations of base 10 and numbers; geometry and measurement; and data. Their results indicated a strong reliability ($\alpha = .85$) for the test and different responses from learners. Similarly, Unal (2019) developed a measure for mathematics vocabulary for learners in Grade 8 to investigate the relationships between different categories of vocabulary and mathematics computation. Learners in this study took three tests, where the mathematics vocabulary was highly reliable when looking at the results. Additionally, there was a correlation and a strong relationship between the vocabulary categories investigated in the study; however, there was no significant relationship between vocabulary computation and general knowledge of vocabulary. Both studies looked at measures developed to study mathematical vocabulary in both the lower grades and middle grade, which revealed strong reliability for mathematical vocabulary knowledge in both studies. However, the inability of the studies to look at which vocabulary is used in their classroom, as well as how it influences learners' thinking, creates a gap in knowledge creation that my study seeks to fill. In my study, the emphasis is not on giving learners a test in order to check their vocabulary knowledge but rather to identify the vocabulary used in the classroom and how learners use it to think mathematically.

4.4 SUMMARY

In this chapter I discussed mathematical vocabulary and what it means to use mathematics vocabulary in a mathematics classroom. Thereafter, I presented an analysis of studies focusing on mathematical vocabulary, raising arguments about their limitations and significance in terms of my study. I further looked at commognitive studies on mathematics vocabulary, discussed how their data was analysed to reach their findings and indicated what contribution they could make to my study. Finally, I argued for the need to explore the use of mathematical vocabulary in the classroom and how it influences mathematical thinking through a commognitive framework.

CHAPTER 5: METHODOLOGY

5.1 INTRODUCTION

In this chapter I present a description of the research process I followed during the course of this study. I provide information concerning the research methods I employed when undertaking this research, as well as justification for using such methods. I discuss in detail the various stages of the research, commencing with research design, selection of participants, data collection process and the process of data analysis. I conclude the chapter with a discussion about validity and reliability in qualitative research. Lastly, I discuss the way in which validity and reliability were met in the study to ensure quality criteria.

5.2 RESEARCH APPROACH

There are three approaches that researchers must decide from, when carrying out a study namely; the qualitative approach, the quantitative approach and the mixed methods approach. Furthermore, as a researcher, one must also decide which research design the study should follow throughout in order to provide direction for procedures specific to the study (Creswell, 2014; Creswell & Creswell, 2017). I will discuss this in the next section. With regard to the qualitative approach, Parkinson and Drislane (2011) define this approach as research that employs participant observation or case studies as research methods that result in a narrative, descriptive account of a setting. On the other hand, quantitative research focuses on using data gathering techniques that present data in a numerical form for analysis using statistical methods in order to explain a phenomenon (Aliaga & Gunderson, 2002). In terms of the mixed methods approach, Creswell (2014) argues that this approach involves a combination of qualitative and quantitation data integrated into a distinct design in order to offer a more meaningful understanding of the problem. In this study, I followed a qualitative approached as define by Parkinson and Drislane (2011), in the sense that, participant observations were used in order to identify the mathematical vocabulary used by teachers and learners in the mathematics classroom. Also, I used narrative and description of data when analysing the collected data using the commognitive theory. I elaborate further in the next paragraph as to why I deemed qualitative approach relevant to my study.

The world view that I adopted in this study is constructivism, as detailed in Chapter 2; hence the approach adopted will be that which is in agreement with constructivists' stance. Such an approach is qualitative research, the sole purpose of which is to explore and develop an understanding of a central phenomenon (Creswell, 2014; Creswell & Creswell, 2017) by using methods such as case studies or participant observation to produce a narrative and a descriptive account of a setting (Parkinson & Drislane, 2011). In this study, I explored teacher's and learners' use of mathematical vocabulary in mathematical classroom discourse as the central phenomenon during classroom observation. Furthermore, to develop an understanding of the central phenomenon, I looked at how the mathematical vocabulary used influences learners' mathematical thinking and communication.

I selected qualitative approach as an umbrella for the methodology used in this study, as guided by my research purpose, which was to explore how mathematical vocabulary is used during mathematical classroom discourse using the lens of the commognitive theory. The use of a qualitative approach to conduct discourse analysis through the commognitive theory is not foreign in mathematics education research. In a study by Supardi (2021), a qualitative approach was employed to describe learners' error analysis and the study used commognitive theory to further analyse the results obtained from the data for interpretation of objects for what they are. Similarly, another study that was conducted by Mbhiza (2021) to obtain empirical data about teachers in their classrooms followed a qualitative approach to gain insights into teachers' practice as a non-participatory observer. Mbhiza (2021) used commognitive theory constructs to provide structure and generality to the discourse of the teachers during observations. Furthermore, in her study on learners' mathematical thinking, Roberts (2016) conducted a small-scale qualitative study and used commognitive theory to describe the discourse of learners during an interview about linear functions as either being ritualised or explorative, or are in the process on transitioning from being ritualised to being explorative, using the four constructs of the theory. The three studies discussed here focussed on discourse analysis of teachers and learners using commognitive theory to describe the classroom discourse in their research. Most of all, the researchers have deemed qualitative approaches as significant for employment in their studies because they needed descriptive data. Similarly, it is for this reason that I also deemed it significant to use a qualitative approach to obtain insights into the teacher's and learners' classroom discourse, in order to explore their used mathematical vocabulary and produce the descriptive data that I used in reference to commognitive theory constructs and analysed the data. This allowed for an interpretation of an understanding of how mathematical vocabulary was used during classroom discourse to influence learners mathematical thinking.

5.3 STUDY DESIGN

There are various research designs within the qualitative research approach that are prevalent in literature. Some of these designs, among others, are narrative enquiry (Connelly, 2000), phenomenology (Moustakas, 1994), grounded theory (Corbin & Strauss, 1990), ethnography (Fetterman & Del Rio-Roberts, 2010) and case study (Merriam, 1998; Stake, 1995; Yin, 2009; 2012). In terms of phenomenology, Creswell (2012) argues that the research focuses on describing what the meaning of the phenomenon is, based on lived experienced of the individuals. This is achieved by looking at what is common about the participants based on their experience of the phenomenon with the aim of ensuring individual experiences are minimising and that they become a description of the essence that is universal (Moustakas, 1994). Furthermore, data is collected only from participants who have experienced such a phenomenon and generalises the results for all participants (Creswell, 2007) in a form of descriptions about what and how the phenomenon was experienced (Moustakas, 1994). This is not in line with the intentions of my study. In this study I explored how learners communicate in their classroom using words in order to create meaning, compared to what phenomenologist do as outlined above.

In contrast to phenomenology focusing on people's lived experiences, narrative inquiry focusses on finding a way to understand that experience. In their definition of narrative inquiry, Connelly (2000) talk about it as a path down which one goes in order to understand a specific experience. Furthermore, the researcher and the people participating in the research must collaborate after some time, while being socially interactive with the environment at a specific place or at various places. They further argue that the researcher enters the research site with the same spirit of living and telling a story, as they conclude the research with the results that narrates about participants' lives based on their experiences (Connelly, 2000). Similarly, the focus of my study was not based on finding a way to understand learners' experiences in order to tell stories about them but the focus was on exploring what learners do in a natural setting in order to make meaning for understanding.

Three prominent researchers have defined what a case study is; for Meriam (1998), a case study is a single unit that has boundaries around it and is characterised by an intensive, holistic and descriptive analysis of a social unit. On the other hand, Yin (2009) defines a case study as an empirical inquiry that investigates a contemporary phenomenon where the context is not clear. In terms of Stake's (1995) definition, a case study is a type of research design that has an interest in a single case, not focusing on the method used for the inquiry. With reference to the three researchers, it sounds as if a case study approach fits well with the intended purpose of my study; however, it only fits with one leg, which is the case. In this study, a case can be referred to the use of mathematical vocabulary in the classroom. However, in order for me to explore how the mathematical vocabulary used influences learners’ mathematical thinking, I need to understand how learners behave and communicate with each other, therefore a case study cannot work in my study.

In contrast to case study, I now present ethnography research design. Fetterman and Del Rio-Roberts (2010) defines ethnography research as research with the aim of observing and analysing the manner in which interactions take place between each other and with their environment in order to gain an understanding of their culture. He also mentions that ethnographers spend most of their time at the research site, among the participants whose culture is being studied. Furthermore, data in this research is collected from a variety of perspectives and sources in order to gather rich, descriptive and detailed data for easier interpretation, understanding and representation (Fetterman & Del Rio-Roberts, 2010). The research design discussed here is in agreement with what my study is exploring, since the purpose of my study was to observe learners in their mathematics classroom in order to analyse how they interact with each other using mathematical vocabulary. Furthermore, as a researcher I was part of their classroom during data collection, albeit as a non-participating person, in order to gain an understanding of their classroom interaction which I call discourse.

I have adopted ethnography (Fetterman & Del Rio-Roberts, 2010) as the research design used in order to conduct an in-depth study, which in mathematics education research is not foreign and dates back over 3 decades (see, for example, Eisenhart, 1988; Millroy, 1991; Patahuddin, 2010). Ethnography research design is defined as a qualitative research methodology that lends itself to the inquiry of beliefs, social interaction and behaviour of small societies which involves participation and observation for a period of time and the interpretation of collected data (Denzin & Lincoln, 2011; Reeves et al., 2008). Additionally, it has been explicated as a design that is relevant for studying classroom discourse (Tsui, 2012).

Furthermore, Creswell (2012) supports the use of ethnography as a research design because of its ability to function as a strategy of inquiry in which a researcher studies a group in its natural setting. As such, the group in its natural setting in which this study was undertaken was a Grade 9 mathematics classroom.

A researcher who uses ethnography as a research design is referred to as an ethnographer who strides into a culture or social situation to explore its terrain, collect data and analyse it (Fetterman & Del Rio-Roberts, 2010). It is further argued that an ethnographer must rely on all senses, thoughts and feelings of the terrain. This is because ethnography involves a human instrument as the data collection tool that is most sensitive and perceptive, yet it can gather information that can be subjective and misleading (Denzin & Lincoln, 2011; Fetterman & Del Rio-Roberts, 2010). Similarly, this research made use of learners and their educator in a natural setting to collect data looking at their use of mathematical vocabulary during classroom discourse. As an ethnographer, I went in to their Grade 9 mathematics classroom to explore how they use mathematics vocabulary to communicate as they learn mathematics. Additionally, I have used the learners and their teacher as human instruments for data collection, which I analysed using Sfard's commognitive framework as a method of discourse analysis in order to explore how their use of mathematics vocabulary during classroom discourse influenced learners' mathematical thinking and communication.

5.4 CHOOSING PARTICIPANTS

Participants in this study were selected following convenience sampling and purposive sampling (Cohen et al., 2000). With Convenience sampling, I conveniently selected research participants because of their convenient, accessibility and proximity to me as the researcher (Cohen et. al., 2000). I have conveniently sampled a high school that is located in an urban suburb of a small town called Burgersfort in Limpopo Province. The school falls under the Sekhukhune East education district where I was teaching at the time, and I chose the school as the research site because it was easier for me to access the school and I also relate well with the teacher and the learners. The school initially had three Grade 9 classes with a total of 145 learners, with each class having 48, 48 and 49 learners respectively. The three classes were taught by one teacher. However, COVID-19 disrupted our plans and schools had to readjust to the ‘new normal’ and the learners were separated into six classes with five classes having 24 learners and one class having 25 learners. The classes were now taught by two educators, with one educator teaching three classes out of the six and the other teaching the remaining three.

Subsequently, I purposively sampled one Grade 9 mathematics classroom, a class that was dominated by discourse and not by teacher talk or direct instruction. This is supported by the constructivist world view which envisages that the construction of meaning in one’s world occurs through engagement with others (Gordon, 2009). Participants in their classroom were engaging with each other and with their educator during mathematics teaching and learning in order to construct meaning during classroom discourse. Purposive sampling is defined by Cohen et al. (2000) as a non-probability sampling, where the researcher chooses the sample for a specific reason. Fetterman and Del Rio-Roberts (2010) further supports this and asserts that ethnographers are allowed to use their judgement when choosing a sample and, hence, Fetterman and Del Rio-Roberts refers to this as ‘judgmental sampling’ (p. 43).

A week prior to data collection, I conducted a classroom visit to all the Grade 9 classes to offer monitoring and support, thereby evaluating the level of classroom discourse present in each classroom. In most of the classes, learners were not engaging when the teacher asked questions to prompt their thinking, while in the other classes, a different teacher was using a teacher-centred approach to teach. Therefore, I purposively sampled one Grade 9 mathematics classroom with 25 learners, which included the mathematics teacher responsible for facilitation of learning in that class, as well as the learners, based on my judgement of the learners' willingness to engage in interactions with the teacher and with their peers during teaching and learning. The class had 12 male and 13 female learners, all black Africans and Sepedi home language speakers. The arrangement of furniture in the classroom was in the form of rows and columns for compliance with COVID-19 regulations, which served as a limitation during data collection as the regulations did not allow learners to sit in groups or to work in groups. Furthermore, Grade 9 is the class in which inadequate use of mathematics vocabulary was uncovered by the Mathematics Annual National Assessments (ANA) results (ANA, 2014).

5.5 DATA GATHERING TECHNIQUES

In ethnographic research, data collection is done mainly through participant observation, interviews, structured questionnaires and document analysis (Fetterman & Del Rio-Roberts, 2010). Participant observation involves the researcher's participation in the lives of the participants under study, maintaining a professional distance, which will allow for effective observation and data recording (Creswell, 2014; Fetterman & Del Rio-Roberts, 2010) with the sole purpose of learning their language and observing the patterns of their behaviour for a specific period of time (Fetterman & Del Rio-Roberts, 2010). On the other hand, interviews, which can be formal or informal, refer to a stage where the researcher asks one or more of the participants general, open-ended questions and records their answers (Creswell, 2014). Formal interviews (structured or semi-structured) have a generally explicit goal, since they serve to compare and represent responses by putting them into the context of a common group's beliefs and themes (Fetterman & Del Rio-Roberts, 2010). Informal interviews involve casual conversations with structured questions that have a specific and implicit agenda, such as discovering what a participant thinks and how their perception compares to that of other participants (Fetterman & Del Rio-Roberts, 2010).

Fetterman and Del Rio-Roberts (2010) further explains that questionnaires are similar to structured interviews; however, they differ in the sense that, when using questionnaires, the participants and the researcher do not need to be in close proximity to one another and there is no verbal exchange or opportunity for clarification. Documents on the other hand, refers to records that researchers collect or obtain from participant at the research site under study, which can include journals, diaries, letters etc (Creswell, 2014). This study made use of classroom observations, informal interviews and documents to collects data, which I discuss fully in the next section, I further outline the teaching plan that was used by the teacher during data collection.

5.5.1 Observations

Participant observation (Fetterman & Del Rio-Roberts, 2010) was used to collect data in a mathematics classroom in order to identify the mathematics vocabulary that teachers and learners used during mathematics classroom discourse. This was used to craft their endorsed narratives as well as their routines (Sfard, 2016) in order to explore how the mathematical vocabulary used influences their mathematical thinking and communication. I made use of one camera to collect data through video recordings made to capture whole-class interactions during teaching and learning, which was held at the back of the classroom by myself as a non-participant observer. Even though Sfard (2008) recommends for the use of two cameras, where one should record the whole class while the other records the class from the front of the classroom in order to capture learners' interaction, it was not the case for me during data collection because COVID-19 regulations did not allow learners to sit close to each other. However, learners' interactions that occurred in the classroom were influenced by the teacher giving each learner a chance to respond to another learner's answer, which I could still capture from the back of the class. Sfard (2016), also supports the use of video recording by arguing that video recording provides dense, authentic data and captures all the visual events that are happening during the lesson.

My role as a non-participant observer is supported by Creswell (2014), who argues that a non-participant observer only visits the research site and records notes without involving themselves in the activities of the participants. As such, I did not take part in the discussions in the classroom, nor did I take part in the everyday lesson preparation of the teacher or during selection of activities to hand out to learners in the classroom. My role was simply that of an outsider who comes into the classroom, sits at the back of the classroom and starts recording the video, while taking notes on the participant observation tool (**Annexure G**). This approach is further supported by Gately and Gately (2001) and Creswell (2012), in that observation can be carried out with or without the researcher as a participant.

5.5.2 Document analysis

Document analysis in ethnographic research is described as a form of collecting data that saves time and is most valuable (Fetterman & Del Rio-Roberts, 2010). In this study, documents refer to learners' classwork books, which I had access to and collected after each lesson (Creswell, 2014) in order to make copies of the responses to the activities given on that specific day. The selection of documents followed a purposive sampling method; that is, I collected learners' books for a specific purpose, which is, to gather their classroom interactions during the lesson (Cohen et al., 2000). Their written responses played an important role during data analysis when I had to compare and explore how they represented, in writing, the mathematics vocabulary used in the classroom and how their thinking was influenced as they communicated through writing. During classroom teaching and learning, learners were given a mathematics activity to which they had to provide written responses during the lesson and immediately discuss. However, sometimes they were given an opportunity to write responses at home, which would be discussed during the next lesson. Whichever the case was, I would immediately after discussion collect the learners' books because this additional information was important for adding more insights into the research (Fetterman & Del Rio-Roberts, 2010).

5.5.3 Interviews

In this study, I made use of interviews as a secondary source of data collection tool to support participant observation. However, when selecting interviewees, I looked at the learners' written responses and their ability to solve the given problem in the activity so that I could be able to seek more information from them in terms of the mathematical vocabulary used during the classroom discourse, and how they represented their word use visually in their written responses. Sfard (2008) supports this way of selecting participants for interviews as she emphasises the point that the selection should be based on the participant's academic level that qualifies them as a representation of the sample of interest in the study. Furthermore, unstructured questions were used during the interviews in a way that they allowed me to further question the participant based on the responses they provided. During the interviews, I had one-on-one sessions with the selected participants in my office, with their classwork books with me, and I made use of a voice recorder to record our interactions. The interviewees were given a new blank sheet which they could use if they felt the need to express their explanation on paper, which I also collected at the end of the interview. The interviews were conducted in my office at the school during break time, as per my appointment with the concerned learner.

I have attached **Annexure H** as a guideline of how questions should be asked and which direction they should take during the interview but this guide did not limit which questions were asked and how I asked them during interviews. This is because informal interviews with unstructured questions give participants the opportunity to freely express their own understanding in their own terms, as well as providing me, as a researcher, with confirmation that what they were explaining in class is really what they really meant (Genzuk, 2003). This is supported by Fetterman and Del Rio-Roberts (2010), who indicates that informal interviews are convenient throughout the ethnographic study in a sense that they help the ethnographer discover participants' thinking and how their perceptions compare with those of others. In this study, interviews are only used as a secondary data collection technique; therefore, participants were interviewed only when there was anything new that surfaced during the data collection period that needed to be clarified so that, during data analysis, I did not misrepresent learners' use of mathematical vocabulary. Hence, not all lessons that were observed had follow-up interviews. Similarly, transcriptions of the interviews conducted are attached as **Annexure I** and did not form part of the analyses but are referred to in the discussions, where necessary. This is so because the study focused on the observations to generate the primary data required to answer research questions.

5.6 TEACHING PLAN

I received ethical clearance from the Limpopo Department of Education in March 2021, and I immediately engaged with the teachers responsible for the class I observed in order to fully discuss the purpose and aim of the study. Observations were done during the normal class teaching time and I did not tamper with the everyday running of the school. Unfortunately, the Grade 9 learners only come to school for three weeks in a term, one week a month. Of the three weeks, one week was used mainly for assessment and one week for remedial work, with only one week, and in some cases two weeks, available for teaching. The Department of Basic Education has issued an Annual Teaching Plan (ATP) for recovery as a result of COVID-19; however, because the learners' time table was rotational, educators were allowed to pick the topics that they felt would serve as a foundation for learners in the next grade and did not follow the ATP per se. However, it should be noted that the topics selected were in line with the ATP and the Curriculum Assessment and Policy Statement (CAPS) for Grade 9 (Department of Basic Education [DBE], 2011).

TABLE 3: EDUCATOR'S RE-ARRANGED ANNUAL TEACHING PLAN

DATE learners were coming to school	TOPIC	DURATION as per ATP	Lesson focus
TERM 1 (3 weeks)			
01 – 05 Mar 2021 22 – 26 Mar 2021 12 – 16 Apr 2021	1. Whole numbers 2. Exponents	1. 6 hrs 2. 9 hrs	- Multiples and factors integers - Calculations using numbers in exponetial form, revise and extend the general laws of exponents, perform calculations involving all four operations using numbers in exponential form
	3. Numeric and geometric patterns	3. 4,5 hrs	- Investigate and extend patterns looking at relationships betwem numbers including patterns - Describe and justify general rules for observed relationships between numbers in own words or in algebraic language
TERM 2			
10 – 14 May 2021 07 – 11 June 2021 05 – 09 June 2021	1. Algebraic expressions 2. Algebraic equations	1. 16 hrs 2. 13.5 hrs	- Algebraic language, expand and simplify algebraic expressions, factorise algebraic expressions
			- Solve equation by inspection, using factorisation, using additive and multiplicative inverses, using laws of exponens, solve equations by substitution
TERM 3			
02 – 06 Aug 2021 30 – 02 Sep 2021 27 – 01 Oct 2021	1. Geometry of straight lines 2. Geometry of 2D shapes and constructions of geometric figures	1. 9 hrs 2. 9 hrs	- Angle relationships between angles formesd by perpendicular lines, intersecting lines, parallel lines cut by a transversal - Solving geometric problems involving relationships above - Classifying 2D shapes and construction

I put in on record that I started collecting data from March 2021 to October 2021, a period of eight months. According to the school's rotational system, it meant that I have collected data for a period of nine weeks. However, I only managed to spend three weeks with the learners in their classroom, simply because the other weeks were used for the administration of tasks that were sometimes done during the periods reserved for mathematics. Also, at some point I had to attend to my everyday classes, which often clashed with the period for the study class, and so I had to attend to my classes, especially when arrangements with other teachers to swap periods was not successful. **Annexure F** documents the activities that the teacher gave to learners during teaching and learning in the classroom, which are based on some of the topics highlighted on the table above. It should be noted that the activities do not represent one complete lesson but a series of lessons on the topic during the three weeks that I observed their classroom.

5.7 DATA ANALYSIS

In Chapter 2, I demonstrated how commognitive theory can be use as both a framework and a method of discourse analysis. Furthermore, it is argued that a researcher must ensure that the framework chosen for analysis is aligned to the research purpose and the research questions (Kim et al., 2017; Sfard, 2008). With that being said, in this study I used the commognitive framework as a method of data analysis because it is useful for analysing mathematical discourse (Kim et al., 2017 p. 484). The interplay between the socialisation process, meaning making and language use can be identified through discourse analysis, which can yield findings that are not developed by other methods of discourse analysis (Kim et al., 2017 p. 482). This is because discourse analysis has the ability to explore: (1) the significant role the use of language plays in cognition and meta cognition by revealing how the use of the same word by learners has an impact on meta-discursive rules; (2) how participants' discourse can offer authentic opportunities that connect theory and practice (coherent in mathematical thinking); (3) the inquiry of how routines or mathematical norms are associated with mathematical objects; and, lastly, (4) the opportunity to look at the broader picture about thinking and learning (Kim et al., 2017 p. 482). However, in this study the purpose of the discourse analysis was to reveal how mathematical vocabulary is used during classroom discourse for: (1) meaning making; and (2) also how the used vocabulary influences learners' mathematical thinking and communication. Below I explain how I went through the collected data systematically, synthesising it into coherent arguments to be used for interpretation.

5.7.1 Data arrangement and selection of segment for transcription

During the first step, when I was preparing for analysing the collected data, I arranged the data into corpus using folders that I coded by the month (e.g., 202105) that the data was collected. I also coded the files I had put into the folders by the date (e.g., 20210511) on which data was collected. The purpose of having corpus coded was for easier identification and to be able to use the same corpus code on the transcriptions. This would make things easier for me during transcription verification so that I could know which video to look for and when, as well as from what time on the excerpt was recorded on the video.

I used the coded corpus in their order to watch the video recording as many times as possible to familiarise myself with the series of events that took place in the classroom. Chuene (2011) argues that watching the videos repeatedly, a number of times, stops immediately when you accept the image portrayed by the videos. Furthermore, Moore and Llompart (2017) assert that it might suit researchers to work through their audio or video files and take notes about interesting fragments for later transcription. In that way, I have watched every lesson video recorded at least four times so that I could be able to select valuable segments to transcribe. Similarly, the decision about what to transcribe was determined by looking at the research questions the study seeks to answer and each transcript was produced for a specific purpose (Ayaß, 2015), that is, to answer the research questions. I coded all the segments that I was to transcribe using the date of the file followed by the exact time in the video where the segment starts to play for easier location purposes (e.g., Segment 1_202105001, 03:12). I did this because deciding what data to transcribe is an important part of the analytical process (Moore & Llompart, 2017).

The data that I present in this report does not capture all the interactions that took place in the classroom, but showcases data that was selected for transcription based on the data's ability to answer the research questions. This is supported by Ayaß (2015), who insists that the decision about what to transcribe can be determined by looking at the research questions that the study seeks to answer and each transcript can be produced for a specific purpose. Mooree and Llompart (2017) further assert that the decision about what to transcribe is an important part of the analytical process. I, therefore, saw it fit not to transcribe all the interactions that took place during the lesson because not all interactions in the classroom where about mathematics and, as such, I considered only interactions that were about mathematics. Furthermore, analysis of discourse in chapter 6 and 7 will only focus of Algebraic expressions and equations and geometry of straight lines as they proved to provide rich data during selection of segments that can answer the research questions (Ayaß, 2015).

5.7.2 Data transcription of selected segments

There are specific responsibilities that a researcher must exercise in commognitive research. In addition, the discourse is taken as 'a unit of analysis and principal object of attention' (Gcasamba, 2014 p. 64). When the data transcription commenced, one of my responsibilities was to look at the data as is and ensure that the fidelity (Sfard, 2008) of learners' interactions in the classroom is maintained in its verbatim utterances, while acknowledging that I can only analyse or transcribe what I observe in terms of gestures and speech (Roberts, 2016; Sfard, 2009).

Similarly, the transcription report indicates what was said and what is done, so that I do not revoice the participants but allow them to use their own voice to interact (Gcasamba, 2014; Kim et al., 2017).

Transcribed data captures the interactions between participants, who include both the educator and the learners on video recordings, with researcher (myself) as a non-participant. The data also indicates the sequence of turns in the conversations, speaker, spoken language and non-verbal language (gestures, facial expressions and other body language) (Sfard, 2008, 2009). Sfard also mentions that it is important to order each turn in the conversation because the numbers serve as a reference point in the discussion of data and also provides an indication of the frequency of interactions in the discourse. Each speaker is represented by a code that identifies the participant. In the data, learners are indicated by the letter L, while T indicates the teacher and R indicates the researcher. In this way, L1 and L2 represents two learners in a discourse, while ALL represents all learners. Sfard (2008) further advises that transcribed data must also show nonverbal communication, including silence during the conversation represented on the Excerpts as a column for 'what is done'.

5.7.2.1 Video transcription

From the segments that I selected for transcription, I only transcribed the classroom interactions involving talk that was about mathematics and not any other talk that took place in the classroom; that is, the transcription will not show, for example, if there was an interruption by a learner seeking permission to go to the bathroom. These interactions were ignored during transcription. I transcribed data sequentially, based on how I selected segments that were based on the different topics taught during the lesson observation. This was done to allow for each segment to be transcribed at once. During transcribing, I sequentially indicated the following on the transcription table (e.g Table for Excerpt 1): turns on conversations; speaker; language spoken; and non-verbal language (Kim et al., 2017). This approach is also supported by Sfard's commognitive framework, which makes it possible to capture interaction between participants.

Furthermore, I developed data codes for each speaker so that they serve as a reference point during data discussion and also because speaker coding indicates the frequency of interactions in the analysed discourse (Kim et al., 2017; Sfard, 2009). Each speaker was provided with a unique letter that included a number for easy identification and to maintain the anonymity of the participant involved. Questions asked during interviews are also allocated codes for easy analyses of the patterns that would be observed (Kim et al., 2017). It should, however, be noted that L1 (which represents Learner 1) in Excerpt 1 and L1 (which represents Learner 1) in Excerpt 2 are not necessarily the same learner but the first learner to be involved in an interaction in that except.

5.7.2.2 Audio interview transcription

Transcribed interview recordings were coded using the same coding as the observation data and, immediately after coding, transcription of the whole interview followed, which also identified the speaker, what was asked or replied and what was being done at the time. All interview recordings for the conducted interviews were transcribed because not all lessons involved follow-up interviews. Also, not all learners whose classwork books were collected were interviewed, as discussed in the previous section. It should also be noted in the interview transcriptions that L1 in Excerpt 1 and L1 in the interview transcripts for Excerpt 1 refers to the same learner. That is, when L1 from Excerpt 1 is sampled for interview, the same code (L1) will be used during the interview for the purpose of data triangulation. Correspondingly, a learner's analysis from the interviews was compared to their corresponding classroom interaction analysis from the segments in order to determine whether there is a link, a pattern or themes between their thinking and communication and to triangulate the data.

5.7.2.3 Documents analysis

As discussed in the previous section, documents were selected for analysis based on the learner's ability to interact during classroom teaching and learning, I selected a snapshot of learners' written responses from their classwork books for the corresponding activity that was discussed in the excerpt concerned. The learner's written response to a classwork was represented as a visual mediator on the excerpts where I was discussing about such a learner. That is, when I was analysing Excerpt 1 and discussing the visual mediator for learner 2 (L2), the visual mediator represented there would be that of L2 as a snapshot from the classwork book. However, in the case where the visual mediator was from the board it will be re-typed and will act as a visual mediator that represents the excerpt being analysed with a heading indicating it is from the board. Similarly, interviewed learners' written responses from the interviews were also used to act as a visual mediator that represents the leaner's word use. Eriksson and Kovalainen (2015) argue that an analysis of learners' responses must be done by firstly marking learners written responses in order to check similarities, patterns and connections. Each script was coded for the purposes of easier identification so that identified key wording be matched with its corresponding code in the document (visual mediator).

5.8 QUALITY CRITERIA

A qualitative study is assessed using the following criteria: credibility; transferability' dependability; and confirmability. With respect to credibility, the focus is on the establishment of a match between participants' constructed realities and those represented by the researcher (Guba & Lincoln, 1989; Sinkovics et al., 2008). This is supported by Anney (2014), who argues that credibility can be ensured by representing and interpreting the results as original views of the participants' collected data.

Transferability helps to ensure that the findings of a study fit into contexts other than the research site, which is determined by the degree of similarity or goodness of fit between the two contexts (Krefting, 1991). Dependability is a criterion that is concerned with the stability of the results over time and, as such, the results must represent the collected data (Sinkovics et al, 2008). Lastly, Ghauri (2004) argues that, for research to pass the confirmability test, the data and its interpretations must be coherent and logically assembled and it must be demonstrated that they are not rooted in the researcher's own imagination.

5.8.1 Credibility

5.8.1.1 Prolonged engagement

Credibility in my study was achieved by prolonged engagements with participants at the research site. I spent three weeks with the participants in their classroom and, because of the school's rotational system, I only collected data for one week per term, from Term 1 to Term 3. Additionally, credibility requires the researcher to adequately submerge themselves in the research setting to enable recurrent patterns to be identified and verified. This is supported by Kielhofner (1982), who argues that the importance of intense participation is to enhance research findings through intimate familiarity and discovery of hidden fact.

5.8.1.2 Persistent observation

Persistent observation in my study was achieved by identifying those characteristics and elements in the classroom during mathematics discourse that are most relevant to the research problem I am studying in order to focus on them in detail. Lincoln and Guba (1985) agree by emphasising the notion that, if prolonged engagement provides scope, then persistent observation provides depth.

5.8.1.3 Data triangulation

This study satisfied the evaluation technique for credibility through triangulation, where data that was collected using documents, video tape and interviews were compared (Knafl & Breitmayer, 1989). This was done in order to maximise the range of data that might contribute to a complete understanding of the research problem. Correspondingly, data collected through interviews, observations (video recording) and documents were triangulated in order to ensure credibility of the results.

5.8.2 Dependability

Participants were given an opportunity to explore the interpretation, findings and recommendations of the research and determine whether the results support the data in the interviews transcription during the study (Tobin & Begley, 2004). This was done by providing the school with a copy of the research report that will be made available to all stakeholders in the school. This approach is also supported by Anney (2014), who argues that the results can be verified by using stepwise replication in order to ensure dependability. During this evaluation process, my co-supervisor was exposed to the same pieces of data to analyse separately, the findings of which were then compared to my findings to determine whether they are similar and, if so, then dependability has been achieved (Anney, 2014).

5.8.3 Transferability

Transferability was maintained by purposively sampling Grade 9 mathematics learners and their educator as participants in this study. This was done because they were the area of interest and they displayed the characteristics the study sought to explore (Creswell, 2012). Also, the purpose, nature and the entire research process was well explained in detail to the participants in order to ensure transferability of the study (Krefting, 1991). Additionally, the final report allows for replication of the study by other researchers because of the thick descriptions contained in the research methodology (Anney, 2014).

5.8.4 Confirmability

Confirmability in my study was achieved through an audit trail in order to confirm that the findings are derived from the collected data (Tobin & Begley, 2004). Additionally, during audit trail, I validated the data by accounting for research processes that were followed for collection, analysis and interpretation of data (Anney, 2014). As such, I have made available all collected data and the transcriptions to any reader who seeks to audit all segments or snapshots that were used when reporting data analysis.

5.9 ETHICAL CONSIDERATIONS

In ethnography, the following are regarded as ethical areas of concern: transparency about data gathering; protection of participants; preserving participants' dignity; and ensuring the participants' privacy (Madison, 2005). In order to adhere to the ethical issues, I obtained ethical clearance from the Turfloop Research Ethics Committee to ensure the compliance of the research to the ethical requirements of the university (**ANNEXURE D**). Additionally, I also obtained ethical clearance from the Limpopo Provincial Research Ethics Committee, ensuring compliance of this study to be carried out with the participants as outlined (**ANNEXURE B**). Similarly, I also secured permission to conduct the study from the school that I sampled as the research site (**ANNEXURE C**).

5.9.1 Informed Consent

The study involved both the educator as well as learners in the sampled classroom and, as such, formal consent letters were sent to parents requesting permission to use their children as participants in the study. It was clearly explained to the learners that participation in this study was voluntary and that they were allowed to withdraw from the study at any stage. It was also conveyed to the learners that their opinions would be respected (AcSS, 2013).

The mathematics teacher responsible for the mathematics class that was used to collect data was also provided with a consent form to give permission to use her class to collect data and her consent to serve as one of the participants in the study. Lastly, learners' consent forms were distributed a week before data collection commenced; this was done in order to explain fully the purpose of the study and my expectations from them during the course of the study.

5.9.2 Confidentiality

To ensure confidentiality, I have briefed participants before commencing with data collection in order to ensure that we all had a clear understanding of their confidentiality regarding the researcher's results and findings (Burns, 2000; Maree, 2013). As such, to ensure honesty and confidentiality for both the educator and learners in this study; videos recorded during the lesson were only used to retrieve the mathematics vocabulary used in the classroom and to complete the required information on the observation tool. This was done using pseudonyms that were allocated to those individual learners who were identified to have used mathematics vocabulary. Furthermore, interviews were recorded using audio recording and learners' names were not recorded during the interview to maintain anonymity. Similarly, the interview transcripts were recorded using pseudonyms to be able to use them in the research texts when reporting on the study. I have stored learners' written response documents in a secure electronic individual folder labelled using their pseudonyms.

5.10 SUMMARY

In Chapter 5, I discussed the research approach that I followed in this study and gave reasons for choosing a qualitative approach for my study. Additionally, I clarified the reason for choosing ethnography as the research design for this research. I also explained how participants were chosen and the sampling criteria I used. Data gathering techniques were also dealt with at length, which was followed by detailing how data was analysed in the study. Similarly, I discussed how I would ensure quality criteria in the study, as well as the ethical considerations adopted.

CHAPTER 6: ANALYSIS OF DISCOURSE ON ALGEBRAIC EXPRESSIONS AND EQUATIONS

6.1 INTRODUCTION

In this chapter, I present the first part of my analysis of data following the commognitive framework developed by Sfard (2008), which I discussed at length in Chapter 2. During the analysis of data, I focused on the four constructs of Sfard's (2008) commognitive theory that I adopted as the lens through which I viewed the data. The construct are as follows: word use; visual mediators; routines; and endorsed narratives. However, during the analysis I focused on two constructs at once, that is, word use and visual mediators and endorsed narratives and routines.

The analysis focused on two sub-research questions, namely (1) what mathematical vocabulary do teachers and learners use during classroom mathematical discourse? and (2) in what way(s) does the used mathematical vocabulary during mathematics classroom discourse influences learners mathematical thinking and communication? Furthermore, answering the two sub-research questions assisted in answering the main research question, namely how do teachers and learners use mathematics vocabulary during classroom mathematics discourse?

6.2 FOCUS OF ANALYSIS IN THE STUDY

During the analysis, I focused on two issues: firstly, I looked at the mathematical vocabulary that learners and the teacher use in the mathematics classroom discourse, and here I considered the word use and their intended visual mediators in order to argue how the words are used for meaning making. Secondly, I focused on how the words identified are used to influences learners mathematical thinking and communication. The answer to how mathematical vocabulary used influences learners' mathematical thinking was looked at when discussing how I decided about the discourse of the teacher and learners being objectified or if they are acting with mathematical objects, that is, when I considered whether the narrative constructed justified the relationship between a realisation and its original signifier is endorsed or not. Furthermore, I used the teacher's and learners' discourse to construct their realisation trees in order to obtain a visual representation of their discursive objects, which helped me to identify whether their routines are explorative or ritualistic (Sfard, 2016; Roberts, 2016). This led to the analysis of how teachers and learners use mathematical vocabulary in a discursive mathematics discourse.

I present the data here according to lessons focused on the topic of algebraic expressions and equations, that is, only data generated from a single item in an activity was considered at a time. The lesson focusses are as follows: (1) expand and simplify algebraic expressions; (2) solve equations by factorisation; (3) solve algebraic equation by using additive and multiplicative inverses; and (4) solve algebraic equations using the laws of exponents. I chose to present the data this way so that coherence of the mathematical classroom discourse that took place in the classroom as learners and the teacher interact in the activities is maintained.

It should be noted that not all interactions are captured here, only excerpts that have the potential to showcase aspects of the mathematical classroom discourse considered important for me to answer the research questions were selected. Additionally, Ayaß (2015) advices that the decision about what to transcribe should be determined by looking at the research questions that the study seeks to answer so that each transcript is produced for a specific purpose.

The analysis is presented in four parts, in this chapter, each part represents data generated from an item in an activity. Each part of the analysis is presented in two subheadings, with the first subheading focusing on word use and visual mediators and the second subheading looking at the endorsed narratives and routines from a ritualisation tree.

6.3 ARRANGEMENT OF ACTIVITIES AND EXCERPTS

The data that is presented and analysed here emanates from three activities that were either done in class with the teacher or given as a homework and discussed during the lesson, all based on algebraic expressions and equations. The activities do not represent a single lesson, instead data were captured over a period of five hours during a period of an hour each. Lesson assessment activities are attached as annexures and item (a) and (b) from the first lesson activity were chosen for analysis based on the selected segments for transcription for subheading 6.4 of the analysis. The activity here required learners to expand algebraic expressions.

The analysis under subheading 6.5 is based on item (a) of the second activity that required learners to factorise algebraic equations. For subheadings 6.6 and 6.7, the analysis focused on item (a) and (b) of the third lesson activity in that week. The focus of the analysis here was on the use of words by the teacher and learners, as well as how the used words influenced learners mathematical thinking.

6.4 ANALYSIS ON EXPANDING AN EXPRESSION

Below I report on interactions that focused on the teacher's usage of mathematical vocabulary when teaching learners how to expand algebraic expressions, while learners are paying attention and responding to questions only when asked by the teacher. Similarly, I report on how a learner used mathematical vocabulary when elaborating on how she expanded an algebraic expression on the board. Excerpt 1 below demonstrates such an episode; it should be noted that the segments here address what the study seeks to answer. The teacher demonstration was based on the expression: $(7-3x)(2+x)$ and the learner's demonstration was on the expression: $4(2x-1)+2(3x)$.

Excerpt 1

Turn	Speaker	What is said	What is done
1.1	T	We are going to do an activity together as a class first. The question says expand, let me start by asking you which mathematical operation do you use when we talk about expansion?	Writing the questions on the board
1.2	L1	Multiplication	After raising a hand and being picked by the teacher to respond
1.3	T	Multiplication, so be reminded that … let me do the first one to show you how multiplication is done in expansion and I will request one of you to come and do the same for the class	Explains looking at the learners and returning to the board to do an item
1.4	T	If you are saying expansion means multiplication then what we are doing here is, this … each and every term in this bracket is going to multiply each and every term in that bracket.	Pointing the first term and the second term
1.5	T	So, the seven and minus three x is going to multiply two and multiply x that side, so in view of that fact, let us do it together	Drawing arrows to demonstrate which to multiply which term
1.6	T	Seven times two we get?	
1.7	ALL	Fourteen	
1.1	T	Then we go plus, seven times x what do we get	
1.9	ALL	Seven x	
1.10	T	Seven x, I think you have seen that seven has multiplied each and every term that side, now we should go to minus three x. Minus three x times two you get … minus …	Looks at the learners expecting an answer
1.11	ALL	Six x	
1.12	T	Then you go minus time a plus is a minus the three x times x you get three x squared, after that you collect like terms together. I think you can see we have got seven x and we got six x, they are what we call like	Writing on the white board

		terms. We have fourteen then plus seven x minus six x we get one x then minus three x squared	

A moment later after learners have responded to the activity, one learner goes to the board to represent her answer explaining to the other learners

1.13	L1	We are going to multiply this number; we are going to say four times two equals to eight x	Pointing at the expression
1.14	L1	Then four times one, we use this minus then we are going to say minus four plus two this one, then four times three x equal to twelve x	Writing the answers on the board
1.15	L1	Then we are going to ... to solve this (*referring to the answer)*	She then solves it quietly on the board
1.16	L1	eight x plus twelve x equals to twenty x and minus four plus two equals to …	Writing the final answer on the board and sits down

6.4.1 Word use and visual mediators

Reporting on the words used by the teacher when teaching learners how to expand an algebraic expression, Line 1.1 showcases the teacher's discourse when asking learners about the mathematical operation that must be used when they are expanding. In Line 1.3, the teacher further elaborates by saying that 'I will show you how multiplication is done when we expand: $(7-3x)(2+x)$'. The choice of words used by the teacher in lines 1.1 and 1.3 indicates words that are mathematical in nature, that is, mathematical vocabulary found in the mathematics literature. However, in Line 1.4, the teacher used 'this' and 'that' to refer to the first bracket $(7-3x)$ and second bracket $(2+x)$, respectively, when explaining to learners that 'each term in the first bracket will multiply every term in the second bracket'. The teacher used pronouns to refer to mathematical expressions or entities in an expression.

Additionally, in lines 1.5 and 1.6, the teacher uses the word 'multiply' and 'times' interchangeably to mean the same thing, that is, to find the product; however, these are two words that are used in mathematics to mean the same mathematical process. In Line 1.12, the teacher introduces a new phrase, 'like terms', when mentioning that they must collect like terms together because they have $7x - 6x$ as 'like terms'. The word 'collect' is a process verb that is used to explain a process that should occur and, in this case, the teacher used the word to mean the process of 'simplifying' the like terms. From the teacher's interactions, I have identified the following vocabulary: (1) words that are mathematical in nature, i.e., *expand, mathematical operation, multiplication, term, bracket, times, plus, squared* and *like terms*; and (2) words that are not mathematical in nature but have the same meaning or are used to mean the same thing in mathematics (colloquial words), i.e., *this, that, we get* and *collect*.

After demonstrating to learners, the teacher gave them an expression to expand individually $4(2x - 1) + 2(3x)$. As she walked around the class to see how learners are responding to the task, one learner was requested by the teacher to explain her working on the board when expanding the given expression. This interaction helps this study to explore the learner's thinking when expanding the given expression, as well as how the learner communicates her thinking, the communication of the learner and herself to the entire class (Sfard, 2008). Line 1.13 showcases the learner's word use when expanding the expression and she started by mentioning that 'I must first "solve this number"' and by 'this number' she was referring to multiplying the outside parentheses number with those inside the bracket $4(2x - 1)$. The use of the word 'must' and 'this' indicates the learner's use of strong verbs of high modality to indicate obeying the teacher's instruction, as well as using pronouns to refer to entities, respectively.

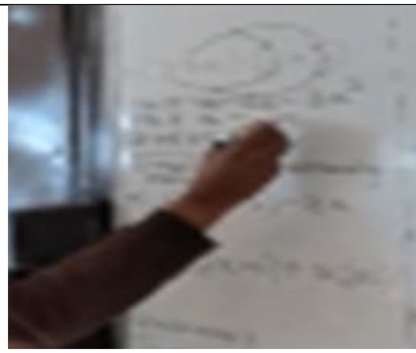
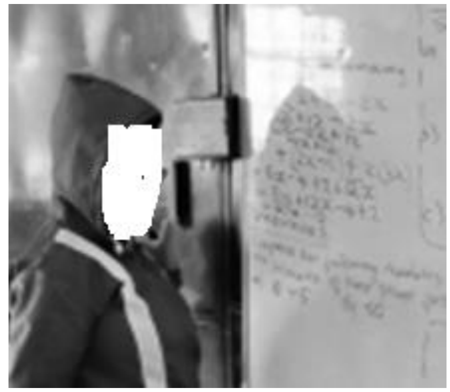
In Line 1.14, the learner uses the words 'times' and 'minus' when explaining how she is doing the expansion with the operations involved in the process. However, in Line 1.15, the learner uses the word 'to solve this' referring to the 'like terms', while in Line 1.16. the phrase 'equals to' was used by the learner. The repetition of the pronoun 'this' by the learner when talking about mathematical entities should be noted. The words used by the learner can be categorised as: (1)

words that are mathematical in nature, i.e., *equals, multiply, solve, times, plus* and *minus*; and (2) words that are not mathematical but used in mathematics to refer to mathematical terms (colloquial words), i.e., *must* and *this'.* According to Miller (2007), learners need to manipulate vocabulary in a variety of ways in order to bring about an increased retention of such words and, as such, repetition is crucial for the retention of new vocabulary. However, in Excerpt 1 this was not the case. The teacher did not instil the idea of repetition of the new vocabulary into the learners thinking, as in the case on Line 1.12 when introducing the vocabulary 'like terms'. It is also not surprising that in lines 1.13 to 1.16, even though the learner had like terms, she did not make use of the mathematics vocabulary that was used by the teacher but instead opted to use the phrase 'to solve this' (Line 1.16) when referring to the process pronoun of simplifying like terms. This is supported by Stahl and Fairbanks (1986) who indicated that learners must be exposed to mathematics vocabulary at least seven times over spaced intervals in order for retention to effectively occur.

Below I present the teacher and L1's visual mediators on the board (learner's written responses to the question discussed in Excerpt 1) that accounts for the mathematics vocabulary used in Excerpt 1. **Table 4** showcases both the teacher's and learners' written responses during their interaction while explaining how to expand the expression to the learners. This will explain how word use accounts for learners' mathematical thinking (Kim et al., 2017; Lestari et al., 2020; Sfard; 2015).

Table 4: Teacher and L1's written responses transcribed from the board

Line	Visual mediator Teacher	Visual mediator Learner
V1.1	$(7-3x)(2+x)$	$4(2x-1)+2(3x)$
V1.2	$14+7x-6x-3x^2$	$8x-4+2+12x$
V1.3	$14+x-3x^2$	$8x+12x-4+2$

V1.4		$20x-2$ 

Looking at the teacher's word use and visual mediators, the type of words used by the teacher can be accounted for in the visual mediators. In Line 1.1, the teacher talks about a question that instructs the class to expand and the teacher asks which mathematics operation is used for expanding. The choice of words here automatically informs involved parties that whenever the question says you must expand then you must think about the mathematical operation called multiplication. However, the teacher did not talk about what type of a question will require the learners to expand, it was supposed to be clearly indicated that we expand expressions (as in V1.1 in **Table 4**).

In V1.2, a result of what the teacher was talking about when she was saying 'each and every term in "this bracket" will multiply each and every term in "that bracket"' is showcased. Mathematically, this would mean using the distributive property to expand the terms in the two brackets. In Line V1.3, it was imperative for the teacher to explain intensively what 'like terms' are and further elaborate on the process of simplifying like terms, that is, to group them first before simplifying them. However, the teacher opted to use the words collect like terms to mathematically mean simplifying like terms. L1's word use differs from the teacher's word use; this is observed where L1 talks about solving 'this number', referring to using distribution property of multiplication to expand the expression $4(2x-1)$ before using the same property to expand the second one $+2(3x)$. However, in V1.2 the learner used the outside term of the first expression to expand the second expression, which was mathematically wrong. In V1.3, the learner referred to simplifying like terms as ‘solve this’.

6.4.2 Endorsed narratives and routines

In this section I use word use and their intended visual mediators from the teacher’s and learners’ discourse in order to categorise their narratives as descriptions of entities and the relations between them or narratives about actions with or by entities. Furthermore, I use the teacher’s discourse to construct a visual representation of the discursive object in order to argue whether the teacher’s discourse is objectified or not. I then look at word use, visual mediators, narratives and the realisation tree in order to identify the teacher’s routine, which will help me to conclude whether the teacher’s narrative is endorsed or not. Below I present the realisation tree (**Table 5**) for the teacher only. I chose to use the teacher’s discourse because the learner’s written response to the task did not give her the correct answer and, as such, it will be discussed in the next paragraphs with referent to the learners’ discourse and visual mediator, also substantiating what the realisation tree would look like.

The teacher’s discourse from Excerpt 1 in Line 1.1 and the corresponding visual mediator in V1.1 produced narratives that are endorsed. Similarly, V1.2 and Line 1.3, where the teacher explains that each term in the first bracket must multiply each and every term in the second bracket, results in the construction of endorsed narrative; that is, the narratives are about the description of entities and the relations between the entities. This is also supported by the realisation tree on Table 5 (nodes 1a to 1f) that the teacher’s discourse was objectified. The application of the distributive property of multiplication indicates that there is a relationship between the entities and this would mean that the narrative originates from explorative routines because the teacher’s discourse was objectified. However, in lines 1.8 and

1.12, the teacher talks about the ‘plus’ and the ‘seven’ as well as the ‘minus’ on the three x $(-3x)$ times a ‘plus’ on the x $(+x)$ in a disobjectified way (node 1e) and, hence, the narrative produced is not endorsed. It is also because the teacher talks about minus ‘–‘ and ‘$3x$’ as two different parts of an entity, which signifies learners’ interaction producing narrative about actions with or by entities (Sfard, 2008, 2016), and, hence, the routines are ritualised originating from spatial arrangement. In Line 1.12, the teacher talks about collecting like terms, which in V1.2 and V1.3 indicates narratives that are endorsed. This is because the teacher’s discourse shows that the origin for the narratives is from horizontal equivalence and, hence, the teacher’s interactions are about a description of entities and the relations between the entities. This signifies the routines as explorative (Node 2a).

Table 5: The representation of the teacher’s realisation routine and tree for the solution to the expression $(7-3x)(2+x)$

Nodes	Realisation Routine	Visual mediators	The solution to the equation $(7-3x)(2+x)$	
1a	Scan the expression to gain sense of which entity should multiply which other entity	$(7-3x)(2+x)$	$7(2+x)-3x(2+x)$	1a
1b	Multiply 7 and 2 and write ‘14’ as first term on line 2	$(7-3x)(2+x)$	(7×2)	1b
1c	Multiply 7 and x and write $7x$ as the second term on line 2	$(7-3x)(2+x)$	$14+(7\times x)$	1c
1d	Multiply $3x$ and 2 and write $6x$ as the third term on line 2	$(7-3x)(2+x)$	$14+7x-3(x\times 2)$	1d
1e	Calculate – times a plus	$(7-3x)(2+x)$	$14+7x-$	1e
1f	Multiply $3x$ and x and write $3x^2$ as the fourth term on line 2	$(7-3x)(2+x)$	$14+7x-6x-3(x\times x)$	1f
			$14+7x-6x-3x^2$	**Realisation 1**
2a	Simplifying like terms $+7x$ and $-6x$	$14+7x-6x-3x^2$	$14+x(7-6)-3x^2$	2a
	Calculating $+7x$ *and* $-6x$ **(Realisation 2)**	$14+x-3x^2$	$14+x-3x^2$	**Realisation 2**

For the learner's discourse, I used Excerpt 1 and the corresponding visual mediator to argue about the narratives and routines of the learner. Furthermore, I did not construct the realisation tree for the learner because the learner's discourse did not warrant realisation from the visual mediators. Looking at L1's interaction in lines 1.13 and 1.14 and the visual mediators in V1.1 and V1.2 to construct narratives, I argue here why I conclude that the narratives produced are not endorsable. This is because the learner explains that she will be solving 'this number', referring to applying the distributive property of multiplication to expand the expression, even though the second expression was not correctly expanded. The inability of the learner to expand correctly signifies that the narratives are produced as a result of ritualised routines. The learner was unable to recognise the unendorsed narratives and also did not correct them. In lines 1.15 and 1.16, the learner was able to account for what she means when she said 'solve this', which, according to the visual mediators in V1.3 and V1.4, showcases the learner's ability to simplify like terms. This indicates that the learner's interaction produces narratives that are endorsed. In this case, the origin of the narrative is horizontal equivalence, which signifies explorative routine; however, the overall narrative is not endorsed because of the learner's inability not to recognise unendorsed narratives when expanding.

6.5 ANALYSIS ON FACTORISATION

In this section I showcase interactions that took place when the teacher was questioning learners on how they would factorise the equation $x^2 + x = 12$. I use Excerpt 2 below to represent the learners' discourse that took place during this lesson; however, it should be noted that the excerpt is not a representation of the entire lesson. I use Excerpt 2 to represent the discourse that took place during the classroom teaching and learning (Setati, 2003). The teacher's focus in this lesson was to get learners to explain how they factorise an equation. The teacher started the lesson by writing the equation on the board and initiated the discourse by questioning the learners how they will factorise it.

Excerpt 2

Turn	Speaker	What is said	What is done
2.1	T	We have x squared plus x is equal to twelve, then from there we have x squared plus x minus twelve is equal to zero then we factorise, somebody must come and factorise, who can factorise? Who can come and factorise?	Writing the equation on the board
2.2	ALL	Silence!!	Looking at the teacher silently
2.3	T	Who can factorise? Eh! Okay tell me how to factorise	Looking around the class
2.4	L1	We open brackets	Loudly
2.5	T	What are we putting inside the brackets	
2.6	L2	x	Says without raising a hand
2.7	T	x where? The first bracket I put what?	Pointing at the bracket
2.8	L2	x	
2.9	T	And also?	
2.10	ALL	The second bracket we put x	Pointing at the bracket
2.11	T	Then from x ?	
2.12	L1	We find the factors of twelve	Looking at the learner next to her
2.13	T	What are the factors of 12 let's mention the factors of 12	Looking at the learners
2.14	L1	Two and six	Says it lower
2.15	T	Then	
2.16	L2	Three and four	
2.17	T	The next one?	
2.18	L3	One and twelve	Scratching head
2.19	T	Then we have how many factors? Three, isn't it?	
2.20	ALL	Yes	
2.21	T	The which one suitable for this expression?	Ask looking around for a learner to raise a hand
2.22	L2	Three and four	
2.23	T	Why three and four	
2.24	L2	Two and six	
2.25	T	So you are changing now? Why two and six?	Asks L2
2.26	L2	Because two is small	Responding while rolling eyes
2.27	L3	Because it comes first	Looking at L2

2.28	T	Remember, the factor that you have to get must satisfy the	Pointing to the board factors and the middle number
2.29	ALL	Middle number	
2.30	T	The middle number, how can two n six satisfy the middle number,	Ask loudly
2.31		Silence!	
2.32	T	It means two and six is wrong, then which factors are we supposed to use?	Looking for a hand
2.33	L2	Three and four	Says it louder smiling
2.34	T	Three and four do we agree that the factors are three and four?	Uses hand to questions
2.35		Silence	
2.36	T	Then what are the signs inside the brackets, three and four, three minus four you can get one so what are the sign inside the brackets? Are the signs the same?	Pointing at the brackets and 3 and 4 on the board
2.37	L1	No	
2.38	T	If the signs are not the same, which sign will be negative or positive?	Points at L2 and L3
2.39	L2	The first one?	
2.40	L3	The second one	
2.41	T	Which one is the first one and which one is the second one?	
2.42	L1	Four	Pointing at L1
2.43	T	Why four is positive?	
2.44		Silence!	
2.45	T	The bigger number will take the sign of the middle number, the here will be x minus three and x plus four equals to zero	Writing on the board

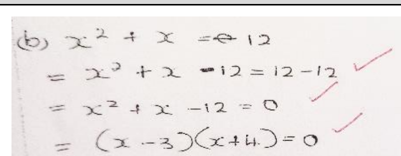
6.5.1 Word use and visual mediators

The classroom discourse in Excerpt 2 begins with the teacher (lines 2.1 and 2.3) interacting with learners about the given equation that requires them to factorise. In Line 2.4, L1 interacts with the teacher by explaining that we need to open brackets in order to factorise, which, in Line 2.5, the teacher seems to agree with and supports L1's answer by asking what are we putting in the brackets. The teacher uses the process verb 'putting' when talking about placing each x in each bracket. In lines 2.7 and 2.10, the teacher and learners talk about the first and second bracket, respectively, which is where x must be put in each case. At Line 2.12 the learners talk about finding the factors of twelve, at this point the teacher (Line 2.13) questions the number of factors we have for twelve.

Additionally, in Line 2.21, the teacher mentions that there are factors that are suitable for the expression they are solving, however, in Line 2.28, she insists that the factors 'must' satisfy the 'middle number' and the learners agree with their teacher's statement (Line 2.29). The use of the high modality verb 'must' indicates the teacher's role as a person of authority. In lines 2.36 and 2.38, the teacher talks about whether the 'signs' in the brackets must be the same or which should be negative or positive. The teacher here uses a colloquial noun to refer to mathematical operations. From Excerpt 2, I can argue that the teacher and learners use the following vocabulary: (1) words that are mathematical in nature, i.e., *factorise, equal, squared, factors, expression, plus, minus, brackets, negative* and *positive*; and (2) words that are not mathematical in nature (colloquial words), i.e., *middle number, suitable, satisfy* and *signs*.

For meaning making, I present below the visual mediators that demonstrate the working of the problem in order to account for learners' and teacher's thinking through word use. **Table 6** showcases such visual mediators.

Table 6: Visual mediator for the teacher on the board and L2's written documents

<table>
<tr><th>Line</th><th>Visual mediator</th><th>L2's visual mediator</th></tr>
<tr><td>V2.1</td><td>$x^2 + x = 12$</td><td rowspan="3"></td></tr>
<tr><td>V2.2</td><td>$x^2 + x - 12 = 0$</td></tr>
<tr><td>V2.3</td><td>$(x - 3)(x + 4) = 0$</td></tr>
</table>

The teacher's conversation in Line 2.1 talks about the 'equation', however, in V2.1, she simply mentions that we have a new equation that we 'must' factorise (V2.2). The interaction with the learners never provided a mathematical explanation of why we no longer have twelve on the right-hand side, which is now on the left-hand side and is negative, where now the equation is equal to zero. She then concludes by saying that this equation (V2.2) allows us to factorise, referring to the equation as 'this'. However, L2's visual mediator begins with an equal sign just below the provided equation which demonstrate that the learner is only focusing on finding a solution to the provided equation. The learner further demonstrates two equal signs on one equation before demonstrating that an additive inverse operation being applied to obtain a standard equation $x^2 + x - 12 = 0$. It is evident that to the learner, there is no meaning attached to the equal sign when solving the equation provided. Mathematically the teacher was supposed to explain the mathematical reason of changing the equation to a standard form so that we could be able to factorise and, hence, without that explanation, the teacher's thinking was not communicated to the learners.

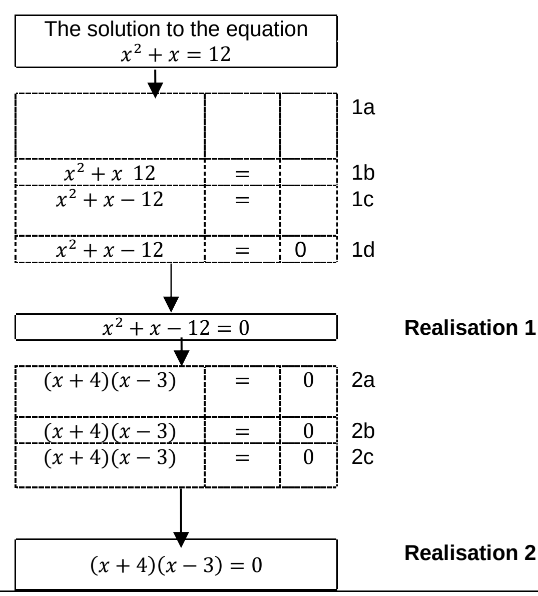
L1 in Line 2.4 talks about opening brackets in order to factorise, which V2.3 showcases as two brackets being equal to zero. However, the teacher agrees with L1 in Line 2.7 where she asks 'what are we putting in the brackets?' L2 seems to be well aware that we put x and x in the brackets, however, it is not mathematically substantiated why two brackets and, mathematically, it means that we are doing a reverse process of expanding a double bracket expression that gives a quadratic equation. In Line 2.12 the learner (L1) mentions that we need to find the factors of twelve in order to also put them in the brackets and L1, L2 and L3 managed to provide the factors. However, it was challenging to decide on which factors to use. In Line 2.21 we see the teacher asking which factors are 'suitable' for the 'expression' and also substantiates that the factors 'need' to 'satisfy the middle number' (lines 2.28 to 2.30). Lastly, from Line 2.36 to Line 2.45, we see the argument being that now the factors that satisfy the middle number are three and four, then which of the two numbers will be positive or negative or whether both will be negative or both be positive. L2 argues that three (Line 2.39) must be positive, however, L3 and L1 (in lines 2.40 and 2.42) disagree, saying that four must be positive, which the teacher in Line 2.45 agrees with when concluding the discussion by arguing that the bigger factor will take the sign of the middle number, as perceived in V2.3.

6.5.2 Endorsed narratives and routines

I also report on the identified words used and their related visual mediators in Excerpt 2 and show how the interactions constructed narratives that are endorsed or not. I then use the literature to argue whether the narrative is regulated by routines that are explorative or ritualised (Roberts, 2016; Sfard, 2008, 2016). Below I present a realisation tree (**Table 7**) for the teacher and learner discourse in order to validate the narrative they have constructed, and use the realisation tree, visual mediator and word use to identify their routines.

Table 7: The representation of the teacher's and learners' realisation routine and tree for the solution to the expression $x^2 + x = 12$

Nodes	Realisation Routine	Visual mediators
1a	Scan the expression to gain sense of which entities belong to the RHS and which belong to the LHS	$x^2 + x = 12$
1b	Write $x^2 + x$ and 12 on the LHS line 2	$x^2 + x = 12$
1c	Change the sign in front of 12 from + to –	$x^2 + x = +12$
1d	Write 0 on the RHS line 2 **Realisation 1**	$x^2 + x - 12 = 0$
2a	Opening two brackets for reverse expanding	$(x+4)(x-3)$
2b	Putting x in each bracket	$(x+4)(x-3)$
2c	Putting in factors of 12 that satisfy the middle number	$(x+4)(x-3)$
3a	Putting signs that will ensure that the factors give us a + middle number **(Realisation 2)**	$(x+4)(x-3)$



The teacher's interaction in Line 2.1 (V2.1 and V2.2) produces narratives that are not endorsed seemingly because the teacher demonstrates moving 12 from the right-hand side to the left-hand side, and changing it from being positive to become negative (nodes 1b to 1c). This narrative originates from spatial arrangements. In fact, for the equation to be in standard form, an additive inverse operation was supposed to be applied. The teacher's interactions on spatial arrangement signifies routines that are ritualistic because the narratives are about actions with or by entities. Learners' interactions from Line 2.4 to Line 2.11, when they talk about opening two brackets as well as putting x in each bracket, produces narratives that are endorsed, originating from vertical equivalences (Nodes 2a to 2b). This also signifies explorative routines because the learners talk about the relationship between the entities they are narrating about. They apply a reverse process of expansion in order to understand that the quadratic equation needs two brackets to be factorised.

Similarly, from Line 2.12 to Line 2.34, when learners and their teacher were interacting about finding the factors of twelve in order to look for one that satisfy the middle number on the equation, produced narratives that are not endorsed. This is because the origin of the narrative is spatial arrangement, which signifies ritualistic routines. The interaction produces narratives that describe how the constant 'twelve' can be related to 'middle number' 'x' by using factors of twelve (Node 2c). This narrative separates '–12' in to two parts of '12' and '–', and '$+x$' as two parts because they were supposed to look for factors of '–12' that will give them '$+1x$'. From Line 2.38 to Line 2.45, the interactions produce narratives that are not endorsed due to the fact that the origin of the narrative is a spatial arrangement because learners talk about entities in a disobjectified way (Node 3a). The interactions about the signs and the factors to be put into the brackets are described as two different parts and not as one. This signifies that the routine produced by the narrative is ritualised and it is an indication that learners' interaction produces narratives about actions with or by entities (Sfard, 2008, 2016).

6.6 ANALYSIS ON FINDING THE VALUE OF x

Excerpt 3 below captures classroom interactions that emanated when the teacher was solving an algebraic equation that involved a fraction as part of the equation with the learners. I chose Excerpt 3 to represent the classroom interaction because it has the ability of contributing to answer the research question, and the discussion expected learners to make use of two important mathematical vocabulary, namely ‘additive inverse’ and ‘multiplication inverse’ to explain their thinking when solving the equation. The classroom discussion was initiated by the teacher who gave the learners an equation $\frac{3x+7}{2} = 5$ and informed the learners that they should collectively solve for x as a class, however, they must come up with the first step. Discussion that followed is captured in Excerpt 3 below.

Excerpt 3

Turn	Speaker	What is said	What is done
3.1	T	We have three x plus seven divided by two equals to five, lets solve for x, what is the first step, remember two is the denominator, what is the first step if two is the denominator	Writes on the board while talking to learners
3.2	L1	We take two *ra multiplaya* (*and multiply)* five *yela kwa (there*)	Explains while Pointing on the board
3.3	T	*Bare* (*they say)* two must multiply five, two multiply five why? This is the denominator, *gora gore* (*it means*) the denominator must multiply five, two multiply by five?	Pointing at the denominator on the board
3.4	L3	Two multiply three x and five	
3.5	T	Bare two will multiply three x and will also multiply five and omongwe are (someone is saying) two must multiply five, which one is correct?	Looking at learners from all sides
3.6	L3	Two must multiply three x and also multiply five	
3.7	T	Oh! two must multiply three x and also multiply five, serious? This is the denominator and the denominator must go and multiply because on this one (*referring to left hand side of the equation*) we have three x plus seven equals to two times five, then raba (*we have*) le three x plus seven equals to ten, then what will happen now?	Points and circles the denominator two, demonstrating to where it must multiply and working on the board while explaining
3.8	L1	Three x plus seven minus seven	

3.9	T	Doing the same here (right hand side of the equation) we will have three x plus seven minus seven equals ten minus seven, then *ra feleletja re naleng* (*we end up having*) three x equal to? what is 10 -7?	Working on the board
3.9	L3	Three	
3.10	T	Three *akere* (*right)*, remember we are solving for x, then what will happen now? Because we do not have x we have three x then what will happen now?	Point to the 3x
3.11	L2	We add	
3.12	L1	The three will go to the other side and be minus	Pointing on the board
3.13	T	We add? Or This three will move to the other side and become negative? Serious? What will happen now? We solve for x, now we have three x equal to three what will happen	Questions again
3.14	L3	x is one mam	Says louder looking at the teacher
3.15	T	Remember when we solve for x and we find that the coefficient of x is more than one, is not one we divide by the coefficient of … we divide by the coefficient of what?	Writes on the board demonstrating the explanation
3.16	ALL	x	
3.17	T	We divide by the coefficient of what? We divide by the coefficient of x and three is the coefficient of x, whatever you do on the left-hand side …?	Divides on the board
3.18	ALL	You must do on the right-hand side	
3.19	T	You also do on the right-hand side, the wen we divide by three here, three divide by three, then x is equal to	Solves the problem on the board
3.20	ALL	One	

6.6.1 Word use and visual mediators

From Line 3.1 to Line 3.7, the teacher's discourse begins with an instruction where by learners are expected 'solve for x'. The word 'solve' is a mathematical word that gives learners direction about what they should do with what is provided to them. The word itself communicates with learners' thinking about their next mathematical procedure. However, the teacher also reads the equation to learners as 'three x plus seven divided by two equals to five' and further clarifies that the use of the word 'divide' by two in the equation means that two is a 'denominator'. Furthermore, the teacher uses the phrase 'must go' as a mathematical process that involves a verb with high modality and this is observed where she says 'two must go and multiply five because two is the denominator' (Line 3.7). Also in Line 3.7, the teacher refers to the phrase 'this one' as the fraction '$\frac{3x+7}{2}$'. The colloquial word 'this' was used as a noun to refer to a fraction.

In Line 3.9 we see the teacher referring the left-hand side of the equation as 'here' and in Line 3.12 she refers to the word 'move' as a mathematical process when saying 'this three will move to the other side and become negative'. I also note the use of the phrase 'the other side' when referring to the right-hand side of the equation. Similarly, during the discourse of the teacher made use of adverbs and nouns to refer to mathematical objects. Lastly, in lines 3.14 and 3.16, I note the use of the word 'coefficient; when referring to the coefficient of x. I can argue conclusively that the teacher's interaction with the class constitutes a mathematical classroom discourse. However, the mathematical vocabulary that was used here is as follows: (1) words that are mathematical in nature, i.e., *solve, divide, equals* and *coefficient, denominator*; (2) words that are not mathematical but were used during the discourse for the same mathematical purpose, i.e., *must go, this one, here, other side* and *move.*

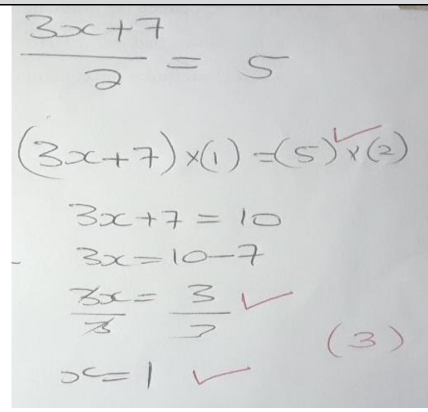
Learner's interaction with the educator started in Line 3.2, where by L1 started her interaction with the teacher by saying 'we take two and multiply five there'. Similarly, L1 here also uses the same vocabulary as her teacher in referring to the right-hand side of the equation as 'there' and used the word 'take' to refer to the process of moving two, which are adverbs. From Line 3.4, L3's conversation included the use of the word 'multiply', which was similar to the words used by the teacher. However, in Line 3.8, L1 introduces the word 'minus' when explaining how to solve $3x + 7 = 10$, while in Line 3.12 L1 mentions that 'three will go to the other side and be minus'. What interests me here is how L1 uses the words 'will go' and the 'other side' when explaining how to solve $3x = 3$. The former was used to refer to the right-hand side, while the latter was used as a mathematical process, thereby disagreeing with L2 in Line 3.11 when saying ‘we must add’ in order to solve for x. Similarly, ‘will go’ symbolises a verb for a process and ‘other side’ as a place.

Learners’ interactions with each other and their teacher constitute a mathematical discourse because the choice of words that learners used in their discourse were aimed at explaining mathematical processes. I can, therefore, report that learners used the following vocabulary in their mathematics discourse: (1) words that are mathematical in nature, i.e., *minus, multiply, add* and *equals to;* (2) words that were not mathematical in nature but are used in mathematics to fulfil the same mathematical purpose, i.e*., this, here, there*, *will go* and *other side.* These categories of vocabularies, if used for the same mathematical purpose, constitute what we call mathematical vocabulary and learners and teachers use them to explain what they think should happen to solve the mathematics problem at hand.

I now use visual mediators to make sense of words used by both learners and the teacher so as to account for their meaning of the words. Later, I argue whether the teacher and the learners word used constitutes what I call mathematical vocabulary. It is with visual mediators that I can be able to argue whether the word use, when looked at with its corresponding visual mediators, does contribute to the learners mathematical thinking and communication. I use **Table 8** to demonstrate my understanding of the learners thinking and communication based on the words used from their discourse in Excerpt 3.

The classroom discourse in Excerpt 3 begins with the teacher reading the equation that learners need to solve aloud, as represented in V3.1, and immediately informs learners that the number ‘two’ in the equation is a denominator and challenges learners to think and communicate what they think the next step is in order to solve for x. However, the manner in which the teacher spoke about the two as a denominator does not support learners’ mathematical thinking in a sense that whenever one talks about a 'denominator' one must also talk about a 'numerator' and linking the two as belonging to a 'fraction'.

Table 8: Visual mediator that was presented by the teacher on the board during discussion and L1's written documents

Line	Visual mediator	L1's written documents
V3.1	$\frac{3x+7}{2} = 5$	
V3.2	$2 \times \frac{3x+7}{2} = 5 \times 2$	
V3.3	$3x + 7 = 10$	
V3.4	$3x + 7 - 7 = 10 - 7$	
V3.5	$3x = 3$	
V3.6	$\frac{3x}{3} = \frac{3}{3}$	
V3.7	$x = 1$	

From Line 3.2 to Line 3.7, we can see L2 trying to explain her thinking that in order 'to solve the equation they must take two and multiply five', but L3 disagrees in Line 3.4 and insists in Line 3.6 that the 'two must multiply $3x$ and 5'. We also see the teacher clarifying the learners understanding by saying that the 'denominator must go and multiply five' and also multiply 'this one', and by 'this one' the teacher was referring to the fraction '$\frac{3x+7}{2}$'. I therefore, argue that, even though the words used correspond to the correct mathematical procedure (V3.2), the use of words does not justify the mathematical procedure used, which was supposed to be a multiplication inverse operation. From the mathematics literature, in order to solve an equation that involves a fraction, we must first solve the fraction by multiplying every variable term in the equation by the multiplication inverse of the denominator.

The visual mediator in V3.4 demonstrates an additive inverse operation being applied to simplify the equation. However, in Line 3.8 the learner only mentions that ‘we say three x plus seven minus seven’ and we see the teacher validating learner’s talk by saying ‘and we do the same ‘here’, referring to the right-hand side of the equation as ‘here’. The word ‘here’ was used mathematically to mean the right-hand side of the equation, even though it is not a mathematical word and it does not have the same meaning but yielded the correct answer, as is visualized in V3.5. In conclusion, V3.5 required learners to again apply a multiplication inverse operation to solve $3x = 3$ but the teacher used a different vocabulary to explain the process of this operation. In Line 3.12 the educator explains this way: ‘when we solve for x and we find that the coefficient of x is more than one, divide by the coefficient x’ (V3.6), which visually is mathematically correct.

6.6.2 Endorsed narratives and routines

I now present a realisation tree (**Table 9**) for the teacher and learners when, together, they were working on solving the equation below, which I use to argue about their narratives and routines. In Excerpt 3, the teacher identifies ‘two’ in the equation (V3.1) as a denominator after instructing the learners that they should solve for x. However, the teacher’s inability to communicate that the left-hand side of the equation has a fraction with a denominator of two renders the teacher’s narrative not endorsed. This is because the teacher talks about the denominator as a different part of the fraction on the left-hand side of the equation and, hence, the source of the narrative is spatial arrangement (Table 8, Node 1a), which signifies a ritualised routine (Berger, 2013; Sfard, 2016).

Table 9: The representation of the teacher's and learners' realisation routine and tree for the solution to the expression $\frac{3x+7}{2} = 5$

Nodes	Realisation Routine	Visual mediators
1a	Scan the expression to gain sense of which entities belong to the RHS and which belong to the LHS	$\frac{3x+7}{2} = 5$
1b	Write $3x + 7$ on the LHS	$\frac{3x+7}{2} = 5$
	Write 5 in the RHS	$\frac{3x+7}{2} = 5$
1c	Write 2 X 5 on the RHS	$\frac{3x+7}{2} = 5$
	Calculate 5 X 2 on the RHS **(Realisation 1)**	$3x + 7 = 10$
2a	Subtract 7 on the LHS	$3x + 7 = 10$
	Subtract 7 on the RHS **(Realisation 2)**	$3x + 7 = 10$
3a	Subtract 7 and 7	$3x + 7 - 7 = 10 - 7$
	Subtract 10 and 7 **(Realisation 3)**	$3x + 7 - 7 = 10 - 7$
4a	Identify the coefficient of x (3) as a divisor	$3x = 3$
4b	Divide $3x$ by 3	$\frac{3x}{3} = 3$
4c	Write 3 with divisor 3	$x = \frac{3}{3}$
	Calculate 3 divide by 3 **(Realisation 4)**	$x = \frac{3}{3}$

The solution to the equation $\frac{3x+7}{2} = 5$			
			1a
$3x + 7$	$=$		1b
$3x + 7$	$=$	5	1c
$3x + 7$	$=$	5×2	1d
$3x + 7 = 10$			**Realisation 1**
$2x + 7 - 7$	$=$	10	2a
$3x + 7 - 7 = 10 - 7$			**Realisation 2**
$3x$	$=$	$10 - 7$	3a
$3x = 3$			**Realisation 3**
$\frac{3x}{3}$			4a
x			4b
x		$\frac{3}{3}$	4c
$x = 1$			**Realisation 4**

From lines 3.2 to 3.7, learners' explanation of how the problem should be solved constitutes narratives that are not endorsed. I defend my argument using L3's explanation, when L3 says the 'two', which in this case is a denominator, must multiply three x and five, while L1 argues that the two must multiply five. The use of the word 'must' indicate that the routine originates from a person of authority, in this case the teacher. Similarly, the narratives demonstrate that learners are not able to recognise that there is a relationship between the left-hand side and the right-hand side of the equation but both learners talk about $3x + 7$ and 5 in a disobjectified way

(Nodes 1b to 1c). The source of the narrative here is a spatial arrangement because the learners use verbs with high modality and this demonstrates obedience to a person of authority. This was supposed to indicate their realisation that they must apply a multiplication inverse operation on both sides of the equation in order to solve the equation (Roberts, 2016). Therefore, the way the learners talk about the words signifies ritualistic routines, hence, the constructed narratives are about description of entities with or by entities (Roberts, 2016; Sfard, 2008).

I use lines 3.6 and 3.7 to look at word use of L1 and the teacher, as well as their corresponding visual mediator in V3.4 to defend that the interaction produces narratives that are endorsed. This is because the discussions (lines 3.6 and 3.7) are about the description of objects and the relations between the objects (Sfard, 2008). We observe this when the teacher and L3 use their own words to explain that 'to be able to solve the equation $3x + 7 = 10$, seven must be subtracted on both sides of the equation', which is an application of additive inverse operation (Node 3a). Furthermore, the origin of the narratives is horizontal equivalence, which renders the routines for this discourse explorative (Berger 2013; Roberts, 2016; Sfard, 2016). Similarly, the narratives constructed in Line 3.14 are endorsed narratives originating from horizontal and vertical equivalence. Here, the teacher uses her own words to explain that we apply the multiplication inverse operation in order to solve $3x = 3$, visualised in V3.6. The words that the teacher uses in this narrative describe the relationship between the entities as well as their description, which signifies routines that are explorative in the discourse (nodes 4a to 4c) (Roberts, 2016; Sfard, 2016).

6.7 ANALYSIS ON ALGEBRAIC EXPONENTIAL EQUATIONS

In this section, I start by presenting an analysis of Excerpt 4, which resulted from the lesson I observed that was on the laws of exponents. I used Excerpt 4 in the analyses to represent learners' discourse that took place in the classroom. However, it should be noted that the discourse in Excerpt 4 is not a representation of the entire lesson but captures parts of the lesson where the interactions emanated from one question in an activity given to learners that was based on laws of exponents. The teacher's focus in this lesson was to get learners to account with reasoning by explaining how they used the concept of 'like bases' to solve exponential equations. From the mathematics literature, I can argue that the activity required learners to realise that, when an exponential equation has the same bases on each side of the equal sign, the exponents must be equated. This also applies when the exponents are algebraic expressions (as was the case of Excerpt 4). In order for learners to account for their answers, the teacher started the lesson by writing the following exponential equation on the board: $3^{x+1} = 81$. In Excerpt 4, I showcase the interactions that resulted in the classroom as learners were accounting for their understanding as they were solving the exponential equation.

Excerpt 4

Turn	Speaker	What is said	What is done
4.1	T	We are solving the equation, this is an exponent isn't it, 3 to the exponent x plus one is an exponent, we are solving for… we are solving the equation using the laws of exponents. *Aker* (isn't it)?	Pointing at the equation on the white board
4.2	ALL	Yes!	
4.3	T	Remember when solving the equation using the laws of exponents what … what are we supposed to do? What are we supposed to do to get rid of that 81? Huh!	Explaining to learners and pointing at 81 on the equation
4.4	AL	Silence!	Looking at the teacher surprised!
4.5	T	Remember we want to change 81 to have which base?	Points out at 81 on the board
4.6	L1 & L3	Three	
4.7	T	To have base 3, how can we write 81 in terms of base 3?	Ask the question twice
4.8	AL	(Learner's whispering) …….	Learner's whisper with each other
4.9	T	3 to the exponent?	Asks the question twice
4.10	ALL	Silence!!	Learners look at their books
4.11	T	3 to the exponent?	

4.12	L3	Four	Shouts louder after looking at her book
4.13	T	Then from there, if the base are the same what is happening? Hugh!	
4.14	L3	We calculate the exponents	Learner says it louder
4.15	T	Oh! We calculate the exponents?	astonished
4.16	L3	Yes	confidently
4.17	T	Then what will happen to the base? Huh! We just calculate the exponents? What will happen to the base? What is happening to the base? We calculate the exponents, then what will happen to the base?	
4.18	L4	We add the base	
4.19	T	We add the base? What do we do to the base?	Loudly, Questions the learners
4.20	L4	We subtract the base	Answers with a lower voice
4.21	T	We multiply the base, we add the base, we subtract the base? If the base are the same we add the base? What are we doing to the base?	Points at the bases on the board
4.22	L5	We finish the bases	Looking down
4.23	T	We finish the bases?	Looking at the learner's side by side
4.24	T	What do we do to the base? Now the base are the same. Why do we do to end up looking at the exponents? Like we said we end up looking at the exponents? Like we said we end up looking at the exponents. Then what happened to the base? What did they do? They subtracted each other? They divided each other? What did they do?	Circles the exponents and underlines the bases
4.25	L3	They vanished	
4.26	L5	We change them into x + 1 and 4	
4.27	T	We change them in to x+1 and 4? *jebaan!* what happened to the base?	
4.28	L3	They vanished!	Uses hand to demonstrate they vanished
4.29	L2	We say x plus one is equal to four	Answers the teacher
4.30	T	Yes! We can say x plus one is equal to 4, then what happened to this 1 (*underlining the bases*)?	The teacher pointing at the number bases
4.31	L2	They subtract each other mam	Say it and lough
4.32	L3	They subtracted each other	Says looking at L2
4.33	L1	NO! ……..	Raises her hand
4.34	ALL	Laugh	

4.35	T	What happened to the base?	Pointing at L1
4.36	L1	Vanished!	Says looking down
4.37		They vanished? *Emongwe are* (*someone says*) they subtracted each other; we add the base? What really happened to the base?	
4.38	L1	They vanished	Uses hand to demonstrate vanish
4.39	T	They vanished *neh*? What happened to the base? Meaning if they vanish *retlo didirang* (*what will we do to them*)? *Retlodi ignora* (*are we going to ignore them?*) or what?	Explains with an unbelievable look
4.40	L2	*Retotlodi ignora* (*are we going to ignore them?*)	
4.41	L5	*Disepetje* (*the left)*	
4.42	T	*Retlodi ignora goba* (*are we going to ignore them?*)? *di vanishitje* (*they vanished*)? *Di dirile eng* (*what did they do*)? *Radi ignora akere* (*we ignore them right*)? *Bare* (*they say*) we ignore the base like after ignoring the base *re shala le eng* (*what will we be left with*)?	Questions learners walking around the classroom
4.43	L3	x plus 1 equal to 4 … (*the teacher interupts*)	Answers looking up
4.44	T	The exponents, x plus 1 is equal to 4, remember we are solving for x, let's go to the second step where you said we ignore the base, then from there we have x plus one equal to 4. Then what will happen?	Writing ting on the board explaining to the learners
4.45	L2	X plus 1 minus 1 is equal to four minus 1	
4.46	T	Then from there what will happen, 1-1 *ke* (*is*) 0 then on the other side what is 4-1 is …	Writing on the board
4.47	All	3	Says louder
4.48	T	Three, that's correct	Underlines the 3

6.7.1 Word use and visual mediators

The interactions in Excerpt 4 started with the teacher initiating a discourse in the classroom by asking the whole class a question. The discussion started with the teacher instructing learners (Line 4.1) that they are 'solving the equation' by referring to $3^{x+1} = 81$ as the equation which learners must solve. Subsequently the teacher also reminded learners that 'three to the exponent x plus one' visually as 3^{x+1} is an exponent. In Line 4.3, the teacher explains that, for the learners to solve the equation using the laws of exponents, they need to 'get rid of eighty-one', the word 'get rid' here was used as an action verb with high modality. Also in Line 4.5, the teacher repeats the statement but make use of a new word and mentions that eighty-one must now be ‘changed into a base’, the word ‘must’ is also an action verb with high modality.

Furthermore, the teacher repeats (Line 4.7) the statement by saying ‘we need to write eighty-one in terms of base 3’. The teacher’s choice of words during the classroom interaction influences how thinking and communication occurs during teaching and learning. From lines 4.1 to 4.7, I report the teacher’s used vocabulary as: (1) words that are mathematical in nature (colloquial words), i.e., *plus, solving, equation, exponent* and *base*; and (2) words that are not mathematical in nature but have the same meaning in mathematics, i.e.*, must, get rid, change* and *in terms of* (Lamb, 1980; Powell and Nelson, 2017; Shepherd, 1973).

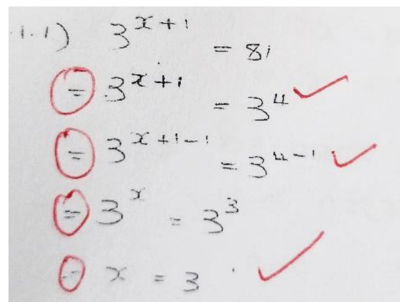
When looking at the learners’ responses during the classroom discourse, it was evident that learners word use was influenced by how the teacher was posing questions to them. Focusing on Line 4.14, I observed L3 responding to the teacher’s question ‘what do we do when the bases are the same?’ by saying ‘we calculate the exponents’. The choice of using the words ‘calculate’ the exponents was accounted for by the learner in Line 4.43 where the learner (L3) clarifies his meaning of the word by stating that we calculate the exponents by solving ‘x plus one equals to four', represented visually as $x + 1 = 4$. However, the statement does not explain what happened to the bases, as asked by the teacher. This statement (Line 4.43) also substantiates the learner’s use of words such as ‘the bases vanish’ (Line 4.25 and 4.28), as well as ‘they subtracted each other’ (Line 4.32).

To explain their understanding of the law of exponents, learners continued to use words such as we ‘add’, ‘subtract’ (L4, lines 4.18 and 4.20) and ‘finish the base’ (L5, Line 4.22), which are action verbs used to explain mathematical processes. Surprisingly, in Line 4.29, L2 agrees with L3 when saying ‘we say x plus one is equal to four’ ($x + 1 = 4$). However, his reasoning is different to that of L3. L2 retaliates that the bases have subtracted each other (Line 4.31) but later changes his mind to say ‘we ignore the bases’ (Line 4.40). However, L5 insists that ‘we finish them’ (Line 4.22) and later changes his statement to say that ‘the bases have left’ (Line 4.41).

From the learner’s interactions I report that the vocabulary used in Excerpt 4 can be categorised as follows: (1) words that are mathematical in nature, i.e., *exponents, add, subtract, divide, calculate, plus, bases* and *equal to*; and (2) words that are not mathematical but that are used in mathematics to mean the same thing (colloquial words), i.e., *ignore, vanish, left* and *finish,* (Lamb, 1980; Powell and Nelson, 2017; Shepherd, 1973). Similarly, as with the teacher’s word use of everyday English language, learners have also used words in order to make a meaning on the visual mediators that correspond to what has happened to the bases when they are now the same (Peng & Lin, 2019).

In order for me to understand how the identified mathematics vocabulary was used in the classroom, I present below L2’s visual mediators from documents (learner’s written responses to the question discussed in Excerpt 4), which accounts for the mathematics vocabulary used in Excerpt 4, as well as the class visual mediator on the board when working with the teacher. Word use and visual mediators serve as a medium for meaning making and I use them (words used and visual mediators) in **Table 10** to account for the learners’ mathematical thinking (Lestari et al., 2020; Sfard, 2008). I now look at how learners’ and the teacher’s use of mathematical vocabulary to accounts for learners’ mathematical thinking during mathematics teaching and learning. Table 10 showcases L2’s written responses (documents) and the visual mediator for the whole class discussion on the board as they engaged with their teacher.

Table 10: L2's written response to the question discussed in Excerpt 4

Line	Visual mediator (board)	L2's Document analysis
V4.1	$3^{x+1} = 81$	
V4.2	$3^{x+1} = 3^4$	
V4.3	$x + 1 = 4$	
V4.4	$x + 1 - 1 = 4 - 1$	
V4.5	$x + 0 = 3$	
V4.6	$x = 3$	

In Line 4.1, the teacher talks about 'Solving the equation, this is an exponent isn't it, "3 to the exponent x plus one' is an exponent", referring to '$3^{x+1} = 81$' as the equation to be solved and there is also an exponent to be solved at the same time. Additionally, in V4.1 the teacher further refers to 3^{x+1} on the left-hand side of the equation as an exponent. The teacher's meaning of an exponent in this case contradicts its related visual mediator and I use **Figure 1** below to substantiate my claim.

The Grade 7 – 9 Curriculum and Assessment Policy Statement (CAPS) states the following (P43 and P81):

> Learners need to understand that in the exponential form a^b, the number is read as a to the power of b, where a is called the base and b is called the exponent or index and b indicates the number of factors that are multiplied.

A visual mediator is as follows:

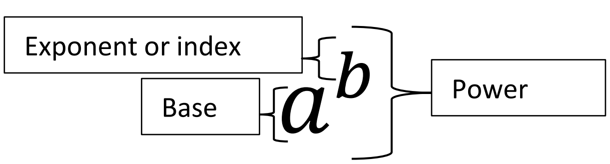


Figure 1: Visual mediator of an exponential form explained

Figure 1 illustrates a demonstration of the representation of the word use and visual mediators from the CAPS document, which I use to support the claim I made about the meaning of the mathematical vocabulary used by the teacher in Line 4.1. I defend my argument that the word ‘exponent’ (Line 4.1) is not a representation of its corresponding visual mediator in V4.1, as stated in mathematics literature. However, what this means is that the teacher’s thinking about the word ‘exponent’ and how it is communicated meant one and the same thing as its corresponding visual mediator. In lines 4.3, 4.5 and 4.7 I report about the teacher’s use of the words ‘get rid of’, ‘change’ and ‘write’ interchangeably when trying to make learners understand what the question asked means in a sense that the words have the same meaning (Ripardo, 2017). V4.2 indicates that the teacher’s meaning of the words is the same and, hence, we see 81 in V4.1 written as a power of three to the exponent four (3^4) in V4.2. The words (‘get rid of’, ‘change’ and ‘write’) have different meanings in everyday English language compared to how the teacher used them in Excerpt 4 to mean the same thing (Powell & Nelson, 2017). Hence, they have been used in a mathematically meaningful way.

In line V4.3, new words were introduced, as viewed in Excerpt 4 when learners were explaining why we equate the exponents ‘$x + 1 = 4$’ when the bases are the same. My interest during analysis was on the manner in which learners used the words ‘subtract’, they ‘left’, we ‘finish’, ‘vanish’ and ‘ignore’ bases to answer the question of what happens to bases when they are the same. Learners used any word that can explain the disappearing of the bases in V4.3 in order to substantiate why they have equated the exponents. For them to explain the bases disappearing in Line 4.3, they used words such as ‘divide’ and ‘subtract’ which in their thinking meant that, for the bases not to be there, they must either be subtracted or divided. Surprisingly, it was not the case for L3 (Line 4.28), L1 (Line 4.36), L2 (Line 4.40) and L5 (Line 4.42) because their thinking was that they vanished (L3 and L1) or they left (L5) or we ignored them (L2), which, in his documents analysis, we see him calculating the exponents without tampering with the bases. Although L1, L2 and

L3's thinking seemed to be a correct explanation of what the visual mediators showcases, the choice of words used have a different meaning in everyday English language, which in mathematics does not warrant a correct mathematical procedure, even though the explanation led them to correct answers. Furthermore, mathematics literature (one-to-one property of exponents) states that, when the bases are the same in an equation, the exponents are equal to each other (Abrahmson, 2014), which is the reason the learners were supposed to provide when asked what happened to the bases. It should however be noted that as it was the case in section 6.5.1 table 6, it is also the case here in table 10 for L2's visual mediator. The learner here started with an equal sign just below the provided equation ($3^{x+1} = 81$) which demonstrate that the learner is only focusing on finding a solution to the provided equation. The learner further demonstrates two equal signs on one equation before demonstrating that vertical equivalence is being applied to obtain the same bases. It is evident that to the learner, there is no meaning attached to the equal sign when solving the equation provided and the learner is only thinking about the closing condition (Sfard, 2008; Roberts, 2016).

6.7.2 Endorsed narratives and routines

The word use (in my case mathematical vocabulary) identified and their corresponding visual mediators guides the production of narratives and routines from Excerpt 4 in this section. The type of words that learners used in Excerpt 4 encouraged me to have an interest in order to understand what they really mean when using words such as 'vanish' and 'we "ignore" the bases' to substantiation what happened to the bases when equating their exponents, and I purposefully sampled L2 and L3 for interviews after the lesson (**Annexure I**), which I report on later in this section. Below I report on how the word use I extracted in Excerpt 4 and their identified related visual mediators are used to produce narratives (Sfard, 2008: Roberts, 2016). Below I present a realisation tree (**Table 11**) that represents the classroom discourse between the teacher and the learners as the solve the exponential equation.

Table 11: The representation of the teacher's and learners' realisation routine and tree for the solution to the expression $3^{x+1} = 81$

Nodes	Realisation Routine	Visual mediators	The solution to the equation $3^{x+1} = 81$	
1a	Scan the expression to gain sense of which entities belong to the RHS and which belong to the LHS and which entity needs to be transposed	$3^{x+1} = 81$		1a
1b	Write 3^{x+1} on the LHS	$3^{x+1} = 81$	3^{x+1} $=$	1b
1c	Transpose 81 to 3^4 on the LHS	$3^{x+1} = 81$	3^{x+1} $=$ 3^4	1c
1d	write $x + 1$ on the LHS	$3^{x+1} = 81$	$x + 1$ $=$ 4	1d
	write 4 on the RHS **(Realisation 1)**	$3^{x+1} = 81$	$x + 1 = 4$	**Realisation 1**
2a	Write $x + 1$ and –1 on the LHS	$x + 1 - 1 = 4$	$x + 1 - 1$ $=$	2a
2b	Write 4 and –1 on the RHS	$x + 1 - 1 = 4 - 1$	$x + 1 - 1$ $=$ $4 - 1$	2a
2c	Calculate 1 – 1 on the LHS	$x + 1 - 1 = 4$	x $=$ $4 - 1$	2c
	Calculate 4 – 1 on the RHS **(Realisation 2)**	$x = 4 - 1$	$x = 1$	**Realisation 2**

Looking at the word use and visual mediators from lines 4.1 to 4.7, the teacher's narrative explains that, when given an equation to solve and that equation has an exponent (3^{x+1}) on the left-hand side (LHS) and a whole number (81) on the right hand (RHS) side, in order to solve it 'we must get rid' or change or write the whole number in terms of a base, which is a use of a process verb with high modality. The teacher's narrative signifies number (81) and algebraic terms (3^{x+1}) as mathematical objects. This discourse, which is sourced from horizontal equivalences, signifies that the teacher talks about mathematical objects in an objectified way (Roberts, 2016; Roberts & Le Roux, 2019; Sfard, 2008). Similarly, from Line V4.1 to Line V4.2 we observe 81 being transposed to 3^4, which also signifies that the discourse as objectified because the teacher's narrative indicates the vertical equivalence of 81 and 3^4 (Roberts, 2016). I, therefore, argue that the source of the narrative of the teacher from lines 4.1 to 4.7, which is also visually mediated in V4.1 and V4.2, is from horizontal and vertical equivalences, which signifies endorsed narratives (nodes 1a to 1c). The teacher's use of the horizontal and vertical equivalences demonstrates the teacher's ability to use routines that are explorative when solving equations. Furthermore, Sfard (2016) argues that mathematics discourse that produces endorsed narratives is regulated by explorative routines because their constructed narratives are about description of entities and relations between entities.

I now showcase narratives that are constructed looking at the words used by learners and also looking at their related visual mediators as well as an interpretation of the learners' discourse showcased in Excerpt 4. Learners' choice of words in Excerpt 4 is used to explain the corresponding visual mediator for the mathematical procedure they applied, which is observed from step V4.2 to step V4.3 when the learners were solving the equation that was influence by the question posed by the teacher, as the person of authority, in Line 4.13 (Sfard, 2008). I then used the learners' interaction to construct the following narratives about their arguments, 'when the bases are the same, we ignore the bases and calculate the exponents' (Node 1c).

From the interview I conducted with L2, much of the word use was clarified, that is in Line I.1.10, he mentions that, when they say solve the equation according to his understanding, they actually wanted him to find the value of x, which, if substituted on the Left-hand side, will be equal to 81. This is an indication that the learner uses mathematical vocabulary objectively. Similarly, in Line I1.10 he further brings in the word 'ignore' the bases simply because they are the same and his reasoning was that it is because 'we equate the exponents, we make them an equation' (Line I1.12). The learner here again uses mathematical vocabulary objectively as he talks about the descriptions of mathematical objects and the relationship between them. Lastly, in Line I1.11, I introduce the word 'vanish' in order to get his understanding about the manner in which it was used in the class, and he agreed that vanishing means equating the exponents and making them an equation because we only talk about the exponents.

However, when L3 in Line 4.15 said 'we calculate the exponents', he was actually, according to L5, narrating that we change the bases into their exponents (Line 14.6). This is an indication that L3 and L5, in their narratives, talk about the base (3) and algebraic exponent ($x + 1$) independently in a disobjectified way, where they separate the integer (3) and algebraic terms ($x + 1$) (in this case, an exponent) in to two parts, while they were supposed treat the two as one mathematical object 3^{x+1}. I, therefore, make a claim that the source of the narrative in this case is spatial arrangement of the power. The narrative I constructed in Line 4.45 from L2's discourse supports the claim I made because L2's narrative indicates that the bases have left, i.e., they are separated from their exponents (Node 1c). This narrative indicates that the learner (L2) talks about and acts on entities in a disobjectified way, which signifies that the narratives is not endorsable. The learners use of spatial arrangement is an indication that learners made use of ritualised routines when solving the equation and this signifies that the constructed narratives are ritualistic (Sfard, 2016; Roberts, 2016; Roberts & Le Roux, 2019). Furthermore, ritualistic narratives are also an indication that learners' interaction produces narrative about actions with or by objects (Sfard, 2008).

However, in lines 4.29 and 4.40, I looked at L2's usage of words to explain the corresponding visual mediators and constructed the following narrative: 'when the bases are the same, we say the exponent on the RHS is equal to the exponent of the RHS and ignore the bases' (Node 1c). The source of this narrative is from horizontal equivalence because the learner talks about the powers (3^{x+1} and 3^4) in an objectified way (Roberts; 2016). This source of narrative indicates that the mathematical discourse is explorative and is an endorsed narrative, which categorises the narrative as a narrative about description of entities and relations between entities (Sfard, 2008, 2016).

Moving on with the learners' discourse, I showcase how I used L2's words use (Line 4.45) and the intended visual mediator (V4.4) to construct the learner's narrative when solving the equation $x + 1 = 4$. The source of the narrative is from horizontal equivalence because the narrative is about the description of entities and relations between entities. L2 narrates that, in order to solve the equation in V4.3, we must subtract one on the right-hand side and left-hand side, which leaves the answer as $x = 3$.(nodes 2a to 2c). The manner in which L2 talks about mathematical objects renders the discourse objectified, which categorises the discourse as explorative that produces endorsed narrative. The source of the endorsed narratives (in Line 4.45) is what Sfard (2008) refers to as narratives for the realisation of signifiers. I, therefore, argue that the equation $x + 1 = 4$ signifies the realisation that $x = 3$ after the learner applied additive inverse operation. My argument then proves that the property of horizontal equivalence as the source of the narrative was preserved in the equation, rendering the narrative explorative (Roberts, 2016; Roberts and & Le Roux, 2019; Sfard, 2008;).

6.8 SUMMARY

In Chapter 6, I have presented data using excerpts, looking at the three activities that focused on algebraic expressions and equations, from which a few items were chosen for analysis of the activities. I analysed and interpreted the data in order to identify the mathematics vocabulary that teachers and learners use in the classroom during mathematics classroom discourse, as well as how the vocabulary used influences learners mathematical thinking and communication. To do this, I first focused on the word use and their intended visual mediators for meaning making. Secondly, I focused on endorsed narratives and routines using word use, visual mediators and realisation trees constructed form the teacher and learners' discourse.

During analysis I also accounted for the features of language that are imbedded in the teacher and learners' mathematical vocabulary and the origin of their narratives in order to identify the regulated routines. As acknowledged during the introduction of this study, learners use every day English language as well as mathematical language in the classroom to communicate about mathematical objects. Therefore, both the use of everyday English language and mathematical language was looked at as mathematical vocabulary as long as the words are used to make meaning in mathematics discourse. It is very clear that fluency in mathematics language largely depends on one's mastery of the mathematics vocabulary which in fact initially originates from one's everyday spoken English language.

CHAPTER 7: ANALYSIS OF DISCOURSE ON GEOMETRY OF STRAIGHT LINES:

7.1 INTRODUCTION

In this chapter, I present the second part of my analysis of data following the commognitive framework developed by Sfard (2008), which I discussed at length in Chapter 2. Similarly, as in Chapter 6, the analysis of data here also focuses on the four constructs of Sfard's (2008) commognitive theory that I adopted as the lens through which I viewed the data. However, in this chapter, learners' activities concerned the geometry of straight lines.

7.2 FOCUS OF ANALYSIS IN THE STUDY

As stated in Chapter 6, the analysis focuses on two issues: first, the mathematical vocabulary that learners and their teacher use during mathematics classroom discourse; and secondly, on how the words identified are used to influence learners' mathematical thinking and communication. The data here is presented according to discussion activities on the topic of geometry of straight lines. The first discussion activity focused on solving geometric problems involving angles in triangles and quadrilaterals using known properties of triangles and quadrilaterals, as well as properties of congruent and similar triangles. The second discussion activity focused on solving geometric problems using the relationship between pairs of angles formed by perpendicular lines, intersecting lines and parallel lines cut by a transversal. I choose to present the data in this way so that coherence of the mathematical classroom discourse that took place in the classroom, as learners and the teacher interact in the activities, is maintained.

It should be noted that not all interactions were captured here, however, only excerpts that have the ability to showcase aspects of the mathematical classroom discourse considered important for me to answer the research questions were selected. Additionally, Ayaß (2015) advices that the decision about what to transcribe should be determined by looking at the research questions that the study seeks to answer so that each transcript is produced for a specific purpose. The analysis is presented in two parts, each part represents data generated from a discussion activity. Each part of the analysis is presented under two subheadings, with the first subheading focusing on word use and visual mediators and the second subheading looking at the endorsed narratives and routines from ritualisation trees.

7.3 ARRANGEMENT OF ACTIVITIES AND EXCERPTS

The data that is presented and analysed here emanates from two discussion activities that were done in class with the teacher and discussed during the lesson all based on geometry of straight lines. The activities do not represent a single lesson, instead they were captured over a period of two hours, with a period of an hour each. In subheading 7.4, the analysis is based on solving geometric problems involving triangles using known properties of triangles and similar triangles. Subheading 7.5, on the other hand, focusses on solving geometric problems using the relationship between pairs of angles formed by perpendicular lines, intersecting lines and parallel lines cut by a transversal. The focus again is to analyse the use of words by the teacher and learners as well as how the used words influence learners mathematical thinking.

7.4 ANALYSIS ON GEOMETRY OF STRAIGHT LINES 1

Excerpt 5 below showcases interactions during classroom teaching and learning when learners and their teacher were discussing and activity on Space and shape, with a focus on angles in a triangle. **Figure 2** below is the activity that was discussed during the lesson and I used it to represent the classroom discourse that took place in the class (Setati, 2003).

DISCUSSION ACTIVITY 1

1. The diagram below shows a right-angled triangle ΔABC with $\hat{A} = 38°$. In ΔBCD $BD = CD$ and $AB||CD$. Find x, y and z, with reasons

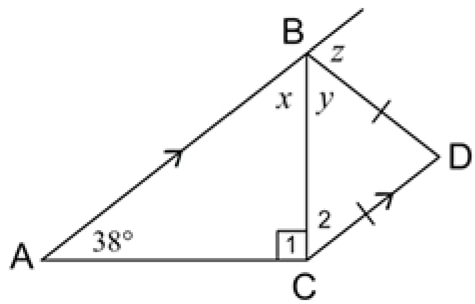


Figure 2: Class activity that learners and the teacher worked on

Excerpt 5

Turn	Speaker	What is said	What is done
5.1	T	You are provided with two triangles, ABC and BCD, the question says ADC is a right-angled triangle and angle A is 38. Also, they say triangle BCD has BC equal to CD we must find x why and z and provide reasons for our answers. So, let's find x why and z.	Explaining to the learners drawing the rider on the board
5.2	L1	x is 38	
5.3	T	x is 38 why?	Asking with hands
5.4	L2	No mam it's not 38	Interrupts the teacher
5.5	T	Let us here why he says is 38	Pointing at L1
5.6	L1	If A is 38 then B is also 38 I just see	Pointing at the angles
5.7	T	Okay L2 tell us why you say is wrong? Remember ABC is a right-angle triangle so what is x?	Pointing at L2
5.8	L2	Mam x is 52 because 180 minus 90 minus 38 is 52	Showing which angles we are subtracting
5.9	T	What is your reason for that answer	
5.10	L2	Is 52 because when we say 180 minus 90 minus 38 we get 52 that is why mam	Saying it aloud with laughter
5.11	T	Yes, it is 52 because all angle must add up to 180, 90 plus 38 plus x equal 180 and making x the subject of the formula we have x equals 52. Now lets us find y, what is y.	Demonstrating how angles add up to 180 and Points at y

5.12	L1	Eish maybe 52 because x is 52 and x and y are next to each other	Scratching head
5.13	L3	Yea it is 52 but the reason is that it is equal to C2 which is equal to x	Pointing at the angles
5.14	T	L3 how do you know C2 is equal to x?	Points at L3
5.15	L3	Because there is Z and you said when we have Z the angles inside is equal which is x and C2	Demonstrate the Z on the rider and points at the inside angles
5.16	L1	Owoo! Is that one of Z but I was right the answer is 52	Whispers to L3
5.17	T	Now that we have x and why can we find z? How much is z?	
5.18	L2	This is easy mam we say 180 minus 52 minus 52 which is 76	Pointing at the board
5.19	T	Why do you say 180 minus 52 minus 52? What is your reason	Looking at L2
5.20	L3	Mam when we have parallel line and all angle are on the line, we subtract all angles from 180 to get z	Demonstrate the parallel line and angles to subtract
5.21	T	Yes, but why do we subtract from 180?	
5.22	L1	Because that how it is mam, we do it like that	Says louder jokingly

7.4.1 Word use and visual mediators

The mathematical discourse in Excerpt 5 begins with the teacher, in Line 5.1, explaining to learners that they are provided with ‘two triangles ABC and BCD’ and also that ABC is a ‘right-angled triangle’. The teacher also explains to learners that angle A is 38 degrees and, in triangle BCD, BC is equal to BD. To initiate classroom discourse among the learners, the teacher poses a question for learners to find x, y and z on the given triangles and provide reasons for that. L1 in Line 5.3 says $x = 38°$, to which when the teacher asks why? L2 immediately disagrees, imitating L1 and says x not $38°$. Moving on, the teacher insists that L1 be given a chance to explain the mathematical reasoning for his answer and L1 substantiates his answer by arguing that ‘when A is 38 then I just see that $x = 38°$’. L1, in this case, uses the visual appearance of the rider to reason for the answer and does not provide mathematical reasons. Immediately, the teacher gave L2 his chance to explain why he disagreed. Line 5.8 showcases L2’s explanation. He argues that ’$x =$

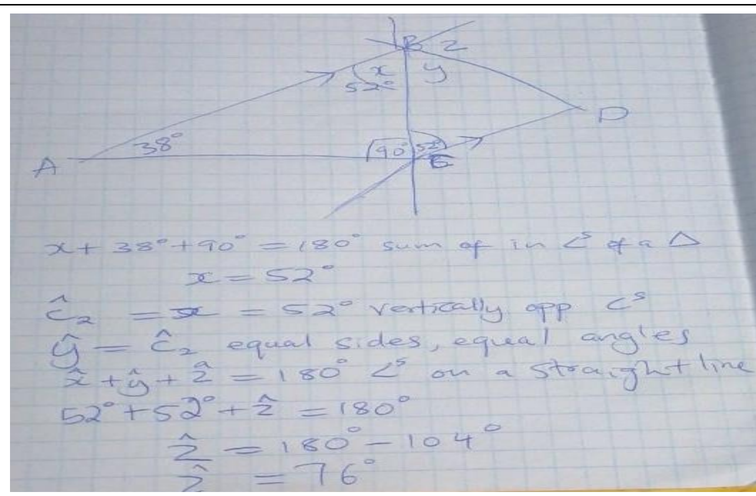
$52°$ because $180 - 38 = 52$', to which, when he was asked what the reason for that is, he just said 'because when we minus thirty-eight from one-eighty we get fifty-two'. In Line 5.11, the teacher agrees with L2 but corrects his reasoning and mentions that the reason is that 'all the angles must add up to 180'.

Moving on with the conversation, learners are now supposed to find y and L1 says 'maybe y equals fifty-two ($y = 52$) because x equals fifty-two ($x = 52$) and x and y are next to each other'. The use of the word 'maybe', which is an adverb for sequential action, and also the use of the words 'next to each other', were used to indicate a place where they (x and y) are situated. The use of the word 'maybe' represents an assumption that L1 made in his mind, which L3 also agrees to (Line 5.13), which is 'y being equal to fifty-two ($y = 52$)' but her reasoning is different. In her reasoning, L3 argues that y is fifty-two because it is equal to C_2 and C_2 is equal to x. The teacher asked L3 to account for the statements made that C_2 is equal to x, how does she know? In response to which, in Line 5.15, she (L3) argues that there is 'Z' and 'the teacher told them that when there is Z the angle inside (referring to x and C2) Z are equal'. Immediately, L1 says 'Owoo! It is that one of "Z"', which is an indication that the learner remembers this concept but still insists that he was right because his answer of fifty-two is still correct, regardless of his reason being wrong (Line 5.16). The use of the visual cue 'Z' is used to refer to parallel lines that are cut by a transversal.

Finalising classroom discourse, learners were now required to find z after finding x and y. Here we see L2, in Line 5.18, saying 'this is easy we just say one-eighty minus fifty-two minus fifty-two and get seventy-six ($180 - 52 - 52 = 76$)'. L3 argues that L2's working is correct because, when we have parallel lines and all angles are on the line, we do exactly as L2 is saying (Line 5.20). In Line 5.21, the teacher repeats the question and ask why do we subtract all the angles from one-eighty? L1 argues that 'it is how it is mam, we do it just like that'. From the excerpt above I argue that the teacher's and learners' use of words from this excerpt indicates the following categories of vocabulary (1) words that are mathematical in nature, i.e., *triangle, right-angled triangle, minus, angle A, equal, add, subtract* and *parallel line*; and (2) words that are not mathematical but are used to refer to the same meaning in mathematics (colloquial words), i.e., *find, next to each other, angles inside* and *on the line*.

For meaning making in terms of the words used by both learners and the teacher, I used L2 and L3's visual mediators from the classroom discourse to analyse how the use of words were represented on their visual mediators in comparison to the teacher's visual mediator from preparation book (**Table 12**).

Table 12: L2 and L3's visual mediators vs teacher's visual mediator

Line	L2 and L3's Visual mediators	Teacher's Visual Mediators
V5.1	$x = 180 - 38 - 90$ $x = 52$	
V5.2	$x = C_2 = 52$ $y = C_2 = 52$	
V5.3	$x = 180 - 52 - 52$ $x = 76$	

Learners' choice of words when explaining how they have calculated the value of x is not mathematically substantiated, even though the process yields the correct answer. L2, in Line 5.8, explains why $x = 52°$ by saying that it is because we must subtract ninety and thirty-eight from one-eighty, visually represented as $x = 180 - 38 - 90$, instead of giving a valid reason that the interior angles of a triangle add up to one hundred and eighty degrees. Therefore. a right-angled triangle has one angle that is ninety degrees, substituting thirty eight degrees and solving further, we use additive inverse operations to get fifty-two degrees (V5.1). Furthermore, L3 also reasons that $y = 52°$ because there is ‘Z’ and the teacher said that, when there is ‘Z’, the angles inside are equal, which would mean that x and angle C_2 are equal (Line 5.15). From the mathematics literature, the argument of the learners was supposed to be: when having parallel lines joined by a diagonal line that forms a visual cue of the letter Z or N, the alternating angles are equal and, also, the opposite angles of an isosceles triangle are equal (V5.2). Lastly, L3 argues that, when we have a parallel line and all angles are on that line, we subtract all angles from one-eighty to get seventy-six (Line 5.20). Mathematically, the right explanation will be that angles on a straight line add up to 180^0 and after substituting the given angles, we use the additive inverse operation to solve the question and get 76^0 as the answer (V5.3). When looking at the words used by learners in Excerpt 5, I can argue that the words used during the discourse were used to explain the mathematical procedure (e.g., Excerpt 5, Line 5.5 and Table 12, L2 and L3’s visual mediators) that learners would follow when solving a mathematical problem, rather than explaining the mathematical process (e.g., Excerpt 5, Line 5.11 and Table 12, teacher’s visual mediators) that is required to solve such a problem. Their visual mediators also indicate that learners only focus on what is required to get to answers and not on why they are doing the procedure to get to the answers.

7.4.2 Endorsed narratives and routines

I used learners’ interaction in Excerpt 5, looking at their word use and visual mediators to argue whether the constructed narratives produced are endorsed or not, and used the constructed realisation tree from their discourse (**Table 13**) to argue whether the narratives produced are regulated by rituals or explorations. L2’s interaction in Line 5.10, when talking about subtraction of two angles that are known from 180^0 in a triangle to get the unknown angle, demonstrates ritualistic routines that produced narratives that are not endorsed (Nodes 1a to 1d). The origin of the routine here is spatial arrangement because of the learner's ability to remember that, when given a triangle, the sum of all the interior angles is 180^0 and to apply additive inverse operations to get the remaining unknown angle from 180^0. Hence this signifies ritualistic routines (Roberts, 2016; Sfard, 2016).

Table 13: Realisation routine and tree for learners' discourse when solving x, y and z on the rider in Activity 1

Nodes	Realisation Routine	Visual mediators	The solution to the equation $x+y+z=180$ on straight line AB		
1a	scan triangle ABC and realise the interior angle add up to 180 degrees	$x+90+38=180$		1a	
1b	Write x on the LHS	$x+90+38=180$	$x =$	1b	
1c	Write 180 on the RHS	$x+90+38=180$	$x = 180$	1c	
1d	write -90 on the RHS	$x+90+38=180$	$x = 180-90$	1d	
	write -38 on the RHS **(Realisation 1)**	$x+90+38=180$	$x=180-90-38$		**Realisation 1**
2a	Subtract 90 from 180	$x=180-90-38$	$x = 90-38$	2a	
	Subtract 38 from 90	$x=180-90-38$	$x=52$		**Realisation 2**
3a	Scan the rider and identify parallel lines cut by a transversal	$x=C_2$	$x =$		
3b	Identify co-interior angle on triangle ABC	$x=C_2$	$x = C_2$		
	Equating co-interior angle on triangle with BCD that on ABC	$x=C_2$	$C_2=52$		**Realisation 3**
4a	Scan triangle BCD and realise opposite sides are equal and so is their opposite angles	$C_2=52$			
4b	Write C_2 on the LHS	$C_2=52$	$C_2 = 52$		
4b	Replace 52 by y on the RHS	$C_2=52$	$C_2 = y$		
	Replace C_2on the LHS by y	$C_2=52$	$y=52$		**Realisation 4**
5a	Scanning the equation to substitute x, y and find z	$52+52+z=180$			
5b	Write z on the LHS	$52+52+z=180$	$z =$		
5c	Write 180 on the RHS	$52+52+z=180$	$z = 180$		
5d	Write – 52 on the RHS	$52+52+z=180$	$z = 180-52$		
	Write – 52 on the RHS (**Realisation 5**)	$52+52+z=180$	$z=180-52-52$		**Realisation 5**
6a	Subtract 52 from 180	$z=180-52-52$	$z=128-52$		
	Subtract 52 from 128	$z=180-52-52$	$z=76$		**Realisation 6**

In Line 5.15, L3's interaction produces narratives that are not endorsed because he refers to a person of authority as the reason for using the routine, who used an explorative routine to teach the concept prior to assessment. This is so because the learner used the person of authority (the teacher) to substantiate his answer. However, the person of authority's routine, which was adopted by L3, produced narratives that are endorsed and regulated by explorative routines (Roberts, 2016; Sfard, 2016). The learner's ability to recall that, when there are parallel lines that form the visual cue of letter Z then we have angles that are equal, proves that the learner was thinking ritualistic, it is also because the discourse depends on the visual appearance of the rider or entities when solving the question (Nodes 4a to 4b). Similarly, in Line 5.22, L3 again makes use of ritualistic routines when linking the straight line and angles that are on the straight line, because this routine originates from spatial arrangement of subtracting the angles from 180^0 which regulates narratives that are not endorsed (Nodes 5a to 6a). Here again, the origin of the routine is spatial arrangement and the learner talks about mathematical entities in a disobjectified way (Roberts, 2016; Sfard, 2008).

7.5 ANALYSIS ON GEOMETRY OF STRAIGHT LINES 2

Below I showcase Excerpt 6 where learners were provided with a rider and are supposed to use relevant theorems to determine the value of x and y. The interactions here are used to represent the discourse that took place in the classroom (Setati, 2003). Activity 2 (**Figure 3**) was discussed as presented below. The activity required learners to use the theorems on geometry of parallel lines and properties of triangles in order to determine the value of x and y which would be true for the theorems applied.

DISCUSSION ACTIVITY 2:

1. In the figure below, determine with reasons the value of x and y.

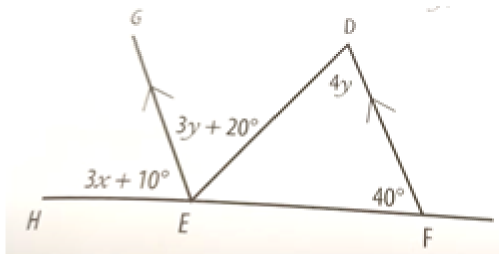


Figure 3: Class activity that leaners and the teacher worked on

Excerpt 6

Turn	Speaker	What is said	What is done
6.1	T	We need to determine the value of x and y in the given figure, in geometry of straight lines, remember whenever you see parallel lines you need to know whether you using the F for corresponding angles, $\sqcup$ for co-interior angles or the N or Z for alternate angles so let us see what we have here in order to determine the value of x and y, can we name the types of lines we having	Draws the diagram on the board and demonstrate parallel lines on the rider
6.2	L1	We have straight line HEF and we can say 180- 3x+10-3y+20	Reading on the classwork book
6.3	T	Okay someone what are you saying	
6.4	L2	We have N mam so the angles inside are equal	
6.5	T	Someone what do you see?	
6.6	L3	I see F maam so 3x+10 equals 40	Shows the class F on the diagram
6.7	T	Is that all? Anyone who see something else which was not said?	Looks at the entire class
6.9	L4	I see 3x+10 equal 4y+40, the angle outside a triangle is equal to the two angles inside	Demonstrate it sitting down
6.10	T	Okay is that all?	
6.11		Silence!	
6.12	T	Now remember that you cannot make a statement in geometry without telling us a reason for the statement, so let's start with L1, what is you reason for the statement	Points at L1
6.13	L1	*Eehe! Mam nna* I just know that if we have a straight line, we say 180 and subtract all the angles	Explain with hands

6.14	T	Okay any one to help here, do you agree that his reason and statement are correct?	
6.15	L3	Mam, I think the reason is that all angles on a straight line *geredi hlakancha di refa* (*when we add them we get*) 180	
6.16	T	Yes, that is true but can you see that we don't have the other angle on the straight line from the triangle?	Pointing at the diagram
6.17	ALL	Yes,	
6.18	T	Then we cannot use this statement because we well have lot of unknowns, x, y and the angle in the triangles, so L2 tell us your reason for the statement	Points at L2
6.19	L2	Mam *akere* (isn't it) you said when we have N or Z the angles inside are equal so this is the N and this angle is equal to this one in here	Pointing on the diagram on the board
6.20	T	Okay but we need to have a maths reason for that, 3y+20=4y and the correct reason is alternate angles with line GE parallel to GF so the parallel lines are the ones giving us the N but we must know that the angles equal is what? Is alternating angles so this one we can solve, what do we do next	Demonstrate on the board while explaining
6.21	L3	We say 3y-3y+20=4y-3y then 20=y	Reading from his working on the book
6.22	T	Correct our y is 20 degrees. So, we need to solve x let's look and L3's statement and see if it cannot help us calculate x, L3 give us your reason	
6.23	L3	When we have parallel lines and they form the letter F the angles below the parallel lines are equal mam so 3x+10=40 and then x is 10	Showing the below angles and looking at his answer
6.24	T	X is 10 yes but these angles have names we say GE is parallel to DF so corresponding angles are equal, so we must use the correct names for the angles.	Points at the angles and make an arrow and write the name
6.25	L4	So, mam why it did not give me 10?	Looking confused
6.26	T	Okay yours is that the exterior angle of a triangle equals the sum of the two opposite angle of the triangle in this case we must have 4y+40=3x+10+3y+20 because the exterior angle is not only 3x+10 but 3x+10+3y+20 so be careful when you use that and if we solve that we will have y+40=3x+30 then 60-30=3y then y equals 10 you see they are the same?	Demonstrate on the board while writing
6.27	ALL	Yes	

7.5.1 Word use and visual mediators

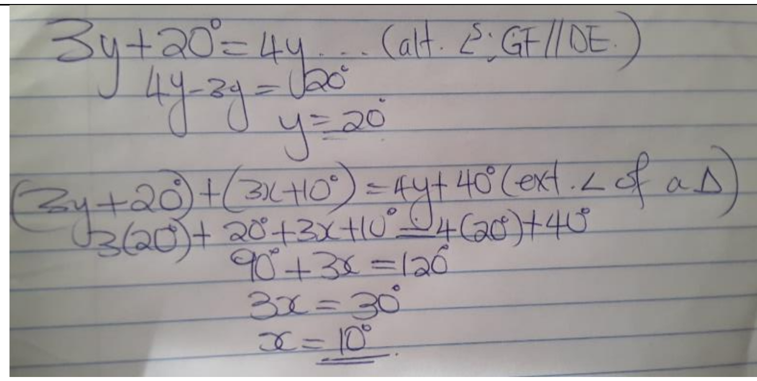
The classroom interactions in Excerpt 6 commence with the educator reminding learners that in 'geometry of straight lines', whenever you have parallel lines, you must look for the visual cues N, Z, F and ⊔, which you can use to check for alternate angles, co-interior angles or corresponding angles, respectively. To initiate mathematical discourse, the teacher, in Line 6.1, asks learners to give the different types of lines they see on the provided figure, which she believes could help them to solve for x and y. L1, in Line 6.2, mentions that there is a straight line where one can solve by saying $180 - 3x + 10x - 3y + 20$. However, in Line 6.12 we can see the teacher asking L1 to provide a reason for the statement he made. The teacher further argues that in geometry you cannot make a statement without providing a reason. Here I argue that words that L1 used to justify his statement are not mathematically justified. This is observed in his reply (Line 6.13) when he says he just knows that in a straight line all angles involved must be subtracted from one-eighty, the words used here were used looking at the visual appearance of the mathematical entities. His reasoning led to the teacher clarifying that in a straight line all angles add up to one-eighty degrees and not that all angles are subtracted from one-eighty (Line 6.16), which is also supported by L3 in Line 6.15. Here again, it is evident that L1, in his reasoning, was using the word 'minus' in order to explain the mathematical procedure he does to obtain the final answer.

Moving on with the conversation, L2 in Line 6.4 recognises the visual cue letter N and argues that the angles 'inside' are equal. L2's choice of colloquial word 'inside' to refer to alternating angles being equal demonstrates the use of nouns as colloquial words. Similarly, in Line 6.19, L2 talks about 'this' angle is equal to 'that angle there' referring to alternating angles using pronouns. This is observed in Line 6.20, where the teacher reiterates that we can only have the letter N if we have two lines that are parallel and, if that is the case, the angles that are equal are alternating angles and not 'inside angles. L2 here used the words 'inside angles' to refer to alternating angles but both words have different meanings because we could have inside angles that are not alternating angles (Powell & Nelson, 2017). In Line 6.6, L3 recognises the visual cue letter F from the figure and gives the following as his reason for the statement provided 'whenever we have parallel lines that form the letter F in a given figure the angles "below" the parallel lines are equal' (Line 6.23). However, the teacher insists that it is important not to say the 'angles below the parallel lines' but to call them corresponding angles as this is the name given to such angles (Line 6.24). The word 'below' is used as an adverb of the place where the angles are located.

Lastly, L4 in Line 6.9 recognises 'an angle out of a triangle known' mathematically known as the exterior angle of a triangle but he mentions that 'two angles inside a triangle are equal to the outside angle'. The use of the word 'out' is an adverb of place for explaining where the angle is located and, according to the teacher in Line 6.26, was uncalled for. The statement was supposed to be 'the exterior angle of a triangle is equal to the sum of the two opposite interior angles of the triangle'. Also, what made L4's answer wrong was that he did not understand what an exterior angle was as he used only one part of the exterior angle, while he was provided with two angles that form the exterior angle of the given triangle. From the excerpt, the teacher makes use of mathematical words to make statements in geometry, i.e., *exterior angles, parallel lines* and *alternating angle*, however, the learners make use of a combination of mathematical words, i.e., *parallel lines* and colloquial words, i.e., *inside angles* and *outside angles*.

Word use has intended visual mediators that help in meaning making for better understanding of what the learners or the teacher mean when using certain words during mathematics discourse. In this discussion, I use words used by the teacher or learners and their corresponding visual mediators to account for leaners' mathematical reasoning in **Table 14** below.

Table 14: L2 and L3 Visual mediators from their discourse and teacher's visual mediator from preparation activity 2

Line	L2 and L3's Visual mediators	Teacher's Visual Mediators
V6.1	$3y + 20 = 4y$ $3y - 3y + 20 = 4y - 3y$ $20 = y$	
V6.2	$3x + 10 = 40$ $x = 10$	

In Line 6.1, the teacher used the word 'figure' to refer to the given 'rider', I can, therefore, argue that the teacher spoke about the given rider not as a mathematical object that learners must engage with mathematically to extract more information as she called it a 'figure'. However, she further requests learners to identify the different lines that they can see on the figure and now the teacher talks about the figure as having some elements that are mathematical on it. In Line 6.9, L4 refers to the 'exterior angle of a triangle' as 'angle outside a triangle', which he visually represents as $3x + 10$ instead of $(3x + 10) + (3y + 20)$. In Line 6.21, L3 supports L2's statements that alternating angles are equal and that we can solve for y using additive inverse (V6.1). L3 further refers to corresponding angles, in Line 6.23, as angles below the parallel lines visually represented as $3x + 10$ and 40 (V6.2). Looking at how learners use words to explain mathematical entities, I can, therefore, argue that the learners are using these words to explain the visual appearance they see on the provided diagrams or visual mediators in order to make sense of what they think they mean and not based on their mathematical understanding (Sfard, 2015).

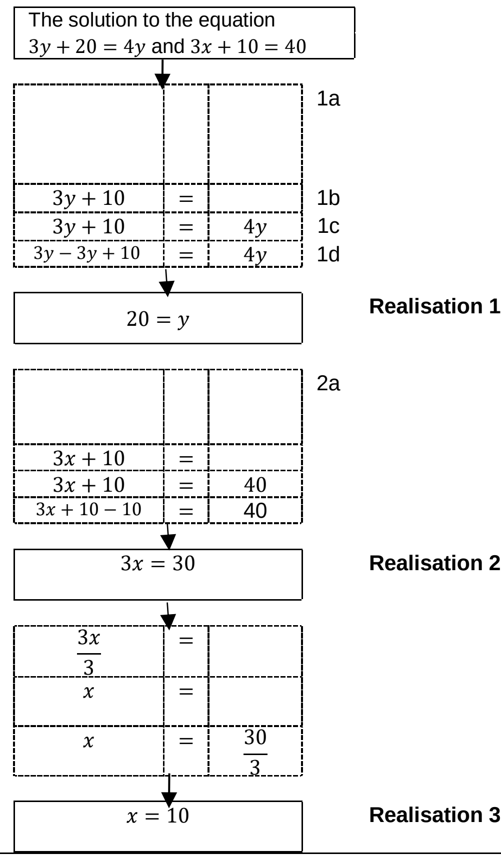
7.5.2 Endorsed narratives and routines

In Excerpt 6, I use learners' and teacher's interaction in order to argue whether the narrative produced is endorsed or not and use the realisation tree (**Table 15**) constructed below using the classroom discourse, specifically looking at the learners to argue whether their narratives are regulated by explorative routines or ritualistic routines. The type of routines that learners use to construct narrative through word use, visual mediators and realisation trees can be used to determine how learners think when communicating mathematically (Roberts & Le Roux, 2019; Sfard, 2015).

In Line 6.1, the teacher's interaction produces narratives that are endorsed because of the routine of the teacher that is regulated by the ability to look for the relationship between the angles on the provided rider, looking at different lines we are having and the polygons that formed the rider. Similarly, in line 6.2, L1 demonstrates a ritualised routine when trying to show the relationship between a straight line and the angles on the straight line ($180 - 3x + 10x - 3y + 20$), which he repeats in Line 6.13. However, L1's explanation is an explanation of how to work out the theorem to get the final answer after substitution, because the theorem talks about the sum of all angles and not the difference in all the angles. Although, the routine, if applied, can result in a narrative where in the final answer becomes correct, provided values of x and y are known, which L3, in Line 6.15, also clarifies, L1's statement as not being mathematically explained well.

Table 15: learners and teachers discourse used to construct realisation routine and tree for activity 2

Nodes	Realisation Routine	Visual mediators
1a	scan the rider and realise the alternating angles are equal and which entity belong on the RHS and LHS	$3y + 20 = 4y$
1b	Write $3y$ and 20 on the LHS	$3y + 20 = 4y$
1c	write $4y$ on the RHS	$3y + 20 = 4y$
1d	Subtract $3y$ on the LHS	$3y + 20 = 4y$
	Subtract $3y$ on the RHS **(Realisation 1)**	$3y + 20 = 4y$
2a	scan the rider and realise the co-interior angles are equal and which entity belong on the RHS and LHS	$3x + 10 = 40$
2b	write $3x$ and 10 on the RHS	$3x + 10 = 40$
2c	write 40 on the RHS	$3x + 10 = 40$
2d	Subtract 10 on the LHS	$3x + 10 = 40$
	Subtract 10 on the RHS **(Realisation 2)**	$3x + 10 = 40$
3a	Identify the coefficient of x (3) as a divisor	$3x = 30$
3b	Divide $3x$ by 3	$\frac{3x}{3} = 30$
3c	Write 30 with divisor 3	$x = \frac{30}{3}$
	Calculate 30 divide by 3 **(Realisation 3)**	$x = \frac{30}{3}$



L2's narrative in Line 6.4, when he talks about the angles inside the letter N (alternate angles) are equal, is regulated by routines that have been used before and yielded correct answers. However, the word use does not build the mathematical proficiency of the learners. The routine used here demonstrates that the learner was thinking exploratively about the angles involved when recognising the visual cue letter N and think about the relationship between the visual cue N and the angles involved there. Hence the narrative is endorsable (Node 2a). Similarly, in Line 6.6, L3 also produces endorsed narratives because the learner talks about the angles in an objectified way, that is, the description of entities and the relations between the entities, which signifies explorative routines. In Line 6.9, L4 talks about the angle outside the triangle (exterior angle of a triangle) as being equal to the two angles inside the triangle (two opposite interior angle of the triangle), which is classified as a theorem. This indicates the learner's ability to use meta rules when solving mathematical problems, which signifies endorsed narratives. Even though learners have used different words that are not mathematical in nature, the idea of the theorems that must be applied can be regarded as endorsable. This is also observed in Line 6.23, when L3 talks about recognising the visual cue letter F formed by parallel lines. L3 used meta rules to give reasons for his mathematical statement, which indicates explorative routines (Roberts, 2016; Sfard, 2016). His interaction produces narratives that are endorsed.

Looking at L3 and L2's realisation tree for obtaining the value of the y, as indicated on the visual mediator (V6.1), the learners' narrative in Line 6.19 and in Line 6.21 produces narratives that are endorsed because of the use of additive inverse operation to obtain the value of y. Similarly, the origin of the narrative is horizontal equivalence, which is regulated by explorative routines (Sfard, 2016) (Nodes 1a to 1d). Furthermore, in Line 6.23 visually mediated in V6.2, the narratives produced are not endorsed because the narrative is only concerned with the procedure for producing $x =$ … and not with the closing condition goal. This narrative is regulated by ritualistic routines (Roberts & Le Roux, 2019; Sfard, 2016) (Nodes 2a to 3c).

7.6 SUMMARY

In chapter 7, I presented data using excerpts to look at the two activities that focused on geometry of straight lines. I analysed the excerpts and interpreted the data in order to identify the mathematics vocabulary that teachers and learners use in the classroom during mathematics classroom discourse, as well as how the vocabulary used influences learners' mathematical thinking and communication. To do this, I first focused on the word use and their intended visual mediators for meaning making. Secondly, I focused on endorsed narratives and routines using word use, visual mediators and realisation trees constructed from the teacher's and learners' discourse.

During the analysis, I also accounted for the features of language that are imbedded in the teacher's and learner's mathematical vocabulary and the origin of their narratives in order to identify the regulated routines. As acknowledged during the introduction of this study, learners use every day English language and mathematical language in the classroom to communicate about mathematical objects. Therefore, both the use of everyday English language and mathematical language was looked at as mathematical vocabulary as long as the languages are used to make a meaning in mathematics discourse.

CHAPTER 8: CONCLUSION AND RECOMMENDATIONS

8.1 INTRODUCTION

In chapters 6 and 7, I presented an analysis of data generated through the use of the data collection methods explained in Chapter 5. I used the theoretical framework discussed in Chapter 2 to analyse the data. Chapter 8 is the concluding section of this dissertation and, therefore, I answer the research questions that guided the research in this section in order to achieve the research purpose. I further outline limitations of the study and conclude the chapter by giving recommendations for further areas of research that focus on mathematical vocabulary usage during mathematical classroom discourse.

The following two sub-research questions guided this study:

- What mathematical vocabulary do teachers and learners use during mathematics classroom discourse?
- In what way(s) does the used mathematical vocabulary influence learners' mathematical thinking and communication?

The above sub-research questions were used to guide this study in order to answer the main research question below:

1. How do teachers and learners use mathematics vocabulary during mathematics classroom discourse?

To present the conclusion, I do not follow the traditional way of answering research questions separately. However, one subheading focuses on the mathematical vocabulary used and how mathematical vocabulary is used during mathematics classroom discourse. Answers to the way(s) in which the mathematical vocabulary used to influence learners' mathematical thinking and communication are presented in the second subheading. In both subheadings, I link the results to seven features of explorative and ritualistic discourse from Sfard's (2008) commognitive framework, looking at both the teacher's and learners' discourse in the mathematics classroom namely: (1) degree of objectification; (2) endorsed narratives; (3) closing condition; (4) for whom the routine is performed; (5) by whom the routine is performed; (6) level of flexibility; and (7) level of correctability. The features of the discourse will, at the same time, answer the main research question.

8.2 CONCLUSION

Mathematics vocabulary is reported as not usually taught in schools in a sense that, when learners are not exposed to a good textbook, they will not have a place to read the vocabulary (Fletcher & Santoli, 2003). This could be the reason why learners do not know the necessary vocabulary to express their mathematical ideas (Blessman & Myszczak, 2001). Adams (2003) also advises on the importance of knowing the meaning of the vocabulary that learners are using when trying to make sense of mathematics because using everyday language may confuse their mathematical understanding. Similarly, learners' problem-solving skills improve when learners are aware of the vocabulary they use and how to communicate mathematics effectively (Schoenberger & Liming, 2001).

The mathematics curriculum advocates for mathematics teaching to develop essential mathematical skills so that 'the learner should develop the correct use of the language of mathematics, develop number vocabulary, number concept and calculation and application skills, to learn to listen, communicate, think, reason logically and apply the mathematical knowledge gain' (CAPS, 2011, p. 8–9). In order for learners to develop the correct use of the language of mathematics, they must engage with mathematics in the classroom and such engagements should be facilitated for correct use of mathematics language. Similarly, for learners to engage, they must communicate and be able to reason and think mathematically. Hence the study I proposed here sought to explore how learners' use of mathematical vocabulary influenced their mathematical thinking and communication.

8.2.1 Mathematical vocabulary and how it is used

In reporting the results, I need to acknowledge that, during classroom teaching and learning of mathematics, the mathematical vocabulary that is used consists of both mathematical language and everyday English language (Peng & Lin, 2019). During my discussion in Chapter 4, I indicated that this study would look at both languages as mathematical vocabulary as long as they are used for mathematical meaning and understanding. From the analysis, there is evidence to show that during classroom discourse the teacher and learners use words that are found in mathematics textbooks (Fletcher & Santoli, 2003), which Lamb (1980), Powell & Nelson (2017) and Shepherd (1973), refer to as words that are mathematical in nature. Similarly, there is also of evidence of words that learners use which are not found in the mathematics textbook but we use them in our everyday language of communication (English), However, in this case, learners use them for mathematical reasoning. This is what Peng and Lin (2019) emphasise as the everyday English language that learners encounter in mathematics.

Similarly, Adams (2003) argued that the use of everyday language may confuse learners to develop mathematical understanding. This was never the case in this study because learners managed to get correct answers when using everyday language. From this I conclude that both teachers and learners make use of mathematical vocabulary that comprises of mathematical words and everyday English language words that are used to explain mathematical meaning when solving mathematical problems. This means that the everyday language must be used in a mathematically accepted way in order to talk about mathematical objects in an objectified way (Sfard, 2008) because the everyday language has proven to provide a fertile ground upon which mathematics vocabulary can be re-inforced.

The focus of the study on the mathematical vocabulary usage was on how learners and their teacher use mathematical vocabulary to talk about entities in an objectified way, which is what Sfard (2008) talks about when discussing the degree of objectification. I report here that, from the analysis, there is evidence that indicate learners' use of pronouns, nouns, adverbs, verbs etc, which are colloquial words, to refer to mathematical entities and mathematical processes (Mpofu & Pournara, 2018). For example, in Excerpt 1, the teacher talks about 'collecting' like terms as a mathematical process that requires learners to simplify the like terms. Also, in Excerpt 3, learners and their teacher insisted that the denominator 'must go' and multiply five 'that' side (Line 3.7). This is also an indication of using words in a routine-driven way (Sfard, 2008, 2016). The pronouns 'must go' used indicates the material process of personality. The action nouns and verbs 'this' and 'that' when used to refer to entities, is an indication in the learners' discourse, that the verbs and nouns are used in a phase driven way to talk about mathematical objects in an objectified way (Ripardo, 2017; Roberts; 2016, Sfard,2008). In Excerpt 4, learners made use of the words 'vanish and they left' in a routine-driven way (lines 4.28 and 4.41) simply because they made use of the visual appearance of mathematical objects to explain their mathematical arguments (Roberts, 2016, Sfard, 2012), which, in my case, I argue that the talk is disobjectified, even though it produces correct answers.

Furthermore, in Excerpt 2, the operations and the whole numbers on '-3' and '$+4$' were separated into two separate entities during factorisation (Line 2.45), which is an indication of talking about the entities in a disobjectified way (Roberts, 2016; Sfard, 2008). However, from the realisation trees, we have only one algebraic branch with more than one realisation, which, in this case, would mean learners work with mathematical objects in an objectified way (Roberts & Le Roux, 2019). For example, learners are able to scan the equation and decide which entity belongs to the right-hand side of the equation and which belongs to the left-hand side of the equation (Table 7, Nodes 1a to 1d). However, this was not the case in the geometry of straight lines because learners were only interested in finding the value of the unknown and not treating the angles as mathematical objects. This is observed in Excerpt 5, where learners are saying that ‘we subtract 90 and 38 from 180 to get the remaining angle’ (Table 13, Nodes 1a to 1d). Similarly. The learners here, were only concerned with what Sfard (2008) calls the final answer of the realisation and not the production of endorsed narratives.

From the few examples that I provided from the analysis, I argue that, even though learners talk about mathematical objects in an objectified way in a few instances, in most cases where they involve negative entities, learners, and also the teacher, operate them as two different parts in a disobjectified way (Sfard, 2008). This is also the case during expansion of an expression in Excerpt 1 (Line 1.12) when multiplying ‘a minus and a plus’. This brings me to conclude that learners operate with mathematical objects in an objectified when they involve positive entities. This was also what Roberts (2016) found in her study, i.e., that learners regularly produce endorsed narrative with mathematical objects that involve positive integers. It was also evident that the routine for operating entities as two different parts originated from the teacher’s routine (Line 1.12). I therefore recommend that teachers need to be cautious when operating with entities and not separate operations from their mathematical terms. It should also be stressed during mathematics teacher trainings that, operations belong to the mathematical terms next to them and they should be treated as such so that learners will also be able to treat mathematical object in an objectified way for explorative thinking to be achieved (Sfard, 2016).

8.2.2 Mathematical vocabulary as an influence on learners' mathematical thinking and communication

In this section I report on how words used and their visual mediators contribute towards learners' mathematical thinking and communication. Sfard (2008) talks about explorative thinking as being observed on learners' ability to access a variety of ways to finding a solution to the same problem. However, this was not the case in my study because, in algebra and geometry of straight lines, learners are taught to work with only one way to finding a solution, which is an algebraic algorithm. So, the realisation trees I report on only have an algebraic branch and I used this to analyse how visual mediators act as signifiers between one entity and the other (Roberts, 2016). To report on the findings here, I show how learners' discourse satisfies the remaining six features of explorative and ritualistic discourse, as outlined by Sfard (2016).

The results indicate that learners produce endorsed narratives when working with whole numbers and this occurs through the use of additive operation inverse in Excerpt 3 (Lines 3.8 and 3.9) and also in Excerpt 4 (Line 4.45). Similarly, in Table 11 (nodes 1b to 1c), the realisation tree shows the teacher and learners working with vertical equivalence, which is a source for endorsed narrative. From the reported results on endorsed narrative, I can conclude that the teacher's and learners' thinking in this case was explorative. However, in her study, Roberts (2016) did not find any evidence of vertical and horizontal equivalence. In addition to endorsed narratives, learners produce narratives with disobjectified entities that are not endorsed, mostly in geometry of straight lines, even though their answers, in most cases, are correct. The origin of the narratives here is mainly from spatial arrangements, which can be observed in excerpts 5 and 6 when learners are required to find the third angle in a right-angle triangle when one angle is 38^0, they spatially arrange their working to subtract all the known angles from 180^0 to get the unknown as x (Line 5.8, V5.1 and V5.3). Also, in Excerpt 3, learners and the teacher talk about two as the denominator going to multiply five on the right-hand side and do not talk about multiplication inverse operation, produced narratives that are not endorsed, in which visual appearance was the source of the narrative (Table 9, nodes 1a to 1c). With this being reported from the analysis, the learners' thinking, in this case, can be said to be ritualistic (Sfard, 2016).

There is no evidence from the data analysis to indicate that learners see the closing condition in a given equation as to produce endorsed narratives but, in all cases, learners are working on an equation in order to give the value of the unknown. This is observed Excerpt 6 (Line 6.23) when saying '$3x + 10 = 40$ then $x = 10$' (V6.2), there is no evidence of the eagerness for the learners to produce endorsed narrative as long as the answer is correct. Thus, if the narrative to obtain the answer is not known, the realisation tree of this case will have only one realisation, which is not enough for rendering learners' thinking as explorative (Roberts & Le Roux, 2019; Sfard, 2016). As such, I conclude that learners thinking about the closing condition is ritualistic.

In some cases, the results from the analysis indicate that the learners make use of high modality verbs in their discourse, which is an indication that they do not work on mathematical objects independently but rely on other learners or the teacher for their working. In some cases, learners refer to the person of authority to substantiate why they perform a specific routine. This can be observed in Excerpt 5, line 5.15 and Excerpt 6, line 6.19, where learners keep on referring to the teacher when justifying their statements to say 'mam "you" said when we have parallel lines, we do this …' as their supporting argument for the routines used. Also, the use of high modality verbs such as 'we "must" find the factors of 12 that satisfy the middle number' indicates that the learners depend on other learners in order to solve their mathematical equations or expressions. When this happens, their thinking is regarded as ritualistic (Roberts, 2016; Roberts & Le Roux, 2019; Sfard, 2016). It was also evident in Roberts's (2016) study that the learners perform ritualistic routines, depending on other learners or a person of authority.

It should also be reported that quite a few learners are very flexible about using the horizontal equivalence, which can be observed in Table 10 for L2's documents analysis. The application of the horizontal equivalence by the learner is a good indication of how flexible the learner is about mathematical objects to produce endorsed narrative. The learner's operation with the visual mediators indicates that the learners think exploratively about the mathematical entities. However, from the analysis, most learners do not display such flexibility, which was also the case in Mpofu and Pournara's (2018) study where they also came to the same finding about the level of flexibility in learners' discourse to produce endorsed narrative. They further proposed for the teaching and learning of mathematics to focus of explorations of mathematical object, similarly, I also advocate for the teaching of mathematics to focus on the mathematical object in an explorative way rather than the ritualistic way of only being concerned with obtaining the correct final answer.

Furthermore, in Excerpt 6, Line 6.25, when L4 asked the teacher why he did not get 10 as the correct is an indication that the learner is willing to correct his narrative, which, in Line 6.26, the teacher clarified. In this case, the learners will be able to transition from ritualistic thinking to explorative thinking with the support of the teacher and other learners (Sfard, 2008). This was also supported by the findings in the Mpofu and Mudaly (2020) study that have proposed that, mathematics learning could be enhanced by allowing learners to fully participate in the mathematics discourse, especially to encourage teachers to request reasons from learners for their responses, as the provision of reasons is responsible for transitioning learners from ritualised to exploratory routines. Furthermore, I make a conclusion that learners' ability to provide reasons for every mathematical move they make during problem solving can improve their ability to think exploratively about the answers they provide and recommend that as teachers we need to do away with just accepting correct answers from the learners and start to demand explanations for how they have managed to produce the correct answers as a way of promoting explorative thinking.

8.3 LIMITATIONS

The interpretation of the classroom discourse in this study depended on me as the researcher and my interpretation of the teacher's and learners' discourse on how the use of mathematical vocabulary during mathematics classroom discourse cannot be entirely accurate. My findings in this research pertain to one Grade 9 classroom that was taught by one mathematics teacher in the class that I managed to observe during COVID-19. As such, the findings cannot be a generalisation of what is happening in other Grade 9 classrooms. Similarly, the manner in which the teacher planned for her lessons, how she planned the assessment activities and how she posed questions to learners during classroom observation could have influenced their responses, and as such I cannot assume or conclude that other learners in other Grade 9 classes would respond the same and their teachers would pose questions the same way. My presence in their classroom as an observer could have influenced their natural setting in the classroom, which could have influenced how they behaved and responded to questions in my presence. And for that, I cannot make conclusions that in my absence they would behave and respond the same way.

Another limitation to my study is the fact that classroom discourse that I analysed was based only on the learners who were participating during lesson observation, which means I have no indication of what those who were not participating were saying in their minds. It could have also been of great contribution if two or more educators who both teach Grade 9 were observed in order to analyse how the learners' thinking and how they use mathematics vocabulary is influenced by the teacher’s approach of teaching and how they pose questions during teaching and learning.

8.4 RECOMMENDATIONS

Research on mathematics vocabulary is a very popular way of evaluating learner performance or mathematics vocabulary knowledge and such analyses forms the backbone of research reports on learner performance in national assessment tasks (Roberts, 2016). Most researchers, as outlined in the literature review. have used the commognitive framework to study mathematics vocabulary but there is minimal research on how mathematical vocabulary is used in the classroom during teaching and learning. Most research focused on the knowledge and on how the mathematics vocabulary knowledge affects the performance of learners in mathematics. I suggest further research that will focus mainly on a broader large scale and focus on, not only one class and one teacher in one school, but on a sample representative of an education circuit or education district. I have a belief that, if most schools and teachers were engaged, the findings would tell a much richer story about the way in which teachers and learners use mathematical vocabulary in the classroom during teaching and learning and how the use of such mathematical vocabulary influences learner’s mathematical thinking and communication.

Furthermore, it will also be of great importance for future research to focus on distinguishing with examples derived from activities that are exemplified in a study, ways in which knowledge construction is showcased. This should be done by drawing directly or indirectly on commognitive perspective and cement connections or disconnections through how learning is defined commognitively or in participatory metaphor.

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ANNEXURES

ANNEXURE A: Letter for Requesting Permission to Conduct Research

Enq: Sihlangu SP
079 629 1767
Email: spsihlangu@gmail.com

P O BOX 2798
BURGERSFORT
1150

LIMPOPO DEPARTMENT OF EDUCATION
Private bag X94890
Polokwane
0700

ATTENTION: HEAD OF DEPARTMENT

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above matter refers.
2. I am a Mathematics Education masters' student at University of Limpopo – Turfloop Campus, as part of the requirements to the fulfilment of the degree I need to conduct a research and produce a dissertation.
3. This letter serves to request permission to conduct research **high school** in the **Circuit** of **Sekhukhune East district**. The school has been purposively selected as it is where I am currently teaching.
4. The proposed research title is **"A Discursive Analysis of the use of Mathematics Vocabulary in a grade 9 mathematics classroom"**. Even though the research will be classroom based, my research agenda will not affect the day to day academic activities of the classroom and the school. Relevant research ethics will be adhered to.
5. To support no 6 above, the university's ethical committee has issued a certificate for ethical clearance on the proposed study venting its compliance with relevant research ethics, and its ethical clearance certificate number is **TREC/364/2020:PG**
6. I will be glad if you can grant me the permission to conduct the research.

Regards

Mr. SP Sihlangu

ANNEXURE B: Clearance Certificate from Limpopo Provincial Research Ethics

CONFIDENTIAL

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

OFFICE OF THE PREMIER

Office of the Premier

Research and Development Directorate

Private Bag X9483, Polokwane, 0700, South Africa

Tel: (015) 230 9910, Email: mokobij@premier.limpopo.gov.za

LIMPOPO PROVINCIAL RESEARCH ETHICS

COMMITTEE CLEARANCE CERTIFICATE

Meeting: February 2021

Project Number: LPREC/21/2021: PG

Subject: A Discursive Analysis of the Use of Mathematical Vocabulary in a Grade 9 Mathematics Classroom

Researcher: Sihlangu SP

Dr Thembinkosi Mabila

Chairperson: Limpopo Provincial Research Ethics Committee

The Limpopo Provincial Research Ethics Committee (LPREC) is registered with National Health Research Council (NHREC) Registration Number **REC-111513-038.**

Note:

i. **This study is categorized as a Low Risk Level in accordance with risk level descriptors as enshrined in LPREC Standard Operating Procedures (SOPs)**

ii. **Should there be any amendment to the approved research proposal; the researcher(s) must re-submit the proposal to the ethics committee for review prior data collection.**

iii. **The researcher(s) must provide annual reporting to the committee as well as the relevant department and also provide the department with the final report/thesis.**

iv. **The ethical clearance certificate is valid for 12 months. Should the need to extend the period for data collection arise then the researcher should renew the certificate through LPREC secretariat. PLEASE QUOTE THE PROJECT NUMBER IN ALL ENQUIRIES.**

ANNEXURE C: Permission Letter from the School

[redacted]

Enq.

18/03/2021

Email: @gmail.com

SIHLANGU SP
P O BOX 2798
BURGERSFORT
1150

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above matter refers.
2. It is with pleasure to inform you that permission to conduct research has been granted as requested for the research titled **"A Discursive Analysis of the use of Mathematics Vocabulary in a grade 9 mathematics classroom"**.
3. Throughout your research period it is recommended that you ensure that your research does not interfere with the academic activities of the school.
4. You are once again reminded to adhere to all relevant ethical considerations issues as recommended by the Department of Education and your institution.

Regards

DEPARTMENT OF EDUCATION
PRINCIPAL
2021 -03- 18
LIMPOPO PROVINCE

ANNEXURE D: Research Ethical Clearance Certificate

University of Limpopo
Department of Research Administration and Development
Private Bag X1106, Sovenga, 0727, South Africa
Tel: (015) 268 3935, Fax: (015) 268 2306, Email: anastasia.ngobe@ul.ac.za

TURFLOOP RESEARCH ETHICS COMMITTEE

ETHICS CLEARANCE CERTIFICATE

MEETING: **18 November 2020**

PROJECT NUMBER: **TREC/364/2020: PG**

PROJECT:

Title:	A discursive analysis of the use of mathematical vocabulary in a Grade 9 mathematics classroom
Researcher:	SP Sihlangu
Supervisor:	Mr LL Mokwana
Co-Supervisor/s:	Prof KJ Masha
School:	Education
Degree:	Master of Education in Mathematics Education

PROF P MASOKO
CHAIRPERSON: TURFLOOP RESEARCH ETHICS COMMITTEE
The Turfloop Research Ethics Committee (TREC) is registered with the National Health Research Ethics Council, Registration Number: **REC-0310111-031**

Note:

i) **This Ethics Clearance Certificate will be valid for one (1) year, as from the abovementioned date. Application for annual renewal (or annual review) need to be received by TREC one month before lapse of this period.**

ii) **Should any departure be contemplated from the research procedure as approved, the researcher(s) must re-submit the protocol to the committee, together with the Application for Amendment form.**

iii) **PLEASE QUOTE THE PROTOCOL NUMBER IN ALL ENQUIRIES.**

Finding solutions for Africa

ANNEXURE E: Confirmation Letter from Language Editor

Andrew
Scholtz
Editing

Associate Member of:
Professional EDITORS Guild

Andrew Scholtz
Associate Member

Membership number: SCH018
Membership year: March 2022 to February 2023

224, Raptors View, Hoedspruit, 1380
PO Box 1172, Hoedspruit, 1380

084 602 1938 • 076 079 0214 • atjscholtz@gmail.com

Affidavit

Date: 30 June 2022

To Whom it May Concern

I hereby confirm that I have edited the Med dissertation entitled *A Discursive Analysis of the Use of Mathematical Vocabulary in a Grade 9 Mathematics Classroom* written by Siphiwe Pat Sihlangu and have suggested a number of changes that the author may or may not accept, at their discretion.

Each of us has our own unique voice as far as both spoken and written language is concerned. In my role as editor, I try not to let my own 'written voice' overshadow the voice of the author, while at the same time attempting to ensure a readable document.

Please refer any queries to me.

Andrew Scholtz

Qualifications:

- MA (Digital Media in Education) – University of Kwazulu-Natal (2006)
- Accreditation of Assessors in Higher Education (Short Course) – Rhodes University (2007)
- Postgraduate Diploma in Dispute Settlement – University of Stellenbosch Business School (2013)
- SLP Family Law (Short Course) – North West University (2013)
- Strengthening Postgraduate Supervision (Short Course) – Rhodes University (2019)
- UCT Copy-editing Online Short Course – University of Cape Town (2020)
- Approved freelance editor and proofreader for Juta & Company (Pty) Ltd

Evidence of qualifications are available on request.

ANNEXURE F: Class Activities for Discussion

ALGEBRA ACTIVITY 1

1. Expand the following
   a. $(7-3x)(2+x)$
   b. $(2x-1)+2(3x)$
   c. $(x+2)(x-4)$
   d. $2(2x+3)$

ALGEBRA ACTIVITY 2

1. Factorize the following
   a. $x^2+x=12$
   b. $x^2+11x+18$
   c. $x^2-8x-20$
   d. $(x+4)^2-9$

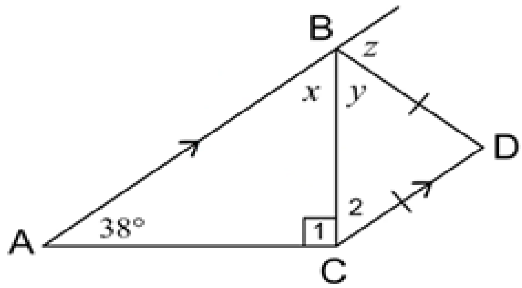
ALGEBRA (EXPONENTIAL) ACTIVITY 3

1. Solve the following equations
   a. $3^{x+1}=81$
   b. $\frac{3x+7}{2}=5$
   c. $4^{4x-8}+1=257$
   d. $12x-10=2(2x+3)$

GEOMETRY OF STRAIGHT LINES ACTIVITIES

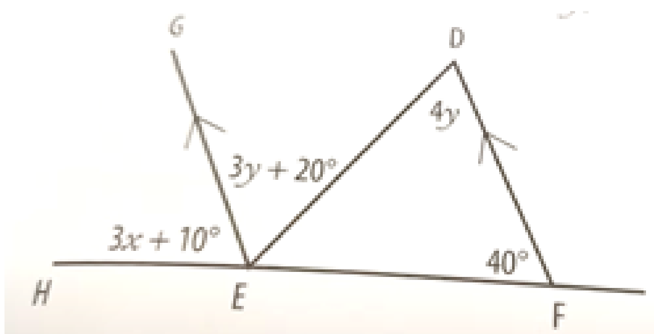
DISCUSSION ACTIVITY 1

1. The diagram below shows a right-angled triangle ΔABC with $\hat{A} = 38°$. In ΔBCD $BD = CD$ and $AB||CD$. Find x, y and z, with reasons



DISCUSSION ACTIVITY 2:

1. In the figure below, determine with reasons the value of x and y.



ANNEXURE G: Example of Observation Tool

Section A: Background information ***(this information will not be used in reporting of results, in the case it is used only pseudo names will be used)***

Name of Observer: _______________ Date of Observation: ____________

Duration of Observation: _________ Total no of Participants: ______

Presenter: _________________________

Section B: Background ***information (This section provides a brief overview of the session being observed)***

Explanation of the classroom setting in which the observation was conducted, nature of the classroom as well as its conduciveness to teaching and learning:

____
____
____
____
____
____
____
____

Section C: Research observation

1. Identify the different mathematics vocabularies used in the classroom during the classroom discourse? (Word use)

Vocabulary	Communicated by	Vocabulary	Communicated by

2. Which visual mediators are used in the classroom in order to explain or understand the vocabularies used (Visual mediators)

Vocabulary	Explanation (visuals)

3. Is there any pattern in how they explain the used vocabulary? If so how? (Routines)

Vocabulary	Patterns in explanations

4. What are the narratives that were used during the classroom discourse?

Item	Narrative

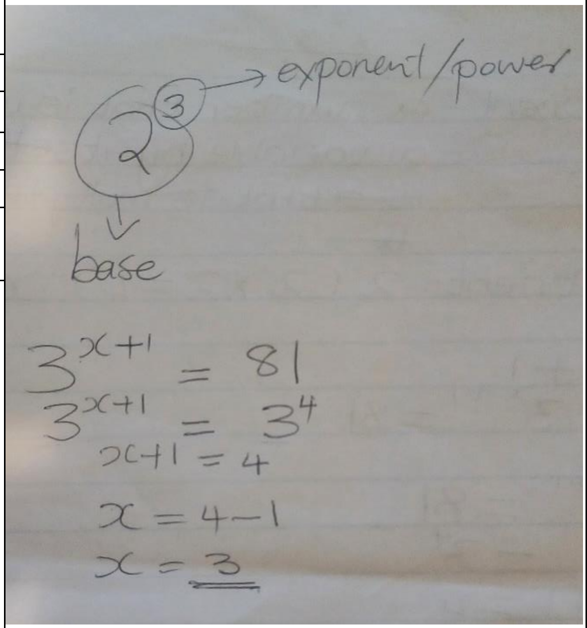
ANNEXUTE H: Example of Interview Guideline Tool

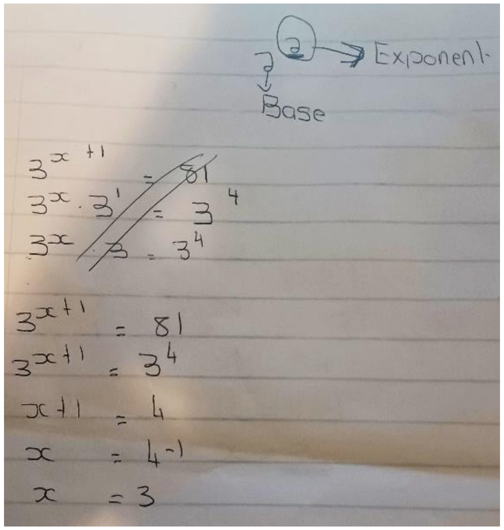
GUIDELINES FOR INTERVIEW QUESTIONS:

The nature of the questioning technique that will be used will follow the sequence below:

1. What is your understanding of the word ……………….
2. How can you explain the concept to another learner?
3. Can you explain it in another way?
4. Can you provide an example?
    a. In explaining we will be looking at the following:
        i. Word use – different words the learner is using to explain
        ii. Visual mediators – how they explain it visually (drawing) or algebraic equations (use equations)
        iii. Routines – the patterns of answering the 2 questions above
        iv. Narratives – observed from all the above

ANNEXURE I: Interview Transcription

INTERVIEW WITH THE L2 AFTER LESSON FOR EXCERPT 1			
Turn	**Speaker**	**What is said**	**Visual Mediator**
I.1.1	I	What do you understand by the word exponent?	This is what the learner (L2) was writing when answering the interview questions 
I.1.2	L2	An exponent is a number that we can raise in the form of a power to another number	
I.1.3	I	How can you explain the concept on exponent to another learner?	
I.1.4	L2	I will say an exponent is a number that you use to multiply that base raised to for example two exponent two means you multiply two twice	
I.1.5	I	Do you have another way which you can explain the word exponent?	
I.1.6	L2	No	
I.1.7	I	Can you provide we with an example of an exponent	
I.1.8	L2	Two exponent 3, where the three is an exponent and the two is the two is an exponent	
I.1.9	I	What do you understand when they say solve the exponent 3 exp x+1 = 81 or solve the equation	
I.1.10	L2	They actually want the value of x for which you raise 3 as the sum of x+1 as the exponent so that it can give you 81. you just keep 3 exp x+1 on the LHS and go to the RHS where you have 81, you have to simplify it to a base that is the same as the LHS that means your base must be 3 and be raised to a certain exponent which must give you 81 which the exponent is 4. When you have 3 exp x+1 equals to 3 x 4, we have the same bases and when we have the same bases you go to the exponents and ignore the base you go to x+1 = 4, when the bases are the same, we simply equate the exponents. So that means x+1 will be equal to 4 and the you take the 1 to the RHS and transpose it and it will be x=4-1 and that will be equal to 3 This means x here will be 3 and 3+1 is 4 which is equal to 81.	
I.1.11	I	Let’s go back to where the bases are the same you said we are actually going ignore or rather say they vanish? I want you to explain that	

I.1.12	L2	When I say the vanish, I mean it's a law of exponents when bases are the same in an equation you equate, you make the exponents an equation, same applies to when you have the exponents being the same you equate the bases. It means if I have x to the power 3 is equal to 2 to the power 3 it means x = 2 because the powers are the same.	
INTERVIEW WITH THE L3 AFTER LESSON FOR EXCERPT 1			
Turn	**Speaker**	**What is said**	**Visual Mediator**
I.2.1	I	What do you understand by the word exponent?	This is what the learner (L3) was writing when answering the interview questions 
I.2.2	L3	An exponent is a number which tells us how many times does a number multiply it self	
I.2.3	I	How can you explain the concept on exponent to another learner?	
I.2.4	L3	An exponent is shown by a repeating number that multiply is self	
I.2.5	I	Do you have another way which you can explain the word exponent?	
I.2.6	L3	no	
I.2.7	I	Can you provide we with an example of an exponent	
I.2.8	**L3**	Two to the exponent 2, 2 up here is our exponent and the two below is a base	
I.2.9	I	What do you understand when they say solve the exponent 3 exp x+1 = 81 or solve the equation	
I.2.10	**L3**	one of the laws of exponents is that when two numbers multiply each other and they have exponents and they are the same, we keep the bases and add the exponents. We can also write 81 terms of 3 to the exponent something and t will be 81 written the same way as 3 exponent 4. Since the bases are the same, we can equate the exponents and we have x+1 equals to 4 and x is equal to 4 minus 1 which is 3	
I.2.11	I	Let's go back to where the bases are the same you said we are actually going ignore or rather say they vanish? I want you to explain that.	
I.2.12	**L3**	When the bases are the same and we are multiply, we keep the bases and add the exponent. What I have learner and what they have taught me is that when the bases are the same, we equate the exponents	
I2.13	I	Then what happened to the bases because they are now no longer there when you equate the exponents	
I2.14	L3	Ah! Sir I don't know	