INVESTIGATING GRADE 4 LEARNERS' ENACTMENT OF UTTERANCES EMBODIED IN A MATHEMATICAL CLASSROOM DISCOURSE

MASTER OF EDUCATION

TA MOKWANA

2023

INVESTIGATING GRADE 4 LEARNERS' ENACTMENT OF UTTERANCES EMBODIED IN A MATHEMATICAL CLASSROOM DISCOURSE

by

TEBOGO ABIE MOKWANA

[redacted]

DISSERTATION

Submitted in fulfilment of the requirements for the degree of

MASTER OF EDUCATION

in

MATHEMATICS EDUCATION

in the

FACULTY OF HUMANITIES

(School of Education)

at the

UNIVERSITY OF LIMPOPO

SUPERVISOR: DR DJ MUTHELO

2023

DECLARATION

I declare that the dissertation hereby submitted to the University of Limpopo, for the degree of Master of Education in Mathematics Education has not previously been submitted by for a degree at this or any other university; that it is my work in design and in execution, and that all material contained herein has been duly acknowledged.

____________________ __________________

Mokwana, TA (Mr) **Date**

DEDICATION

I dedicate this dissertation to the following persons:

- My parents, Mr Jerry Thabana Mokwana and Mrs Grace Mankopodi Mokwana, for their love, support, and encouragement.
- My brother, Dr Lekwa Mokwana, for his love, care, support, inspiration, and encouragement to study further.
- My sister, Ms Baratang Miriam Mokwana, for her love and encouragement.
- My partner, Ms Keneilwe Mogale, for her love, support, and encouragement.

ACKNOWLEDGEMENTS

- Primarily, I thank God as I believe he gave me strength to persevere and work hard throughout the study.
- Next, I thank my supervisor Dr Dimakatjo Muthelo for his dedicated support and guidance. Dr Muthelo always gave his all in any way possible to assist and advise throughout the research project. Furthermore, thank you for making time for us to meet on weekends to make sure that I complete this study.
- Further, I thank Dr Paul Mutodi for his guidance and providing insightful knowledge in conceptualising the study.
- Moreover, I thank Dr Lekwa Mokwana for valued inputs when conceptualising the study. Thank you for our chats, which made a tremendous contribution to the study and your personal support of my academic endeavours.

ABSTRACT

The study aimed to investigate Grade 4 learners' enactment of utterances embodied in mathematical classroom discourse. I adopted teaching experiments as a design for the study and I observed learners' utterances and interactions in their attempts to develop mathematical knowledge; hence I adopted a teaching experiment design. I designed the research process, and engaged with and reported on all 73 Grade 4 learners in the classroom. The learners were grouped in groups of four in order for them to engage with each other and with me. Furthermore, I only questioned and guided learners when they were learning in groups. In teaching experiment design, the researcher's role is to organise, listen and observe and, in this study, I organised, listened to and observed the learners, who I viewed as 'students of mathematics'. I collected data by making use of observations, interviews, video recordings and note-taking to capture learners' interactions among themselves and with my. Furthermore, I used preliminary and retrospective analysis, as prescribed by the teaching experiment design (Steffe & Thompson, 2000). As a result, I captured the results from each teaching episode and analysed the learners' interactions preliminarily. Afterwards, I returned to the teaching episodes and analysed the learners' interactions in order to undertake a retrospective analysis. I used tenets of the dynamic system theory of cognitive development to analyse the interactions.The key findings from the study were that learners portray different utterances depending on the concept of engagement. These utterances influenced their learning of mathematical concepts. The positive utterances that learners portrayed enhanced their development in mathematical thinking. On the other hand, negative utterances did not influence the development of learners' mathematical thinking. Furthermore, learners portrayed positive utterances when engaged in questions involving mathematical shapes and negative utterances when engaged in word problems.

TABLE OF CONTENTS

CHAPTER 1: INTRODUCTION

1.1 BACKGROUND

There is an attempt in research on how learners make use of utterances to understand mathematical concepts (Alibali & Nathan, 2012; Nemirovsky & Ferrara, 2009; Desutter & Stieff, 2017). The attempts are caused by the belief in research that learners will better understand mathematical concepts when they make use of these utterances when learning mathematical concepts (Alibali & Nathan, 2012). There is a concern about how learners make use of utterances to express mathematical ideas, and that teachers do not take this into consideration (Lester & Newman, 2018). There is a lack of teacher understanding of how the body and mind work together to build a mathematical idea (Alibali & Nathan, 2012). Furthermore, there are attempts at trying to understand how learners make use of the utterances when learning concepts to embody mathematical concepts (Vasc & Ionescu, 2013),

The human mind is not only formed by our brain but is also an aspect of our body (Tran, Smith & Buschkuehi, 2017). The brain internally communicates with the body when engaging with ideas (Nguyen & Larson, 2015). This is referred to as embodied cognition (Nemirovsky & Ferrara, 2009). Communication can be physically noticeable and/or inhibited in the mind (Nemirovsky & Ferrara, 2008). Learners are engaged in learning that makes the communication abstract and become inhibited in the mind or the communication become physically noticeable (Alibali & Nathan, 2012). Communication becomes inhibited in the mind when learners are engaged in rote learning, where they are passive recipients of knowledge (Alibali & Nathan, 2012). On the other hand, communication becomes physically noticeable in discourse learning, where learners actively create knowledge (Vasc & Ionescu, 2013).

Learners exhibit utterances such as body motion, eye motion, hand gestures, tone of voice and sound productions when learning mathematical concepts in a classroom (Alibali, 2011; Alibali & Nathan, 2012). The utterances portray whether learners are engaged in ‘offline cognition’ or ‘online cognition’ as modes of cognition concepts in the classroom (Nemirovsky & Ferrara, 2009). Learners are engaged in offline cognition when they can embody an idea for problem-solving purposes (Nemirovsky & Ferrara, 2009). Furthermore, learners are engaged in online cognition to deal with more straightforward activities, usually exercises after introducing an idea (Nemirovsky & Ferrara, 2009).

Learners in their early education (ages 6–9 years of age) find it challenging to engage in the offline cognition of mathematical ideas (Desutter & Stieff, 2017). They are unable to use embodied concepts for problem-solving purposes (Nemirovsky & Ferrara, 2009). However, according to Vasc and Ionescu (2013), learners in early education use utterances when learning mathematical concepts. Therefore, it is not known how they understand mathematical concepts when using the utterances (Bjomebye, 2019; Nemirovsky & Ferrara, 2009; Soylu, Lester & Newman, 2018; Shimeng, 2021).

1.2 PROBLEM STATEMENT

Embodied cognition enables learners to portray and use utterances in a mathematics classroom discourse (Alibali & Nathan, 2017; Nathan, Williams-Pierce *et al*, 2021). In the classroom, learners should portray and use utterances when explaining ideas and asking questions, and when interacting with other learners and a teacher (Alibali, 2011; Abrahamson & Tminic, 2015; Desutter & Stieff, 2017). This improves the learners' ability to connect to mathematical concepts and develop mathematical competence, particularly at the primary level (Selvianiresa & Prabawanta, 2017). It is believed that learners between the ages of 6 and 9 years express and learn ideas through utterances (Vasc & Ionescu, 2013; Weisberg & Newcombe, 2017). They use utterances to enhance their verbal interactions during whole-class discussions (Margutti, 2010).

Studies on utterances focused on analysing teachers' and learners' actions embodied in the mathematics classroom (Alibali, 2011; Askew, Abdulhamid & Mathews, 2014). Studies on teachers' actions focused on how teachers demonstrate mathematical concepts for learners to embody mathematical cognition (Askew, Abdulhamid & Mathews, 2014; Alibali, 2011; Alibali & Nathan, 2012; Askew, 2012). Teachers demonstrate simulated activities to learners to embody mathematical cognition (Abrahamson & Lindgren, 2018). As a result, learners capture the idea but find it difficult to cope with abstract activities (Abrahamson, 2017; Soylu et al., 2018). This is caused by learners only focusing on imitating the teacher (Benedek, 2019; Roth, 2010).

On the other hand, studies on learners' actions focused on learners' embodiment of mathematical concepts in a discourse classroom (Abrahamson, 2017; Alibali, 2011; Roth, 2010). In a mathematics classroom discourse, learners portray utterances to show their ability to make sense of mathematical concepts (Landy *et al*, 2014; Soylu et al., 2018). However, learners also portray utterances that show their inability to link ideas learnt when dealing with abstract mathematical concepts (Askew et al., 2014; Murphy, 2006; Roth, 2010).

The studies do not indicate how young children understand mathematical concepts through the utterances they portray (Alibali, 2011; Desutter & Stieff, 2017; Saleh, Muhammad & Abdullah, 2020; Selvianiresa & Prabawanta, 2017; Vasc & Ionescu, 2013); hence, they find it difficult bring their mental representations into action to deal with abstract mathematical concepts (Bjomebye, 2019; Liu, 2020; Soylu et al., 2018). Furthermore, the lack of understanding of how learners understand concepts through their utterance will prevent teachers from helping learners to understand the concepts (Vasc & Ionescu, 2013).

The proposed study intends to investigate Grade 4 learners' enactment of utterances in a mathematics classroom discourse.

1.3 PURPOSE OF THE STUDY

The study aimed to investigate Grade 4 learners' enactment of utterances in a mathematical classroom discourse.

1.4 RESEARCH QUESTIONS

This study was guided by the following research questions:

- What utterances do Grade 4 learners portray in a mathematics classroom discourse?
- What do the utterances reveal about learners' understanding of mathematical concepts?

1.5 SIGNIFICANCE OF THE STUDY

The study will contribute knowledge on how young mathematics learners gain an understanding of mathematical concepts through their utterances. Therefore, primary school mathematics educators will be able to pay attention to learners' utterances and know what they reveal about their understanding of mathematical concepts. Furthermore, the teachers will be able to interpret learners' utterances and develop a suitable instruction for the learners in the classroom.

Primary school educators will be aware of the value of learners' embodied actions and their meaning. The educators will be able to see the body's contribution to learning mathematical concepts and how it works with the mind in constructing knowledge.

Teachers will be able to learn new ways of instruction to consider, which will take note of learners' utterances. They will be able to see that utterances carry meaning and therefore infuse that understanding into new classroom designs.

1.6 TEACHER RESEARCHER

I have taught mathematics Grade 4 from 2019 to date, and have therefore been involved in preparing and administering lessons. I always strived to make use of the teaching strategies that I learnt during undergraduate studies. The strategies involve building concepts from learners' prior knowledge and allowing them to discover new knowledge through questioning. This has made learners adapt to a new way of learning mathematics as they thought mathematics is all about getting procedures and examples and then calculating, looking at the examples. Furthermore, as I developed in teaching Grade 4 learners, it made me realise that they are curious about learning mathematics.

Fortunately the methodology that I adopted in the study allows me as a teacher to give learners activities and observe their understanding of mathematical concepts, rather than giving them procedures on how questions should be answered. I assumed the dual role of being a teacher and a researcher, which gave me an opportunity to rigorously reflect on the teaching and learning of mathematical concepts. Learners had become used to the teaching strategy that I use and, as a result, how my lessons proceeded it was not surprising to them. Furthermore, I did have access to the whole classroom as I was interacting with learners in their groups.

1.7 SETTING

The study took place in a school in Pietersburg education circuit, Polokwane, South Africa. The school is a quintile 5 school and therefore parents are paying for the learners' tuition. Furthermore, the school consists of only 417 learners. This is the school where I teach learners intermediate and senior phase mathematics daily. The school is situated on one of the farms on the outskirts of Polokwane, where there is no immediate community to service. Therefore, some learners travel to the school daily, while others stay in the school's hostel facilities. The school consists not only of learners who come from places around Polokwane but also from around the province of Limpopo at large.

I conducted the study with my Grade 4 learners in the school, 73 learners in total.

1.8 OVERVIEW OF THE DISSERTATION

In Chapter 1, I provide an introduction of the overall study as follows: First, I present the background to the study by highlighting literature that talks about embodied cognition and utterances. This makes it possible for me to highlight the notion that young children learn through those utterances. Secondly, I present the research problem statement, where I identify a gap in the literature for the purposes of this study. Thirdly, I outline the purpose of the study and the research questions that emanated from the problem statement. After that, I outline the significance of the study, which shows the importance of the study in teaching and learning. Lastly, I outline details of the setting of the study.

In Chapter 2, I provide an introduction to the literature review, a detailed discussion of literature that focuses on studies related to the embodied cognition utterances that learners portray and the learning environment of engagement. Furthermore, I also outline the theoretical framework I used as a lens for the study. This theoretical framework guided me when I analysed the data by using its construct. The theory adopted for use in studies on embodiments is the dynamic system theory of cognitive development, which indicates that the body and brain work together to construct knowledge (Thelen & Smith, 1995). The constructs of the theory are: 'thought as an in-the-moment event, thought as being open to a continually changing world, and thought as cognition that is not stationary' (Smith, 2005). Lastly, I present a summary of the chapter,

In the third chapter, I introduce the methodology and described the research approach, and how and why I adopted a qualitative research approach for the study. After that, I describe the research design I used and the teaching experiments I employed, starting by describing prominent research designs in the qualitative research approach. Furthermore, I describe the participants of the study and how data was collected and analysed, as prescribed by the teaching experiments. After that, I outline quality criteria and ethical considerations to describe how I ensured the quality of the study and how issues around ethics were dealt with correctly, respectively. Lastly, I present a summary of the chapter.

In Chapter 4, I present the results and an analysis of the results I obtained from the study in the form of three teaching experiments. I present the results for each teaching experiment and discuss the preliminary analysis. Thereafter, I used the constructs of the dynamic system theory of cognitive development (Thelen & Smith, 1995) to describe the retrospective analysis of the results. After that, I provide summary of the chapter.

In the fifth chapter, I draw conclusions to the study, based on the results and an analysis of the results. After that, I identify the limitations of the study, based on the conclusion, and present recommendations for further research. Lastly, I present a summary of the findings.

1.9 SUMMARY

In Chapter 1, I presented the background to the study, followed by the problem statement, which situates the study among other studies on embodied cognition. The purpose of the study was deduced from the problem statement. Furthermore, I came up with research questions for the study. Thereafter, I described the study's significance and the study's setting.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, I present studies on embodied cognition in mathematics classroom discourse and how utterances come to the surface. After that, I present the relationship between utterances and embodied cognition, and how primary school learners learn through the use of the utterances they portray. I then elaborate on how the learners embody mathematical cognition through utterances and strategies for teaching and learning. After that, I elaborate on the theory – the dynamic system theory of cognitive development (Thelen & Smith, 1995) – that I adopted for the study. Lastly, I elaborate on how the theory is used for data analysis (Smith, 2005).

2.2 EMBODIED COGNITION

Embodied cognition refers to learners' ability to engage in bodily activities during learning and provides teachers with a deep understanding of learners' perceptions of mathematical knowledge and actions with respect to the learning of mathematical concepts (Abrahamson & Tminic, 2015). The embodiment of mathematical knowledge is rooted in learners' perceptions and actions when learning mathematical concepts (Anada & Prabawanta, 2020). The perceptions refer to learners' ideas about mathematical concepts and actions refer to what learners do when learning the concepts (Nemirovsky & Ferrara, 2008). These perceptions and actions are made explicit through utterances that learners portray in a mathematical classroom (Nemirovsky & Ferrara, 2008). Furthermore, these perceptions and actions depend on the learning environment in which teaching and learning occurs (Desutter & Stieff, 2017). Therefore, learners' utterances can have a huge effect on how they think, and how they think can influence their actions (Tran, Smith & Buschkuehl, 2017). However, a contributing factor is the learning environment in which engagement with mathematical concepts takes place (Weisberg & Newcombe, 2017).

2.3 MATHEMATICAL CLASSROOM DISCOURSE

Discourse refers to the verbal communication that is used to share ideas (Walshaw & Anthony, 2008). However, communication takes place when there is an idea that individuals are trying to interrogate and exchange ideas on (Alibali & Nathan, 2012).

According to Nguyen and Larson (2015), when individuals share ideas verbally, some of the communication is non-verbal.

Mathematical classroom discourse is the use of communication in the mathematical classroom when learning concepts (Walshaw & Anthony, 2008). Learners make use of the communication to interact with each other and with the teacher when learning mathematical concepts (Wagner, 2007). Enculturation into such learning environments enables learners to develop mathematical knowledge. However, teachers have a responsibility to ask leading questions, to ask students specific questions and to encourage learners to present and share ideas in order to support the learners (McCarthy et al*.,* 2016). Learners develop an understanding of mathematical concepts and become aware of more advanced questions when they are engaged in a mathematical classroom discourse (Wagner, 2007).

Furthermore, in mathematical classroom discourse learners, can freely engage in perceptions of mathematical ideas and actions with respect to their bodily movements (Nemirovsky & Ferrara, 2009). This learning environment allows learners to create memory traces as they relate mathematical thinking to their body movements in the process of understanding mathematical knowledge (Tran, Smith & Buschkuehl, 2017). As a result, learners embody mathematical knowledge when learning mathematical concepts (Abrahamson & Tminic, 2015). On the other hand, in an environment where learners are not engaged in bodily actions, learners do not develop an understanding of mathematical knowledge (Nemirovsky & Ferrara, 2009). This causes a mismatch between their bodily actions and the conception of mathematical knowledge (Noble, Nemivrosky, Wright & Tierney, 2001).

In mathematical classroom discourse, learners use non-verbal utterances to represent ideas when learning concepts (Weisberg & Newcombe, 2017; Munson & Dyer, 2020). The use of utterances reflects embodiment and helps develop thinking when learning mathematical concepts (Vasc & Ionescu, 2013). Learners embody mathematical concepts by linking linguistic and sensorimotor representations (Weisberg & Newcombe, 2017). However, communication is often imaginary and abstract (Alibali & Nathan, 2012).

Learners deal with abstract mathematical concepts by linking the knowledge to their bodily actions to make it less abstract (Tran, Smith & Buschkuehl, 2017). This involves the use of hands to imagine mathematical knowledge when learning mathematical concepts (Anada & Prabawanta, 2020). This helps learners to organise their thoughts as bits of information, rather than trying to understand abstract mathematical information as a whole (Tran, Smith & Buschkuehl, 2017).

2.4 EMBODIED COGNITION AND STRATEGIES FOR TEACHING AND LEARNING

Learners embody mathematical concepts by making use of various methods of learning as strategies for teaching and learning, which include motor learning (Abrahamson & Tminic, 2015), collaborative learning (Desutter & Stieff, 2017), problem-based learning (Saleh, Muhammad & Abdullah, 2020), discovery learning (Sung et al., 2017) and gaming (Casano, Tee, Agapito, Arroyo & Rodrigo*,* 2016). These methods of learning engage learners in a learner-centred classroom to embody cognition (Lestari & Surya, 2018). Hence, the embodiment of mathematical concepts focuses on learners rather than on the teacher. However, the learners' attitude towards the teaching method influences their understanding of mathematical concepts (Andamon & Tan, 2018).

Learning mathematical concepts by playing games is effective since it affords learners an opportunity to be hands-on, participate actively and communicate (Casano et al*.,* 2016). Hence, when playing gaming, learners demonstrate a greater understanding of mathematical concepts (Burte et al*.*, 2017). This shows an embodiment of mathematical concepts accumulated in a technological way of learning (Cangelosi & Stramandinoli, 2017). Learners face the challenge of focusing on the game itself, rather than the concept that should be learnt (Cangelosi & Stramandinoli, 2017). On the other hand, learners face the challenge of being flexible with mathematical concepts learnt and fail to deal with abstract mathematical concepts using pen and paper (Dove, 2015). These challenges are caused by learners being enculturated into learning using games but having to us pen and paper when being assessed (Menary, 2015).

Embodied cognition through motor learning engages learners in the use of existing schemas to make sense of mathematical activities through smooth transition from less complex problems to more complex problems (Abrahamson & Tminic, 2015). These self-exploratory activities enable learners to discover mathematical concepts in a learning environment (Sung et al., 2017; Truxaw & DeFranco, 2008), which is called a mathematical classroom discourse (Walhow & Anthony, 2008). In this classroom, learners think about mathematical concepts coherently, from simple to complex, rather than thinking about concepts as being isolated (Abrahamson & Tminic, 2015). Furthermore, learners are actively engaged and there is a noticeable growth in learners' development of mathematical learning (Walshaw & Anthony, 2008). However, learners need to be enculturated into this learning environment (Walshaw & Anthony, 2008), thus, the environment will allow researchers to listen to and observe learners' autonomous thinking and idea generation when learning (Truxaw & Anthony, 2008).

In the mathematical classroom discourse, educators play an important role in the learners active engagement in developmental learning (Walshaw & Anthony, 2008). The educator must give learners an exploratory activity, encourage learners to work independently of their educator, observe and listen to learners' ideas, and regulate feedback (Truxaw & Anthony, 2008). Furthermore, educators need to construct meaningful questions to help learners to understand mathematical concepts (Truxaw & Anthony, 2008). Thus, the educators will be acknowledging the value of learners' ideas (White, 2003).

2.5 ROLE OF UTTERANCES

Utterances are verbal and non-verbal communications that learners portray when engaged in learning of mathematical concepts (Alibali & Nathan, 2012). The utterances are shown by learners to express their understanding of and confusion with mathematical concepts (Nevskyemir & Ferrara, 2008). The utterances that learners portray to show understanding are regarded as positive utterances and utterances that learners portray to show confusion are regarded as negative utterances (Alibali, 2011). Learners show positive utterances, such as head movements, hand movements and smiles, to show their understanding of concepts and movement of shoulders, rolling of eyes and making 'uhm' sounds to show their confusion (Nevskyemir & Ferrara, 2008).

The role of utterances in a mathematical classroom discourse is to enable learners to recall information, stimulate action and represent thoughts (Weisberg & Newcombe, 2017). In the classroom, learners engage in perceptions and actions as the process of discovering mathematical knowledge through utterances (Nurihsan & Agustin, 2016). This engagement can either be with the self, with other learners and/or with the teacher (Nemirovsky & Ferrara, 2008). Therefore, successful engagement enables learners to create strong memory traces for problem-solving (Tran et al., 2017).

Learners use gestures when asking questions, explaining and reasoning, and during interactions in the classroom (Nathan & Walkington, 2017). This helps learners reveal important information about their thinking and express their perception-based knowledge verbally (Nathan & Walkington, 2017). Furthermore, learners' utterances provide valuable information to their teachers on how best to engage with or intervene in their learning (Weisberg & Newcombe, 2017). On the other hand, learners seldom get an opportunity to create meaning using the utterances they portray in mathematical classroom (Weisberg & Newcombe, 2017).

Teachers make use of utterances themselves to explain mathematical concepts to learners and, as a result, learners seldomly create meaning when learning the mathematical concepts (Weisberg & Newcombe, 2017). The study undertaken by Arani (2017) shows that the teachers' use of utterances in mathematics classroom take precedence over the learners' use of utterances. Furthermore, 'novice' educators develop same culture as 'expert' educators, hence there is no role of utterances which learners depict (Arani, 2017)

2.6 CLASSIFICATION OF GESTURES

Gestures contribute to the comprehension of spoken language in both adults and young children (McNeil, Alibali & Evans, 2000; Parton & Edwards, 2009). In adults, they help a little because adults have a developed understanding of the world, but in young children they contribute a great deal to the development of an understanding the world (McNeil, Alibali & Evans, 2000). According to Gullberg (2006), gestures are developing systems in understanding subject matter because young children make use of hands to denote messages and/ or support speech.

In a classroom, young learners make use of gestures to support their thinking and communication so that they can take action when representing their ideas in writing (Lim, Wilson, Hamm, Phillips, Iwabuchi Carballis & Thomas, 2009). Learners make use of gestures as tools to reflect ideas they have about a problem and for transferring knowledge to generalisations about the idea in order to understand abstract concepts (Goldin-Meadow, 2015). Furthermore, these gestures can equip teachers with information about whether learners are on task or not, and/or if learners find difficulties with understanding a problem (Lim et al*.,* 2009). Teachers need to check whether the gestures portrayed by learners portray the same message as their speech, whether the gestures add to their speech and whether there is a mismatch between the gesture and the speech (Goldin-Meadow, 2015). Therefore, teachers will gain an informed understanding of the learning of concepts and also make informed decisions about how they can best assist learners to enhance their understanding and embody the concepts using gestures (Gullberg, 2006).

2.7 DIFFERENT TYPES OF GESTURES

Learners exhibit different gestures as a representation of the meaning they attach to concepts in a mathematical classroom discourse (Abrahamson, Nathan, Williams-Pierce, Walington, Ottomar, Soto & Alibali, 2020). According to Alibali and Nathan (2012) gestures show the level of embodied knowledge. The embodied knowledge is grounded in the kind of learning environment that learners experience. Learners embody mathematical knowledge when they engage in collaborative learning (Munson & Dyer, 2020). Learners can use pointing, representational and/or metaphoric gestures to show the level of embodied knowledge when learning (Nathan & Watkins, 2017).

2.7.1 Pointing gestures/within moments

Ponting gestures are the perceptual memories and symbols that help learners create meaning when learning (Alibali & Nathan, 2012; Nathan & Walkington, 2017). Pointing gestures are the immediate thinking that learners portray when learning mathematical concepts (Alibali & Nathan, 2012). Learners use pointing gestures when trying to attach meaning to mathematical concepts in a classroom and teachers make use of pointing gestures to guide students' attention and to influence speech, especially when a verbal message is disregarded in the classroom (Nathan & Walkington, 2017).

2.7.2 Representational gestures/across moments

Representational gestures involve simulation activities that allow learners to experience a particular action (Alibali & Nathan, 2012). These simulation activities involve a learner making mental images of an idea they are trying to learn (Alibali & Nathan, 2012). Nemirovsky (2008) supports the notion that learners create mental images through mathematical imaginations. The imaginations engage them in the depiction of utterances that symbolise a mathematical idea learnt (Kosmos & Zaphiris, 2018).

In mathematical classroom, learners should be engaged in a learning activity that allows them to make use of mathematical utterances (Alibali & Nathan, 2012). The utterances support learners' mental images, which they create when they think about mathematical concepts (Munson & Dyer, 2020). This helps learners to create their own meaning and become successful at solving complex mathematical problems (Nathan & Walkington, 2017).

On the other hand, learners can imagine mathematical concepts but fail to solve these complex mathematical concepts (Munson & Dyer, 2020). This involves application activities that require learners to connect with previously learnt concepts (Munson & Dyer, 2020). Learners' mental imaging can be left as imaginary information and lack connection to the bodily movements (Munson & Dyer, 2020). As a result, learners lack the connection of previously learnt mathematical knowledge to new mathematical concepts (Nemivrosky, 2008)

2.7.3 Metaphoric gestures/beyond moments

Metaphoric gestures are simulation activities that involve learners' use of schemas to create meaning when engaged in abstract concepts (Alibali & Nathan, 2012). Learners make use of metaphoric gestures when they think deeply about mathematical concepts, especially during problem-solving (Munson and Dyer, 2020). Therefore, learners do not end up having fixed cognitive schemas and become successful at problem-solving (Schoever, Leseman, Slot, Bekker, Keijzer & Kroesbergens, 2019).

Learners' use of metaphoric gestures is evident when they make use of different contexts to create mathematical knowledge (Schoever et al*.,* 2019). The learners make use of their 'offline cognition' of related mathematical contexts to successfully solve complex mathematical problems (Nemivrosky & Ferrara, 2008). Therefore, the creation of the mathematical knowledge to solve complex mathematical problems is dependent on learners' embodiments of the mathematical knowledge (Alibali & Nathan, 2012).

2.8 UTTERANCES AND PRIMARY EDUCATION

According to Ionescu (2013), learners at the primary level use utterances to enhance their verbal communication when learning. Some learners may offer more advanced explanations through non-verbal communication than when using verbal communication (Ionescu, 2013; Saleh, Muhamad & Abdulla, 2020). However, teachers in the classrooms focus on learners' participation and beliefs rather than reasoning (Hwang, 2018; Bakker & Schoevers, 2019) and, as a result, some learners will lose interest in the sharing of their ideas on the subject matter (Bjomebye, 2019).

Learners who have lost interest in sharing ideas are often disregarded by the teacher over learners who share their ideas because teachers only focus on learners who participate (Bjomebye, 2019). Therefore, educators disregard utterances that the learners portray (Roth, 2010). It is of the outmost importance not to disregard young children's interactions as they contain utterances that create meaning in mathematics learning (Roberts & Venkat, 2016). Disregarding these interactions simplify mathematical concepts for learners and they will be able to deal with abstract mathematical problems (Askew et al*.,* 2014; Murphy, 2006). As a result, learners will develop a strong connection ability to deal with complex mathematical concepts and therefore succeed in problem-solving (Askew et al*.,* 2014; Selvianiresa & Prabawanta, 2017).

On the other hand, teachers believe that demonstrating mathematical concepts to elementary learners helps them make sense of mathematical concepts (Abrahamson & Lindgren, 2018). However, this enables learners to learn without interacting with each other but to only listen, observe and imitate what the teacher does in solving mathematical problem (Nathan & Walington, 2017). Benedek (2019) argues that allowing learners to imitate teachers' utterances helps them solve problems. However, the challenge is rooted in learners' only thinking about the utterances of the teacher and the meaning attached to them (Soylu et al., 2018). These learners are not allowed to create their own meaning attached to their own utterances (Shimeng, 2021).

2.9 UTTERANCES AND MATHEMATICAL COGNITION

Embodied mathematical cognition enables learners to engage offline instead of online cognition (Novskyemir & Ferrera, 2008). Offline cognition refers to the embodied knowledge available to learners to deal with future mathematical problems, while online cognition is the embodied knowledge that is only available in a current situation (Novskyemir & Ferrera, 2008). Learners are engaged in bodily activities to embody mathematical cognition, enabling them to engage their offline cognition (Abrahamson & Tminic, 2015). Therefore, learners who can engage their offline cognition cope with complex mathematical problems and can reason mathematically (Saleh et al., 2020). However, learners should be engaged in meaningful learning that arouses their interest in concepts in order to embody mathematical knowledge (Anada & Prabawanta, 2020; Saleh et al., 2020).

On the other hand, learners use utterances to derive an understanding of mathematical concepts in a collaborative classroom (Desutter & Stieff, 2017; Mahon, 2015). In the collaborative classroom, learners are engaged in sensory motor activities that allow them to use their perceptual memories while interacting with other learners (Kosmas & Zaphiris, 2018). Therefore, learners use their offline cognition to encode abstract information and engage in mathematical reasoning through their interactions (Mahon, 2015; Saleh & Prabawanta, 2020; Schoever, Leseman, Slot, Bekker, Keijzer & Kroesbergens, 2019; Weisberg & Newcombe, 2017;). This supports research on science, technology, engineering and maths (STEM) as these subjects require learners to use reasoning when learning (Weisberg & Newcombe, 2017).

Learners' engagement in reasoning when learning improves their attention limits and enables them to manage the complexity of demanding tasks (Nathan & Walkington, 2017). Weisberg and Newcome (2017) believe that learners develop routine procedures for dealing with similar mathematical problems to reduce cognitive load in a mathematical classroom discourse (Weisberg & Newcombe, 2017). This enables learners to cope with problem-solving questions when dealing with abstract mathematical concepts (Weisberg & Newcombe, 2017).

Teachers over use utterances when explaining mathematical concepts for learners to embody mathematical cognition in a teacher-centred classroom (Rueckert, Church, Avila & Trejor, 2017). However, learners become passively engaged in such a learning environment because they listen to what the teacher says, observe what they do and receive direct answers when asking questions, instead of the teacher using questioning to initiate discussions (Rueckert et al., 2017). As a result, learners lose mathematical connections along the way and fail to activate offline cognition for problem-solving (Rueckert et al., 2017). Therefore, these learners face problems with generalising mathematical concepts and being flexible in solving mathematical problems (Healy, 2000; Dove, 2015).

2.10 THEORETICAL FRAMEWORK

I have adopted the dynamic system theory of cognitive development (Smith, 2005; Thelen & Smith, 1995) for this study to make sense of and describe embodied cognition as I observed it in my mathematics classroom. The theory states that cognition combines processes in the mind, the body, and the world (environment) (Smith, 2005). This is referred to as an embodied approach to cognition, which offers an understanding of how body and mind work together to enable action and cognition in the world (Spencer et al., 2012). The body contributes to cognition and influences information processing through action (Soltz & Shapiro, 2019). The theory argues that cognition is a process that consists of three dimensions: thought as an in-the-moment event; thought as being open to a continually changing world; and thought as cognition that is not stationary (Smith, 2005).

Thought as an in-the-moment event involves the creation of constructs in the mind for any given activity (Smith, 2005). The constructs created indicate learners' success or failure in doing an activity. However, this is characterised by forming a plan of action when learners encounter an activity. This plan uses memories to determine the best way to attempt the activity. These memories are called perceptual memories, which occur when learners interact with the subject matter (Alibali & Nathan, 2012). In the study context, I analysed learners' immediate utterances as they engaged in activities for learning. The immediate utterances involved learners' reactions as they immediately encountered mathematical questions, both positive and negative.

Thought as being open to a continually changing world refers to observable cognition linked to the physical world beyond the body (Smith, 2005). This can be observed as learners engaging in a sensory-motor activity, which involves experiencing a particular action (Alibali & Nathan, 2012). As learners were engaged in activities, I analysed their mathematical cognition and the utterances they portrayed. These are the utterances that learners portray to show the development of an understanding or misunderstanding of mathematical concepts.

Lastly, thought as cognition that is not stationary refers to a growth in learners' cognition (Smith, 2005) that results from engagement in different activities that require different cognitive levels (Smith, 2005). This is supported by embodied mechanisms of change, which include the multimodal sensory-motor system, exploration and dynamic grouping of social entities in order for learners to use schemas to deal with abstract concepts (Alibali & Nathan, 2012; Smith, 2005)

The multimodal sensory-motor systems involve systems of interactions in the classroom, exploration involves learners' engagement in activities to explore a concept, and dynamic grouping of social entities refers to the social environment in which learners should be engaged. These will enable knowledge to emerge due to learners' interactions with an environment resulting from sensory-motor activity (Smith, 2005). Therefore, in this study I analysed learners' interactions and the utterances they portrayed in groups when learning mathematical concepts. This involved an analysis of the growth from low-order to high-order activities when learning mathematical concepts.

I used the dynamic systems theory of cognitive development because it shows how learners develop an understanding of mathematical knowledge in stages. This helped to evaluate learners' utterances through each stage of the development of mathematical cognition and the theory has been widely used in studies on cognitive development.

The dynamic systems theory of cognitive development has been widely studied in language acquisition (De Bot, Lowie & Verspoor, 2007; Verspoor, Lowie and Van Dijk, 2008). The study of Verspoor, Lowie and Van Dijk (2008) made use of the dynamic systems theory of cognitive development (Thelen and Smith, 1995) to study the development of second language learners. They found that learners' development in understanding the second language is non-linear and is dependent on the variation of resources for teaching and learning (Verspoor, Lowie & Van Dijk, 2008). Furthermore, Freeman and Cameron (2008) support the notion that engagement of learners in meaningful interactions stimulate mental imaging and enhance development of complex language understanding. However, the complex language understanding take time to develop (De Bot, Lowie & Verspoor, 2007). The cognitive developments were not specifically researched in contexts specific to mathematics education, but it ought to yield fruitful results in a suitable learning environment (Geert & Steenbeek, 2005).

2.11 SUMMARY

In this chapter, I presented a review of the literature on embodied cognition and its relationship to the utterances that learners portray in mathematics classrooms. The literature on embodied cognition and utterances shows how primary school learners learn. I then presented literature on the teaching and learning of primary school learners and the utterances they portray. After that, I presented the different kinds of utterances that learners may portray to show their level of learning. Lastly, I presented a section on how the theory of dynamic systems was adopted for my study.

CHAPTER 3: METHODOLOGY

3.1 INTRODUCTION

In this chapter, I present the research paradigms that justify the choice of a qualitative research approach. After that, I present different research designs prominent in qualitative research design. The data collection methods and how data was analysed are also described. Furthermore, I describe how I ensured the quality of the research and took ethical considerations into account.

3.2 RESEARCH APPROACH

Researchers are driven by a belief system called a paradigm (Guba & Lincoln, 1994). This paradigm provides meaning on why and how things are the way they are, and the way things are done (Guba & Lincoln, 1994; Kivunja & Kuyini, 2017; Patton, 1975). A paradigm is based on an epistemological view and an ontological view. The epistemological view deals with 'how individuals come to know and the things they regard as knowledge' (Kivunja & Kuyini, 2017, p. 27). Knowledge can be based on faith, beliefs, or intuition, gained from reading books or from people who are knowledgeable, be based on reasoning, or be derived from sense experiences (Kivunja & Kuyini, 2017). On the other hand, the ontological view is concerned with the 'assumptions that we make to believe that something makes sense' (Kivunja & Kuyini, 2017, p. 27)

The epistemological and ontological views give rise to different paradigms in research (Guba & Lincoln, 1994). I acknowledge the existence of different paradigms – positivist, constructivist, critical and pragmatic (Kivunja & Kuyini, 2017) – the positivist paradigm is about a single reality that is possible to measure and understand, while pragmatics believe that reality is continually changing and is interpreted against new situations. Lastly, critical theory is about understanding how communication is used to provide ways to foster positive social change (Kivunja & Kuyini, 2017). I discuss the constructivist paradigm in detail as the adopted paradigm. My discussions focus on the adoption of constructivism as a referent to how I teach and also as a referent to research.

At a more general level, constructivism is viewed as a theory of knowing. Knowledge, from this perspective, is constructed/derived from a meaning-making search, where learners engage in the process of individual interpretations of their experiences (Applefield, Huber & Moallem, (2000). Constructivism focuses on the idea that social interactions create meaning when learning, and realities are multiple and socially created (Guba & Lincoln, 1994). These ideas informed the teaching in the three teaching experiments reported on in this study.

My thoughts about what it means to do research can be accounted for within the constructivist paradigm. Constructivists suggest that there are multiple realities. These realities are 'social constructions of the mind, and there exist as many such constructions as there are individuals (although clearly many constructions will be shared' (Guba & Lincoln, 1989, p. 43). According to this assertion, the thesis of this study will remain my interpretation of my own experiences and that another researcher might have arrived at different interpretations.

Using constructivism as a referent to research also suggests a different way of looking at data. This is brought about by the subjective interrelationship between the researcher and the participants, and the co-construction of meaning (Mills, Bonner & Francis, 2006). The researcher should develop a reciprocal relationship with the participants (Mills, Bonner & Francis, 2006). This suggests that it is almost impossible to separate the researcher and the researched (Mills, Bonner & Francis, 2006). Consequently, this counteracts the power imbalances between the researcher and the participants (Mills, Bonner & Francis, 2006). The interaction between the researchers and the participants precisely creates the data that will emerge from the inquiry.

In this study, I adopted the qualitative research method because I constructed meanings about my learners' development of mathematical concepts based on the processes and structures of their social setting (Flick, 2004; Merriam & Tisdel, 2015). I engaged learners in mathematical classroom discourse to make sense of the concepts taught and worked directly with them (Creswell, 2009). The learners attempted to create meaning in mathematical concepts while interacting with each other and with me (the researcher) (Creswell & Miller, 2000; Maxwell, 2012). Kivunja and Kuyini (2017) support the notion that the researcher in a qualitative research approach inevitably interacts with the participants.

Furthermore, the purpose of this study also situates the study under qualitative research, as I investigated enactment of utterances in a mathematical classroom. I made use of objectives and hypothesis to observe learners' growth as they learnt mathematical concepts, and observed the utterances that they portrayed when they created meaning of the concepts (Devers & Fankel, 2000). According to Haven and Van Grootel (2019), qualitative research make use of data to generate hypotheses and to make use of predictions to construct knowledge of the participants.

Below is a discussion of the research design that I adopted. The research design referred to should be understood within the constructivist paradigm.

3.3 RESEARCH DESIGN

3.3.1 Research designs

I adopted a teaching experiment as the design in the study as one the design approaches available in qualitative research (Creswel, 2009; Kivunja & Kuyini, 2017). Teaching experiments focus on understanding how learners create meaning at different levels of learning (Steffe & Thompson, 2000). Therefore, the design helped to analyse what learners' utterance meant for mathematical teaching and learning as they developed mathematical understanding at different levels of learning.

3.3.2 Teaching experiment design

In this study, I adopted a teaching experiment as a research design (Steffe & Thompson, 2000). Teaching experiments enable the researcher to experience learners' first-hand mathematical knowledge and reasoning (Steffe & Thompson, 2000). As a result, I experienced learners' first-hand mathematical knowledge of concepts and became an integral part of the study by rigorously interacting with the learners as they learnt those mathematical concepts (Steffe & Thompson, 2000; Cobb, Confrey, DiSessa, Lehrer & Schauble, 2003).

The teaching experiment design adopted in this study consists of a series of teaching episodes and was conducted by one researcher teacher (Cobb et al., 2003; Steffe & Thompson, 2000). Each teaching episode consisted of an objective, a hypothesis and a conjecture (Steffe & Thompson, 2000). Therefore, in this study, I formulated an objective and devised a hypothesis for each teaching episode in order to reach a conjecture about learning mathematical concepts. The conjecture I reached informed the objective and hypothesis of the next teaching episode in a mathematical discourse (Molina, Castro & Castro, 2007). The teaching episodes were guided by objectives and hypotheses for each mathematical learning concept. Hence, the mathematical knowledge and ideas from the first teaching episode informed the teaching and learning of the next teaching episode. However, the recordings of the interactions in the classroom assisted with the objectives and hypothesis of subsequent teaching episodes.

According to Steffe and Thompson (2000), learners have their own perceptions about learning mathematical concepts, and teachers have perceptions about learners learning mathematics. These are referred to as ‘mathematics students’ and ‘students of mathematics’, respectively. However, learners’ perceptions are influenced by teachers’ engagement with the learners in a classroom. In the study, I engaged in a discourse learning environment to influence the ‘mathematics students’. Furthermore, I observed their interactions with the ‘students of mathematics’ as part of the data gathering technique. The mathematics students were an important part, which helped record what learners do and portray.

Furthermore, Steffe and Thompson (2000) affirm that in teaching experiments, learners must take the initiative in their learning. Learners should interact with mathematical concepts independently, without having their educator explain them (Steffe & Thompson, 2000). This is referred to as having ‘students of mathematics’ and allows researchers, through the implementation of teaching experiments, to see learners’ views and reasoning through interactions. However, it is important for the researcher to view learners as able to do mathematics by themselves, without having their teacher explain concepts to them (Steffe & Thompson, 2000). As a result, in the teaching experiments, I viewed learners as being ‘students of mathematics’.

Learners possess the following characteristics as ‘students of mathematics’, firstly, learners can create mathematical knowledge through interaction with previous mathematical concepts and daily experiences (Steffe & Thompson, 2000). Furthermore, learners are ontogenetic in that they can develop knowledge through interactions. However, the important part is drawing conclusions and providing guidance based on what they do and speak when doing mathematical activities (Steffe & Thompson, 2000). This is referred to as a conceptual analysis, whereby the researcher looks closely at learners' conceptions and misconceptions to guide their learning (Steffe & Thompson, 2000).

In conducting the teaching episodes, there should be a teaching agent, one or more students and a witness of the teaching episode (Steffe & Thompson, 2000).

a. Teaching agent

A teaching agent is the researcher teacher who is engaged in the planning and execution of daily activities (Cobb et al., 2003). In the context of this study, I was the teacher researcher. I engaged in formulating an objective and hypothesis for each teaching episode, guided by Annual Teaching Plan (ATP), Curriculum Assessment Policy Statement (CAPS) (2014) and mathematics textbook. In each teaching episode, I inevitably intervened in the teaching and learning in the classroom by posing questions to enhance participation, and to check learners' understandings and confusion. This helped to check whether the objectives and the hypothesis of the first teaching episode had been fulfilled (Molina, Castro & Castro, 2007). Therefore, learners portrayed utterances before and after every classroom intervention, and I was able to reach conjectures throughout the teaching episodes (Molina, Castro & Castro, 2007). The engagement that I had with learners enhanced their interactions with each other to explore their thinking of mathematical ideas (Steffe & Thompson, 2000)

b. One or more students

In the context of this study, one or more students refers to learners who will participate in the research proceedings (Steffe & Thompson, 2000). I collected the data from two Grade 4 classrooms, A and B. The classrooms consisted of 32 and 41 learners, respectively. Thus, I used both classes for the research since I engage them in daily teaching and learning.

I engaged learners in reasoning rather than simply giving them correct answers to gain insights into their understanding and enhance the interactions. Reasoning helps learners create mental images of the mathematical ideas that they learn (Steffe &

Thompson, 2000). Therefore, reasoning played a vital role in the study since it helped learners create mathematical meaning.

c. Witness to the teaching episode

A witness to the teaching episode refers to someone who helps the researcher teacher to interpret learners' utterances and gives opinions on teaching and learning mathematics (Steffe & Thompson, 2000). However, it is the researcher–teachers' choice to witness the teaching episodes (Steffe & Thompson, 2000). In the context of the study, I did not have any witness because I already had a prolonged engagement with the learners and, as such, interactions were already established as part of everyday learning. Also, I am the only mathematics educator at this school and, therefore, no one would be objective in mathematics teaching and learning. I only relied on methods of recording what transpires in the classroom.

3.3.2.1 Teaching Experiment 1

Background

In this teaching experiment, I allowed learners to work on activities in groups to observe their interactions among themselves and with me. Each activity comprised an objective and hypothesis, as prescribed by the teaching experiment (Steffe & Thompson, 2000). The objectives for each activity were guided by the CAPS document and the ATP prescribed for the daily teaching and learning of the learners. The hypothesis for the teaching experiments was guided by learners' responses to a given activity. However, the objectives of teaching episode 1 were based on learners' prior knowledge of counting forwards and backwards, tessellations and using mathematical operations involving three-digit numbers in teaching experiments 1, 2 and 3, respectively.

Learners were enculturated into teaching and learning, allowing them to construct mathematical knowledge. In the first school term prior to data collection, I taught learners to create mathematical knowledge through engaging in activities. However, I assisted at their desks whenever I saw a misconception. Furthermore, learners would ask questions as I moved around their desks. Therefore, they were used to the teaching approach, making it easier for the learners to learn in their respective groups.

In this teaching experiment, I experienced learners' first-hand mathematical learning and reasoning without providing them with any experience of mathematical concepts. I therefore observed their interactions and gave a conceptual analysis of utterances they portrayed when learning the mathematical concepts. Furthermore, the successes and constraints that learners encountered helped me to come up with a conclusion about their development of mathematical knowledge and assisted in the planning for the next hypothesis.

According to Steffe and Thompson (2000), learners are considered to be students of mathematics when they are given an opportunity to interact with mathematical ideas by themselves. As a result, I considered learners as students of mathematics when evaluating the utterances they portrayed when learning mathematical concepts. This enabled the learners to portray utterances independently of what I would say to them with regard to what they are learning.

a. Teaching Episode 1.1

In this episode, the focus was on numeric patterns as a topic. However, on the first day we focused on extending and recognising simple numeric patterns based on Activity 1 below. Learners were arranged into groups of six. This was intentionally done to allow effective classroom interactions.

Objective: Learners should be able to recognise and extend numeric patterns and describe the patterns in words.

Hypothesis: Learners will be able to recognise, extend patterns and describe patterns in words as this involves counting forward and backwards and using mathematical operations.

Activity 1

1. *Recognise and extend each pattern*
2. *Describe each pattern in words*

a) 1; 2; 3; 4; ___; ____; ____
b) 2; 4; 6; 8;___; _____; _____
c) 3; 5; 7; 9; ____;_____;_____
d) 13; 11; 9; ____; _____; _____

e) 1; 2; 4; 8; ____; _____; _____

f) 64; 32; 16; ____; _____; _____

b. Teaching Episode 1.2

In this teaching episode, learners were given further questions on numeric patterns. This time, the questions involved the use of repetitions, and the use of addition, subtraction and multiplication of different numbers as the pattern grows. The assumption is that they will use their understanding of patterns with a constant difference to patterns that require a different interpretation of how patterns grow. I also wanted to observe learners' actions as they explained how the pattern grows.

Objective: Learners should be able to recognise, extend numeric patterns and describe the patterns in words

Hypothesis: Learners will portray the use of fingers to count when they extend patterns. Furthermore, learners will move their heads up and down and utter 'owoo' to show an understanding of recognising, extending and describing the numeric patterns as, during the consolidation of Activity 1, I made them aware that not every pattern requires them to make use of constant difference.

Activity 2

Extend and describe the following patterns

1. *3; 3; 4; 4; 5; ___; ____; ____*
2. *99; 88; 77; ____; ____;_____*
3. *291; 282; 273; 264; ___; ____; ___*
4. *1; 3; 6; 10; ____; ____; ____*
5. *19; 18; 16; 13; ___; ____; ____*
6. *1; 4; 9; 16; ___; ____; ____*

3.3.2.2 Teaching experiment 2

Background

Learners have already learnt the skills to solve numeric patterns by recognising, extending and describing the patterns in words. In this teaching experiment, learners used their knowledge of numeric patterns to solve questions on geometric patterns.

Geometric patterns resemble numeric patterns, requiring similar recognition, extension and description. However, they differ because geometric patterns involve the 2D shapes that the patterns form as they proceed. As a result, it is a pattern that learners may easily recognise because an image is created for them. In addition, learners know about different types of 2D shapes (both regular and irregular) and their properties. Therefore, this also serves as additional knowledge about geometric patterns because they should be able to describe how geometric patterns extend.

I presented three teaching episodes focused on learners' ability to recognise, extend and describe patterns that form different shapes. Learners worked on patterns of triangles, squares and any other 2D shape in teaching episodes 1, 2 and 3, respectively.

a. Teaching episode 2.1

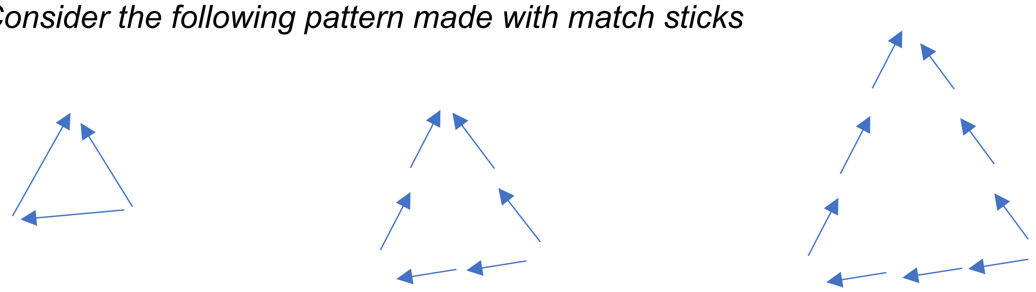
In this teaching episode, I gave learners an activity (see activity 3 below) to work on. I intentionally did this to observe the difference from the previous activities. Furthermore, I did not explain anything to the learners; I wanted to observe whether they could make sense of the activity using their knowledge of numeric patterns and observe their utterances.

Objective: Learners should be able to investigate and extend geometric patterns by looking for relationships between patterns.

Hypothesis: Learners will portray utterances such as smiles, use of fingers to count and move their heads up and down to show understanding of patterns. Furthermore, learners will stare at each other, move their heads sideways, hold their chins and cover their eyes to show a misunderstanding of number patterns as they investigate and extend geometric patterns because they had established similar relationships with numeric patterns in the previous teaching experiment.

Activity 3

Consider the following pattern made with match sticks



Pattern 1 *Pattern 2* *Pattern 3*

1. *Draw pattern 4*
2. *How many matchsticks does pattern 4 have?*
3. *What geometric shape does each pattern make?*
4. *How does the pattern grow?*
5. *How many matchsticks will pattern 10 have?*
6. *Copy and complete the table that follows:*

Pattern Number	*1*	*2*	*3*	*4*	*10*	*25*
Number of match sticks						

b. Teaching episode 2.2

In this teaching episode, I continued with geometric patterns and gave learners a question that involved a pattern of squares. I did this to observe learners' utterances as they interpreted patterns of squares. However, this pattern interpretation is similar to the interpretation of the patterns of triangles in that learners need to focus on the number of matchsticks on each side. Therefore, I wanted learners to interpret a different shape and observe whether they could apply their knowledge of the patterns of triangles.

Objective 2: Learners should be able to investigate and extend geometric patterns by looking for relationships of patterns.

Hypothesis: Learners will be able to investigate and extend geometric patterns because in the previous activity successfully solved geometric patterns of triangles. The learners will portray utterances such as making use of fingers to recognise how the pattern grows, and learners will visualise the pattern to draw the next one.

Activity 4

Look at this pattern of squares

1 *2* *3*

1. Build shape 3 with match sticks

2. Describe how you made the pattern

3. Copy and complete the table and the flow diagram

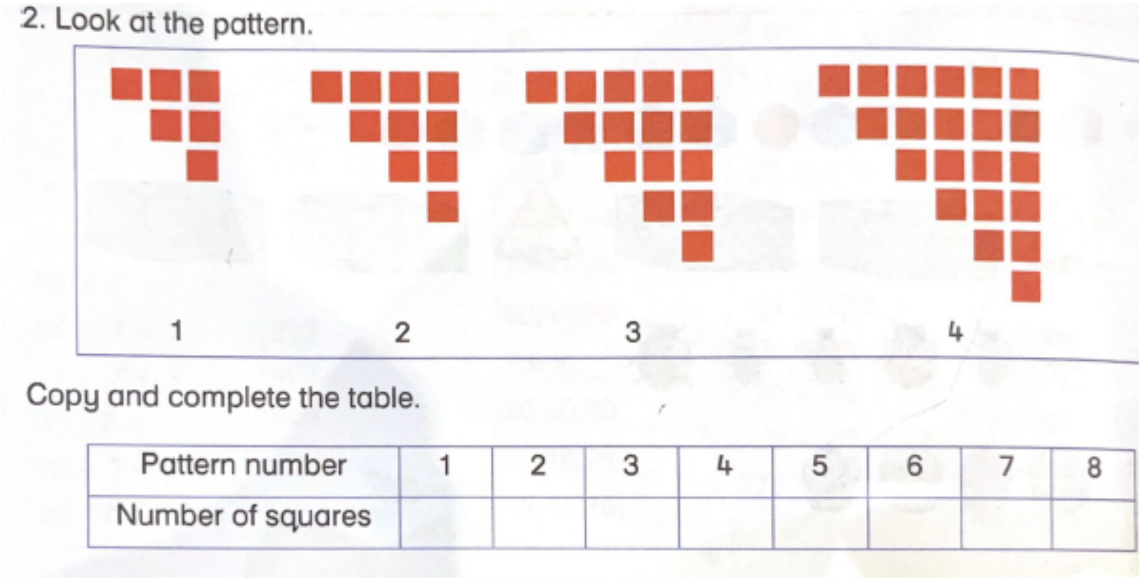
Pattern Number	*1*	*2*	*3*	*4*	*5*	*10*
Number of match sticks						

c. Teaching episode 2.3

In this teaching episode, I thought of giving learners a different geometric pattern from the patterns they had in the above two episodes. I did this on purpose to observe their utterances as they engaged in unusual activities.

Objective: Learners should be able to investigate and extend geometric patterns by looking for relationships of patterns.

Hypothesis: Learners will be able to recognise the pattern because they were previously engaged in patterns that required them to add one matchstick on each side of the shape, but not all the learners will be able to keep the number of squares constant for the entire pattern. Learners will portray utterances such as opening their eyes wide open and moving their heads sideways when they cannot recognise the pattern. Furthermore, learners will portray utterances such as using hand gestures to count the number of squares and visualising to extend the pattern.



2. Look at the pattern.

Copy and complete the table.

Pattern number	1	2	3	4	5	6	7	8
Number of squares								

Figure 2.4: Taken from Viva Mathematics learners books Grade 4

3.3.2.3 Teaching experiment 3a

In this teaching episode, I gave learners one question on word problems involving the use of addition and subtraction. The question involved three-digit number by three-digit number. I did not explain to learners how they should interpret the word problem. I did this intentionally to observe their first-hand mathematical knowledge and understanding. Furthermore, this will enable me to experience the learners' immediate utterances when engaged in word problems.

Objectives: Learners should be able to interpret and solve three digit by three digit word problems involving addition and subtraction.

Hypothesis: Learners will be able to interpret word problems because there are pointers in a word question that indicate the mathematical operations that learners should use. Furthermore, learners will be able to solve the problems after identifying the mathematics to use because they have dealt with the use of mathematical operations.

Question: Calvin released 965 copies of music, he sold 243 copies and later sold 482 copies. How many copies remain to be sold?

a. Teaching episode 3.1

In this teaching episode, I gave learners questions involving the use of one mathematical operation. It was evident from teaching episode 1 above that they could not use two mathematical operations at a time to solve word problems.

Objective: Learners should be able to interpret and solve word problems involving one mathematical operation

Hypothesis: Learners will be able to solve word problems using one operation because this means that the words will be fewer and hence, it will be less of a struggle for learners to try and interpret the questions. Learners will portray making use of fingers to calculate and fold their arms when they do not understand. Furthermore, learners will read word questions for understanding.

Questions:

Amo bought soccer boots for R1296 and Khura bought his for R1125.

1. *How much money was spent by Amo and Khura altogether?*
2. *By how much money was Amo's boots more expensive than Khura's?*
3. *If Amo got a discount of R200 and Khura got a discount of R20, whose boots will be more expensive?*

b. Teaching episode 3.2

In this teaching episode, I engage learners in questions that involve the use of any mathematical operations. These mathematical operations included multiplication and division, over and above the addition and subtraction used in teaching episodes 1 and 2 above. Therefore, this will challenge the learners to interpret the questions, enabling me to observe the utterances that they portray when attempting such questions.

Objectives: Learners should be able to interpret and solve word problems involving any mathematical operation.

Hypothesis: Learners will find it challenging to interpret word problems. They will find it difficult and hold their chins when a question uses complex mathematical language. Some learners will feel excited when they can solve the problem, while others will hold their chins when they find the problem complicated.

3.3.2.4 Teaching experiment 3b

Background

In this teaching experiment, learners worked on word problems using mathematical operations involving up to four-digit numbers. These mathematical operations were addition, subtraction, multiplication and division. Therefore, learners used these operations to solve mathematical problems. These mathematical problems required learners to interpret the questions, decide on a mathematical operation to use and execute the use of the operation. Hence, I wanted to observe learners' utterances as they engage at every stage of the process of solving word problems.

Learners had solved word problems involving two-digit numbers using mathematical operations in Grade 3. Furthermore, I also worked with the learners on the addition and subtraction of three-digit numbers and multiplication and division of three-digit numbers by one-digit numbers. Therefore, this served as prior knowledge that the learners could use to solve word problems.

In teaching experiment 3, I presented three teaching episodes where learners worked on the word problems. I engaged learners in the use of addition and subtraction in teaching episode 1, the use of multiplication and division in teaching episode 2 and I engaged learners individually in teaching episode 3. However, I did not let learners know which operations they were dealing with. I did this intentionally to observe the learners' utterances as they chose a correct mathematical operation.

First classroom interaction

Question

(a) *The mass of 7 boxes of bananas is 175 kilograms. What is the mass of each box of bananas?*

Second classroom interaction

In the second classroom interaction, I interviewed learners individually depending on whether their group completed the following questions. I decided to interview them because some learners hide behind other learners' answers and do not say anything when a question is posed to their group. Therefore, I was able to observe learners' utterances individually.

Objective: Learners should be able to interpret and solve word problems

Hypothesis: Learners will be able to interpret word problems because they know that the questions require them to think differently to attach meaning. However, there will be an exception for learners who have a negative attitude towards the questions. The learners will just sit and fold their arms.

Questions

(b) A farmer wants to plant 189 apple trees. He plants 9 in a row. How many rows of apple trees does he plant?
(c) How many children could each get 5 sweets out of a packet of 152 sweets? How many sweets will be left over?

3.4 PARTICIPANTS IN THE STUDY

In a teaching experiment design, the teacher–researcher is directly involved in the teaching and learning of the research participants (Steffe & Thompson, 2000). I therefore adopted convenient sampling since I was engaging with the learners who I teach daily (Taherdoost, 2016). The learners were readily available for participation in the study (Taherdoost, 2016). Convenient sampling is a non-probability method where researchers collect data from conveniently available participants (Patton, 2005; Dilshad & Latif, 2013).

In teaching experiment design, learners are sampled as they actively participate in learning mathematical concepts (Steffe & Thompson, 2000). However, in this study I only focused on proximity. Hence, I observed all the learners' opinions, viewpoints, and reasoning in relation to the utterances that they portrayed.

Teaching experiments involve one or more student participants (Steffe & Thompson, 2000). I therefore engaged 73 learners who were in two different Grade 4 classrooms, A and B. The A class consisted of 33 learners and the B class consisted of 40 because of the different classroom sizes. Learners were seated in groups of four in both class (A and B) to allow for the interactions of these students of mathematics. Therefore, I had 18 groups from both classes combined. I engaged learners in learning that allowed them to imagine mathematical concepts. This made it possible for the learners to create schemes by drawing on their embodied knowledge. Hence, I attached meaning to the learners' utterances as they were engaged in teaching and learning mathematical concepts.

3.5 DATA COLLECTION

In the study, I used video recording, notetaking, observations and interviews to capture each teaching episode, as prescribed by the teaching experiments approach (Molina, Castro & Castro, 2007). Steffe and Thompson (2000) argue that video recordings are recommended to capture learners' interactions while learning mathematical concepts. As such, I used video recordings as the main method of gathering data. I then used notetaking to reflect on teaching and learning mathematical concepts after every classroom intervention and, lastly, I used interviews to capture data on individual learners to get their thinking about concepts. Researchers should make use of observations and interviews to capture data using notetaking and video recordings, respectively (Maree, 2011)

3.5.1 Video recording

Video recording captures learners' non-verbal and verbal interactions in a classroom when learning concepts (Maree, 2011). In the context of the study, I captured learners' non-verbal interactions by using video recordings to capture their non-verbal utterances when learning mathematical concepts. On the other hand, I used video recordings to capture verbal interactions during interviews.

I used video recordings to capture learners' interactions among themselves and as I interacted with them to check for their creation of mathematical knowledge. I video recorded learners as they learn mathematical concepts in groups to capture the interactions and utterances they portrayed (Shnederman & Plaisant, 2006). The video recordings were done for 15 teaching episodes. I then intervened in their interactions by questioning and probing to get insights into their understanding of mathematical concepts and to assist learners to build models of understanding the concepts for mathematical development (Steffe & Thompson, 2000).

3.5.2 Notetaking

Notetaking involves recording learners' verbal and non-verbal communications by observation, using a reflective journal (Maree, 2011). I made use of notetaking to reflect on the utterances that learners portrayed as they interacted with other learners and to check whether they were engaged in creation of mathematical knowledge per the dynamic systems theory. Through the use of notetaking, I recorded these actions in a reflective journal to gain insight into the learners' behaviour towards mathematical concepts.

3.5.3 Observation

Observation involves the recording of the behaviour portrayed by participants (Maree, 2011). The researcher can act as a 'complete participant', 'participant as observer' or 'complete observer' (Creswell, 2009). Complete participation involves having an assistant who observes the researcher and, similarly, complete observation involves the researcher observing participants' interactions without asking questions (Maree, 2011). On the other hand, with participants as observers, the observer is engaged in the participants' interactions but still observes (Maree, 2011).

In the study, I acted as a participant observer since I observed the learners while interacting with them when learning mathematical concepts. I observed 15 teaching episodes, which were captured in the results and analysis. Furthermore, I took note of learners' non-verbal and verbal actions during all the teaching episodes. I observed learners' interactions when learning mathematical concepts and took note of critical moments before and after I interacted with them. Critical moments are the interactions that learners undergo to create mathematical knowledge, where there is an observable cognition of a mathematical idea and where there is growth in mathematical knowledge, including the utterances they portray.

3.5.4 Interviews

The interview is a method of collecting data based on verbal responses to a given set of questions (Maree, 2011). The interviews resulted from the interventions I made by questioning to get insights into learners' understanding of mathematical concepts when they interacted in groups (Patton, 2005). These are regarded as group interviews (Dilshad & Latif, 2013; Patton, 2005).

I interacted with learners by questioning as part of the interviews conducted. There were 15 interviews, which were conducted as I engaged learners in the teaching episode. Furthermore, I questioned a few of the learners individually about how they managed to get their answers. There were 12 interviews that were conducted with individual learners; these learners were representatives of different groups, and they were chosen by learners in the groups themselves. I did this to gain insights into their thinking as individuals from what they have discussed in their groups and to check for growth in mathematical knowledge.

3.6 DATA ANALYSIS

Teaching experiment data is analysed in two forms, namely preliminary and retrospective (Steffe & Thompson, 2000). Preliminary analysis is done after every classroom intervention and retrospective analysis is the overall analysis of data gathered during the research process (Steffe & Thompson, 2000). The preliminary analysis helped make decisions about future interventions, hypothesis revision and conjecture (Molina, Castro & Castro, 2007). On the other hand, I used retrospective analysis to read through and make sense of the entire data collected throughout the research process (Molina, Castro & Castro, 2007).

3.6.1 Preliminary analysis

I undertook a preliminary analysis by looking at the data I collected from the notetaking and video recordings of each teaching episode. I watched the video recordings and read through the reflections from notetaking to check for the moments where learners were engaged in the creation of constructs when engaged in learning activities. The constructs emerged as learners planned how they would attempt the learning activity. Furthermore, these constructs indicated learners' ability or inability to deal with the activity.

I intervened in both groups of learners who showed either 'ability or inability to deal with the activity'. I asked questions like 'why' they were thinking that way and 'how' they arrived at their thinking, for learners who portrayed an ability to deal with the activity. I did that to check whether my observations reflected the learners' understanding of the concept. I did the same for learners who could not deal with the activity. However, I asked them probing questions to get them to understand the mathematical concept.

Preliminary analysis assists in reflecting on and planning for the next teaching episode (Steffe & Thompson, 2000). Thus, I planned teaching episodes by reflecting on the data from video recordings and notetaking. This helped me to adjust lessons, looking for areas where learners were struggling and were successful in solving mathematical problems

3.6.2 Retrospective analysis

A retrospective analysis involves looking back at the transcripts of the video recordings to do an overall analysis of the data (Steffe & Thompson, 2000). In this study, I read through the transcripts of the video recordings to create categories (Liampotong, 2009). I then gave names to the categories, looking at instances where learners portrayed utterances when learning mathematical concepts and the constructs of the dynamic theory of cognitive development (Liampotong, 2009). This is referred to as coding (Liampotong, 2009).

The process of creating and naming categories emerged as follows: I first looked at instances where learners created constructs, indicating that they were engaged in forming a plan to solve mathematical problems and the utterances they portrayed. Therefore, I looked for instances where learners engaged in sensory-motor activity and the utterances they portrayed. Lastly, I looked for the instances where learners were showing growth in cognition, which indicated learners' use of schemas to solve problems. I repeated the entire process for each teaching experiment.

3.7 QUALITY CRITERIA

Qualitative research should address credibility, transferability, dependability and confirmability to ensure the study's trustworthiness (Bitsch, 2005; Shenton, 2004). These are regarded as prominent criteria in qualitative research (Guba & Lincolm, 1989). In this study, I applied these criteria to ensure the trustworthiness of the study

3.7.1 Credibility

Credibility is the extent to which the results of the study are believable and confirm the research questions and objectives of the study (Bitsch, 2005; Maree, 2014; Shenton, 2004). To ensure the credibility of the study, the researcher should have a prolonged engagement with the participants (Bitsch, 2005). This is regarded as spending time with participants before data collection (Bitsch, 2005). In the study, I spent one school term engaging with learners to ensure prolonged engagement. I, therefore, started collecting data one month after the second school term began. Hence, the learners had an opportunity to get used to the way I teach and to me as their educator. On the other hand, Shenton adds that the participants should also be identified in the study (Shenton, 2004). Hence, I also stated the participants of the study.

Furthermore, researchers should use multiple data sources to ensure the study's credibility (Shenton, 2004). Therefore, in this study I used data collected from observations, using notetaking, and group interviews, using video recordings. Bitsch (2005) refers to this as the triangulation of data

3.7.2 Transferability

Transferability is the extent to which the findings of the study can be applied in different contexts (Shenton, 2004). Transferability can be ensured by describing the setting of the study in detail (Seale, 1999). Therefore, I ensured transferability in this study by describing the participants and the setting of the study in detail. This ensures that the results of the study are applicable in similar contexts. There can be a variation in contexts transferability due to the sampling method of choice (Bitsch, 2005). This variation is limited to sampling methods that use a limited number of learners in a large group (Bitsch, 2005). However, for this study I used convenience sampling, involving all Grade 4 learners in our school. As a result, I also addressed the variation resulting from the sampling methods.

3.7.3 Dependability

Dependability is the correlation of the study's results if the study was to be conducted again (Bitsch, 2005; Shenton, 2004; Maree, 2014). The dependability of the study could be ensured by following methodological steps as outlined in the study (Bitsch, 2005; Mandal, 2018). Therefore, I followed the methods of collecting and analysing data correctly, as planned for in the study. I therefore documented this process to ensure the dependability of the study. Furthermore, there were no deviations that took place in the process of the study. However, deviations are acceptable, provided the researcher indicates them (Bitsch, 2005).

3.7.4 Confirmability

Confirmability is the extent to which the results reflect the participants' responses (Shenton, 2004). This also prevents researchers from being biased towards the participants' results (Bitsch, 2005). In the study, I used not-taking and video recording to collect data, and the data analysis was based on what transpired in the classroom. Furthermore, the video recordings are available for an audit trail to ensure I did not make decisions against the procedure of the research. (Bitsch, 2005). This will ensure the confirmability of the study.

3.8 ETHICAL CONSIDERATIONS

In a qualitative study, the following ethical measures were addressed for the protection of participants and the setting of the study: informed consent; voluntary participation; anonymity; and confidentiality (Arifin, 2018).

3.8.1 Informed consent and voluntary participation

'Consent should be given freely, subjects should understand what is being asked of them, and involved persons must be competent to consent' (Arifin, 2018, p30). In this study I employed ethical measures to protect the participants' and school's identities (Mandal, 2018). These ethical measures involved allowing learners to participate voluntarily in the project (Maree, 2014). I adequately informed the learners about the research proceedings and I let them know that they could withdraw from the research or cease their participation in the project (Arifin, 2018). Furthermore, I requested ethical clearance from the university, which was granted and this I used to get clearance from the provincial department of education. Subsequently, I used the clearance received from the department of education to obtain permission from the school to conduct the research. I also issued consent forms to the parents of the learners, requesting permission for the voluntary participation of their children (Mandal, 2018).

3.8.2 Anonymity and confidentiality

Researchers need to ensure that participants and the research venue are protected from harm through the implementation of measures to ensure privacy, confidentiality and anonymity of the data (Maree, 2014). In this study, I ensured confidentiality by keeping the participants’ and the school's names private. Furthermore, I used pseudonyms during the data analysis to ensure that participants' names remain anonymous (Anderson, 2017). ‘This ensures that the participants are preserved by not revealing their names and identity in the data collection, analysis and reporting of the study findings’ (Arifin, 2018, p30). However, I have taken security measures to ensure that the data is protected from being viewed by outsiders (Arifin, 2018).

3.8.2.1 Classroom interactions

The classroom interactions were video recorded in the classroom where the learners interacted with the teacher (me), without interruptions and without allowing any outsiders to form part of the class. These video recordings were protected using passwords and the contents of the recordings were only shared with the supervisor of the study for audit trail purposes.

3.8.2.2 Data analysis

The transcriptions of the video recordings were done in a quiet room and involved only me in order to protect the data from being viewed by any outsider. I did this to ensure that the participants' interactions and faces were protected. However, as they form part of reporting, the transcriptions were only shared in a study report with the supervisor.

3.9 SUMMARY

In this section, I described the research approach and design adopted in the study. I also described the study's participants as informed by the study's design. Furthermore, I described the study's data collection methods and design. Lastly, I described how I ensured quality of the study and the ethical issues that I took into consideration.

CHAPTER 4: RESULTS AND ANALYSIS

4.1 INTRODUCTION

In this chapter, I present an analysis of the results captured in the form of teaching episodes as prescribed by the teaching experiments approach (Steffe & Thompsons, 2000). The teaching episodes comprised several classroom interventions as activities. Therefore I undertook a preliminary analysis for each classroom intervention. And overall analysis of the teaching episodes follows as a retrospective analysis of the teaching episodes. Furthermore, I used the dynamic system theory of cognitive development as a lens to guide both preliminary and retrospective analysis (Thelen & Smith, 1995)

4.2 TEACHING EXPERIMENTS

4.2.1 Teaching experiment 1

4.2.1.1 Teaching episode 1.1

Classroom interactions

Learners were so hesitant, and most just looked at each other, showing a little confusion because I had not explained anything to them. However, after five minutes they seemed to have started working on the questions, which allowed me to move through the classroom, group by group, to check their progress.

Learners in their respective groups continued to attempt the activities and showed great interaction within those groups. Furthermore, the learners used their hands to count to find numbers that followed the pattern. They explained using multiples, for example, counting in ones, twos and so on. However, a group caught my attention when I saw them using a ruler to attempt question (c). The ruler clearly indicated that they skipped one number to get to the next number. Hence the ruler was assisting them to complete the pattern. As I continued to observe the groups, it seemed as if all the groups avoided describing the patterns in words after extending them.

Learners in most of the groups had not described the patterns in their own words. However, when I asked them, they indicated that the word ‘describe’ confused them.

Some of the groups thought 'to describe' meant writing the numbers in words and others opened dictionaries to look for the word 'describe'. One group, group A, had described the patterns correctly. Thus, I asked Disebo to share her group's answer with the rest of the learners in the classroom. She described the pattern as: 'For us to get the next numbers in the pattern, we counted forward in twos'. The whole class said 'owooo', expressing their understanding of how they should describe patterns. Then, as we continued, the group that explained the description gave another description for question (c) that they extended the pattern by adding odd numbers. However, the group became confused when I asked whether the number two was an odd number. Later, the learners made me realise that they meant adding two to the odd numbers as they used their fingers to count.

The following is how the conversation went with learners in group A who successfully answered question (c).

Teacher: How did you get the answers?

Disebo: We added odd numbers to get to the next numbers in the pattern.

Mongatane: So, we added 2.

Teacher: So, is 2 an odd number?

Disebo: No, we counted like this 9 then 11, we skip 12 because is an odd number then write 13 (counting using fingers) … .

Teacher: Oh, okay it makes sense.

From the interactions above, Disebo and Mongatane explained how they extended the pattern. Mongatane saw it as adding two to the previous term, while Disebo saw it as adding odd numbers. This means that the learners we able to interrogate the concept and managed to reach the correct answer. However, I wanted more clarity on Disebo's explanation. Therefore, she explained using her fingers that from nine, which is an odd number, we skip 10, which is an even number and write 11, which is an odd number and so on. This was not necessarily to add odd numbers but rather continuing the pattern by writing only odd numbers and skipping the even numbers. Learners in the group understood the concept of adding two to the previous number, hence they were stunned by Disebo's explanation when they realised that she is also correct. The use of fingers in Disebo's counting shows how the learner made use of hand gestures to support her answers.

Conclusion

In the interactions above, learners were using fingers as a gesture and some uttered 'owoo' to show their understanding of how they should describe patterns. Disebo used her fingers to support her explanation of how she described the pattern in question (b). Similarly, she used the fingers to show how she extended the patterns in question (c). She had the number nine in her mind and then represented the number 9 with one finger, then started counting as follows: 'we say 9, then we skip 10 and write 11'. On the other hand, most learners in the classroom had nine in mind and showed two by using two fingers. Therefore, they skipped two to get to 11 and did the same for the entire pattern. As a result, regardless of the different interpretations, learners were able to recognise recognise, extend patterns and describe patterns in words.

Next day …

The next day I did not say anything to the learners. I wanted them to continue with the questions. As they discovered how they should describe patterns, we continued with the questions. The learners successfully answered question (d) and described the pattern correctly. They made use of their fingers and some of them were using a ruler to count backwards in twos. However, learners struggled with question (e) because it did not involve an addition or subtraction of constant terms. Therefore, I engaged learners in their groups regarding question (e) to check their thinking processes. The following is an extract of how the conversations with the learners went:

The question was for the learners to extend and describe the pattern in words.

(e) 1; 2; 4; 8; ____; _____; _____

The learners in Group B responded 1; 2; 4; 8; <u>12; 16; 20</u>.

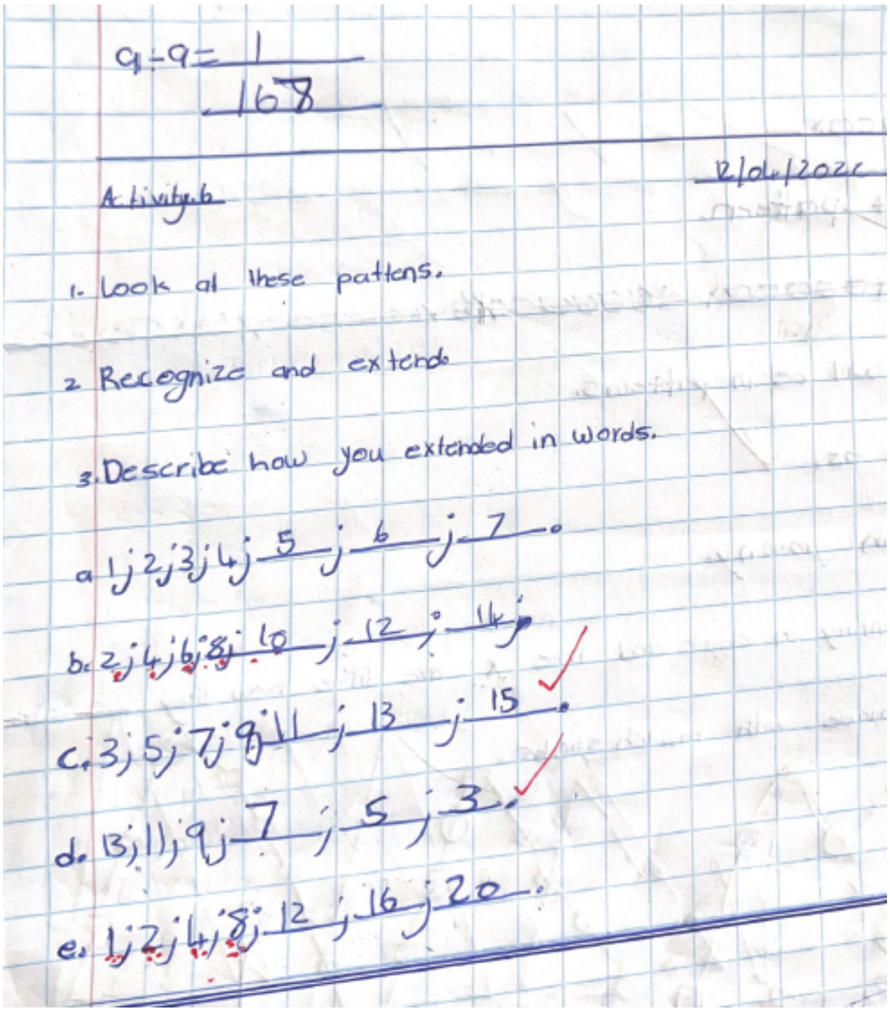


Figure 1.1: Calculations

Teacher: How did you get the answers?

Mosima: Sir, we did like this aker it says 1; 2; 4; 8 then from 4 to 8, I saw that we are counting in 4s (showing using fingers).

Teacher: But what about from 1 to 2? Did we also count in 4s?

Group: No sir (opened their eyes wide, while others are holding their chins).

Tumelo: We started counting here in 4s (Referring from 4 to 8, see figure 1.1 above)

Teacher: But we are not counting in 4s from the beginning.

Group: (They started to calculate again, trying to study the pattern again.)

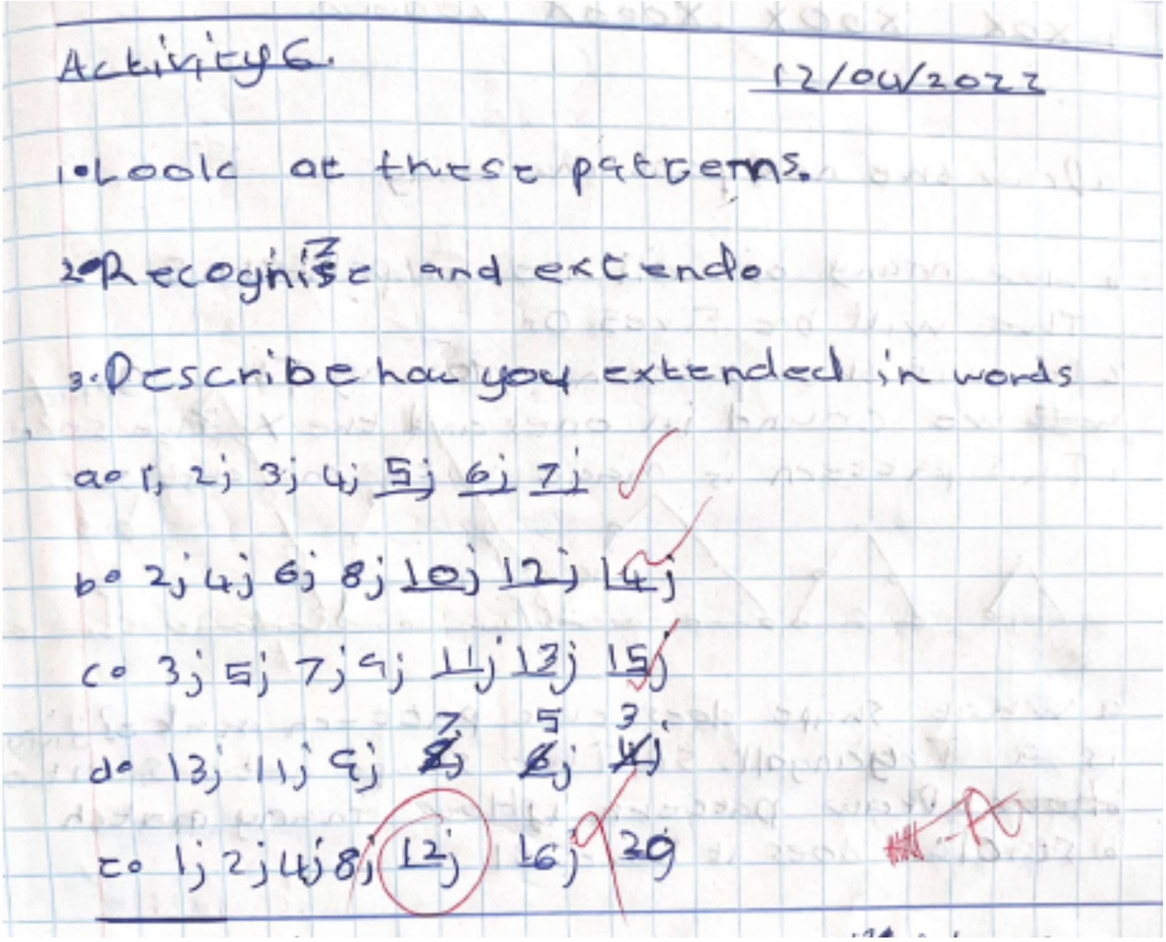


Figure 1.2: Calculations

Teacher: Why did you start where 4 is? And disregard other numbers.

Tumelo: Because they are close to each other.

Teacher: Is that the reason for you all?

Group: Yes.

From the above interactions, learners in group B used their fingers in support of how they extended the pattern on question (e). These learners showed four fingers and added the four to the previous number, starting with eight. However, they got confused because the pattern did not extend by four from the beginning. Consequently, learners used constant difference from four to eight, which was not supposed to be the case because, from the beginning, the pattern is not constant. Hence, they opened their eyes wide and held their chins with their hands to indicate that they saw that they had misinterpreted the question. This showed a mismatch of how they have used hand gestures that had led to misinterpretation of the question.

I then took the learners back to the previous questions we did and asked them to count together with me. At this stage, I showed them that, with the other questions, they had started from the first number to recognise the pattern. I then left the group to continue interpreting the patterns and went to the next group (group D).

With question (e), I found that group D had done the same thing as group C. They insisted that they had extended by adding four because from one to two, four cannot get in; hence they had the same answers as group C (see figure 1.2). Learners could not conceptualise how the pattern was created from the beginning and could only start counting from four because this made sense for them. Thus, I discussed with the group the nature of the pattern and that the pattern does not grow by a specific number. The learners noticed that and immediately started working on the question again.

In group C, there was a smooth transition from how learners made use of their prior knowledge of counting forward and backward to answer the questions. I dwelt much on question (e) as most learners in the classroom struggled to interpret the pattern. I had the following interactions with group C regarding question (e), as it seemed that they got to the correct answer:

Learners in group C responded 1; 2; 4; 8; 16; 32;…

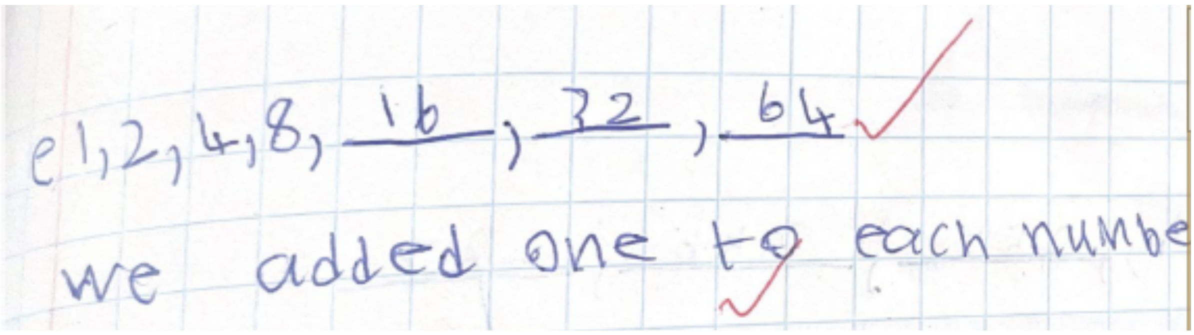


Figure 1.3: Calculations

The following conversation emerged with the group.

Teacher: How did you get 16? (Asking the group.)

Odirile: We said 8 plus 8 is 16.

Teacher: So, you just started with 8 plus 8 being 16?

Group: Yes.

Teacher: So how was the 4 in the pattern found? (Trying to check the idea that they used.)

Odirile: 1 plus 1 is 2, 2 plus 2 is 4 … (which of course was okay, but I had to challenge to see if they all are standing their ground).

Teacher: But how about the 1? How come we are not saying 0 plus 0 is 1 since we adding same numbers?

Odirile: Uhm (they all seemed confused).

Thasha: No, the answer will be 12 not 16.

Teacher: How is it 12?

Thasha: (Smiled.)

Odirile: No, it will be 16 because going forward, it makes more sense (she continued counting to 16 plus 16 is 32).

Learners in group C had correct answers to the question and they described the question correctly. The learners used their fingers to count when calculating the answers. However, learners got confused when I asked how pattern 1 was found. Odirile uttered 'uhm' because it would not make sense to say zero plus zero to get one, and Thasha wanted the answer to change to 12 (meaning that from eight, we extend by adding four). After that, Odirile insisted that it must 16 and even showed that it makes sense as the pattern continues. I then moved to the other groups to observe how they were attempting the same question. The learners in the group thought their answers were wrong because I ask questions; however, Odirile had a conceptual grounding in how she answered the questions, which indicated that she has transferred knowledge to a generalisation of an idea.

Some groups had the idea that after eight there should be 12 because they insisted that we count in fours. However, I sensitised them to the fact that, as with the rest of the patterns, we had always start from the first number to recognise the patterns and left them to think about it. On the other hand, other groups were adamant that the pattern did not make sense and insisted that I might have made a mistake. The learners kept pointing fingers at each other, saying they did not understand, but then

I asked them to think of how the individual numbers were found in the pattern. This means that how learners used their fingers to count indicated that the learners were not trying to understand the pattern, which resulted in wrong generalisations.

(d) As I manoeuvred through the groups, I encountered an interesting answer for question € from group E. They had described the pattern and extended it as follow€*(e) 1; 2; 4; 8; ____; _____; _____*

The learners in Group E responded 1; 2; 4; 8; 9; 10;12.

Teacher: How did you extend the pattern?

Thabo: For us to get these three numbers we counted in 1s and 2s.

Teacher: What gave the idea that we should count in 1s and 2s?

Tiny: We saw this (pointing at 1 and 2; 2 and 4), from 1 to 2 its 1 and from 2 to 4 its 2, then we saw that we have three spaces … .

Teacher: Then what about from 4 to 8?

Thabo: From 4 to 8 we need 4.

Teacher: Does the number you add keep on increasing or decreasing?

Group: Increasing.

Teacher: But why add 1 from 8 to get 9, if the number is increasing?

Group: (Seemed confused, with one learner scratching her head.) Aowa!! Sir.

Thabo: But we can add 5 (of which I believe she just answered looking at 'increasing' that we spoke about).

In the above interactions, learners used their hands to extend the pattern in ones and twos. They put one finger on the table from one hand and, as they counted with the other fingers from a different hand, they skipped the one all the time. The learners did not study the pattern correctly because they had created their own pattern, which could only make sense to them. In the pattern, they assumed they should add ones and twos, as the first three numbers in the pattern did that. The difference between one and two adds one, and the difference between two and four is two. Hence, learners interpreted it like that. Therefore, when I engaged with them, they became confused and uttered 'aowa sir', and did not want to accept that what they did was wrong, as it made sense to them. However, Thabo thought that, since from four to eight they can add four, this means that the next number they can now add is five. She was only thinking about the next number after four, which is five. As a result, the group did not thoroughly think through my engagement with them and I decided to allow them to ponder this without distractions.

Thus, I let learners continue the work and checked on other groups. However, most of them seemed off task because they were not doing anything, and I later decided to do a whole class discussion, discussing question (e) so they could keep up with the other learners. I let group C share their response as part of the discussion. Eventually, learners managed to understand how the question should be tackled

Conclusion

Learners interpreted the pattern differently and portrayed different utterances as such. Learners who interpreted the question both correctly and incorrectly both showed the use of fingers to extend the patterns. Some learners used their fingers to count from eight to 16 to extend the number patterns, while others used their fingers to add one and others four to eight when continuing with the pattern. Furthermore, these learners could also describe patterns in words, regardless of whether their answer was correct or not. On the other hand, during interactions in their groups, learners tended to move their heads up and down to show that they understood where they went wrong. Learners would roll their eyes to show confusion and did not give further explanations to stand their ground. Furthermore, learners scratched their heads when I questioned the way they interpreted patterns because they thought that they had interpreted the patterns is incorrectly

Analysis of teaching episode 1.1

Teaching episode 1 was focused on learners' ability to extend and describe numeric patterns. Learners could extend and describe numeric patterns, but their knowledge was limited to simple counting forward and backwards. This was evident as most of them could not succeed in answering question (e) but were able to do the other questions with minimal to no assistance from the teacher. Therefore, this is further evidence that learners can recognise patterns with constant differences but, because they struggled with question (e) except for one group, they struggle when patterns do not have constant differences.

Interactions in teaching episode 1 show that when learners encountered a challenging question, they either tended to give up or came up with an easy way out. Learners tended to give up when the question is unusual to them. They tried to find ways through the question but eventually still gave up. This was evident when learners looked at each other when asked questions like 'what do you think about the question?' because none of them wanted to answer. On the other hand, learners came up with a way out because they believed that the way they were attempting the question was correct. This is shown by how they interpreted the question in activity 1. Group B had 12 as their first answer (1; 2; 4; 8; 12) because they assumed the pattern would extend in fours. The learners disregarded how the pattern was extended from pattern number 1.

In teaching episode 1, learners portrayed the following utterances to show whether they were successful or unsuccessful in solving mathematical questions. Learners slowly moved their heads up and down to show that they understood explanations and their activities. Furthermore, learners smiled when doing mathematical activities to indicate that they enjoyed and understood the activity. On the other hand, learners tended to smile while holding their heads with both hands when they could not explain how they extended patterns. Furthermore, they often rolled their eyes when their ideas were questioned with facts, which made them feel that they had given their all, but their workings were being questioned negatively.

4.2.1.2 Teaching episode 1.2

Classroom interactions

I wrote this activity on the board for learners, and I did not say much as they already knew what to do. Immediately after I completed writing the number six on the board, one learner in group D, Tekele, raised a hand, having written an incomplete answer, and she asked to tell me the answer to question (1). She had not yet discussed this with her group members at this stage (see Figure 1.4 below).

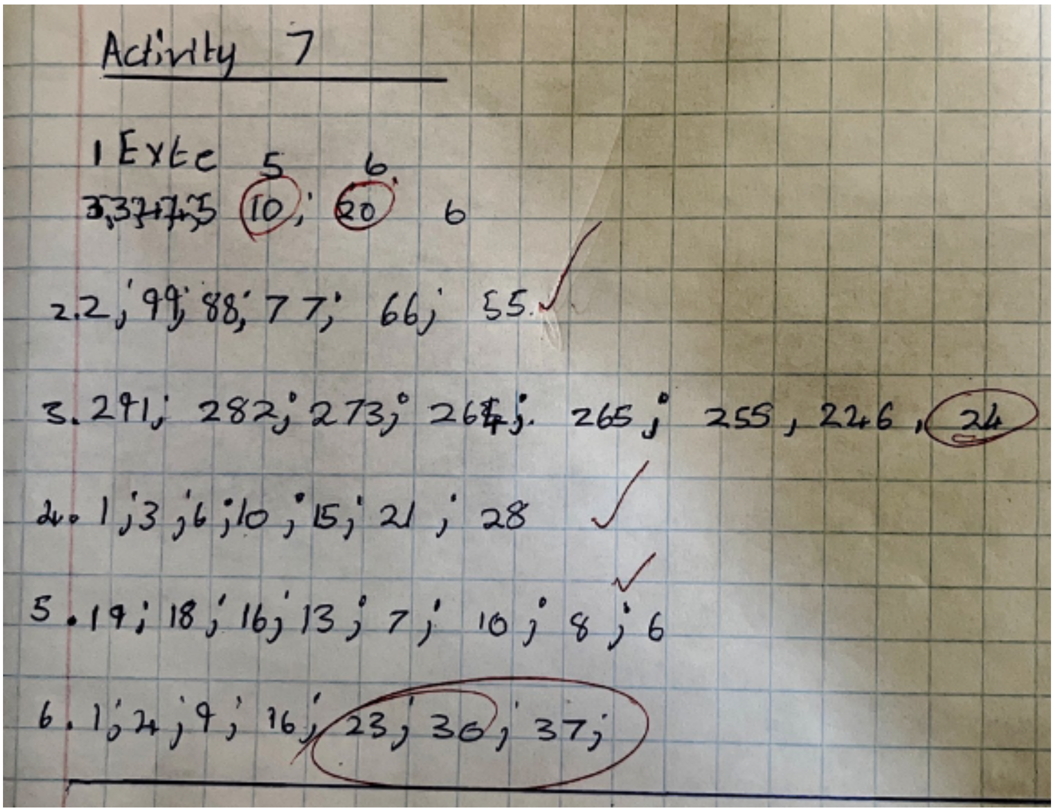


Figure 1.4: Calculations

Tekele: Sir 3 plus 3 is 6, 4 plus 4 is 8 and 5 plus 5 is 10, so the first answer is 10, then 10 plus 10 is 20 and 20 plus 20 is 40 (showing using fingers).

Teacher: Just like that?

Tekele: Yes (expressing how easy it was with a smile and pointing into her book with both hands open).

Teacher: Check how the pattern grows and discuss with your group to check if they agree with you (knowing well that she is wrong).

Tekele was confident in her answer and did not believe that her answer was wrong. After she made use of her fingers to calculate how she got the answers, she expressed confidently, by pointing in her book using two hands, wide open, to say ‘yes’, and smiled to show how easy the question was. Smilling, Tekele covered her answers, after seeing her group doing something different, and she started engaging with them. Tekele wanted an easy way out of the question, which resulted in misinterpretation of the question.

There was great interaction between learners and the activity in the classroom. Learners got the correct answer; however, they said they could not explain it in words. One group (group A) said: ‘the pattern grows in one, one and repeats itself’, but they said they were still discussing it and were not completely sure. I then let them continue, but I knew they were getting somewhere. Thus, learners continued working on the questions, but there was one other group in the classroom (group B) that was ahead of the other groups, and I was interested in how they attempted the question. The group showed great interaction with each other and kept pointing at their books to convince each other. Thus, I had the following interactions with the group:

Group B: (Counting together) 291; 282; 273; 264; 255….

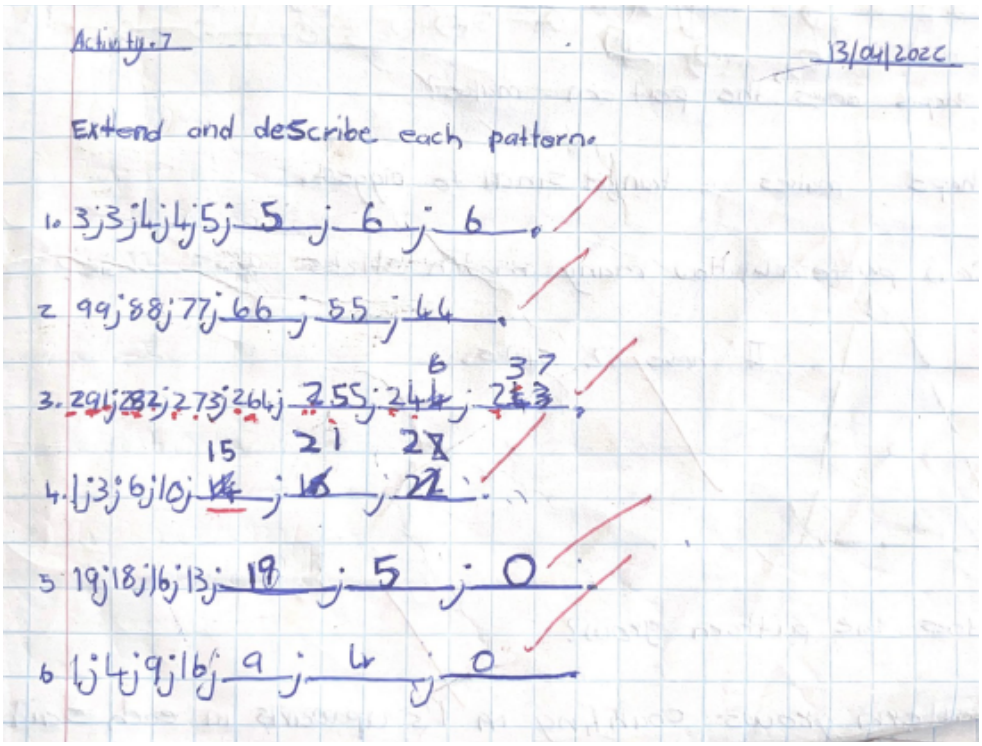


Figure 1.5: Calculations

Teacher: What question is that? Let me see and you were saying?

Mosima & Kgethi: (Starting from the beginning of the pattern and counting with their fingers) 291; 282; 273; 264; 255; 244; 243.

Tumelo: Its 246 after 255.

Teacher: You are saying it's supposed to be what?

Tumelo: (Low tone and showing a face of being unsure with a smile) 246.

Teacher: Why?

Tumelo: (Closed his face with hands and smiled.)

Teacher: Why 246, because Mosima here is having 244 so which one is it supposed to be?

Mosima: It is this one (pointing at her answer) because we are subtracting.

Tumelo: This one (pointing at Mosima's 243 answer) must be 233.

Teacher: Look at individual digits like from 264, then check their tendency as the patterns continues.

(I pointed at hundreds, tens, and the units, asking about what they observe as they seemed confused.)

Mosima: Owoo (responding in Sepedi) Mo ke 6, mo ke 7 (meaning the last 2 answers should be 246 and 237).

In group B, learners counted together as a group with excitement, indicating that they enjoyed extending the patterns. Mosima and Kgethi used their fingers to move one digit up on the units, and one digit down on the tens digit from 264 to get 255. However, the learners had 244 after 255, which did not follow the same interpretation as when they had 244. They subtracted one digit from both the units and the tens digits. Tumelo said the answer should be 246, looking at how they interpreted it, but he became shy to explain his answer because he covered his face with his hands and smiled. Mosima uttered 'no', waved her hands and pointed at 244 as the correct answer. However, Tumelo still did not want to argue his point. Tumelo's point was overpowered by Mosima's confidence and, as a result, he decided to keep quiet.

Furthermore, Mosima realised that Tumelo was right all along, and she uttered 'owoo' to show that she now understood the question. Their interpretation was that, since they now have 255, they therefore subtract one unit from the unit and tenth-place values.

Furthermore, they also insisted that they made a mistake with 243, which they corrected to 233. The group was thrilled, thinking that they had managed to arrive at the correct answer, but they lost their interpretation after getting 255. This showed that the learners had given themselves time to think about the pattern and also managed to correct themselves, and they saw that Tumelo's answer was correct.

On the other hand, Tumelo had the correct interpretation and knew the next number after 255, 246. He said it in a low tone because he knew this was not what they agreed on as a group. However, he also fell for 233, which was incorrectly interpreted. Eventually, learners managed to get the correct interpretation after I asked them to look at individual digits to recognise how the pattern extends. This means that learners bottled up their interpretations in a group because the majority agreed on a different answer and, as a result, the learner agreed with what majority, until the teacher confirmed their answers.

Learners in some groups could interpret the question correctly and, in other groups, did not write anything. The learners' utterances were characterised by being active, with smiles, and holding heads with confusion, respectively. Therefore, I did a whole class discussion with the classroom. This means that the learners did not have the mindset to give up on the question until they found the correct interpretation. However, I asked Mosima to explain how they attempted the question and the learners managed to follow. They were able to give a correct interpretation of the pattern after that. Therefore, I asked learners to continue with the next questions.

Learners were excited and ready for the questions because they immediately got to do the activities. I observed that most learners could interpret and describe question number 4 correctly. However, there was group D, which had different answers to the other learners. As a result, I engaged them to observe how they found the answers. The following is how the interactions went with group D:

(4) 1; 3; 6; 10; ____; ____; ____

Group D: (Counting together using their fingers and some using a ruler) 6 then we skip 4 to get to 10, then 10 we skip 4 to get to 14.

Teacher: Let us see, what do you have?

Ofentse: Sir we have 14 as the first answer.

Teacher: What did you skip to end up at 14?

Group D: We skipped 4.

Teacher: Again?

Learners were just staring at each other and myself.

Teacher: (I then asked then to put fingers on the ruler to recognize the numbers that the pattern grows with and advised that they write those numbers aside.)

Khura: Ohh so it grows with 2, 3, 4 and then the last one we add 5 to make the next number to be 15.

Teacher: Then from 15 what are you going to do?

Group: We skip 6 then itssss … . (counting using a ruler and some with fingers) 21.

The learners continued using the ruler and all having L2 taking the lead in their discussion.

Hope: Now we skip 7, let us count from 21.

Group: 22, 23 … then 28.

Learners in group D could interpret the question (question 4) by themselves. They managed to see how the pattern grew. They initially used hand gestures by counting to recognise how the pattern grows and then got confused after seeing that they could not add four again because they all kept quiet and started staring at each other. Furthermore, learners studied the pattern again and eventually interpreted the pattern correctly. The learners showed great excitement as they completed the number pattern. This shows that the learners in the group managed to bring action into mental representations.

On the other hand, other groups had not done anything with the question. They argued that the pattern did not make sense as the number it grows by is still not the same. These learners' utterances were characterised by the learners looking confused, with their eyes squinting and some folding their arms, showing that they could not attempt the question. Therefore, these groups could not describe and recognise the pattern in question number 4 by themselves. As a result, I decided to engage group E, one of the groups that had not done anything with the question, in order to observe their utterances as I questioned them, trying to get them to understand the questions. The learners in this group gave up when they thought that they had explored all their options and could not interrogate each other on what the answer should look like. Thus, they demonstration little understanding of the pattern.

Teacher: Heee, how far are you? (They all just looked at each other) Let us study the pattern. How did the pattern move from 1 to 3?

Tiny: They times 1 with 3.

Teacher: How about from 3 to 6?

Group: (All kept quiet.)

Teacher: Did they also multiply by 3?

Group: No.

Teacher: Let us look at the numbers they skipped as the pattern grows.

Teacher: Let us count from 1 to 3, 3 to 6 and study the numbers we skip each time.

Group: (Working together) we skip 2, then here its 3.

Tiny: So, going forward we are going to skip 4 to get 10 and add 5 to get 15.

Thabo: Sir we got it now thank you.

Tiny in the group was just unsure of the answers she was giving and the rest of the group was just watching, trying to understand what she was saying. Tiny thought that multiplication by three was a possible extension of the pattern but thought it could not be constant as we continued with the pattern. The group became confused because they kept quiet and stared at each other, as Tiny also ran out of ideas. As a result, I asked the learners to look at numbers extended to two numbers, which enabled learners to recognise the pattern. However, this made me realise that the learners only wanted me to talk to them to get an idea of the pattern. After that, I asked Tiny to discuss it with the whole classroom since they struggled to consolidate the question.

Luckily, they all seemed to have been following as they uttered, ‘oh wooo’, indicating that they now understood.

Learners then carried on with the rest of the questions. They seemed to be carrying on well, with a good response to question number 5. Thus, I became more interested in how they would respond to question number 6. The question involved using multiplication and multiplying position numbers twice, which is unusual to them, so I knew that they would struggle a bit to find answers. Therefore, I wanted to observe the groups’ utterances as they engaged with the question.

(6)1; 4; 9; 16; ___; ____; ____

I observed the groups finding ways of how they can approach the question. Some were trying to find the difference and saw that it did not work. These learners left their work on the way and put their hands on their heads, while shaking their heads. On the other hand, some of the groups got stuck from the beginning and asked whether I had captured the question correctly. These learners gave up on the questions and decided to fold their arms before I kept saying to continue thinking about it, knowing that they did not like failing at a maths question. After 10 minutes, one group raised their hands, and one learner from group A, Disebo, said: ‘Sir I think we got the answer’. However, they had not written anything in their books, but I listened to them. The interactions with group A are captured below:

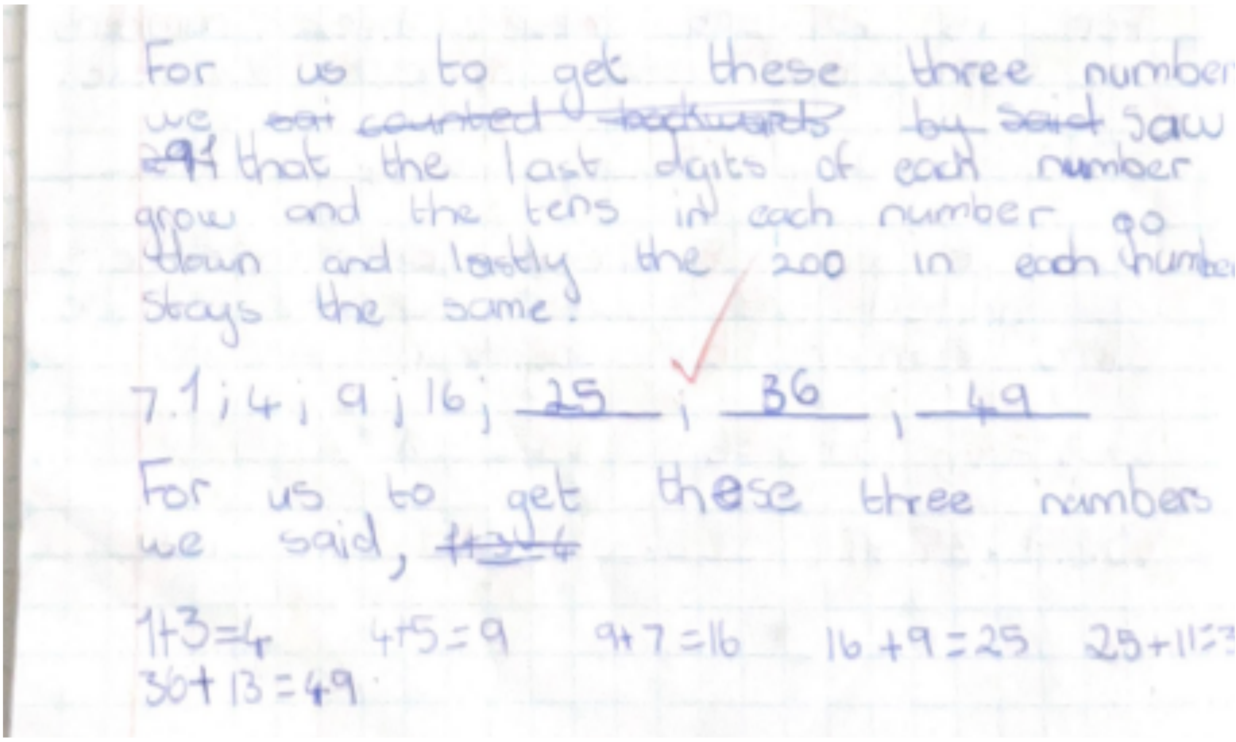


Figure 1.6: Calculations

Teacher: So how does the pattern grow?

Disebo: Sir, we add odd numbers.

Teacher: How?

Disebo: From 1 we add 3 to get 4, from 4 we add 5 to get 9 and from 9 we add 7 to get 16

Teacher: Interesting, continue.

Group: (Answering together) then from 16 we add 9 to get 25, from 25 we add 11 to get 36 and from 36 we add 13 to get 49.

Teacher: Okay, write it and describe it in words.

(See Figure 1.6)

At this point, I was shocked as these learners managed to think about the pattern up to this point. Furthermore, it was interesting to see how she came up with the recognition and she was also convinced that it was correct because she kept showing a smile and using her hands as counters. After that, the group followed as she explained her thinking to me. I then decided to check with other groups and told them that one group managed to do it to encourage them that it was possible. A few minutes later, learners were squinting their eyes to show confusion, while others kept saying that they did not understand. Therefore, I asked Mongatane, a learner in group A, to explain how they recognised the pattern to consolidate the activity.

The group interpreted the question correctly, which indicates that they understood the concept of numeric patterns and they could deal with abstract questions. The learners knew that they had to interpret the pattern from the beginning, which indicates cognition and growth in mathematical knowledge. Furthermore, this indicated that the other questions on numeric patterns helped the learners to enhance their learning using hand gestures to bring action into their mental representations.

Analysis of teaching episode 1.2

Teaching episode 1.2 was focused on learners' ability to recognise and extend numeric patterns. However, in this teaching episode, the patterns not only required learners to make use of constant difference, but learners became excited when I posted the second activity and almost all of them showed a positive response to the activity. The exception was the learners in group, D who thought of adding the numbers instead of continuing with the pattern through repetition. The learners' immediate thoughts took precedence, and they were left unquestioned. It became clear to almost all the learners in the classroom that the pattern repeats, but group D decided to add the numbers. However, after questioning their answers, they interpreted the question correctly.

Learners tended to respond quickly to questions, as if what they got immediately was correct, without questioning it. These learners made use of hand gestures when calculating with excitement. Furthermore, the learners become confident of the answers and excited because they were the first to realise an answer to the question. However, there was a change of mood to show disappointment when the teacher questioned their answers. Learners became gratified by knowing they were the first to realise a correct answer (Hmelo-Silver, 2004). Furthermore, these learners seldomly questioned their answers before considering them correct (Hmelo-Silver, 2004).

On the other hand, some learners did not consistently allow numeric patterns to grow. Learners in group B did not carry on with how the pattern was extending after reaching 255. The group saw that 255 had the same two fives, then thought the next number would be 244, followed by 233. However, this group managed to get 255, which means they could spot how it extends until there. Thus, from 255 on they spotted a different extension and their minds recognised a new pattern altogether. The learners did not notice that the way the pattern was viewed had changed. On the contrary, Tumelo mentioned that after 255, we should have 246, but he could not voice an explanation for this. As a result, his answer was disregarded by the group because they could not get any explanation from him and he then agreed with the group's answers.

Similarly, in group D on question 4, learners took the difference of the last two terms to be constant for the whole pattern to get to their answers. The whole group had agreed on the interpretation of the pattern that, from 10, they add four going forward. However, when asked to explain this, they could recognise the pattern from term one. Learners need continuous guidance from the educator in answering questions (Hmelo-Silver, 2004). The learners find it hard to see their own mistakes when learning. Therefore, it took the learners interacting with the educator to recognise their mistakes.

Learners tended to lose recognition of how a pattern was extended from the beginning, as the pattern continues. The mind creates new thoughts as you encounter a new situation (Alibali & Nathan, 2012). However, these thoughts take precedence over the old ones when left unquestioned. Hence, learners diverted into new thinking without even realising it because they did not question their answers. On the other hand, learners tend not to voice their thoughts because they think other learners will judge them based on their answers (Darragh, 2016). This makes learners fear justifying their responses, hence allowing them to be disregarded.

According to Darragh (2016), learners at an early age of learning fear getting the answers wrong and but they always have something to write down. As a result, learners just write down answers that they cannot explain when encountering a complex question. Utterances of the learners in the classroom were characterised by learners just being mute, just looking, while others said that they did not know how to explain their thinking when they encountered the question (6). However, there is an exception of learners who will try to make sense of the question until they find a way out. In group A, the learners made sense of question 6. They explained that they needed to add odd numbers to get the next term. Furthermore, they showed that 1+3 is 4, 4+5 is 9, 9+7=16, 16+9=25, 25+11=36 and 36+13=49, which makes the pattern to be 1; 4; 9; 16; 25; 36; 49. These learners had already learnt the skill required to analyse as they tried out multiplying by four, then checked and analysed the difference as multiplication did not work.

Analysis of teaching experiment 1

In the teaching experiment 1, learners could recognise, interpret and describe numeric patterns by themselves and with my assistance through questioning. As a result, learners portrayed utterances that revealed whether they successfully solved questions on numeric patterns. Furthermore, learners also portrayed utterances as they were involved in questioning how they attempted questions on numeric patterns. After that, they also portrayed utterances that indicated their success or failure in solving complex numeric patterns.

Learners portrayed immediate utterances like smiles, hand gestures and the use of a ruler to count and become excited, with eagerness to work on the questions, indicating their success in an activity. Similarly, learners portrayed the same utterances when they wrongly attempted a question because they believed that their first interpretations are always correct. On the contrary, the learners squinted their eyes and put their hands on their heads to show that they saw that they were wrong and understood that they misinterpreted the question. Furthermore, learners responded positively by showing eyes wide open, using hand gestures to give explanations and smiling when I engaged them in questioning. On the other hand, some of the learners moved their heads sideways when they did not understand the questions, I posed to them.

Learners who used simpler numeric patterns learnt to apply patterns that did not involve constant difference or were unusual to them were characterised by being excited and showing smiles. These learners showed growth in mathematical cognition as they showed a trend from how these learners dealt with low-order questions to abstract questions. On the other hand, learners who felt challenged by abstract questions on numeric patterns gave up on the questions and folded their arms and closed their mouths because they felt that they had exhausted all possible possibilities of solving the question. Furthermore, only a few in the group managed to follow when I posed questions to help them, while others had already given up because the questions seemed impossible to them. The utterances of these learners were characterised by the learners holding their mouths with their hands and others looking down at their tables.

4.2.2 Teaching experiment 2

4.2.2.1 Teaching episode 2.1

Classroom interactions

I captured the activity (activity 3) on a board for learners to attempt. Learners seemed to be intrigued because the pattern involved shapes. As a result, all learners described the pattern correctly. Furthermore, learners could draw the shape of pattern number 4 with the correct number of matchsticks and the learners responded quickly to the questions. Afterwards, group C asked that I come and check their responses as they were busy with number 5 in the activity. Therefore, I engaged the group on how they attempted the questions because learners were simply doing well with the activity at this point. Furthermore, I also wanted to observe the learners' utterances as I asked them questions to see how the group understood what they had written. As a result, I engaged group C as follows:

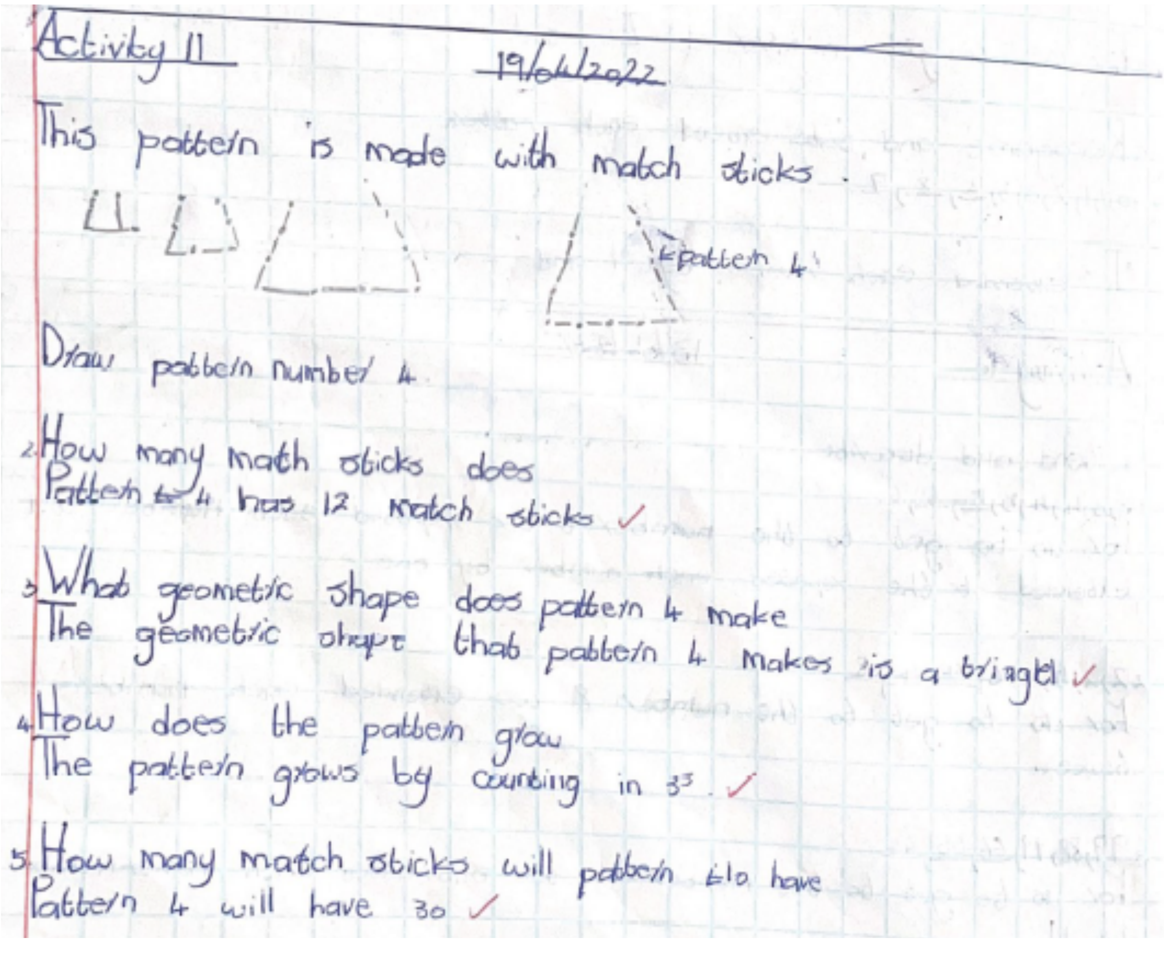


Figure 2.1: Calculations

Teacher: How did you get 30 on number 5?

Odirile: I counted in 3s ten times.

Teacher: Why in 3s?

Odirile: Because everything here goes in 3s.

Teacher; How?

Odirile: (Just looked and became speechless.)

Teacher: Show me how you counted.

Group: We say 3, 6, 9 … (counting using their fingers until they got to 30.

Teacher: what if I say pattern number 50?

Blessing: Ehh we will count in 3s 50 times too.

Teacher: What about pattern number 1000?

Blessing: (Ehh) Sir we can't count in 3s 1000 times.

Odirile: Sir we going to count in 10s.

Teacher: Why 10s?

Odirile: Because it is the biggest.

Learners could interpret the geometric patterns and also find several of the matchsticks that pattern 10 will have (see Figure 2.1). However, I asked them more questions to see how they would interpret the question when I asked about different pattern numbers. Learners realised that, when I keep adding the patterns numbers, it becomes more difficult to count in threes. As a result, learners became confused and opened their eyes wide, while not responding. However, they realise that they cannot count in threes 1 000 times. Hence, Odirile said we would count in 10s without a clear explanation. She argued that 10 is the biggest number, but there has no relationship between the pattern number and the number of sides on a triangle.

I moved on to another group to see whether they could interpret the questions correctly and continued to observe their interactions.

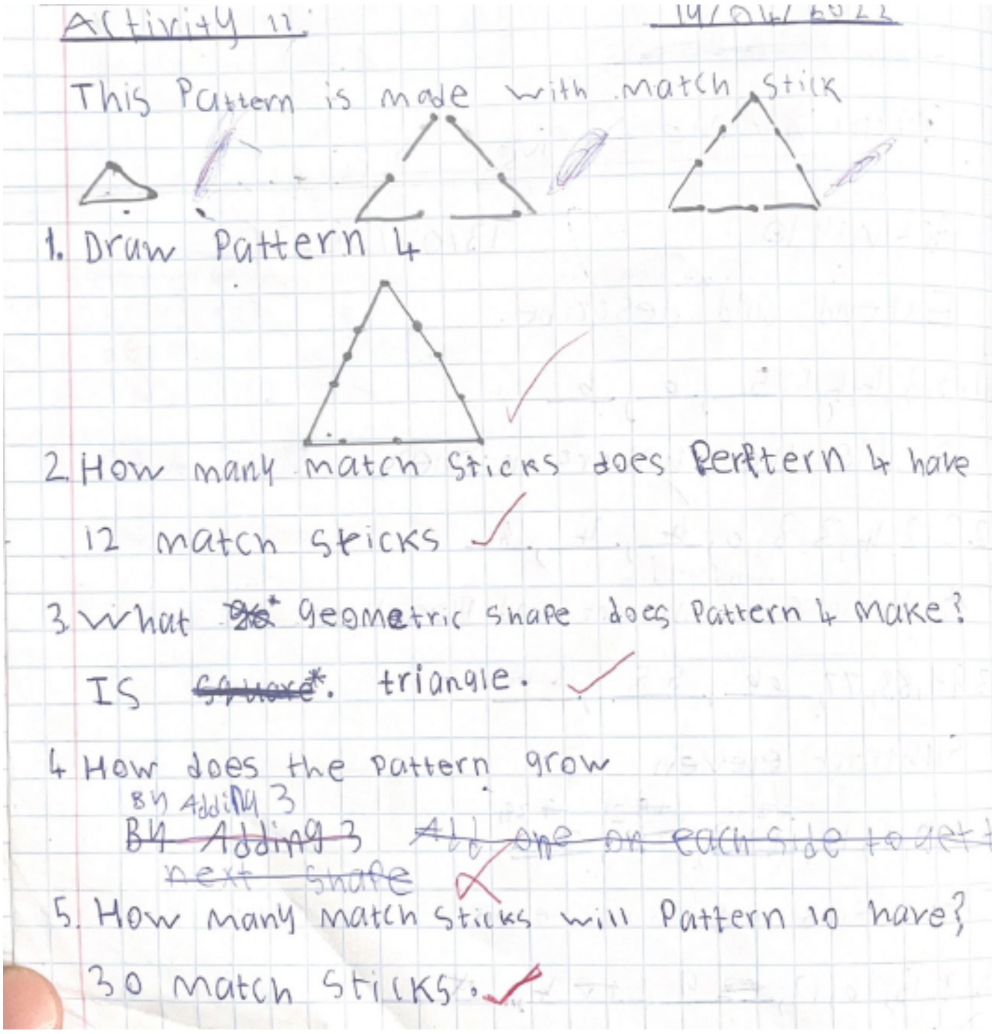


Figure 2.2: Calculations

Teacher: Talk to me about pattern 10, how did you get 30?

Maria: (Pointing at a pattern she drew) It is going to be 10+10+10 which is 30 (pointing the sides of the triangle made by her pattern).

Maria and her group had done well with the pattern. They smiled as they pointed to the triangle's sides to explain how they got 30. I then decided to do a whole classroom discussion with the learners because most of the learners were done and asking that I come and check their workings. Furthermore, during the classroom discussion, I also had an idea of engaging learners on other questions which involve bigger patterns of numbers verbally.

Teacher: Okay, let us talk about how the pattern grows.

Rendani: We add one matchstick on each side on to get to the next pattern.

Teacher: So how did you get 30 for pattern number 10?

Mongatane: I counted in 3s until 10.

Teacher: So, what if I wanted pattern 100? Will you still count in 3s?

Maria: You draw 100 matchsticks one side, 100 on the other side and down you draw 100 to give us 300.

Teacher: What if I wanted matchsticks for pattern 50?

Mongatane: It will be 50 times 3.

Teacher: Which will be?

Class: (Counting together) 50, 100 and 150.

Learners were actively engaging in the whole classroom discussion. Many learners raised their hands for my attention. Learners had two different interpretations, counting in threes and counting in 10s, which eventually led to one interpretation whereby we looked at the number of sides. Thus, eventually, learners realised that they needed to multiply 50 by three (number of sides) and 100 by three to find the number of matchsticks in patterns 50 and 100, respectively. Furthermore, the learners showed their understanding by portraying utterances like smiles and using their hands to explain what a pattern looks like. Maria visualised that pattern 100 would have 100 matchsticks on each side and moved her heads up and down.

Learners found it easy to interpret geometric patterns because they saw the pattern in terms of pictures, which led to correct answers being presented. However, their struggles and successes on numeric patterns also contributed to describing the geometric patterns. Furthermore, this showed how learners can apply concepts into new situations, which is evidence of embodied cognition.

Analysis of teaching episode 2.1

In teaching episode 1, learners found it easy to recognise, extend and describe geometric patterns because they successfully connected to the numeric patterns learnt in teaching experiment 1. Furthermore, learners could also visualise geometric patterns to identify the number of matchsticks they would have. The utterances of the learners were characterised by the learners using their hands to count, visualising and drawing the next pattern, and smiling when doing the activity on geometric patterns.

This shows that learners were enjoying the activity. However, learners in group B and group A interpreted question 5 differently. The groups interpreted the questions as: ‘for pattern 10 we count in 3s ten times’ and ‘for pattern 10 we say 10+10+10 to get 30’, respectively. The explanation from group B had some limitations, as the group insisted that they could not count in threes 1 000 times when I asked more questions because it would be too much work. As a result, they could not connect the relationship between pattern numbers and the number of matchsticks on each side of a triangle. The group did not want to think of anything other than counting in threes.

In addition, many groups thought of counting in threes, hence the whole classroom discussion of question 5. The learners were able to establish a connection between the number of matchsticks and pattern numbers during the whole class discussion. This was shown when they could related their description of the pattern in words to find the number of matchsticks for patterns 100 and 50.

The above means that learners embodied the mathematical concept because most of them managed interpret the question on the number of matchsticks. The learners indicated a success by the use of gestures in comprehension of mathematical knowledge. Furthermore, learners were supporting their thinking by establishing action from their mental representations

4.2.2.2 Teaching episode 2.2

Classroom interactions

I presented the above activity on a board for learners to work on. Learners immediately became interested and got down to do the activity. Hence, learners responded positively to drawing pattern number 3 and uttered smiles, using their hands to visualise what the pattern would be like and most raised their hands, wanting to show me how their next pattern would look. Learners could draw pattern 3 and they kept using their fingers to help them describe how the pattern was made.

On the other hand, in group A, learners seemed to be having conducive interactions as the whole group was engaging. Furthermore, learners were talking in unison when explaining how the pattern grows. They were using their hands to make their point clear, while most learners in other groups focused on completing the table. Thus, I had the following interactions with learners in group A:

Group A: (Talking as a group.) We add one stick to each side of the square (they were answering question 2 at this stage).

Teacher: So how many sticks would you use to build pattern 5?

Disebo: It will be 20 matchsticks.

Many learners could easily say that they added four matchsticks, however, this group mentioned that one matchstick was added to each side of the square, which indicates an understanding of interpretation of geometric patterns, developed knowledge of geometric pattern and growth in mathematical knowledge. Therefore, these learners have embodied the mathematical concept.

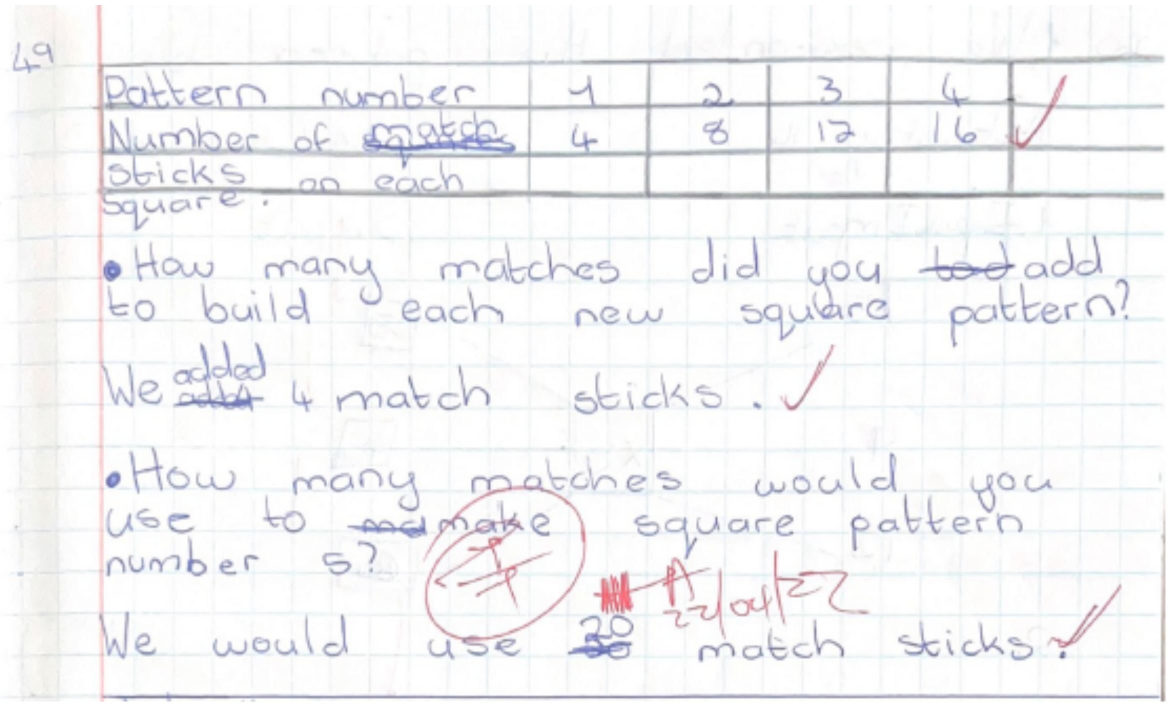


Figure 2.3: Calculations

Teacher: How did you get the answer?

Disebo: We counted 5, 4 times.

After realising that the learners were able to get correct answers, including for pattern 10, I then decided to ask them, verbally about pattern numbers which will have a certain number of matchsticks.

Teacher: Which pattern number will have 24 matchsticks?

Mongatane: Pattern 6.

Teacher: How?

Mongatane: By multiplication, I say (showing with her hands) 6 times 4.

Teacher: 6 times 4?

Disebo: 4; 8; 12; 16, 20 and 24.

Teacher: Which pattern number will have 80 matchsticks?

Disebo: 80 Matchsticks? (She looked surprised.)

Teacher: Yes

Group: (Counting with fingers, trying to find a number that they can multiply by 4 to get 80.)

Disebo: Pattern 20.

Teacher: How did you get 20?

Disebo: I counted in 4s until I got to 80, then it gave me 20.

Learners in group A managed to solve all the questions, so I confirmed the answers required for completing the table. However, I reversed the questions and asked about pattern numbers to check for learners' understanding. I asked learners which pattern number would have 80 matchsticks and learners in the group seemed to be confused. However, they all started counting with their fingers. Disebo quickly responded by saying pattern 20 and the group immediately rolled their eyes because they were still counting. After that, the group understood when Disebo explained how she got to the answer (see the interactions above). Thus, I confirmed with the rest of the learners in the classroom about the activity. Learners in the classroom did well with the activity and actively responded to the questions. Therefore, I moved on to an activity that did not involve triangles and squares.

Analysis of teaching episode 2.2

The activity presented above is like activity 4. Therefore, it became easier for learners to understand the questions. Learners were applying knowledge of the geometric pattern of triangles to the geometric pattern of squares. Hence, it was easy for learners to interpret the geometric pattern of squares and utter positive utterances such as smiles. On the other hand, learners in the classroom could visualise the pattern to find the number of matchsticks for pattern number 10. The learners connected pattern numbers to several sticks on each side of the square. However, group A had more extended questions than the rest of the learners in the classroom as they had finished answering the question. Furthermore, learners uttered smiles and used of hands in finding pattern numbers.

In group A, learners were engaged in using perceptual memories to create mathematical concepts. This was evident when learners thought of question number 6, when asked, ‘which pattern number will have 24 matchsticks?’, which can be multiplied by four to get 24, and they immediately found the answer. On the other hand, when they were asked about the pattern numbers given 80 matchsticks, they felt that it was a big number and used their fingers to count. Furthermore, learners used the same strategy of hands to think about a number that can be multiplied by four to get 80.

4.2.2.3 Teaching episode 2.3

Classroom interactions

In this activity, I wanted learners to interpret geometric patterns that do not involve patterns of triangles and squares. I did this purposefully to observe learners' utterances as they attempted the activity. Most of the groups managed to spot how the pattern grew and could fill in the table, as they could see that they added consecutive numbers from pattern number 4. On the other hand, there was a group of learners, group C, who had different answers as a group, and I wanted to observe how they ended up with these different answers.

One learner had 26 blocks for pattern number 5 and the other one had 28. I then had an interaction with the group regarding their answers.

Teacher: How did you get 26?

Ntombi: I plussed (added) 5 (showing it with a hand).

Teacher: Why did u plus 5?

Ntombi: (Muted.)

Teacher: So, how many should it be? (Asking the other members of the group.)

Thasha: 28?

Teacher: How did you get it?

Thasha: 21 plus 7 is 28.

Teacher: And 34?

Thasha: 28 plus 8 is 34.

Teacher: Confirm if you will get 34 when you add 8 there.

Thasha: (Counting with his fingers) 34.

Teacher: Let me see how you are counting, count from 28.

Thasha: (Counting, starting with the fifth finger) 36.

Teacher: Check with the rest of the group why you got different answers.

Ntombi added five to get pattern 5. However, she could not explain why she got the answer. I assumed she thought of the previous geometric patterns whereby the pattern number described the number of matchsticks on each side of the shape. Therefore, she did not clearly recognise how the pattern grows, looking at the number of squares. She only assumed that for pattern number 5, we should add several squares. However, she kept quiet and felt too ashamed to explain.

On the other hand, Thasha managed to study the pattern correctly, but she could not add 28 and eight correctly on the first trial. She was happy to explain because she knew that her interpretations made sense. Thasha uttered a smile and used her fingers to point and count how she got pattern 5. Similarly, she made use of her fingers to correct pattern number 6. Furthermore, the rest of the group managed to understand the solution after Thasha gave an explanation and, hence, I asked them to engage together as a group. Afterwards, I engaged the next group, group E, seated next to group C.

In group E, learners had the wrong answer for pattern 5, which indicated that the learners were unable to interpret the pattern correctly. Therefore, I engaged them with a question to get an insight into their views about their answers. The interactions with the group went as follows:

Teacher: How did you get 31?

Maria: Sir, we counted in fours.

Teacher: Okay, count in fours so that I can see.

Group: (All counting at the same time) 4 … 32, 36 (became confused when they saw that they have passed 31).

Maria: Sir I understand (she counted from pattern 4, trying to extend the drawing by showing with a pen, however as she was extending the pattern, she left one square in her drawing as she was trying to be quick) it's going to be 27.

Teacher: Okay draw pattern 5.

She drew the pattern and counted the squares.

Maria: 28.

Learners in group E initially thought of adding four to get to the next pattern number. Hence they got 31 for pattern 5. The learners were convinced that it was correct because they were active and used their fingers as a group to find the rest of the pattern numbers. However, Maria gained confidence after seeing that I was not satisfied with their answers and gave a different interpretation. However, her interpretation was based on how the drawings should look. Hence she studied the drawings by redrawing the patterns. She gained the attention of her group mates. Leswene and Lesibana were folding their arms and opening their eyes, while Bokang had her hand on the check listening. Learners in the group were attentive and eventually saw that the number of squares that are being added was changing. After that, Maria gave an answer to pattern 5, but the group was still confused about how she got the 28 for pattern 5.

Furthermore, learners had not understood how the pattern grew. They had not got the numbers the pattern extends with. I then asked the learners to answer the question, 'How does the pattern grow?' Learners could see that the patterns differed in numbers 4, 5, 6, 7 and so on. However, they squinted their eyes to show confusion because they expected the number to be the same. After that, Maria said, ;to get pattern 5, we add 7 squares'. This made the group understand because they uttered 'oh wooo' to indicate that they now understood.

Analysis of teaching episode 2.3

In teaching episode 3, learners became active in explaining their answers when they were confident they were correct, while others did not explain when they were not confident. Learners who were confident in their responses were characterised by using their hands to express their ideas and smile when doing so. On the other hand, learners turned mute, held their chins, scratched their heads and opened their eyes wide when they were not confident in their responses. However, the latter group of learners tended to start talking when asked questions to help them understand the concepts better. After that, they ultimately portrayed positive utterances, such as moving their heads up and down and using their hands to extend mathematical ideas.

In group C, learners did not give reasons why they added seven to find pattern number 5 because they have doubts about the answer. On the other hand, learners gave reasons for their answers when they were confident of their responses. This is when learners simply roll their eyes, while others looked happy to respond, expressing less confidence and more confidence, respectively. Furthermore, these learners tended to think that some learners had different answers to others in the group, which contributed to reduced confidence to give reasons for their answers.

In group E, learners were confused by the question because they had found pattern 5 to have 31 squares. The learners said they counted fours to get to their answer, without having any connections to how the pattern was created. However, they thought of the pattern differently when I questioned their actions. One learner from the group showed the group that they could extend by drawing the next pattern, and then we got to 27, which was corrected to 28. The learners had not studied the pattern to recognise how it grows. Therefore, I realised that the learners were missing a question, which was asking about how the pattern grows. Eventually, the learners were able to recognise the geometric pattern without having to make drawings.

Learners were creating images when working with geometric patterns since the patterns came in terms of drawings. According to Guner and Uygun (2020) learners find it easy to create images when dealing with geometric patterns because the patterns are presented in drawings. The images they created assisted learners to answer questions for extended pattern numbers. However, some learners could not create these images until they were posed a question that asked about how a pattern had been created. On the other hand, some learners could create these images because they already knew that the patterns were connected. Therefore, they visualised what a pattern would look like and the number of squares it would have.

Analysis of teaching experiment 2

In teaching experiment 2, learners could recognise and extend the patterns on teaching episodes 1 and 2, and found it difficult to interpret the patterns. Learners could use their perceptual memories to interpret the patterns in the teaching episodes 1 and 2 because they could realise that the number of matchsticks is added to each side of the shape. However, they could not interpret the patterns in teaching episode 3 because the number of squares that the pattern grew with was not constant. Learners portrayed utterances in the process of interpreting the patterns.

Learners portrayed immediate utterances, such as being excited and active, because they could visualise the geometric patterns. Learners continued with a positive working spirit and portrayed the use of hands to virtually build the next pattern, movements of heads in upward and downward directions and use of fingers to express their explanations when I asked them questions verbally. However, most learners scratched their heads when they encountered an activity in teaching episode 3, while some group members folded their arms and others were busy moving their shoulders in upward and downward movements during their group interactions. The utterances that learners portrayed showed either significant growth in mathematical development or no significant growth in mathematical development.

Learners in the Grade 4 classroom showed growth in mathematical development when they used their hands to virtualise what the next pattern would look like and when they used their hands to discover the number of matchsticks on each side of a shape. Furthermore, Maria virtualised what the next pattern would look like in the teaching episode 3 to find the number of squares that needed to be added to extend the pattern. On the other hand, learners did not show any growth in mathematical development when they gave up on the question in the teaching episode 3 and portrayed negative utterances, like moving their shoulders in an upward and downward direction. This shows that learners may have given up or not know how to answer the question.

4.2.3 Teaching experiment 3

4.2.3.1 Teaching episode 3.1

In the classroom, the learners got excited when they encountered word problems and knew they should use mathematical operations to solve them. However, they seemed to be struggling to interpret the question. A few learners said they did not understand the question. However, I insisted that everyone work on the question in their groups. As they worked in their groups, I heard the groups starting to engage in the questions. Individuals in some groups were asking each other whether they should use addition or subtraction. Thus, I encountered one group, group B, who added all the number of copies released and sold to find the answer. As much as they surprised me, I wanted to hear their understanding of the question. Hence, I interacted with them as follows:

Teacher: Let us see, what does 1620 represent?

Mosima: We added everything to know how many it made.

Teacher: But they gave us 965 as copies which were made, 243 are sold and 482 were sold too. So, they want to know the number of copies that remain.

Mosima: Now I understand.

Teacher: Okay let me allow you to work on it.

Learners in group B misinterpreted the question because they took it as if the 965 copies released were sold as well and they ended up adding all the number of copies sold. However, the utterances of the learners were characterised by using their fingers to count the total number of copies sold. Furthermore, learners in the group claimed to understand the question after I explained that 965 represents the number of copies that were there. They moved their heads in an up-and-down direction to indicate that they understood. However, it seemed like they only said that because they needed some space to work on the question because they kept looking at each other without doing anything about it.

I then moved on to another group and I found them having 725 as an answer, so I decided to engage with them as well:

Teacher: So, from the question, how many copies were released?

Oratilwe: 243.

Onkabetse: 482.

Teacher: Read the question again, so 243 and 482 were sold.

Group: (They all looked confused, looking at each other.)

Teacher: What does the 725 represent?

Group: Is the answer.

Teacher: The number of copies left to be sold?

Oratilwe: (Sigh) No the number of copies sold.

Teacher: Then how many copies are left to be sold?

Oratilwe: Yooh sir!

Teacher: Therefore, you need to calculate the number of copies which are to be sold.

Group: Okay sir.

Learners in the group managed to get the total number of copies sold. However, they thought that this was what the question was asking. The learners could not continue to find the number of copies left to be sold because they felt that they had answered the question. Furthermore, the learners were simply calculating without attaching meaning because they all held their heads and looked at me when I asked what 243 and 482 represented. On the other hand, Oratilwe seemed to have an idea since she could indicate that 725 represented the number of copies sold. Similarly, she could not continue finding the number of copies left to be sold. She just said, 'yooh, sir!' to indicate that she did not know how to get further than 725.

Learners in other groups said that the question did not make sense, with most of them having added all three numbers, and some of them had not done anything with the question.

Learners tended to just use addition when they encountered a question that they did not understand. This was characterised by the learners not being able to say what the answer represents and not knowing what they were calculating. Therefore, learners did not embody such mathematical knowledge, and this did not help them understand more abstract questions.

Analysis of teaching episode 3.1

The question confused all the learners in the classroom as they were using addition without any confidence when attempting the questions. The learners covered their answers with their hands because they did not know what their answers represented. One group of learners added all the numbers that were in the question. In their explanation, they said that they needed to find the number of copies altogether. However, the total number of copies was given in the question.

Similarly, another group gave two different answers when asked about the number of copies sold. Therefore, learners read the questions once and decide on what to do immediately. Learners at an early stage of learning think of addition operations when they are given more than two numbers in a word problem question (Verschaffe, Depaepe & Van Dooren 2020). Hence some of the learners would add everything in question to get the total number of copies made.

Learners in the classroom could not interpret the question without further clarification from the teacher because the learners were just staring at each other without doing the work. I, therefore, clarified with learners that 965 copies were made, and they sold 243 and 482 copies. Learners were moving their heads in an upward and downward direction. The learners could interpret the question because they started using their fingers to calculate the number of copies sold. However, my clarification said the same thing that was in the question. Learners at an early age need clarification of the problem when dealing with word problems (Verschaffe, Depaepe & Van Dooren 2020). However, they added 243 and 482 to get 725 and left it as the answer. Only one group managed to subtract this number from the total number of copies to get the number of copies to be sold.

Furthermore, when I asked what the number represents, of learners who managed to get 725 as their answer, some said the answer was the number of copies sold, while covering their mouths, while others did not say anything. Learners struggle with how the language is used in word problems (Verschaffe, Depaepe & Van Dooren 2020). Hence learners thought that finding the number of copies sold was what the question was asking. On the other hand, learners were done with the question because they did the calculation. Learners are used to questions involving one mathematical operation at a time. Hence almost all of them did one calculation and folded their arms.

4.2.3.2 Teaching episode 3.2

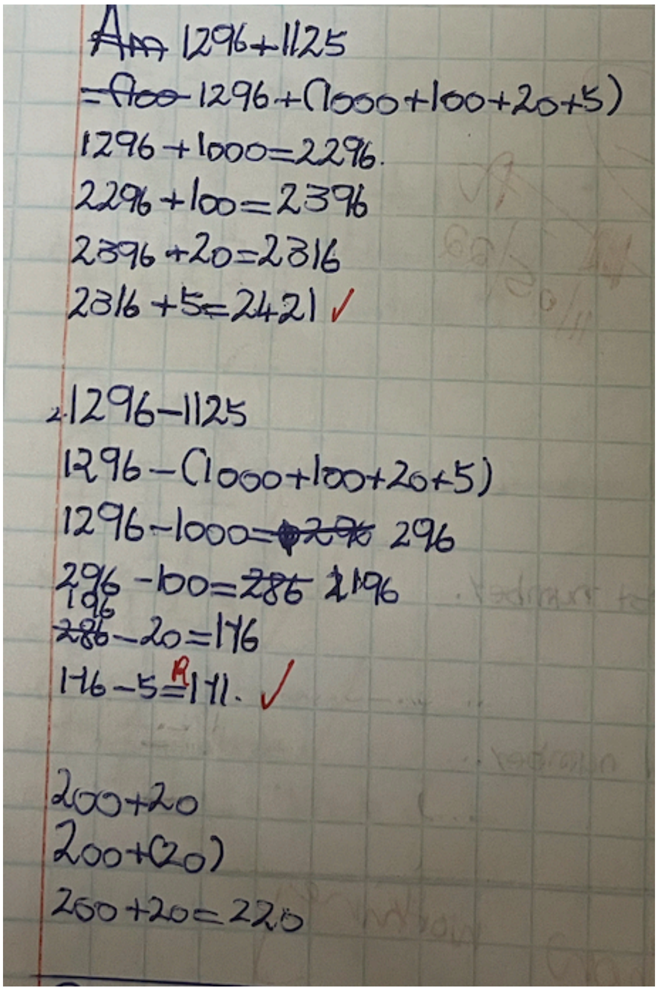


Figure 3.1: Calculations

Learners attempted the questions and I noticed that most of them were using addition to attempt the first question, and executing the use of addition properly. Therefore, I asked the whole classroom why they were using addition. Mongatane confidently raised a hand and said, ‘because they said altogether, altogether means we add’. Then the learners went ahead to do the second question and they continued to do well. As a result, they used subtraction to find the price difference.

Activities of the learners in the classroom were characterised by the learners being active, engaging with one another and learners who were happy about the activity as it made much more sense than what they had encountered in the first teaching episode above.

On the other hand, learners could not understand question 3, some left the question and others tried to attempt it. Learners who did not attempt the question gave up when they saw the word ‘discount’. At the same time, other learners were just adding the discount prices (see Figure 3.1 above). After that, I explained what a discount means and then the learners answered the question verbally to say Khura’s boots would be more expensive. This indicated that learners only did not understand the meaning of a discount.

Analysis of teaching episode 3.2

Learners found it easy to make use of addition and subtraction. As such, they found the questions easy. The learners became excited because they managed to figure it out by themselves. However, they became confused when they had to find new prices for soccer boots for a given discount. Learners were holding their chins with their hands to show their confusion, while others were just calculating without attaching meaning. Therefore, learners could interpret and solve the word problems in the teaching episode 2.

4.2.3.3 Teaching episode 3.3

First classroom interactions

I gave the above question to learners to work on. Learners immediately got to the question, and they all thought about how to approach it. In group B, however, the learners had already figured out how to approach the question. The learners were busy responding to the question, and I decided to engage with them as follows:

Teacher: With this question have you decided on the operation you should use?

Maria: You can say, 175 times 7 or you can say 175 plus 175 seven times.

Teacher: Why are you multiplying?

Bokang: Ehheee (laughing at Maria to think that I’m assuming she is wrong).

Teacher: Don’t say ehhhee, what operation did you use yourself?

Maria: Because (she read the question out loud).

Teacher: What does that mean?

Maria: They mean how many in each box (she repeated the question in way and realised she does not understand the question).

Teacher: (I rephrased the question to) It means you are having 7 boxes of bananas; their mass is 175 kg so what will be the mass of one box.

Maria: Ohh sir, its divide.

Teacher: Why?

Maria: Because you must divide to find how many bananas in a box,

In group B, learners were thinking of 175 kg as the mass of one box of bananas, then multiplying it by seven or adding 175 seven times to give them an answer to the question. Thus, Maria was excited to have explained how they would attempt the question. She was also showing me groups of seven bananas using her hands. However, Maria could not clearly explain why she was using multiplication or addition. Instead, she was reading the question again to answer my question. This also indicates that learners created their own questions without paying attention to the details of the original question.

Furthermore, Maria did not take time to tap her fingers and answered ‘divide’ after I rephrased the question. I did not comment or react after she said divide. I only allowed them to continue working and discussing in a group.

Thereafter, I moved around the groups to see another group's mathematical operations. I observed that learners in the groups were multiplying 175 kg by seven in their calculations. The learners were comfortable using multiplication because they were involved in using their fingers to calculate; some had even expanded 175 to apply the multiplication. One learner raised a hand to ask me to come to their group while I was still moving around. Therefore, I decided to go to the group and interact with them.

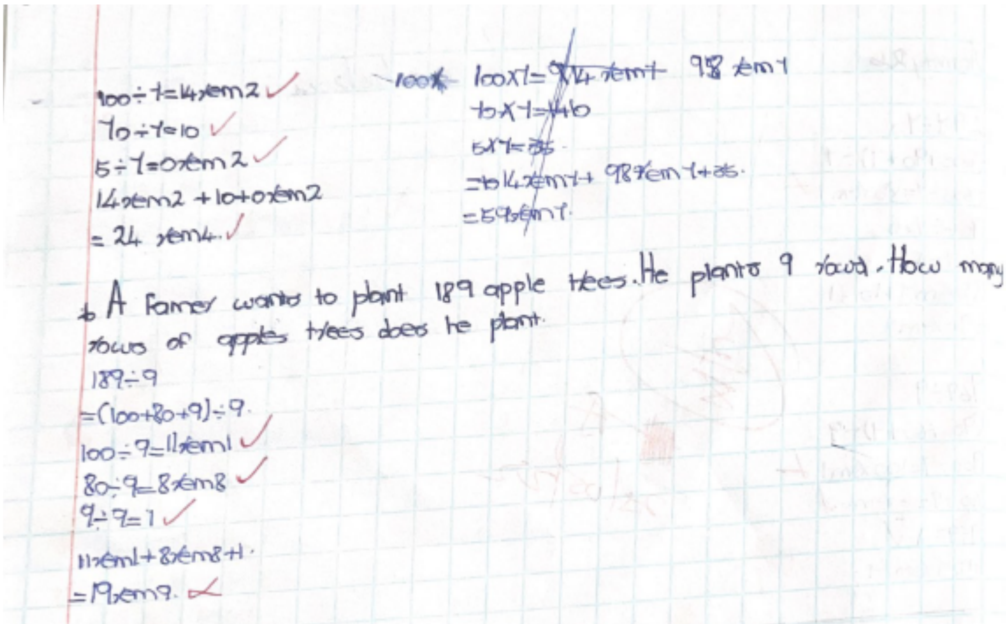


Figure 3.2: Calculations

Teacher: Why are you using division?

Tshego: We are not using division; we are using multiplication.

Rendani: We are making 175, seven times.

Teacher: (I realised that they are doing the same thing as Maria did. Therefore, I tried to immediately correct it) But look, the combination of the 7 boxes has a mass of 175 kg so the question wants the mass of one box.

Rendani: Oh woo (Indicating that she understands).

Tshego: Sir. So, we must count, say 175 until 7.

Teacher: How?

Tshego: So, we count 175 plus 175 until 7.

Rendani: (She laughed at Tshego's explanation) Sir we are still calculating.

In this group, Rendani quickly realised that her interpretation of multiplying was wrong because after she uttered 'oh woo', she started working on the question using division. However, Tshego was still in the dark because she still interpreted the question as multiplication after my explanation. Furthermore, her eyes were wide open while stuttering, indicating that she was not sure of what she was saying.

Moving on through the groups, I found that another group of learners had used addition to add 175 and seven. However, when I asked why they did this, they just looked at each other as a group and did not provide any explanation. Furthermore, others in the group said that they do not know whether they should use multiplication or division. But in their calculations they used multiplication. Seemingly most of the learners opted for multiplication but could not explain why they used it.

I then decided to do a whole class discussion because some groups had already moved to the second question. I presented similar questions so that it made a little bit of sense to the learners who were left behind.

Question: If we are having 3 packets of Simba and their mass is 6 g, what would be the mass of each packet?

The whole class answered “2 g” I then asked how they attempted the question or how they would attempt the question of 175 kg of bananas.

Rendani immediately raised up here hand.

Teacher: Rendani.

Rendani: 25 kilograms.

Teacher: How did you get 25?

Rendani: First I said 15 seven times and it did not give me the answer, until I got to 25 (which made sense to the learners).

When I asked the whole class how they would move from 175 kg to get 25 kg, they, fortunately, responded positively and indicated that they had used division. Learners could interpret and solve the question after I gave them a simpler question to attempt verbally.

Analysis of teaching episode 3: First classroom interaction

In the first classroom interactions, learners thought of multiplication when answering the question. Maria, among other learners, read the question again when asked why they were using multiplication. However, most of them thought the number of bananas altogether, if they are 175 in one box. The learners created their own questions instead of trying to understand the one that was asked. The learners managed to understand the question after I have gave them an example with packets of Simba (chips).

Learners managed to relate to the example of Simba to come up with the use of division operations to answer the question.

Conversely, some learners tended to give answers they could explain, while others tended to give up on the questions they did not understand. One group of learners decided to convince themselves that they needed to add 175 and seven to get the number of bananas in each box. The learners transferred knowledge of previous activities irrelevantly to a new situation. They thought that if they used addition and subtraction previously, it would be the case with the new situation. Therefore, the learners quickly generalised their learning when learning mathematical concepts, before they could attach meaning.

The above means that learners made use incorrect mathematical operations when dealing with word problems, however, in their minds they had the correct interpretation of the question. Furthermore, learners made much more understanding of word problems when they were engaged in simpler mathematical problems. This allowed them to connect bits of information to solve a question they deemed abstract and transferred knowledge to a generalisation of the idea.

Second classroom interactions

An interview with Tiny

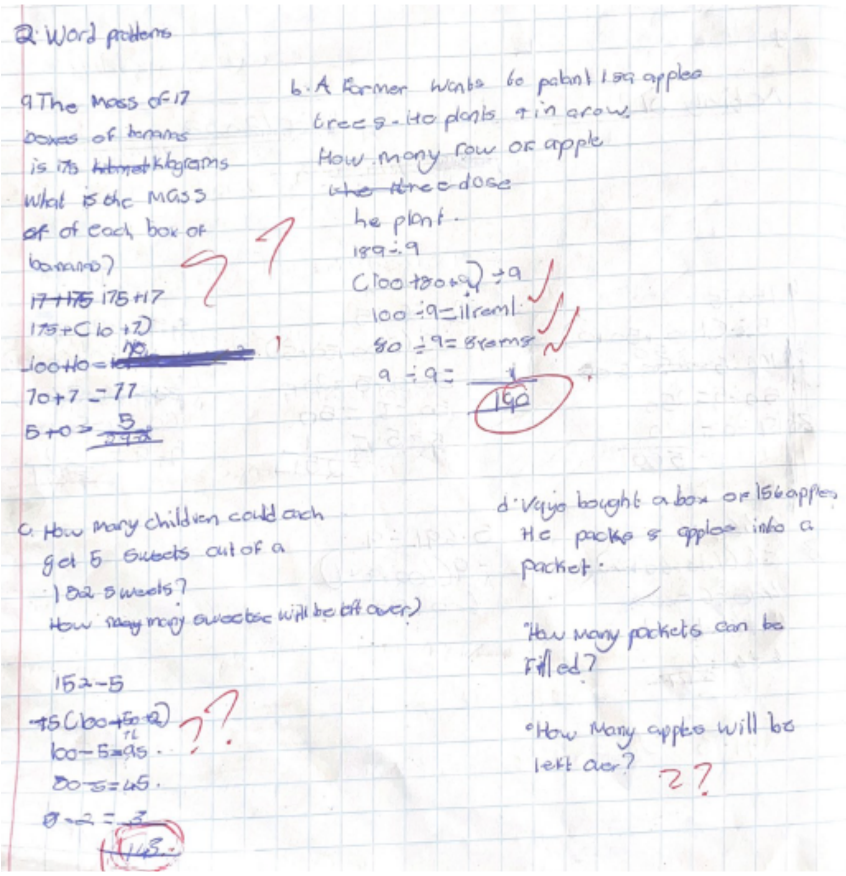


Figure 3.3: Calculations

Teacher: How did you attempt question b? (She had showed correct working, but her answer was 190 in the end).

Tiny: Aker sir, (Pointing in her book to show me) I saw 100 divided by 9 is 11 remainder 1 and 80 divide by 9 is 8 remainder 8 and so when you add these number you get 190?

Teacher: Haa, so when you add these answers how do you get 190?

Tiny: Aker sir I added 100, 80 and 8 and the remainders.

Teacher: Should we add those or the answers?

Tiny: Ohh (Indicating that she understands where she went wrong).

Tiny was confident in giving her explanations. She made use of her hands to express her ideas. Furthermore, she was audible, which showed her confidence in her answer.

Tiny successfully interpreted the question. However, she made the mistake of adding the expanded numbers to get the answer instead of the answers she got when she divided the expanded numbers by nine.

I, therefore, continued talking to Tiny about question c as follows.

Teacher: How did you attempt question (c) (After she read the question again).

Tiny: I used minus (subtraction).

Teacher: Why?

Tiny: Because they want to know how many sweets each child will get and how many will be left over.

At this point, Tiny read the question again to respond to my question.

Teacher: So how many sweets will each child get?

Tiny: Will get 95 and … .

Teacher: No remember they want to know how many children will each get 5 sweets, so if its 95, will all 95 children get 5 sweets from 152?

And this 143, what does it represent?

Tiny: It represents, how many sweets are left.

Tiny: (She started counting using her fingers, but I could see that she is confused.)

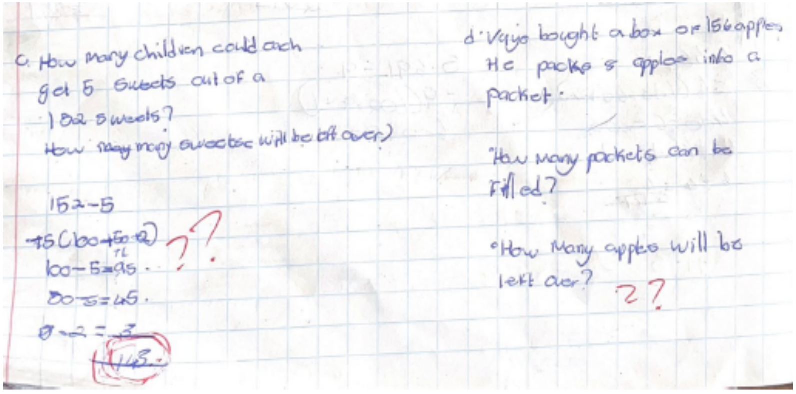
Teacher: Okay think more about it, then we will talk.

Tiny kept reading the question again and again to explain why she used subtraction. She was not focusing on her own understanding of the question. However, she opened her eyes wide and held her head when I rephrased the question for her. After that, she managed to see that her answer was wrong after she saw that 95 children could not be accommodated by 152 sweets if each got 5 sweets. Furthermore, she wanted to recalculate using her fingers. Therefore, I asked her to talk to her group about it.

Hope was next in the queue for the interview; hence I had the following conversation with her.

An Interview with Hope

Hope did well with question a, but did question b the same way that Tiny did. Therefore, I wanted to hear her views and for her to realise that she did question b incorrectly by questioning her.



Teacher: If we are having 10 sweets, how many children would get 3 sweets?

Hope: 3 kids.

Teacher: How?

Hope: Because they are saying 3 sweets sir.

Teacher: How about 4 sweets, if we having 10 sweets?

Hope: okay with 3 children?

Teacher: No, I am looking for number of children.

Hope: 4 (after thinking for a minute) no, 2

Teacher: Why?

Hope: Because there will be remainders.

Teacher: So, let us go back to question (c), how will you work it out?

Hope: Counting in 5s until I get to 152.

Teacher: Okay you will work it out.

The questions I asked Hope at the beginning assisted her to interpret the question.

I, therefore, moved to Karabo, Thasha and Phomelelo for interviews. All kept quiet when I asked them questions regarding their answers. Karabo and Thasha managed to interpret question b and used the multiplication operation on question c. However, Karabo bit his finger when I asked him to explain why he used multiplication and did know why he used multiplication. Similarly, Phomelelo interpreted question b correctly, but she could not properly execute division operations. Furthermore, Phomelelo used subtraction in question c to get 150 children, which would not be correct, but she was quiet when I asked her why she used multiplication.

Thereafter, I moved around the classroom groups to observe how learners addressed the two questions. I realised that most learners used the multiplication operation on question c without knowing why. On the other hand, only a few of them managed to interpret the questions correctly. Dineo was one of the learners who managed to interpret the questions correctly.

Analysis of second classroom interactions

Learners make use of the questions to answer interview questions without attaching meaning. Tiny, rephrased question c to answer the question: 'Why did you use subtraction on this question?' She said she used subtraction because she wanted to know how many sweets each child would get and how many would be left over. Therefore, learners use mathematical operations because they know that with word questions, they will need to use one and then not attach meaning to it.

Similarly, Hope used mathematical operations without attaching meaning. After breaking the question down into simpler questions, she only managed to make sense of question c. However, in the process, she did not make sense of the simpler questions I asked. She answered, '3 kids' to the question: 'How many children could each get 3 sweets out of 10 sweets?' and then answered '4 kids' for the question: 'How many children could each get 4 sweets out of 10 sweets?'. Hope's answers showed that she was only answering the question by looking at the number of sweets each child could get and making that the number of children. Eventually. Hope managed to make sense of the answer herself after she decided to think thoroughly about the question.

On the other hand, some learners in Grade 4 could interpret word questions but find found it difficult to explain their answers when asked why they used certain mathematical operations. Thasha and Karabo were able to interpret question b, while Phomelelo managed to interpret question c, but none of the learners could say a word about why they approached the questions in the way that they did. Similarly, they still could not say anything about the questions they misinterpreted.

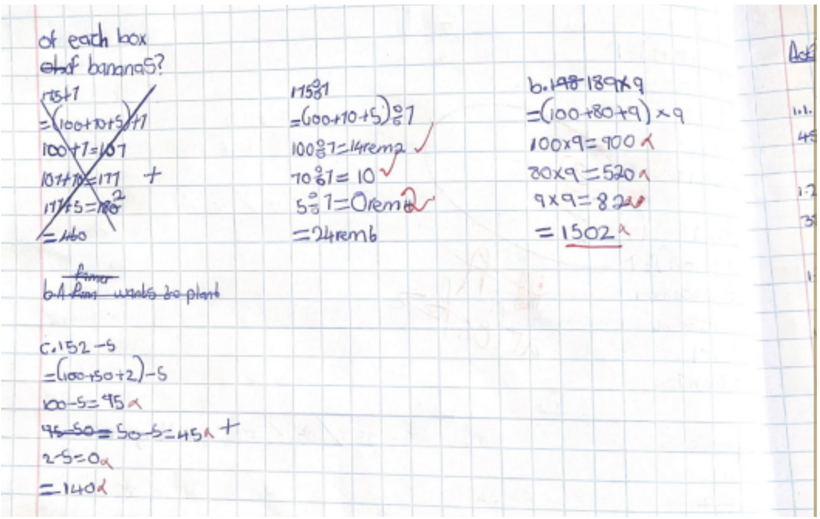


Figure 3.4: Thasha's response

In conclusion, learners who were able to interpret word problems were likely to be successful in solving the question; however, only a few learners could interpret the word problem correctly. Furthermore, learners who did not interpret the word problems correctly became confused when solving the questions and tended to use any mathematical operation. These learners needed to be taken through simpler questions to comprehend the word problem. Learners failed to comprehend word problems that involved bigger numbers because they focussed on the numbers rather than making meaning of the problem (Verschaffel, Depaepe & Van Dooren 2020).

Analysis of teaching experiment 3

Some learners could interpret and solve word problem questions, while others managed to solve some of the word problem questions. The utterances of learners who managed to solve the questions were characterised by the learners reading the questions for understanding. The learners repeatedly asked about the meaning of words like 'discount' to show that they wanted to understand the question. Furthermore, these learners used their fingers to help them calculate and managed to interpret questions involving any mathematical operation. After that, the learners showed great development as they could interpret questions that required simple interpretations to complicated questions.

On the other hand, learners did not show any growth in mathematical development when they answered mathematical questions without understanding the question. Learners in the classroom would only read the questions once and start answering the questions using incorrect mathematical operations. Furthermore, learners covered their answers with their hands. They were staring at each other and resting their chins on their hands. Learners were covering their answers so that I did not question their calculations. They were staring at each other. They needed someone to answer the questions in the group and held their hands on their chin because they could not interpret the question.

4.3 RETROSPECTIVE ANALYSIS

The retrospective analysis involved going through the video recordings again at a microscopic level to analyse learners' thinking on mathematical concepts (Steffe & Thompson, 2000). Therefore, I watched the video recordings again to analyse how learners used mental operations and cognitive and mathematical play. Furthermore, I analysed how these learners interacted and developed schemas when learning mathematical concepts. However, the focus is on the utterances the learners portray when learning mathematical concepts. Therefore, I use the theory of Thelen and Smith (1995) as a lens in this analysis. The theory consists of three tenets: thought as an in-moment event, thought as being open to a continually changing world and thought as cognition that is not stationary.

Application of concepts in isolation

Learners used utterances well when learning mathematical concepts, indicating their success and failure in dealing with mathematical questions. These utterances were portrayed when learners interacted with themselves, the group and the teacher during teaching and learning. The learners' immediate utterances included confusion shown on their faces by looking at each other to see who gets to write the given activity (activity 1 of teaching episode 1 in teaching experiment 1). However, these learners were encouraged by merely looking at other groups having an air of purposeful activity. In the Grade 4 classrooms, the groups of learners usually did not want to lag behind the other groups. Hence, they compared themselves to other groups of learners who seemed to show great working spirit. Learners in the early stages of learning compete with their peers in activities (Vasc & Ionescu, 2013). As a result, the learners had an air of purposeful activity in their respective groups.

In the teaching experiments, learners portrayed positive utterances, such as smiling, to indicate that they could make sense of the activities involving extending numeric patterns. The learners also used their figures to count when extending the patterns, indicating that they had an idea of what they were doing. These were the immediate utterances they portrayed when dealing with patterns involving simple counting forward and backwards, and patterns involving repetitions. Furthermore, they described how the pattern grew successfully as they were confident that their answers were correct. As a result, there was an observable cognition of how the body worked together with the mind to successfully answer the questions.

Learners mostly operated under the ‘thought as an in-moment event’ tenet, whereby they managed to use their perceptual memories to extend and describe numeric patterns involving repetitions and constant terms, and portrayed utterances as such. Similarly, the learners portrayed positive utterances, such as making motions with their heads and using fingers, to show their understanding. However, these learners mostly struggled with patterns involving constant difference instead of constant ratio, while other groups realised that they needed to use the constant ratio. Therefore, learners indicated an observable cognition as they engaged in learning mathematical concepts; however, most of their cognition is rooted in previous knowledge that does not apply to new situations.

Learners who used the previous knowledge of extending patterns involving repetitions and constant difference to patterns involving constant ratios incorrectly were confident in their responses and portrayed positive utterances. However, they did not have the knowledge growth that would allow them to solve problems involving any other pattern, like exponential patterns and patterns that would require them to add different numbers as the pattern proceeded. Similarly, these learners lacked knowledge of interpreting geometric patterns. The learners used hand gestures to correctly interpret simpler geometric patterns, like making a visual drawing to show what the next pattern would look like. However, they tended to fail to use the same drawing to think about how pattern 25 could be made, which made it difficult for them to generalise the pattern.

Furthermore, the same problem persisted when these learners solved word problems using mathematical operations. The learners used the mathematical operations without any understanding and portrayed utterances such as using fingers, rolling eyes, shaking heads and biting fingers. In other words, learners were showing a feeling of being confused about interpreting the word problems.

Learners portrayed either positive or negative utterances when learning mathematical concepts. These utterances indicated the failure of the learners to do mathematical activities. Thus, there were bits of cognition as the learners use the utterances. The positive utterances showed learners engaged in the concept with an idea in mind. However, there was no observable growth in cognition that enabled learners to deal with abstract mathematical concepts. As a result, learners portrayed negative utterances to show their inability to deal with abstract mathematical concepts.

Making use of an idea with a limited understanding

The teaching experiments show the significance of learners' limited understanding of mathematical concepts. Learners portrayed utterances to show that their understanding of mathematical concepts was limited. As a result, they failed to use schemas to apply their mathematical knowledge when doing activities. Learners' utterances included using fingers to visualise patterns, using lines to show how they understood word problems involving mathematical operations and using their heads and hands to show their confusion.

Learners used their figures to visualise patterns to make it possible to interpret the patterns. However, they faced problems when they had to calculate their answers. As a result, they faced problems with mathematical operations, mainly the use of addition and multiplication, when they dealt with patterns they did not see as drawings. This problem prevented learners from completing their interpretations of the patterns. Similarly, learners made use of lines to extend patterns. However, it reached a point where they could not draw the pattern because it involved large numbers, which required them to use generalisation. Therefore, learners ended up shaking their heads because they saw how the patterns grew but could not figure out which mathematical operations they had use to come up with an answer. Hence, the growth of their cognition became stunted as they had to count to come up with several matchsticks for a given pattern.

On the other hand, the teaching experiments showed how learners partially used mathematical operations to solve mathematical problems. Learners pointed out words like ‘altogether’ and ‘difference’, which made it possible for them to deal with word problems involving addition and subtraction. However, with multiplication and division problems, learners would always just choose any of the mathematical operations for any given reason. As a result, learners portrayed utterances such as opening their eyes wide and scratching their heads to show that they did not understand why they used the mathematical operations. Therefore, learners showed partial growth in their mathematical cognition, which prevents them from using their schemas to solve problems that involve making use of more than mathematical operations at a time.

Consequently, learners just did not do anything when they encounter problems requiring them to use more than one mathematical operation. Thus, they operated under the ‘thought as being open to a continually changing environment’ tenet as there was an observable cognition that enabled them to portray some positive utterances as they engaged in mathematical activities. However, their use of schemas to enable them to solve abstract mathematical concepts was limited.

Making use of schemas to solve abstract mathematical concepts.

The teaching experiment showed concrete evidence of learners portraying utterances that involve the growth of mathematical cognition of learners. Knowledge growth indicates how learners deal with interpretation questions that involve patterns requiring them to use constant ratios and exponential patterns, and other patterns that do not require them to make use of constant ratios. Furthermore, the learners could interpret patterns, such that they could even tell a pattern number given for a given number of matchsticks for that pattern. The learners successfully interpreted word problems requiring mathematical operations when the question required more than one mathematical operation. Furthermore, the learners carried out the use of mathematical operations successfully. However, the groups had marginalised learners who had to take directives from the dominant learners. The ideas of the dominant learners made it possible for the marginalised learners to also understand the concept being tackled.

Consequently, learners used schemas to deal with abstract mathematical concepts and indicated utterances such as positive hand gazing and smiles when explaining their answers. The learners used schemas to connect concepts and manage abstract mathematical concepts. The use of schemas shows learners' growth in understanding mathematical concepts and how these learners transcended from understanding low-order questions to understanding high-order questions. In the process of doing activities, learners portrayed an air of purposive activity and were proactive. As a result, the learners are engaged in the 'thought as a cognition that is not stationary' tenet because they could use schemas to deal with abstract mathematical concepts.

Cognition as an individual attribute

Learners were afraid to show their understanding of mathematical concepts as individuals because they mostly rely on other learners in groups. Therefore, learners portrayed utterances such as being shy, while some keep quiet, when asked to explain their answers. However, they tended to say something if a fellow groupmate had already said it. The learners mostly engaged in the 'thought as being open to continually changing world' tenet when they succeeded in using perceptual memories to solving mathematical problems. On the other hand, there is a mismatch between the gain of mathematical cognition and the utterances they portrayed. As a result, they ended up not being able to make use of schemas to deal with abstract mathematical problems.

Some learners identified themselves as smart when dealing with mathematical problems. As a result, they portrayed positive utterances when asked questions individually. These utterances included using fingers and drawings to demonstrate their understanding of the concept. Therefore, they indicated growth in mathematical concepts, which shows how they use schemas to deal with abstract mathematical problems.

4.4 SUMMARY

In this chapter, I presented the results of teaching experiments. I expanded each teaching experiment in terms of teaching episodes. Thus, in each teaching episode, I presented a discussion on classroom interactions as the results and preliminary analysis of the teaching episodes. Furthermore, I have presented the retrospective analysis, the overall analysis of the results using constructs of the dynamic system theory of cognitive development by Thelen and Smith (1995).

CHAPTER 5: DISCUSSION

5.1 INTRODUCTION

In this chapter, I state the research questions and summarise the major findings of the study. Furthermore, I identify relationships in the data from the analysis and contextualise the findings by further analysing the interactions of the teaching experiments to provide interpretations of the results; consequently, the research questions were answered. After that, I present the study's limitations by evaluating weaknesses within the research design that influenced the study's outcomes. Furthermore, recommendations of the study for further research on the topic are presented.

5.2 RESEARCH PROBLEM AND RESEARCH QUESTIONS

The study's research problem was to investigate Grade 4 learners' enactment of utterances in a mathematics classroom discourse. Learners at the Grade 4 level use utterances when learning (Vasc & Ionescu, 2013). These learners portray utterances to show their ability and inability to link ideas when dealing with mathematical problems (Askew et al*.,* 2014). Furthermore, learners express their ideas through the utterances they portray in the mathematical classrooms when learning.

The research questions for the study were as follows.

- What utterances do Grade 4 learners portray in a mathematics classroom discourse?
- What do the utterances reveal about learners' understanding of mathematical concepts?

The first research question focused mainly on the utterances that learners in the Grade 4 classroom portray when learning mathematical concepts. These are all the negative and positive utterances that influenced their learning. On the other hand, the second research question focused on the interpretations that I made about how these utterances influence learners' understanding of mathematical concepts.

5.3 SUMMARY OF KEY FINDINGS

Learners portrayed both positive and negative utterances when learning mathematical concepts. The positive utterances included learners' smiling when doing an activity, confidently tapping fingers when they realised an answer, being active in their groups, showing visuals using their hands, using their hands to make explanations, moving their heads in an upwards and downwards direction and uttering 'oh woo' to show understanding of mathematical concepts. Furthermore, learners portrayed negative utterances such as opening their hands with the palm facing up to show that they did not know the answer to a question, moving their heads sideways, opening their eyes wide to show confusion, moving their shoulders in an upward and downward direction, closing their mouths when responding to verbal questions and uttering 'uhm' to show their misunderstanding. These utterances influenced learners' learning of mathematical concepts. Furthermore, the utterances also revealed important constructs about the learning of Grade 4 learners.

In the Grade 4 classroom, learners were applying prior knowledge irrelevantly to new mathematical concepts. They also believed in the first interpretation of questions and were comfortable with questions that included visuals. The language used when solving word problems tended to be complex and they did not read questions for understanding. These prevented learners from being 'students of mathematics' because they would rely on me for guidance during group interactions. Therefore, learners did not take the initiative to question their own thinking when learning.

5.4 INTERPRETATION OF RESULTS

5.4.1 Utterances when learners connect mathematical concepts.

Learners in the Grade 4 classroom portrayed positive and negative utterances as they learnt mathematical concepts, regardless of misinterpreting a mathematical concept. Learners portrayed positive utterances such as smiling, using hand gestures to make sense of numeric patterns, and moving heads in upward and downward directions to show that the pattern made sense when they applied a concept incorrectly. The learners made use of the constant differences in a pattern that required the addition of different numbers. However, these learners became excited and believed that they had attempted the question correctly.

On the other hand, learners portrayed negative utterances when their answers were being questioned. Learners portrayed utterances such as resting their chins on their hands, opening their eyes wide and uttering the word ‘uhm’ with a feeling of disappointment. However, after the questioning, learners immediately uttered ‘oh woo’ and reworked their responses. This indicated learners’ ability to take criticism positively and keep a positive working spirit. Furthermore, it allowed learners to develop an understanding of interpreting patterns from repetitions and constant differences to patterns that do not require repetitions and constant differences.

Learners portrayed positive and negative utterances to indicate how they developed an understanding of mathematical concepts. The positive utterances indicated that learners could interact with the subject matter by themselves to solve mathematical questions (see interactions with groups A and B). The utterances of these learners were characterised by the learners actively participating in their groups, asking each other questions, using their hands effectively to count and calculate, and having learners move their heads in an upward-downward direction. On the other hand, learners portrayed negative utterances but took criticism positively and applied relevant contexts to mathematical concepts.

5.4.2 Utterances when learners reach conclusions.

Learners in the Grade 4 classroom portrayed negative utterances such as shaking their heads, folding their arms, uttering ‘uhm’ and opening their hands wide and uttering ‘we do not understand the question’ when they encountered word problem questions. The learners read word questions once and immediately attempted the questions without questioning each other on the meaning of the question. This caused learners to not portray any development in interpreting word problem questions because there was no reasoning based on their choice of mathematical operation. Furthermore, learners could not explain when asked about their mathematical operations.

On the other hand, there was evident growth in how learners interpreted word questions. The learners spotted words that were conclusive to the mathematical operation they should use, words like ‘altogether’ and ‘difference’ that they interpreted as addition and subtraction, respectively. Furthermore, learners would visualise the use of multiplication and division by restructuring questions. For example, Disebo made use of fingers to represent number rows and started counting in multiples of nine to demonstrate her understanding of word problems.

5.4.3 Utterances when learners recognise shapes.

Learners portrayed positive utterances when they were working on geometric patterns. The utterances of the learners were characterised by the learners visualising the next shape before drawing, using fingers to count several matchsticks, and moving heads in an upward–downward movement with a smile. Furthermore, learners uttered 'oh woo' in their respective groups to show their understanding of the mathematical concept. This shows that learners were able to recognise and interpret geometric patterns. However, learners encountered a challenge when interpreting patterns in teaching experiment 2, teaching episode 3.

Learners in teaching episode 3 of teaching experiment 2 were engaged in the rigorous interpretation of the geometric patterns. They were engaged in visualising what the next pattern would look like and using their hands to count the number of squares. Furthermore, learners were engaged in observing the difference between the respective geometric shapes to study the number of squares the next pattern would have. As a result, there was an observable development in how learners used simpler geometric patterns to complex geometric patterns.

5.4.4 Utterances when learners encounter complex words.

The utterances of learners in the classroom were characterised by learners being quiet, staring at each other and folding their arms to show that they do not understand word problem questions. This was the case because learners were unfamiliar with some of the words used in word questions. Learners could not interpret word problems because they struggled with words like 'discount' and phrases such as 'how many more or less'. As a result, learners were confused but could not say out loud that the question was confusing them.

On the other hand, some learners were familiar with the words and this was characterised by the use of correct mathematical operations to solve word problem questions. The utterances of these learners were characterised by learners using fingers to calculate and create imaginary visuals, making drawings to interpret the questions and bending their heads sideways to show that they were thinking about the word problems. There was a development of mathematical knowledge as learners were interpreting questions involving addition and subtraction to interpreting questions involving multiplication and division.

5.4.5 Utterances when learners read questions.

Learners' utterances were portrayed by learners making positive utterances when they were working on word problem questions. Learners seemed to simply calculate without having a correct interpretation of the questions. Therefore, used any mathematical operation without understanding because they only stare when asked why. This shows that learners were not reading the word problems for understanding. They were simply using the numbers in their calculations. These learners would show with their hands that they did not know why they used a particular mathematical operation and eventually shake their heads to indicate more confusion. As a result, there was no significant development in the concept.

5.5 LIMITATIONS OF THE STUDY

In the study, I encountered restrictions that posed some weaknesses. Learners were working in groups. Therefore, some of the learners absented themselves from discussions with their group members. As a result, it became challenging to observe and record learners' interactions while trying to encourage everyone to engage with their group members. These learners also have important mathematical utterances that could have been important for the study. Furthermore, the study took place at a small school with only 47 Grade 4 learners. As a result, I interacted with all the learners, but the sample was still small, affecting the study's outcomes.

5.6 RECOMMENDATIONS

From the key findings, I recommend that future studies focus on how learners use utterances to develop mathematical knowledge through different phases of primary education. However, learners should be engaged in mathematical classroom discourse, as this encourages learners to interact with each other and become autonomous thinkers. On the other hand, teachers should observe learners' utterances in their teaching because these utterances carry meaning about how they understand mathematical concepts. Furthermore, teachers should also engage learners in more word questions to make them become familiar with mathematical words and be able to interpret word problems.

5.7 SUMMARY

In this chapter, I have presented a summary of the research problem, research questions and key findings. After that, I presented the interpretation of the results in responding to the research questions. Furthermore, I discussed the study's limitations and provided recommendations for future research on embodied cognition.

REFERENCES

Abrahamson, D. (2017). The SAGE Encyclopedia of Out-of-School Learning. Embodiment and mathematics learning. *SAGE Publications, Inc,* 248-252

Abrahamson, D., & Lindgren, R. (2014). Embodiment and embodied design. U*C Berkely*

Abrahamson, D., & Trninic, D. (2015). Bringing forth mathematical concepts: Signifying sensorimotor enactment in fields of promoted action. ZDM, 47(2), 295-306.

Alibali, M. W., & Nathan, M. J. (2012). Embodiment in mathematics teaching and learning: Evidence from learners' and teachers' gestures. *Journal of the learning sciences*, *21*(2), 247-286.

Anderson, V. (2017). Criteria for evaluating qualitative research. *Human Resource Development Quarterly*, 1-9.

Applefield, J. M., Huber, R., & Moallem, M. (2000). Constructivism in theory and practice: Toward a better understanding. *The High School Journal*, *84*(2), 35-53.

Arani, M (2017). Shared teaching culture in different forms: a comparison of expert and novice teachers' practices. Educational Research for Policy and Practice 16(3)

Arifin, S. R. M. (2018). Ethical considerations in qualitative study. *International journal of care scholars*, *1*(2), 30-33.

Askew, M., Abdulhamid, L., & Mathews, C. (2014). Embodiment in Teaching and Learning Early Counting: Grounding Metaphors. North American Chapter of the International Group for the Psychology of Mathematics Education.

Benedek, A. (2018). Embodied Conceptions of Mathematical Understanding in the Twentieth Century: the emergence of Zoltan P. Dienes's principles and their origin. Research Centre for the Humanities, Hungarian Academy of Sciences, Budapest, Hungary.

Bitsch, V. (2005). Qualitative research: A grounded theory example and evaluation criteria. *Journal of agribusiness*, *23*(345-2016-15096), 75-91.

Bjørnebye, M. (2019, February). Pre-schoolers' ability to synchronise multiple representations of numerosity in embodiment of a counting-on-strategy.

In Eleventh Congress of the European Society for Research in Mathematics Education (CERME11) (No. 04). Freudenthal Group; Freudenthal Institute; ERME.

Burte, H., Gardony, A. L., Hutton, A., & Taylor, H. A. (2017). Think3d!: Improving mathematics learning through embodied spatial training. *Cognitive Research: Principles and Implications*, *2*(1), 1-18.

Cangelosi, A., & Stramandinoli, F. (2018). A review of abstract concept learning in embodied agents and robots. *Philosophical Transactions of the Royal Society B: Biological Sciences*, *373*(1752), 20170131.

Casano, J., Tee, H., Agapito, J., Arroyo, I., & Rodrigo, M. M. T. (2016, December). Migration and evaluation of a framework for developing embodied cognition learning games. In *Proceedings of the 3rd Asia-Europe symposium on simulation & serious gaming* (pp. 199-203).

Castro-Alonso, J. C., Ayres, P., & Paas, F. (2015). The potential of embodied cognition to improve STEAM instructional dynamic visualizations. In *Emerging technologies for STEAM education* (pp. 113-136). Springer, Cham.

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational researcher*, *32*(1), 9-13.

Creswell, J. W. (2009). Mapping the field of mixed methods research. *Journal of mixed methods research*, *3*(2), 95-108.

Creswell, J. W., & Miller, D. L. (2000). Determining validity in qualitative inquiry. *Theory into practice*, *39*(3), 124-130.

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, *93*, 19-33.

De Bot, K., Lowie, W., & Verspoor, M. (2007). A dynamic systems theory approach to second language acquisition. *Bilingualism: Language and* cognition, *10*(1), 7-21.

DeSutter, D., & Stieff, M. (2017). Teaching students to think spatially through embodied actions: Design principles for learning environments in science, technology, engineering, and mathematics. *Cognitive research: principles and implications*, *2*(1), 1-20.

Devers, K. J., & Frankel, R. M. (2000). Study design in qualitative research--2: Sampling and data collection strategies. *Education for health*, *13*(2), 263.

Dilshad, R. M., & Latif, M. I. (2013). Focus group interview as a tool for qualitative research: An analysis. *Pakistan Journal of Social Sciences (PJSS)*, *33*(1).

Dove, G. (2016). Three symbol ungrounding problems: Abstract concepts and the future of embodied cognition. *Psychonomic bulletin & review*, *23*(4), 1109-1121.

Flick, U. (2004). Triangulation in qualitative research. *A companion to qualitative research*, *3*, 178-183.

Freeman, D. L., & Cameron, L. (2008). Research methodology on language development from a complex systems perspective. The modern language journal, 92(2), 200-213.

Goldin-Meadow, S. (2015). From action to abstraction: Gesture as a mechanism of change. *Developmental review*, *38*, 167-184.

Guba, E. G., & Lincoln, Y. S. (1994). Competing paradigms in qualitative research. *Handbook of qualitative research*, *2*(163-194), 105.

Gullberg, M. (2006). Some reasons for studying gesture and second language acquisition (Hommage à Adam Kendon).

Güner, P., & Uygun, T. (2020). Examining students' mathematical understanding of patterns by Pirie-Kieren model. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *35*(3), 644-661.

Healy, K. (2000). Social work practices: Contemporary perspectives on change.

Hmelo-Silver, C. E. (2004). Problem-based learning: What and how do students learn?. *Educational psychology review*, *16*, 235-266.

Kivunja, C., & Kuyini, A. B. (2017). Understanding and applying research paradigms in educational contexts. *International Journal of higher education*, *6*(5), 26-41.

Kosmas, P., & Zaphiris, P. (2018). Embodied cognition and its implications in education: An overview of recent literature. *International Journal of Educational and Pedagogical Sciences*, *12*(7), 930-936.

Kosmas, P., Ioannou, A., & Zaphiris, P. (2019). Implementing embodied learning in the classroom: Effects on children's memory and language skills. *Educational Media International*, *56*(1), 59-74.

L. Haven, T., & Van Grootel, D. L. (2019). Preregistering qualitative research. *Accountability in research*, *26*(3), 229-244.

Landy, D., Trninic, D., Soylu, F., Kehoe, J., & Fishwick, P. (2014). The implications of embodiment for mathematics and computing education. In *Proceedings of the Annual Meeting of the Cognitive Science Society* (Vol. 36, No. 36).

Lestari, L., & Surya, E. (2017). The effectiveness of realistic mathematics education approach on ability of students' mathematical concept understanding. *International Journal of Sciences: Basic and Applied Research (IJSBAR)*, *34*(1), 91-100.

Liamputtong, P. (2009). Qualitative data analysis: conceptual and practical considerations. *Health promotion journal of Australia*, *20*(2), 133-139.

Lim, V. K., Wilson, A. J., Hamm, J. P., Phillips, N., Iwabuchi, S. J., Corballis, M. C., ... & Thomas, M. O. (2009). Semantic processing of mathematical gestures. *Brain and cognition*, *71*(3), 306-312.

Liu, S. (2020). *Intellectual emancipation and embodiment in early mathematics learning* (Master's thesis, Werklund School of Education).

Mahon, B. Z. (2015). What is embodied about cognition?. *Language, cognition and neuroscience*, *30*(4), 420-429.

Mandal, P. C. (2018). Qualitative data analysis and its nature: debates and discussion. *International Journal of Society Systems Science*, *10*(2), 148-158.

Maree, K. (2011) . First step in research. 1064 Arcadia street, Hatfield, Pretoria:

Margutti, P. (2010). On designedly incomplete utterances: What counts as learning for teachers and students in primary classroom interaction. *Research on Language and Social Interaction*, *43*(4), 315-345.

Maxwell, J. A. (2012). *Qualitative research design: An interactive approach*. Sage publications.

McNeil, N. M., Alibali, M. W., & Evans, J. L. (2000). The role of gesture in children's comprehension of spoken language: Now they need it, now they don't. *Journal of Nonverbal Behavior*, *24*, 131-150.

Menary, R. (2015). Mathematical cognition: A case of enculturation. T. Metzinger & J. M. Windt (Eds). *Open MIND:* 25(T). Frankfurt am Main: MIND Group

Merriam, S. B., & Tisdell, E. J. (2015). *Qualitative research: A guide to design and implementation*. John Wiley & Sons.

Mills, J., Bonner, A., & Francis, K. (2006). The development of constructivist grounded theory. *International journal of qualitative methods*, *5*(1), 25-35.

Munson, J., & Dyer, E. (2020). Collaborative sensemaking through side-by-side coaching: Examining in-the-moment discursive reasoning opportunities for teachers and coaches.

Nathan, M. J., & Walkington, C. (2017). Grounded and embodied mathematical cognition: Promoting mathematical insight and proof using action and language. *Cognitive research: principles and implications*, *2*(1), 1-20.

Nathan, M. J., Williams-Pierce, C., Walkington, C., Abrahamson, D., Ottmar, E., Soto, H., & Alibali, M. W. (2021). Embodied Design for Mathematical Imagination and Cognition.

Nemirovsky, R., & Ferrara, F. (2009). Mathematical imagination and embodied cognition. *Educational Studies in Mathematics*, *70*(2), 159-174.

Nguyen, D. J., & Larson, J. B. (2015). Don’t forget about the body: Exploring the curricular possibilities of embodied pedagogy. *Innovative Higher Education*, *40*(4), 331-344.

Noble, T., Nemirovsky, R., Wright, T., & Tierney, C. (2001). Experiencing change: The mathematics of change in multiple environments. Journal for Research in Mathematics Education, 32(1), 85-108.

Ottmar, E. R., Walkington, C., Abrahamson, D., Nathan, M. J., Harrison, A., & Smith, C. (2019). Embodied Mathematical Imagination and Cognition (EMIC) Working Group. *North American Chapter of the International Group for the Psychology of Mathematics Education*.

Parton, K., & Edwards, G. (2009, December). Features of conductor gesture: Towards a framework for analysis within interaction. In *The Second International*

Conference on Music Communication Science, 3-4 December 2009, Sydney, Australia.

Parton, N. (2005). *Safeguarding childhood: Early intervention and surveillance in a late modern society*. Red Globe Press.

Potter, J., & Edwards, D. (1996). Discourse analysis. In Introducing psychological research (pp. 419-425). Palgrave, London.

Roberts, N., & Venkat, H. (2016). Learning from disruptive classroom behaviour in a Grade 2 mathematics lesson. *South African Journal of Childhood Education*, *6*(1), 1-10.

Roth, W. M. (2010). Incarnation: Radicalizing the embodiment of mathematics. *For the Learning of Mathematics*, *30*(2), 8-17.

Rueckert, L., Church, R. B., Avila, A., & Trejo, T. (2017). Gesture enhances learning of a complex statistical concept. *Cognitive research: principles and implications*, *2*(1), 1-6.

Saleh, S., Muhammad, A., & Abdullah, S. M. S. (2020). STEM project-based approach in enhancing conceptual understanding and inventive thinking skills among secondary school students. *Journal of Nusantara Studies (JONUS)*, *5*(1), 234-254.

Schoevers, E. M., Leseman, P. P., Slot, E. M., Bakker, A., Keijzer, R., & Kroesbergen, E. H. (2019). Promoting pupils' creative thinking in primary school mathematics: A case study. *Thinking skills and creativity*, *31*, 323-334.

Schoevers, E. M., Leseman, P. P., Slot, E. M., Bakker, A., Keijzer, R., & Kroesbergen, E. H. (2019). Promoting pupils' creative thinking in primary school mathematics: A case study. *Thinking skills and creativity*, *31*, 323-334.

Seale, C. (1999). The quality of qualitative research. *The Quality of Qualitative Research*, 1-224.

Selvianiresa, D., & Prabawanto, S. (2017, September). Contextual teaching and learning approach of mathematics in primary schools. *In Journal of Physics: Conference Series* (Vol. 895, No. 1, p. 012171). IOP Publishing.

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for information*, *22*(2), 63-75.

Shneiderman, B., & Plaisant, C. (2006, May). Strategies for evaluating information visualization tools: multi-dimensional in-depth long-term case studies. In *Proceedings of the 2006 AVI workshop on BEyond time and errors: novel evaluation methods for information visualization* (pp. 1-7).

Smith, L. B. (2005). Cognition as a dynamic system: Principles from embodiment. *Developmental Review*, *25*(3-4), 278-298.

Soylu, F., Lester Jr, F. K., & Newman, S. D. (2018). You can count on your fingers: The role of fingers in early mathematical development. *Journal of Numerical Cognition*, *4*(1), 107-135.

Spencer, J. P., Austin, A., & Schutte, A. R. (2012). Contributions of dynamic systems theory to cognitive development. *Cognitive Development*, *27*(4), 401-418.

Steffe, L. P., Thompson, P. W., & Von Glasersfeld, E. (2000). Teaching experiment methodology: Underlying principles and essential elements. *Handbook of research design in mathematics and science education*, 267-306.

Sung, W., Ahn, J., & Black, J. B. (2017). Introducing computational thinking to young learners: Practicing computational perspectives through embodiment in mathematics education. *Technology, Knowledge and Learning*, *22*(3), 443-463.

Taherdoost, H. (2016). Sampling methods in research methodology; how to choose a sampling technique for research. *How to choose a sampling technique for research (April 10, 2016)*.

Thelen, E., & Smith, L. B. (1996). *A dynamic systems approach to the development of cognition and action*. MIT Press.

Tran, C., Smith, B., & Buschkuehl, M. (2017). Support of mathematical thinking through embodied cognition: Nondigital and digital approaches. *Cognitive Research: Principles and Implications*, *2*(1), 1-18.

Truxaw, M. P., & DeFranco, T. C. (2008). Mapping mathematics classroom discourse and its implications for models of teaching. *Journal for Research in Mathematics Education*, *39*(5), 489-525.

van Geert, P., & Steenbeek, H. (2005). The dynamics of scaffolding. *New ideas in Psychology*, *23*(3), 115-128.

VanSchaik Publishers

Vasc, D., & Ionescu, T. (2013). Embodying cognition: Gestures and their role in the development of thinking. *Cognitie, Creier, Comportament/Cognition, Brain, Behavior*, *17*(2).

Verschaffel, L., Depaepe, F., & Van Dooren, W. (2020). Word problems in mathematics education. *Encyclopedia of mathematics education*, 908-911.Güner, P., & Uygun, T. (2020). Examining students' mathematical understanding of patterns by Pirie-Kieren model. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *35*(3), 644-661.

Verschaffel, L., Depaepe, F., & Van Dooren, W. (2020). Word problems in mathematics education. *Encyclopedia of mathematics education*, 908-911.

Verspoor, M., Lowie, W., & Van Dijk, M. (2008). Variability in second language development from a dynamic systems perspective. *The Modern Language Journal*, *92*(2), 214-231.

Wagner, J. (2007). Exports and productivity: A survey of the evidence from firm-level data. *World economy*, *30*(1), 60-82.

Walshaw, M., &Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classroom. *Review of educational research,* 78(3), 516-551.

Weisberg, S. M., & Newcombe, N. S. (2017). Embodied cognition and STEM learning: Overview of a topical collection in CR: PI. *Cognitive Research: Principles and Implications*, *2*(1), 1-6.

White, D. Y. (2003) Promoting mathematical discourse with diverse students. *The journal of Mathematical Behavior* 22(1), 37-53

Wilson, M. Six Views of Embodied Cognition. Margaret Wilson University of California, Santa Cruz

ANNEXURES

ANNEXURE A: RESEARCH ETHICAL CLEARANCE CERTIFICATE

UNIVERSITY OF LIMPOPO

University of Limpopo
Department of Research Administration and Development
Private Bag X1106, Sovenga, 0727, South Africa
Tel: (015) 268 3935, Fax: (015) 268 2306, Email: anastasia.ngobe@ul.ac.za

TURFLOOP RESEARCH ETHICS COMMITTEE

ETHICS CLEARANCE CERTIFICATE

MEETING: **23 February 2022**

PROJECT NUMBER: **TREC/27/2022: PG**

PROJECT:

Title:	Investigating Grade 4 Learners' Enactment of Utterances in a Mathematical Discourse Classroom.
Researcher:	TA Mokwana
Supervisor:	Dr. DJ Muthelo
Co-Supervisor/s:	N/A
School:	Education
Degree:	Master of Education in Mathematics Education

PROF P MASOKO
CHAIRPERSON: TURFLOOP RESEARCH ETHICS COMMITTEE

The Turfloop Research Ethics Committee (TREC) is registered with the National Health Research Ethics Council, Registration Number: **REC-0310111-031**

Note:

i) **This Ethics Clearance Certificate will be valid for one (1) year, as from the abovementioned date. Application for annual renewal (or annual review) need to be received by TREC one month before lapse of this period.**

ii) **Should any departure be contemplated from the research procedure as approved, the researcher(s) must re-submit the protocol to the committee, together with the Application for Amendment form.**

iii) **PLEASE QUOTE THE PROTOCOL NUMBER IN ALL ENQUIRIES.**

Finding solutions for Africa

ANNEXURE B: PERMISSION LETTER FROM DEPARTMENT OF EDUCATION

LIMPOPO

PROVINCIAL GOVERNMENT

REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF

EDUCATION

CONFIDENTIAL

Ref: 2/2/2 Enq: Makola MC Tel No: 015 290 9448 E-mail:MakolaMC@edu.limpopo.gov.za

Mokwana TA
Private Bag X 1106
Sovenga
0727

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.
2. The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"INVESTIGATING GRADE 4 LEARNERS ENACTMENT OF UTTERANCES IN A MATHEMATICAL DICOURSE CLASSROOM "**
3. The following conditions should be considered:

3.1 The research should not have any financial implications for Limpopo Department of Education.

3.2 Arrangements should be made with the Circuit Office and the School concerned.

3.3 The conduct of research should not in anyhow disrupt the academic programs at the schools.

3.4 The research should not be conducted during the time of Examinations especially the fourth term.

3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).

3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.

Cnr 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X 9489, Polokwane, 0700
Tel:015 290 7600/ 7702 Fax 086 218 0560

The heartland of Southern Africa-development is about people

4 Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.

5 The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes.

Mashaba KM
DDG: CORPORATE SERVICES

19/07/2022
Date

Cnr 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X 9489, Polokwane, 0700
Tel:015 290 7600/ 7702 Fax 086 218 0560

The heartland of Southern Africa-development is about people

ANNEXURE C: CONFIRMATION LETTER FROM LANGUAGE EDITORS

TransKaroo COMMUNICATIONS

15 December 2022
Pretoria, South Africa

To whom it may concern,

I hereby confirm that I undertook the language editing for the MNur Thesis:

INVESTIGATING GRADE 4 LEARNERS' ENACTMENT OF UTTERANCES IN A MATHEMATICAL CLASSROOM DISCOURSE

by Tebogo Mokwana

Cillié Swart BA (Harvard) MBA (Kuehne)
+27 (0)73 612 0278
pjcswart@transkaroo.net

Andrew
Scholtz
Editing

Associate Member of:
Professional EDITORS Guild
30 1993–2023
Promoting excellence in editing

Membership number: SCH018
Membership year: March 2023 to February 2024
084 602 1938
atjscholtz@gmail.com
www.editors.org.za

224, Raptors View, Hoedspruit, 1380
PO Box 1172, Hoedspruit, 1380

084 602 1938 • 076 079 0214 • atjscholtz@gmail.com

Affidavit

Date: 23 May 2023

To Whom it May Concern

I hereby confirm that I have undertaken a copy-edit of the thesis titled *Investigating Grade 4 Learners' Enactment of Utterances Embodied in a Mathematical Classroom Discourse* written by Tebogo Abie Mokwana. I have suggested a number of changes that the author may or may not accept, at their discretion.

Each of us has our own unique voice as far as both spoken and written language is concerned. In my role as editor, I try not to let my own 'written voice' overshadow the voice of the author, while at the same time attempting to ensure a readable document.

Please refer any queries to me.

Andrew Scholtz

Qualifications:

- MA (Digital Media in Education) – University of Kwazulu-Natal (2006)
- Accreditation of Assessors in Higher Education (Short Course) – Rhodes University (2007)
- Postgraduate Diploma in Dispute Settlement – University of Stellenbosch Business School (2013)
- SLP Family Law (Short Course) – North West University (2013)
- Strengthening Postgraduate Supervision (Short Course) – Rhodes University (2019)
- UCT Copy-editing Online Short Course – University of Cape Town (2020)
- Approved freelance editor and proofreader for Juta & Company (Pty) Ltd

Evidence of qualifications are available on request.

ANNEXURE D: LEARNING ACTIVITIES

Activity 1

Recognise and extend each pattern

Describe each pattern in words

4.2.1.2.1 1; 2; 3; 4; ____; _____; _____
4.2.1.2.2 2; 4; 6; 8;____; ______; ______
4.2.1.2.3 3; 5; 7; 9; _____;______;______
4.2.1.2.4 13; 11; 9; _____; ______; ______

(e) 1; 2; 4; 8; _____; ______; ______

(f) 64; 32; 16; _____; ______; ______

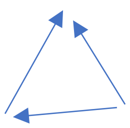
Activity 2

Extend and describe the following patterns

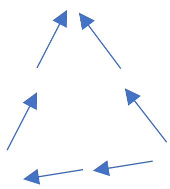
1. 3; 3; 4; 4; 5; ____; _____; _____
2. 99; 88; 77; _____; _____;______
3. 291; 282; 273; 264; ____; _____; ____
4. 1; 3; 6; 10; _____; _____; _____
5. 19; 18; 16; 13; ____; _____; _____
6. 1; 4; 9; 16; ____; _____; _____

Activity 3

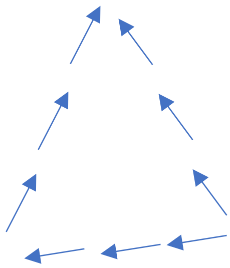
Consider the following pattern made with match sticks



Pattern 1



Pattern 2



Pattern 3

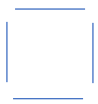
1. Draw pattern 4
2. How many matchsticks does pattern 4 have?
3. What geometric shape does each pattern make?
4. How does the pattern grow?
5. How many matchsticks will pattern 10 have?

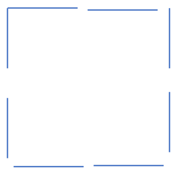
6. Copy and complete the table that follows:

Pattern Number	1	2	3	4	10	25
Number of match sticks						

Activity 4

Look at this pattern of squares





1 2 3

1. Build shape 3 with match sticks

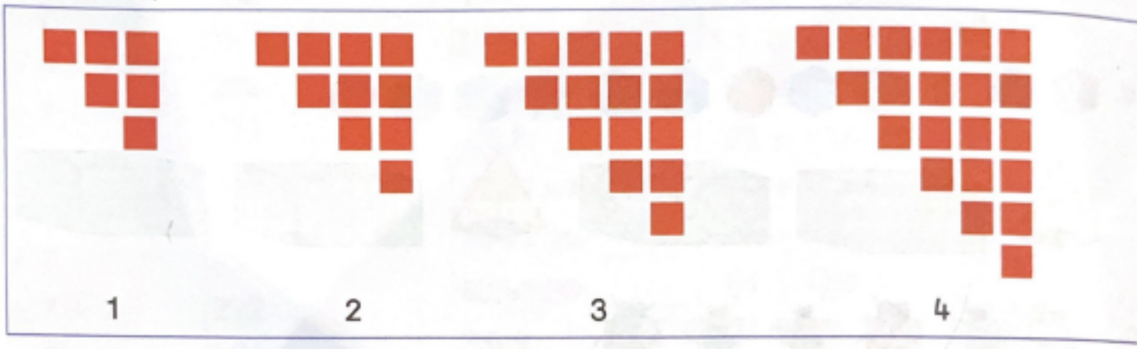
2. Describe how you made the pattern

3. Copy and complete the table and the flow diagram

Pattern Number	1	2	3	4	5	10
Number of match sticks						

Activity 5

2. Look at the pattern.



Copy and complete the table.

Pattern number	1	2	3	4	5	6	7	8
Number of squares								

Figure 2.4: Taken from Viva Mathematics learners books Grade 4

Activity 6

Calvin released 965 copies of music, he sold 243 copies and later sold 482 copies. How many copies remain to be sold?

Activity 7

Amo bought soccer boots for R1296 and Khura bought his for R1125.

1. How much money was spent by Amo and Khura altogether?
2. By how much money was Amo's boots more expensive than Khura's?
3. If Amo got a discount of R200 and Khura got a discount of R20, whose boots will be more expensive?

Activity 8

(a) The mass of 7 boxes of bananas is 175 kilograms. What is the mass of each box of bananas?

(b) A farmer wants to plant 189 apple trees. He plants 9 in a row. How many rows of apple trees does he plant?

(c) How many children could each get 5 sweets out of a packet of 152 sweets? How many sweets will be left over?