Grade 4 Mathematics Teachers' Development of Learners' Mathematical Proficiency Towards Solving Algebraic Word Problems

By

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DISSERTATION

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DECLARATION

I Glen Matji, declare that the dissertation titled: **Grade 4 Mathematics Teachers' Development of Learners' Mathematical Proficiency Towards Solving Algebraic Word Problems** hereby submitted to the University of Limpopo, for the degree of Master of Education in Mathematics Education has not previously been submitted by me for a degree at this or any other university; that it is my work in design and execution, and that all material contained herein has been duly acknowledged.

Matji G (Mr.)

14 January 2024

Date

DEDICATION

This dissertation is dedicated to my late mother, Raisibe Salaminah Mashaba Matji, and my late father, Jesaya Thabarako Matji.

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ABSTRACT

The purpose of this qualitative case study was to explore Grade 4 mathematics teachers' development of learners' mathematical proficiency towards solving algebraic word problems. The purpose of the study was guided by two research questions: 'What are Grade 4 learners' challenges when solving algebraic word problems?' and 'How does a Grade 4 mathematics teacher develop learners' mathematical proficiency towards solving algebraic word problems?' This study was prompted by Grade 4 learners' poor performance when solving algebraic word problems as outlined in the Annual National Assessments report (ANA) reflections on learners' performance from grades 1–6 (Department of Basic Education, 2014). The reflection on ANA demonstrates that Grade 4 learners have poor understanding and little knowledge of solving algebraic word problems. This qualitative case study used convenience sampling to select participants who are conveniently available with regard to access and location (Lopez & Whitehead, 2013). The study sampled a Grade 4 mathematics teacher and 76 mathematics learners in Grade 4 classes (class A consisted of 41 learners and class B consisted of 38 learners). Classes A and B were taught by same teacher at the primary school where the data were collected.

Data were collected by means of observation, written tasks and interviews. A Grade 4 mathematics teacher was observed when attempting to develop learners' mathematical proficiency towards solving algebraic word problems. Written tasks were administered to Grade 4 mathematics learners to determine the challenges they faced when solving algebraic word problems. Interviews were conducted with the Grade 4 mathematics teacher and the Grade 4 mathematics learners. The Grade 4 mathematics teacher was interviewed to get an insight into how he develop learners' mathematical proficiency, while Grade 4 mathematics learners where interviewed to find out on how they experienced the lessons and to identify the course of their challenges when solving algebraic word problems.

Data were analysed using the three steps proposed by the Merriam (1998) case study, guided by Kilpatrick et al. (2001) theory of mathematical proficiency, which

is characterised by five strands, namely conceptual understanding, procedural fluency, strategic competency, adaptive reasoning and productive disposition. The steps followed were consolidating, reducing and interpreting data. Using content analysis as an analytic strategy, data collected from observations, learners' written tasks and interviews were initially analysed separately, guided by the strands of mathematical proficiency indicators, and later reconciled the analysis using methodological triangulation.

Quality criteria of the study were ensured using the four principles of Lincoln and Guba (1985) to ensure trustworthiness of the qualitative research, namely credibility, transferability, dependability and confirmability. Ethical consideration of the study was achieved by way of voluntary participation, informed consent, anonymity and confidentiality.

The findings of the study reveal that in order for the teacher to develop learners' mathematical proficiency toward solving algebraic word problems, teachers should have rich pedagogical content knowledge (PCK); be a reflective practitioner to reflect on the lesson they taught in the classroom before preparing for the new lesson and devise a new plan of delivering content knowledge; and incorporate discourse-based mathematics instruction within a learning-centred classroom to create an effective environment for learners to develop mathematical proficiency. The findings reveal that a learning-centred classroom incorporated with discourse-based mathematics instruction improved Grade 4 learners' development of mathematical proficiency towards solving algebraic word problems. The study also revealed that Grade 4 mathematics learners still face great challenges when solving algebraic word problems as they appeared to lack the necessary mathematical vocabulary and conceptual understanding. It is therefore recommended that teachers should intensify discourse-based mathematics instruction within learning-centred classroom to improve Grade 4 mathematics learners' development of mathematical proficiency towards solving algebraic word problems. It is also recommended that similar studies can be conducted using a sample of more than one mathematics teacher so as to get

the insights into how different teachers develop learners' mathematical proficiency, and drawing on learners from different primary schools to identify other challenges that they may face when solving algebraic word problems.

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CHAPTER 1: OVERVIEW OF THE STUDY

1.1 Introduction and Background

Teaching and learning of mathematics in South African schools is not yielding the intended outcomes of South Africa's education policies and curricula (Department of Basic Education [DBE], 2018). DBE (2018) further outlines that this claim is evident from the findings of various research studies that were conducted by the universities and other research agencies in South Africa. The low learner achievement levels revealed by national assessments such as Annual National Assessments (ANA); regional assessments, such as Southern and Eastern Consortium for Monitoring Education Quality (SACMEQ); and international assessments, such as the Trends in Mathematics and Science Study (TIMSS), are indicative, at least in part, of current 'ineffective' teaching and learning practices (DBE, 2018).

The DBE (2012) aims to increase number of learners mastering the minimum competencies in language and numeracy in grades 3, 6 and 9 to 90 per cent by the end of each year. The DBE proposes that the required competency levels be defined as 50 per cent and above. In other words, 90 per cent of learners in grades 3, 6 and 9 must achieve 50 per cent or more in the ANA in these subjects. As a result, this could afford learners an opportunity to study maths- and science-related courses at university level (DBE, 2012).

The DBE developed a better way to support teachers to deliver mathematics content using DBE workbooks (Fleisch et al., 2010). Fleisch et al. (2010) alluded to the notion that the DBE workbooks provided teachers with lessons that are well structured and the assessment that covers the mathematics curriculum. Based on the experience of teaching mathematics in the Intermediate Phase, it seems that learners enjoy using DBE workbooks to do mathematics more than using the mathematics textbook. The DBE workbooks provide learners with examples and problems that are related to the class activity and the home activity.

Unfortunately, teachers who teach mathematics do not go into their classrooms with the DBE workbooks, they use the mathematics textbook. This is so because there was no significant difference in learner performance between learners who utilise workbooks and those who utilise textbooks as resources (Fleisch et al., 2011).

In contrast, the provision of a resource such as a DBE workbook or a textbook on its own does not guarantee an improvement in the understanding of concepts and meaning making by learners in mathematics (Fleisch et al., 2011). Uttal et al. (1997) share the sentiment that providing learners with mathematical resources in mathematics classes does not guarantee that learners will understand concepts and make the necessary mathematical connections. While the provision of the DBE workbook in the South African context may help to address some teaching and learning needs (Fleisch et al., 2010), this does not seem to happen. I have observed Grade 4 learners facing challenges with solving algebraic word problems. According to Kunene (2019), these challenges could be the result of teaching that does not promote the development of learners' mathematical proficiency towards solving algebraic word problems.

Mathematics teachers in South African schools are experiencing challenges in developing learners' mathematical proficiency towards solving algebraic word problems (Mouton et al., 2013). In addition, the 2014 ANA, which reflects on learners' performance, found that learners in grades 1–6 demonstrate a poor understanding of and little knowledge about solving algebraic word problems. It would therefore seem appropriate to suggest that Grade 4 mathematics teachers should try to address this gap in knowledge and understanding by teaching for the development of mathematical proficiency, so that learners are able to solve algebraic word problems. Thus, the current study intends to explore how Grade 4 mathematics teachers develop learners' mathematical proficiency towards solving algebraic word problems

Kilpatrick et al. (2001) define teaching for mathematical proficiency as comprising several strands that go beyond a simple contrast between knowledge and understanding. They view teaching for mathematical proficiency as a way of adding practical knowledge to factual knowledge. In the classroom setting, teaching for mathematical proficiency includes the preparation of the lesson, an understanding of the content and the use of examples when teaching the learners, so that they can grasp the content taught (Niess, 2005). Teaching for mathematical proficiency requires a great deal of effort because it takes time to develop concepts and connections (Kilpatrick et al., 2001). In addition, teachers should help learners to develop all strands of mathematical proficiency in order for them to be successful in mathematics, given that these strands are interwoven and interdependent. Doğruer et al. (2015) argue that mathematical discourse plays an important role when teaching mathematical proficiency, as it engages learners in classroom interaction. It also affords learners the opportunity to communicate about mathematics and acquire a concrete comprehension of mathematical concepts. A classroom in which learners are afforded opportunities to engage in meaningful mathematical discourse is desirable for the effective teaching and learning of mathematical proficiency (Sfard, 2007). Learners' engagement in mathematical discourse enables them to challenge each other's understanding of mathematical concepts and justify their mathematical arguments (Kersaint, 2015). Furthermore, use of appropriate mathematical vocabulary helps learners to think, learn and communicate mathematics, and master the concept of mathematics (Jourdain & Sharma, 2016). Mathematical discourse is one of the important aspects of developing learners' proficiency in mathematics (Riccomin et al., 2015). It further, affords learners the opportunity to communicate about algebraic word problems and acquire a concrete comprehension of mathematical concepts (Doğruer et al., 2015). Therefore, in supporting learners' growth in mathematical knowledge, teachers should pay attention on learners' use of mathematical discourse in order to provide them with the necessary skills that they need in order to think and talk about mathematics, and absorb mathematical proficiency (Turnuklu & Yesildere, 2007).

1.2 Research Problem

Mathematics teachers in the Intermediate Phase in South African schools are expected to teach mathematics as prescribed by the Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011). Teachers are expected to follow the curriculum and 'teach' the topics in the week-by-week order prescribed in CAPS (DBE, 2011). Furthermore, Intermediate Phase teachers must be conscious of the fact that the DBE workbooks structure the curriculum per week of teaching time, allowing them to ensure that the full curriculum is covered (Spaull, 2013). The main intention of this approach is to enable learners to acquire essential mathematical skills. As a result, teachers could help learners to develop mathematical proficiency by using DBE workbooks as part of their daily preparation for teaching algebraic word problems (DBE, 2012). The DBE (2018) further outlined a framework for empowering mathematics teachers to perform better in their mathematics teaching for developing learners' mathematical proficiency towards solving algebraic word problems. This framework is referred to as The Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding. The framework is based on the five strands of mathematical proficiency developed by Kilpatrick et al. (2001). The strands include conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (DBE, 2018). However, the firth strand of mathematical proficiency (productive disposition) has been replaced with learning-centred classroom. A learning-centred classroom focuses on learning where teachers design learning experiences to help learners to develop mathematical proficiency. In addition, such a classroom is characterised by a culture of interaction between teachers and learners, through the process of teaching and learning. Teachers play an important role in facilitating and nurturing the culture of learning-centred classroom.

The manner in which teachers conduct a classroom depends on the way in which they view mathematics. In most cases, teachers who view mathematics as a body of knowledge that they have to impart to their learners would tell learners what to do, and how to do it. On the other hand, teachers who see mathematics as a body

of knowledge that learners must actively explore and engage with would create a learning environment where learners can make sense of mathematics. Ultimately, learners would have the opportunity to express their ideas, to ask questions from their peers and the teacher, and to discuss their ways of thinking about solving of algebraic word problems (DBE, 2018). This framework has been introduced not as a new curriculum and not replacing the existing curriculum (CAPS), but rather to support the implementation of the current curriculum (CAPS) by means of introducing a model that could help teachers change the way they in which they teach.

The framework model and the supporting exemplars are provided to offer guidance to teachers to transform their teaching. It has been outlined in the framework that the expected transformation should lead to teaching for the development of mathematical proficiency, so that learning for understanding can take place in all mathematics classrooms in South Africa (DBE, 2018). Figure 1 represents the model that was drawn from Kilpatrick et al. (2001).

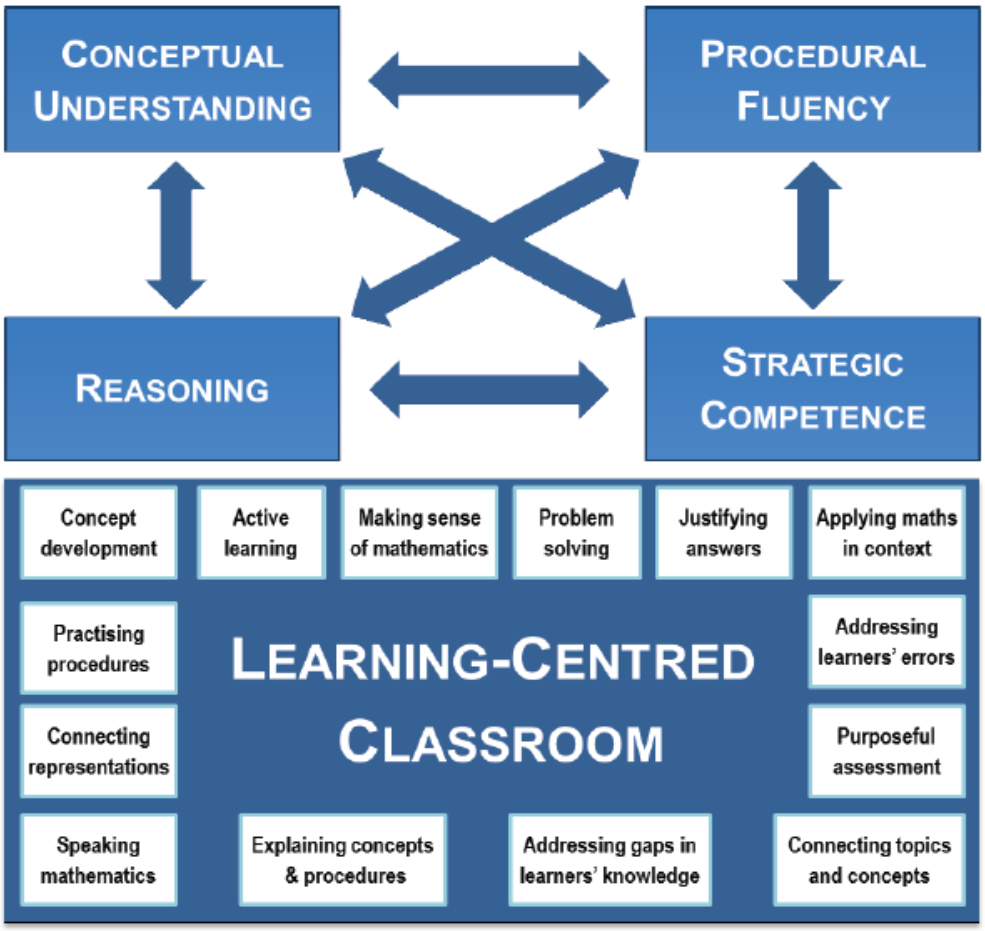


Figure 1: Model for mathematics learning and teaching (DBE, 2018., p. 9)

Studies have highlighted teachers' lack of experience in teaching mathematics to develop learners' mathematical proficiency, as prescribed by CAPS at the Intermediate Phase, (Khoza, 2016; Biyela, 2018). These experiences include teaching strategies or methodologies that seek to develop learners' proficiency in mathematics when solving algebraic word problems. The DBE (2010) introduced the ANA to encourage teachers to improve their assessment practices, make it easier for districts to pinpoint schools that need assistance, encourage schools to celebrate outstanding performance and empower parents with important information about their children's performance. Moreover, ANA provides a national baseline to benchmark annual targets and achievement towards realising the desired 60 per cent threshold of learners mastering the minimum literacy and numeracy competencies by the end of grades 3, 6 and 9 respectively. However, the target has not been realised as a result of teaching that does not

promote the development of mathematical proficiency. The DBE conducted an ANA in February 2011, September 2012, 2013 and 2014 in literacy/language and numeracy/mathematics. The first set of tests were administered to learners who had completed grades 1–6 in 2010. The ANA undertaken from 2012 to 2014 were administered to learners in their current academic year grades; that is, from grades 1–6. Unlike examinations that are designed to inform decisions on learner promotion and progression (summative evaluation), ANA data are meant to be used for both diagnostic purposes at individual learner level and for decision-making purposes at systemic level (DBE, 2011).

In South Africa, the 2014 mathematics ANA report highlighted the fact that learners in grades 1–6 are still unfamiliar with mathematical terminology and properties, and that they often use them incorrectly (ANA, 2014). As a result, learners fail to respond to algebraic word problems. The overall 2014 results for ANA in grades 1–6 point towards an upward movement of test scores. However, as mentioned above, the national baseline benchmarks annual targets and achievement towards realising the desired 60 per cent threshold of learners mastering mathematics were not realised in grades 3 to 6 (DBE, 2014). The mathematics performance of learners has remained at a low level, as was the case in 2012 and 2013. Table 1 shows the average national percentages that learners achieved in mathematics from 2012 to 2014.

Table 1: Summary table for mathematics in 2012, 2013 and 2014 (DBE, 2014, p. 9)

Grade / Year	Average mathematics percentage mark		
	2012	2013	2014
1	68	60	68
2	57	59	62
3	41	53	56
4	37	37	37
5	30	33	37
6	27	39	43
9	13	14	11

From my experience of teaching at Intermediate Phase, one of the contributing factors to learners' challenges in solving algebraic word problems is that teachers lack teaching strategies or methodologies that aim to develop learners' mathematical proficiency towards solving algebraic word problems. Pongsakdi et al. (2020) stated that challenges faced when solving algebraic word problems include understanding the problem and deciding on what operations need to be performed in order to solve the problem. Since teachers lack such teaching strategies or methodologies, this leads to teachers continually failing to help learners to develop strands of mathematical proficiency towards solving algebraic word problems (Kunene, 2019). Thus, this study attempts to deal with this failure by exploring how a Grade 4 mathematics teacher develop learners' mathematical proficiency towards solving algebraic word problems.

1.3 Purpose of the Study

The purpose of this study was to explore a Grade 4 mathematics teacher's development of learners' mathematical proficiency towards solving algebraic word problems.

1.3.1 Objectives

The objectives that guided the study were:

- To identify Grade 4 learners' challenges when solving algebraic word problems.
- To document how Grade 4 mathematics teachers develop learners' mathematical proficiency towards solving algebraic word problems.

1.3.2 Research questions

The research questions were:

- What are Grade 4 learners' challenges when solving algebraic word problems?
- How do Grade 4 mathematics teachers develop learners' mathematical proficiency towards solving algebraic word problems?

1.4 Research Methodology

This study used a qualitative research approach. A qualitative research approach was found to be suitable since the study intended to consider the thoughts, feelings, experiences, meaning of language and processes of both a Grade 4 mathematics teacher and the learners in the classroom setting (Lopez & Whitehead, 2013). A qualitative case study design was adopted as proposed by Merriam (1998). Merriam highlighted that a case can be a person, a programme, a group of people, a specific policy and so on that represents more comprehensive data. In this study, the case was a Grade 4 mathematics teacher and the Grade 4 mathematics learners who he was teaching so as to come up with a rich comprehensive data. The purpose of the study was to explore how the teacher developed learners' mathematical proficiency when solving algebraic word problems. Convenience sampling was used because it allows for the selection of a number of participants who are conveniently available with regard to access, location, time and willingness (Lopez & Whitehead, 2013). This study sampled a Grade 4 mathematics teacher and 167 Grade 4 mathematics learners; however, out of 167 learners only 79 were permitted by their parents to participate in the study.

Data were collected by means of observations, written tasks and interviews. During the data collection process, the teacher was given two learning episodes to teach. The learning episodes were prepared targeting the content area of numbers, operations and relationships. The first learning episode targeted addition and subtraction in the context of finance. The second learning episode targeted addition, subtraction, multiplication and division in a variety of contexts. After the presentation of each learning episode, learners were given written tasks to respond to. These tasks would help me, as the researcher, to crosscheck whether learning had taken place. Subsequently, learners' responses to the given tasks were collected with the intention of later analysing them to identify the challenges encountered. The interview process took take place immediately after the completion of the written task.

Merriam (1998) defines data analysis as the process of making sense of data. The proposed steps used when making sense of data involve consolidating, reducing and interpreting data. Content analysis was employed as an analytic strategy, and data collected from observations, learners' written tasks and interviews were initially analysed separately, guided by the strands of mathematical proficiency, and thereafter reconciled using methodological triangulation.

According to Lincoln and Guba (1985), there are four principles used to ensure trustworthiness of a qualitative research, namely credibility, transferability, dependability and confirmability. Consequently, these four principles were used to ensure trustworthiness of the study.

Ethical consideration in the study was obtained by following proper ethical considerations, namely voluntary participation, informed consent, anonymity and confidentiality.

1.5 Significance of the Study

The rationale behind the study was prompted by the observation I have made in respect of Grade 4 learners facing challenges in solving algebraic word problems. Kunene (2019) stated that these challenges could be faced because of teaching that does not promote the development of learners' mathematical proficiency towards solving algebraic word problems. Therefore, I was then motivated to explore the incorporation of discourse-based mathematics instruction in a learning-centred classroom strategy to improve learners' development of mathematical proficiency towards solving algebraic word problems. Both the theoretical and empirical literature indicate that discourse-based mathematics instruction helps teachers to improve learners' mathematical performance (Legesse et al., 2020; Bradford, 2007). As a result, it was appropriate to incorporate discourse-based mathematics instruction into learning-centred classrooms to develop learners' mathematical proficiency towards solving

algebraic word problems. This study contributes empirical evidence to the body of knowledge about the incorporation of discourse-based mathematics instruction into learning-centred classrooms to promote the development of learners' mathematical proficiency towards solving algebraic word problems, which provides useful insights for school teachers and policymakers to develop discourse-oriented curriculum materials and professional development programmes. This current study afforded me the opportunity to develop professionally, to improve my practice and to examine new methods and ideas for teaching mathematics. The findings of the study are likely to add to the existing body of knowledge concerned with how teachers should develop learners' mathematical proficiency when solving algebraic word problems, since there is limited literature about teachers developing learners' mathematical proficiency towards solving algebraic word problems, particularly at Intermediate Phase.

1.6 Overview of the Dissertation

This study is divided into six chapters. The following are the outlines of each of the chapters. Chapter 1 deals with the background, research problem, purpose of the study, research questions, significance of the study and, lastly, an overview of the study. The background of the study captures what prompted the study and my experience of teaching at Intermediate Phase. The research problem outlines what is expected from the teachers when developing learners' mathematical proficiency towards solving algebraic word problems; the reality of the problem when teachers are teaching learners; and, lastly, what the study intended to achieve. The significance of the study outlines the importance of partaking in this study and what the mathematics community could learn from this study. Lastly, an overview of the dissertation is provided, which outlines how the chapters of the study has been arranged.

Chapter 2 deals with the literature review of this study, focusing on the following subheadings: teaching for mathematical proficiency; what it means to be proficient in mathematics; mathematical word problem; challenges faced by

learners; problem posing; teacher knowledge; mathematics language in word problem-solving; and theoretical framework.

Chapter 3 presents the research methodology that was used in this study. This chapter deals with the research design; how population was sampled; how data were collected; how data were analysed; how quality criteria were ensured; and, lastly, how ethical considerations were ensured.

Chapter 4 reports on the results gathered from observation, learners' written tasks, the teacher interview and the interviews with the learners for learning episode 1 and learning episode 2. Learning episode 1 focused on finance as the context, while learning episode 2 focussed on different contexts.

Chapter 5 reports on responses to research questions. Research question 1 focussed on the Grade 4 learners' challenges when solving algebraic word problems. The following subheadings were used when responding to research question 1: mathematical vocabulary; conceptual understanding; adaptive reasoning; arithmetic skills; and, lastly, reflective practice. Research question 2 focussed on how a Grade 4 mathematics teacher develops learners' mathematical proficiency towards solving algebraic word problems. The following subheadings were used to respond to the question: pedagogical content knowledge (PCK); reflective practice; learning-centred classrooms; and problem posing.

Chapter 6 presents the recommendations from and implications of the study. This study recommend that similar studies can be conducted using a sample of more than one Grade 4 mathematics teacher and using learners from different schools in order to obtain insights into how Grade 4 mathematics teachers develop their learners' mathematical proficiency and the challenges faced by Grade 4 learners when solving algebraic word problems. The implications of the study were that algebraic word problem activities should not only be given to learners as

classwork or homework, but teachers should also strive equip learners with the necessary skills to help them to solve algebraic word problems.

1.7 Chapter Summary

This chapter provided the background, research problem, purpose of the study, research methodology and significance of the study, and an overview of the dissertation. The background described what prompted the study. This chapter also drew attention to The Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding that was meant to empower mathematics teachers with the necessary tools to perform better in their mathematics teaching for developing learners' mathematical proficiency towards solving algebraic word problems (DBE, 2018). The purpose of the study outlined the objectives of and the research questions addressed in the study. The next chapter reviews the literature pertaining to the development of learners' mathematical proficiency towards solving algebraic word problems.

CHAPTER 2: LITERATURE REVIEW

This chapter presents a review of the literature concerning the development of learners' mathematical proficiency. This review includes teaching for mathematical proficiency, studies that focused on teaching for mathematical proficiency, what it means to be proficient in mathematics, algebraic word problems, problem posing when developing learners' mathematical proficiency, challenges faced by learners when solving algebraic word problems, teacher knowledge, teacher knowledge in problem posing, teacher knowledge in problem posing, studies that focused on mathematical language against solving algebraic word problems, the use of assessment to develop learners' mathematical proficiency and theoretical framework that guided the study. The theoretical framework that guided the study is the five strands of mathematical proficiency developed by Kilpatrick et al. (2001).

2.1 What it Means to be Proficient in Mathematics

Teachers play a critical role in the development of mathematics proficiency in learners. As a result, it is of paramount importance for teachers to understand what it means for learners to be proficient in mathematics (Chapman, 2015). Kilpatrick et al. (2001). used the best available research on mathematics learning to identify five strands of mathematical proficiency, namely adaptive reasoning, strategic competence, conceptual understanding, productive disposition and procedural fluency. The fact that there are five strands alerts teachers to the reality that success in mathematics is multifaceted. This is part of what makes mathematics teaching so complex. Teachers may conclude from low scores on formal assessments that some learners have low procedural fluency in a particular content strand in mathematics. However, developing procedural fluency alone is not likely to result in sufficient progress for these learners. According to Kilpatrick et al. (2001). it is important to facilitate opportunities for struggling learners to: (a) strengthen their dispositions toward approaching mathematical tasks; (b) develop conceptual understanding of the target topics; (c) increase their ability to be strategic in problem solving; and (d) improve their

adaptive reasoning skills. Only when all strands of mathematical proficiency are developed would learners have an ample opportunity to solve algebraic word problems. Kilpatrick et al. (2001) stated that it is important to facilitate opportunities for struggling learners to develop mathematical proficiency, however they are not clear on how to facilitate these opportunities. Therefore, this study sought to determine a strategy that teachers can use to afford learners an opportunity to develop mathematical proficiency towards solving algebraic word problems.

Mathematics learning involves active engagement in logical reasoning and sense making, and is not defined by the facts that learners know or even what procedures they can memorise. Complementing the five strands of proficiency (Kilpatrick et al., 2001), the National Council of Teachers of Mathematics (NCTM) (2000) highlighted five process standards that describe the ways in which learners should go about learning mathematics content. Then, in 2010, the National Governors Association Centre for Best Practices and the Council of Chief State School Officers (NGA & CSSO) drew on the five strands of proficiency and the five process standards to create the eight mathematical practices presented in the Common Core State Standards for Mathematics. These eight standards describe the mathematical habits of mind that facilitate optimal learning in mathematics. These practices summarise how learners should go about learning the ‘what’ of mathematics. Figure 2 provides a side-by-side comparison of the strands of proficiency, the process standards and the mathematical practices. In its entirety, this figure supports a robust vision of what it means for learners to do mathematics (Allsopp et al., 2017).

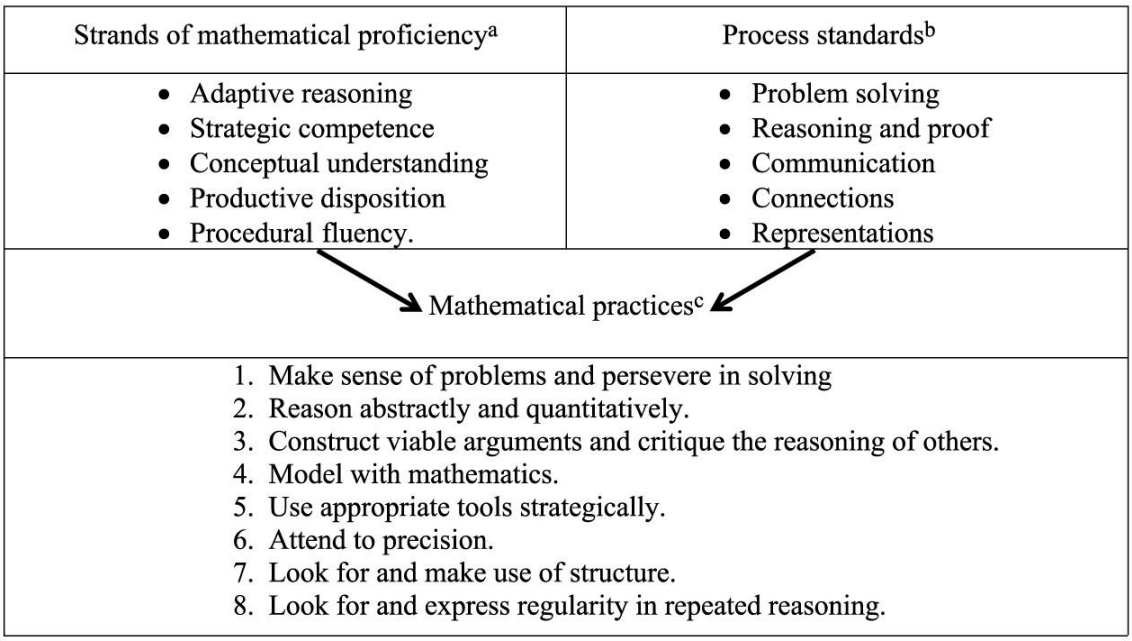


Figure 2: A robust vision of what it means for learners to do mathematics (Allsopp et al., 2017, p. 275)

Figure 2 illustrates a strong vision for how learners should engage in mathematics.

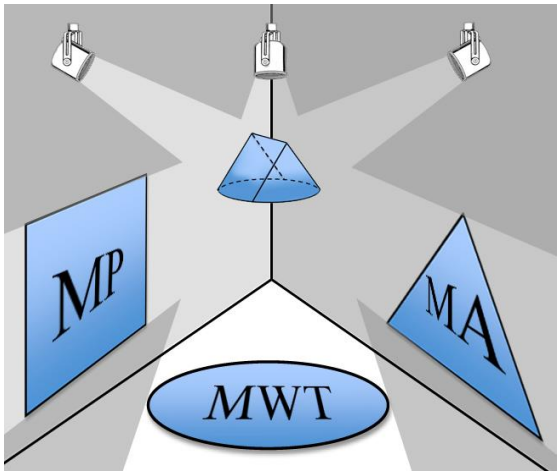
Unfortunately, the vision of what it means for learners to learn and succeed in mathematics established in Figure 2 does not clearly outline how teachers should create an environment that could afford learners an opportunity to engage in mathematics; particularly when learners are solving algebraic word problems. Hence, this study attempts to determine how teachers can create an environment that would result in learners being proficient in mathematics.

2.2 Teaching for Mathematical Proficiency

Hiebert and Grouws (2007) argue that teaching for mathematical proficiency significantly affects the nature and level of learners' learning of mathematics. It is different from the mathematical proficiency needed for engineering, accounting or the medical professions (Heid et al., 2015). It is even different from the mathematical proficiency a mathematician needs. For example, a mathematician

may prove a theorem and an architect may perform geometric calculations. For these users of mathematics, it is sufficient that they have the skills and understanding for the task at hand. However, a teacher's work includes these tasks as well as interpreting learners' mathematics, developing multiple representations of a mathematical concept and knowing where learners are on the path of mathematical understanding using formative assessment (Schoenfeld & Kilpatrick, 2008).

Teaching for mathematical proficiency is viewed as three components, that is mathematical proficiency, mathematical activity and mathematical work of teaching (Wilson et al., 2010). Each component outlines a different view of teaching mathematical proficiency.



MP Mathematical proficiency
MA Mathematical activity
MWT Mathematical work of teaching

Figure 3: Framework for mathematical proficiency for teaching (Wilson et al., 2010, p. 3)

Mathematical proficiency includes aspects of mathematical knowledge and ability such as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition that teachers themselves need, and they seek to foster learners to learn mathematics (Kilpatrick et al., 2001). Teaching for mathematical proficiency goes beyond what one might find in a Grade 4 mathematical classroom. Learners' development of mathematical proficiency usually depends heavily on how well-developed the teacher's proficiency is (Groves, 2012).

Proficiency in mathematical activity can be seen when doing mathematics. Examples include representing mathematical objects and operations, connecting mathematical concepts, modelling mathematical phenomena and justifying mathematical arguments (Rowland & Ruthven, 2011). This facet of teachers' mathematical proficiency is displayed as they engage learners in the day-to-day of learning mathematics (Department of Basic Education [DBE], 2011). Teachers need deep knowledge of, for example, what characterises the structure of mathematics (as opposed to conventions that have been adopted over the centuries) and how to generalise mathematical findings (Wilson et al., 2010). The more a teacher's proficiency in mathematical activity has developed, the better equipped they are able to facilitate learning and doing mathematics.

Proficiency in the mathematical work of teaching diverges sharply from the mathematical proficiency needed in other professions requiring mathematics (NRC & Mathematics Learning Study Committee, 2001). One of its aspects is an understanding of the mathematical thinking of learners, which may include, for example, recognising the mathematical nature of their errors and misconceptions. Another aspect of the mathematical work of teaching is knowledge of and proficiency in the mathematics that comes before and after what is being taught (Wilson et al., 2010). A teacher benefits from knowing what learners have learned in previous years so that they can help them build upon that prior knowledge (Shandomo, 2010). The teacher also needs to provide a foundation for the mathematics they could be learning later, which requires knowing and understanding the mathematics in the rest of the curriculum.

The three components of teaching for mathematical proficiency together form a full picture of the mathematics required of a Grade 4 mathematics teacher to develop learners' mathematical proficiency (Suh, 2007). It is not enough to know the mathematics that learners are learning. Grade 4 mathematics teachers should also possess a depth and extent of mathematical proficiency that could equip them to foster their learners' mathematical proficiency (Mntunjani, 2016). Mathematical activity and the mathematical work of teaching emerge from, and

depend upon, the teacher's mathematical proficiency (Schoenfeld & Kilpatrick, 2008).

Rowan et al. (2002) explain that teachers should know content and use teaching strategies that positively affect learners' learning of mathematical proficiency. In addition, classroom teachers should possess pedagogical content knowledge (PCK) to deliver the content effectively (Shulman, 1987). Shulman (1987) defined PCK as the integration or amalgamation of pedagogy and content that covers the 'what' and 'how' of teaching. Content and pedagogy are viewed as one indistinguishable body of knowledge, whereby content is about what is known and pedagogy is about how to teach it (Shulman, 1987).

Shulman (1987) stated that teachers need seven knowledge bases in order to teach mathematical content effectively and successfully. The seven knowledge bases are (i) content knowledge; (ii) general pedagogical knowledge; (iii) curriculum knowledge; (iv) PCK; (v) knowledge of learners and their characteristics; (vi) knowledge of educational contexts; and (vii) knowledge of educational ends, purposes and values. For the purposes of this study, teachers need to have seven knowledge bases so as to develop learners' mathematical proficiency towards solving algebraic word problems. Furthermore, Smith and Neale (1989) proposed that teachers also need to know the subject matter, content knowledge, instructional strategies knowledge and learners' knowledge to teach to develop learners' mathematical proficiency.

The DBE (2018) developed a mathematics teaching and learning framework for South Africa. The framework is called Teaching Mathematics for Understanding, as mentioned in Chapter 1 paragraph 1.2 The Mathematics Teaching and Learning Framework (DBE, 2018) draws on the five strands of mathematical proficiency described by Kilpatrick et al. (2001) and the 'dimensions represent a contextualisation and adaption of the strands to the South African context' (DBE, 2018, p. 13). DBE (2018) acknowledges that this framework is not a new curriculum and does not replace the existing curriculum. Instead, it supports the

implementation of the current curriculum by introducing a model to help teachers to change the way in which they teach. The framework model and the supporting exemplars are provided to offer guidance to teachers that will enable them to transform their teaching and develop learners' mathematical proficiency towards solving algebraic word problems. This transformation should lead to teaching for understanding, so that learning for understanding will take place in all mathematics classrooms in South Africa. The framework, as illustrated in Figure 1, calls for teachers to take steps to bring about the transformation of teaching and learning of mathematics in South Africa, and to strive to:

- ❖ teach mathematics for conceptual understanding to enable comprehension of mathematical concepts, operations and relations;
- ❖ teach so that learners develop procedural fluency which involves skill in carrying out procedures flexibly, accurately, efficiently and appropriately;
- ❖ develop learners' strategic competence – the ability to formulate, represent and decide on appropriate strategies to solve mathematical problems;
- ❖ provide multiple and varied opportunities for learners to develop their mathematical reasoning skills – the capacity for logical thought, reflection, explanation and justification; and
- ❖ promote a learning-centred classroom that enables all of the above, supported by teachers engaging with learners in ways that foreground mathematical learning for all (DBE, 2018).

The model of the mathematics teaching and learning framework, as illustrated in Figure 1 in Chapter 1, comprises of four key dimensions, namely conceptual understanding, mathematics procedures, strategic competence and reasoning, while each of which is underpinned by a learning-centred classroom. I agree that the model presented in Figure 1 has the potential to develop learners' mathematical proficiency. However, this model is not clear enough on how teachers should ensure that learners are engaged in speaking mathematics in the process of developing learners' mathematical proficiency towards solving algebraic word problems. As a result, teachers fail to ensure that learners speak mathematical language in the process of developing strands of mathematical

proficiency towards them. Therefore, the current study attempts to deal with this failure by incorporating discourse-based mathematics instruction into a learning-centred classroom to develop learners' mathematical proficiency towards solving algebraic word problems.

2.3 Discourse-based Mathematics and Mathematical Proficiency

Research literature in mathematics education recommends different forms of instructional approaches to enable learners to develop mathematical proficiency (Bennett, 2014; Bradford, 2007; Cross, 2009; Manouchehri & Enderson, 1999; NCTM, 2000). Among the recommendations is the use of discourse-based mathematics instruction applied to mathematical tasks that foster learners' understanding of mathematical ideas, communication and problem-solving skills (Bennett, 2014; Cross, 2009; Legesse et al., 2020; NCTM, 2000; Smith & Stein, 2011; Walshaw & Anthony, 2008). Moreover, the literature suggests the engagement of learners in mathematical discursive practices of explaining, justifying, listening to, sharing, comparing, evaluating and interpreting each other's mathematical ideas and reasons, and constructing convincing arguments (Bennett, 2014; Rumsey & Langrall, 2016; Smith & Stein, 2011) to enhance their mathematical proficiency and achievement results (Anthony & Hunter, 2017; Bennett, 2014; Bradford, 2007). However, evaluation studies on the effectiveness of discourse-based mathematics teaching and learning on targeted academic outcomes in different cultural contexts are scant (Bradford, 2007; Rumsey & Langrall, 2016). In particular, there is limited practical effort in using discourse-based mathematics instruction as a teaching strategy in South African school classrooms.

Corrêa (2021) conducted a study that aimed to investigate the mathematical proficiency promoted by mathematical modelling tasks that require students to get involved in the processes of developing mathematical models, instead of just using known or given models. The findings of the study suggested that mathematical modelling has the potential to promote and foster learners'

mathematical proficiency, even when the tasks assigned are not fully completed. The author further outlined that mathematics teachers could benefit from teaching approaches that favour learners' solid learning of mathematics (Corrêa, 2021). The author spoke about teaching approaches that could help the teacher to develop learners' mathematical proficiency, yet failed to mention those teaching approaches. This indicates that there is a knowledge shortcoming in respect of teaching approaches that seek to develop learners' mathematical proficiency particularly when they are solving algebraic word problems.

Discourse-based mathematics instruction is viewed as an approach that teachers use to promote the construction of mathematical understanding for learners' learning of mathematics (Corrêa, 2021). As a result, it affords learners an opportunity to discuss and share their mathematical ideas; to explain their reasoning; to compare and justify problem-solving strategies; to communicate their ideas; to challenge each other's reasoning through questioning; to reflect on and clarify their thinking; and to listen to other's viewpoints (Seeley, 2017). Bradford (2007) views discourse-based mathematics instruction as a 'learning medium' through which learners' construct or develop mathematical understanding and knowledge by explaining how mathematical tasks can be accomplished or how procedures work. The author maintains that it affords learners an opportunity to challenge each other's ideas, compare different solution strategies, share ideas and reasoning about mathematical concepts, and relationships (Bradford, 2007).

Luneta and Legesse (2023) stated that discourse-based mathematics instruction can be characterised by the engagement of learners in cognitively-demanding tasks. These tasks create a platform for teacher–learner and learner–learner interactions over the content topic that are anchored on discourse practices. This practice creates an environment where learners challenge each other's ideas, share ideas, agree or disagree with solution strategies and ideas, compare solution procedures, and explain problem-solving strategies (Legesse et al., 2020; Luoto, 2019; Bradford, 2007).

Discourse-based mathematics instruction is characterised by the following elements:

i) Social interaction and communication: The instruction is based on the interactive discourse over the given tasks between teacher and learner and among learners in which learners' participation in the classroom discourse is structured in independent work, small-group discourse and whole-class discourse (Franke et al., 2015).

ii) Choice of challenging tasks: The tasks should be chosen or designed in such a way that learners gain access to learning mathematics by involving in the discourse practices of explaining, questioning, proving, justifying, agreeing and disagreeing, comparing, exemplifying and making conjectures and generalisations. These are discourse opportunities for learners to foster their mathematical understanding towards development of mathematical proficiency (Franke et al., 2015).

iii) Creating a safe and respectful learning environment in which discourse can occur: There should be learning situations in which learners interact with each other's ideas freely and respectfully (Cobb, 1994; Franke et al., 2015; Steeley, 2017).

iv) Questioning strategy: The teacher should challenge learners' thinking be means of questioning to engage them in 'deeper reflective and integrative thought' (Hiebert & Wearne, 1993, pp. 393-425). The teacher is responsible for planning and crafting a set of different types of questions that can be used to scaffold learners' participation in the discourse (Pourdavood & Wachira, 2015; Walshaw & Anthony, 2008).

Creating discourse-based mathematics instruction affords learners an opportunity to become actively involved in their learning, that helps them to mathematical proficiency towards solving algebraic word problems (Anthony & Hunter, 2017; Rumsey & Langrall, 2016). The role of the teacher in the discourse-based mathematics instruction is to facilitate learners' participation by posing questions, and listening to and monitoring learners' discourse

(Pourdavood & Wachira, 2015). NCTM (2000) describes the role of the teacher as: 'The teacher of mathematics should orchestrate discourse by posing questions and tasks that elicit, engage, and challenge each learners' thinking; listening carefully to learners' ideas; asking learners to clarify their ideas orally and in writing' (pp. 46-56). A teacher's role is to be active in a different way from that in traditional classroom discourse. Instead of doing virtually all the talking and explaining alone, teachers must encourage and expect learners to do so. Teachers must do more listening and learners more reasoning. For discourse-based mathematics instruction to develop learners' mathematical proficiency towards solving algebraic word problems, teachers need to orchestrate the lesson carefully (Legesse et al., 2020).

Luneta and Legesse (2023) reported on the effectiveness of discourse-based mathematics instruction on Grade 11 learners' proficiency in the syllabus topics of algebra and function. This study employed a quasi-experimental study design in an attempt to investigate the efficacy of using discourse-based mathematical instruction as an instructional approach to develop mathematical proficiency in algebra unit topics. The study took place in Ethiopia and the 106 six Grade 11 learners who participated in the study were randomly grouped into an experimental group (52 learners) and a control group (54 learners). Learners in the experiment group were taught algebra using discourse-based mathematics instruction, while learners in the control group were taught algebra using teacher-centred instruction. The results of the study indicate that learners who were taught algebra using discourse-based mathematics instruction demonstrated better performance in attainment of mathematics proficiency than those who were taught the same topics by way of teacher-centred instruction. I agree with the preceding statement because learners who are engaged in teacher-centred instruction received information without being actively involved in their learning. In addition, teachers are the ones who takes full responsibility of the lesson. In contrast, learners who are engaged in discourse-based mathematics instruction become actively involve in their learning. They engage with the course material through activities, discussion or problem solving. The teacher and learners take

equal responsibility of the lesson (Cookson, 2017). I believe that discourse-based mathematics instruction is important to develop learners' mathematical proficiency. However, in South African context, teachers do not have adequate time to implement this approach as they have to finish the syllabus on time (Taole, 2015).

Hong and Choi (2011) conducted a study to investigate the effects of discourse-based mathematics instruction on learners' mathematical attitudes and learning achievements. They provide fifth grade learners with an opportunity to take an active part in learning during mathematics classes and applied discourse-based mathematics instruction, which expanded the speaking experiences as the most fundamental way to express ideas in communication. The findings of the study led to the following results: First, the discourse-based mathematics instruction turned out to have positive influences on flexibility, will power, curiosity, reflection and value of mathematical attitudes. When the results were reviewed before and after the instruction, without considering the sub variables of attitude, there were statistically significant differences ($p < 0.01$), which indicates that the discourse-based mathematics instruction exerted positive effects on learners' mathematical attitudes. Second, there were no statistically significant differences in learning achievement between the experimental and comparative group, but the experimental group, which recorded low mean scores in the pretest, increased their mean scores by 3.81 points in the post-test, which suggests that the discourse-based mathematics instruction had positive influences on them. Third, the subjects' responses to the questionnaire on discourse-based mathematics instruction revealed that the discourse-based mathematics instruction provided them with an opportunity to explore solutions in various ways (Legesse, 2021). In short, discourse-based mathematics instruction had a positive influence on mathematical attitudes and were effective in increasing communication ability towards learners' learning of mathematical proficiency (Legesse, 2021; Yimam & Dagnew Kelkay, 2022).

Bradford (2007) explored the use of student discourse as an instructional strategy for improving low achieving mathematics students'. The findings of this quasi-experimental study, with a treatment–control group design, indicated that student discourse improved mathematics achievement and problem-solving skills in pre-algebra classes among low-achieving high school students (Bradford, 2007). Cross (2009) conducted a quasi-experimental study with a pretest–post-test design to examine the effects of engagement in discursive practices of mathematical argumentation and writing on ninth grade students' mathematical achievement in algebra topics. The study found that students who engaged in mathematical argumentation and writing activities outperformed the control group students, who received the traditional form of instruction (Cross, 2008). A quasi-experimental study by Legesse et al. (2020) examined the effects of discourse-based mathematics instruction on the 11th grade students' mathematical understanding of probability and statistics topics in an Ethiopian secondary school. Legesse et al. (2020) found that students who engaged in mathematical discourse practices of explaining, agreeing and disagreeing, questioning, comparing, and justifying gained a better understanding of probability and statistics concepts and procedures compared to those students who were taught the same topics using the traditional lecture method. Sepeng and Webb (2012) explored the impact of discussion-based teaching on ninth grade students' problem-solving performance. It was found that the discussion-based teaching strategy significantly improved the experimental group's word problem-solving skills when compared to the control group, who were taught using the traditional lecture method (Sepeng & Webb, 2012). Solving word problems allowed students to internalise the mathematical knowledge and understanding they developed during classroom instruction by applying them to real-life contexts (Sepeng & Webb, 2012). Star, Rittle-Johnson and Durkin (2016) found that comparing and explaining multiple solution methods for solving multistep linear equations facilitated the development of mathematical understanding by enabling students to explore the efficiency and accuracy of different solution methods. For example, a comparison of the same algebra problem solved correctly and incorrectly

enhanced the construction of procedural and conceptual knowledge, and avoided common errors (Star et al., 2016).

2.4 Algebraic Word Problems

Algebraic word problems are verbal descriptions of problem situations, typically presented in text, whereby learners need to be in a position to translate or represent the problem mathematically (Carotenuto et al., 2021). Jupri and Drijvers (2016) viewed algebraic word problems as comprising far more than facts, routines and strategies. They further allude to the notion that they include a vast array of interrelated mathematical concepts, ways to represent and communicate those concepts, and tools for solving all kinds of mathematical problems. Moreover, solving algebraic word problems requires reasoning and creativity, providing learners with mathematical knowledge, while also laying a foundation for further studies in mathematics and other disciplines. Wenger (1992) defines algebraic word problem-solving as a process by which learners experience the power and the usefulness of mathematics in a real-life situation. More importantly, algebraic word problems help learners to develop the ability to compute and recognise the need for applying a concept or procedure (Jupri & Drijvers, 2016). Reikerås (2009) alludes to the fact that in order to solve algebraic word problems, calculations are embedded in text, which learners struggle to depict. I share this sentiment, since learners struggle to pick up the calculations that are embedded in algebraic word problems when they are presented with algebraic word problems to solve. Consequently, learners need to develop the five strands of mathematical proficiency so that they are able to solve algebraic word problems effectively and successfully (Sharp & Shih Dennis, 2017).

The use of mathematics in solving algebraic word problems has become widespread, especially because of the increasing computational power of digital computers and computing methods, both of which have facilitated the handling of lengthy and complicated problems (Denning & Tedre, 2019). The process of translating algebraic word problems into a mathematical form can give a better

representation of and solution to certain problems (Nashiru et al., 2018). The process of translation is called mathematical modelling.

Haines and Crouch (2007) describe mathematical modelling as a cyclical process in which algebraic word problems are translated into mathematical language, which is solved within a symbolic system. Furthermore, Corrêa (2021) indicated that mathematical modelling affords learners the opportunity to solve algebraic word problems as they translate algebraic word problems into mathematical language. More generally, mathematical modelling is meant to help learners to better understand algebraic word problems, support mathematics learning (motivation, concept formation, comprehension and retaining), contribute to the development of various mathematical competencies and appropriate attitudes, and contribute to an adequate picture of developing learners' mathematical proficiency (Afram, 2019).

A review of literature illustrates what is entailed to solve algebraic word problems in South African contexts. Sepeng and Sigola (2013) stated that 'algebraic word problems form part of the South African mathematics curriculum and are used as a vehicle to teach learners how to model problems in primary mathematics classrooms' (p. 325). These researchers agree that solving algebraic word problems in mathematics is part of a unit of the text comprising a question and speech, accompanied by an authentic background story, and the syntactical and rhetorical structure that needs to be explicitly clear to enhance understanding. Algebraic word problem-solving is evident when learners are given a problem to solve and the solution of the problem is not immediate (Polya, 2004). Moreover, the current study used algebraic word problems to develop learners' mathematical proficiency, namely conceptual understanding, procedural fluency, strategic competency, adaptive reasoning and productive disposition. For learners to be proficient in mathematics, all strands of mathematical proficiency should be developed, since they are interwoven (Kilpatrick et al., 2001).

Pape and Wang (2003) raise the idea that problem-solving begins when learners read the algebraic word problems text for the solution process that leads to success as the schemas. Furthermore, Pape and Wang (2003) highlight the notion that successful word problem-solvers transfer the text in algebraic word problems to form schemas and evoke internal representations of the problem by using either concrete or semi-concrete external representations to aid in the constructive process. These authors also argue that learners who are less successful problem-solvers do not form cognitive representations, but often translate the problem elements directly to the solution, without creating an image of the problem. In fact, the reading of the text and analysing the problem stimulate the activation of knowledge structures and mathematical problem-solving strategies, creating the mental representations which are an ongoing process (Pape & Wang, 2003).

One can conclude that solving word problems involves exploring learners' abilities to encode a story sum using procedures, structures, strategies and cognitive representations by reading the text to get to the solutions Nur et al., 2022). As learners experience the process, word problem-solving captures the power and usefulness of mathematics in our context (Kunene, 2019). Learners interpret, solve, evaluate and communicate the situational problem that they engage in. It is the language and the calculations inherent to the word problems that pose difficulties for Grade 4 native English language speakers (learners).

2.5 Problem Posing

Problem posing is an effective mathematical activity for improving learners' attainment mathematical proficiency towards solving algebraic word problems (Ulandari et al., 2019). The process of problem posing afford learners an opportunity to understand the complexities of mathematical concepts and the opportunity for learners to interact with their teacher and their peer as they are engaged in meaningful learning of mathematics (Priest, 2009). In addition, problem posing transfers learners' understanding to their concrete knowledge,

applying this knowledge to their posed algebraic word problems (Bernardo, 2001; Mestre, 2002). The literature indicated that this development of concrete, comprehensive mathematical knowledge through problem posing can influence learners' development of mathematical proficiency towards solving algebraic word problems (Van Harpen & Presmeg, 2013). For these reasons, NCTM (2000) and the National Research Council (NRC) (2005) stated that problem posing needs to be incorporated into curriculum to foster learners' development of mathematical proficiency. However, the criteria for categorising problem-posing types has, historically, been highly dependent on the individual preferences (Brown & Walter, 2014). Furthermore, problem posing needs to be strategic and systematic. In his study, Stickles (2006) referred to problem posing as the act of formulating an algebraic word problem. Moreover, problem posing was categorised into two types, namely problem generation and problem reformulation (Pelczer & Rodriguez, 2011; Silver, 1994; Stickles, 2006). Problem generation and problem reformulation differ in their processes of formulation from the start of each respective algebraic word problems. As a result, it was necessary to examine them separately in the current study.

The primary educational purposes of teaching and learning problem generation and problem reformulation are different. Problem generation is a process by which a learner creates a new problem on their own, using a given situation (Stickles, 2011). The given situation in problem generation is more informal than the given situation in problem reformulation. For example, asking learners to formulate problems based on one rectangle-shaped picture could be a problem-generation activity. This informal given situation leads learners to consider their personal experiences, and the learners have the opportunity to connect mathematics with these experiences and real-world situations (Silver, 1994). In the process of applying mathematical knowledge, learners can structuralise their existing schema of mathematical concepts to incorporate new conceptual information and increase their creativity (Grundmeier, 2003). Problem reformulation is similar to problem generation in that the learner can structuralise concepts and increase their creativity in the process of analysing and utilising the

given mathematics problems; however, problem reformulation necessitates a greater focus on reflection of the existing problem (Lee, 2012). A given situation in problem reformulation is also referred to as a given problem. In other words, the use of problem generation primarily offers an opportunity to develop mathematical proficiency towards solving algebraic word and also allows learners to develop their reflective skills.

2.6 Challenges when Solving Algebraic Word Problems

Algebraic word problem-solving remains a difficult concept to understand and to teach because of its dependency on learners' competence in a variety of cognitive skills. Prospective learners are required to possess rich vocabulary, the ability to comprehend the problem and a way to decide which strategy to use to solve the problem (Raoano, 2016). Sepeng and Sigola (2013) explain that learners fail to respond to algebraic word problems simply because they are not familiar with the language used in the question. Sibanda (2017) stated that Grade 4 learners still face challenges in solving algebraic word problems because they lack mathematical vocabulary, which is informed by the transition from Grade 3 to Grade 4. She also mentions that learners in Grade 3 do mathematics in their mother tongue language, which becomes a major problem when they have to change from mother tongue language to the English language as a medium of instruction. Mathematical vocabulary refers to words that label mathematical concepts, for example, quotient, volume, vertex, dividend and hexagon (Sanders, 2007). Grade 4 mathematics learners are likely to be handicapped in their effort to learn mathematics if they do not understand the vocabulary that is used in mathematics classrooms, textbooks and assessment tests (Chinn, 2020). One of the obstacles that make mathematical vocabulary difficult to learn is a lack of opportunity to engage in solving algebraic word problems during instructional practice by instructors (Leavy & Hourigan, 2020). This is because much of the vocabulary used in the mathematics classroom is rarely encountered in everyday life. In addition, mathematics teachers often neglect meaningful vocabulary instruction. In addition, many terms have meanings in the realm of mathematics

that differ from their meanings in everyday usage (Monroe & Orme, 2002). Without appropriate vocabulary instruction, learners are likely to experience difficulties and interference in solving algebraic word problems. Learners need to know the meaning of mathematics vocabulary words, whether written or spoken, to understand and communicate mathematics ideas.

According to Sanders (2007), terms, phrases and symbols are essential for communicating mathematical ideas and becoming fluent with them is vital for Grade 4 mathematics learners in order to learn and develop mathematical proficiency. Research reveals that the knowledge of mathematics vocabulary directly affects achievement in arithmetic, particularly in solving algebraic word problems (Leung, 2005). In addition, the teacher needs to be cautious when operating with entities and not separate operations from their mathematical terms, so that they enrich learners understanding of mathematical vocabulary in solving algebraic word problems (Sihlangu, 2022).

Ní Ríordáin and O'Donoghue (2009) indicated that vocabulary knowledge is strongly related to overall academic achievement in school. Although learners may excel in computation, their ability to apply their mathematics skills will be hindered if they do not understand the vocabulary required to master content and able to apply it in future situations. Thus, teaching vocabulary in the mathematics content area is a critical element of effective instruction. Although mathematics is a visual language of symbols and numbers, it is expressed and explained by means of written and spoken words. Consequently, for learners to proficient in mathematics, they must recognise, comprehend and apply the requisite mathematical vocabulary. Teaching mathematical vocabulary words solely as definitions, as is the practice in South African schools, does not assist learners to comprehend the word when found in mathematics textbooks and examination items (Setati et al., 2002). Learners must be actively engaged in building background knowledge using key content-specific vocabulary. Therefore, it seems proper to suggest that mathematics teachers should use appropriate strategies/methodologies that could incorporate mathematical vocabulary into

their teaching to develop learners' mathematical proficiency towards solving algebraic word problems.

2.7 Teacher Knowledge to Develop Mathematical Proficiency

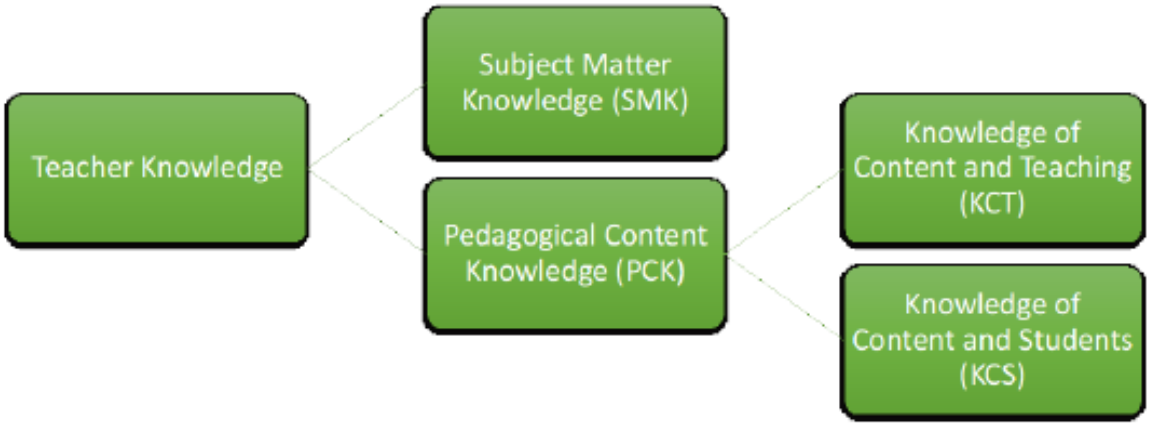


Figure 4: Theoretical framework of teacher knowledge (Even, 1993, p. 94)

Teacher knowledge consists of both subject matter knowledge (SMK) and PCK. These are the basics requirements of the teachers' instructional practices in their classroom in order to develop learners' mathematical proficiency towards solving algebraic word problems. SMK has been defined as 'emphasizing knowledge and understanding of facts, concepts, and principles and the ways in which they are organised, as well as knowledge about mathematics' (Even, 1993, pp. 94-116). Researchers have found that insufficient SMK among teachers has led their learners developing misconceptions, misunderstandings and misinterpretations regarding the development of mathematical proficiency during instructional practice (Valanides, 2000). In addition, teachers who possess higher SMK have more favourable attitudes towards developing learners' mathematical proficiency during their instructional practice than those who lack or have limited SMK (Barlow & Cates, 2006; Quinn, 1997). These findings could indicate that SMK affects not only teachers' teaching practice but also their PCK (Ozden, 2008). Researchers have determined that teachers' PCK is highly related to learners' development of mathematical proficiency (Darling-Hammond, 2000). Therefore,

teachers must develop and maintain sufficient and comprehensive SMK for the betterment of their learners' development of mathematical proficiency towards solving algebraic (NCTM, 2000; Rizvi, 2004; Schmidt et al., 2009).

PCK differs from SMK in the sense that it plays a significant role in characterising and identifying teachers' knowledge regarding the difficulty experienced by their learners with subject matter and the ability to connect mathematical ideas, use examples, provide explanations and apply strategies when encountering mathematical concepts (Wagner et al., 2007). Researchers found that PCK factors, such as teachers' pedagogical preparation, their use of routines in the classroom and their degree content coverage, influenced learners' development of mathematical proficiency (Rowan et al., 2002). Many researchers have attempted to find relationships between teacher knowledge and learners' mathematical academic achievement (Wilson et al., 2001) and have tried to connect PCK to student learning (Ball et al., 2005). Shulman (1987) introduced the term PCK and defined it as a 'special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding'. Since Shulman's (1987) demonstration of the importance of teacher knowledge, attention to teacher knowledge has shifted over time from an analysis of SMK to PCK (Loewenberg Ball et al., 2008). This shift to PCK research has come with the development of various categorisations of PCK.

The subscales of PCK have been categorised differently depending on the researchers analysing them. Since Shulman (1987) introduced PCK, many researchers in mathematics education have described the subscales of PCK by extending Shulman's (1987) concept. Lannin et al. (2013) categorised PCK's subscales for teaching mathematics as teachers' knowledge of the following: curriculum for mathematics; assessment for mathematics; instructional strategies for mathematics; and student understanding within mathematics. In contrast, Hauk et al. (2014) considered PCK to include knowledge of discourse, curricular thinking, anticipatory thinking and implementation thinking. Hill et al. (2008) provided a categorisation of both SMK and PCK subscales, dividing the concept

of SMK into common content knowledge, knowledge at the mathematical horizon and specialised content knowledge; and PCK into knowledge of curriculum, knowledge of mathematical content and learners (KCS), and knowledge of mathematical content and teaching (KCT) (Loewenberg Ball et al., 2008). Although there are several variations of PCK, for the purposes of this study, the researcher used the definition of PCK, its subscales (KCT and KCS), and SMK as provided in the work of Hill et al. (2008). In particular, the researcher primarily focused on two components of PCK, namely, KCS and KCT, in conjunction with SMK, to determine teachers' proficiency in problem posing in relation to these branches of teacher knowledge.

2.8 Teacher Knowledge in Problem Posing

Teachers' competency in and knowledge of using and teaching problem posing are influential factors that can influence learners' conceptual understanding in problem posing. Teachers should have the capacity to generate and reformulate algebraic word problems so as to provide relevant activities for learners' learning. It is critical for teachers to acquire problem posing experiences if they intend to provide new and different forms of learning experiences to their learners (Crespo & Sinclair, 2008; Rowland et al., 2003; Singer & Voica, 2013). Teachers were able to identify learners' mathematical misconceptions by means of problem posing as well as posed problems from the learners (Koichu et al., 2013). Teachers may benefit from problem-posing activities during their professional development experience to develop SMK (Barlow & Cates, 2006). Recognising teachers as critical agents in problem-posing activities for learners is important (Silver, 1994). According to my observation, teachers seem to lack the capacity to generate and formulate algebraic word problems. As a result, they rely on textbooks and previous question papers. They do not strive to generate and formulate algebraic word problems on their own in order to motivate learners to generate and formulate their own algebraic word problems.

Thus, proper training for the teachers is critical. Despite of the importance of the teacher's role, teachers have a narrow concept of problem posing. Since many teachers did not experienced problem posing when they were learners or preservice teachers, they tend to avoid problem posing or provide routinised exercises (Ball, 1990). Crespo (2003) indicated that teachers posed problems that were predictable, undemanding, ill-formulated and unsolvable when they extended a given problem. Most of the posed problems focused on memorisation and procedural understanding rather than on mathematical reasoning and conceptual understanding (Stein et al., 2009). It is, therefore, important to broaden teacher knowledge in problem posing.

Teachers' lack of teacher knowledge can negatively influence not only the quality of the problem-posing activity itself but also learners' problem-posing skills and results (Cankoy & Darbaz, 2010; Rowan et al., 2002; Wilson et al., 2001). Despite the importance of teacher knowledge in problem posing, several teachers have difficulties with building and implementing their SMK and PCK in problem posing. Therefore, the purpose of the study was to explore Grade 4 mathematics teachers' development of learners' mathematical proficiency towards solving algebraic word problems that incorporate the teacher knowledge of SMK and PCK (KCT and KCS) in developing learners' mathematical proficiency. The findings of this study could provide researchers and educators with a better understanding and assessment of teachers' knowledge in relation to developing learners' mathematical proficiency.

2.9 Studies on Mathematical Language

The process of problem-solving, discovery, applying mathematics in real-life contexts and identifying mathematical context all point to the fact that mathematics, as a discipline, has its own language (Wilkinson, 2018). Mathematical language is characterised by its own distinct linguistic and discourse repertoire for cultivating 'reading, writing, speaking, thinking, and reason about the disciplinary content. This embraces precise use of vocabulary,

symbols, representations of mathematical knowledge, argumentation and proof' (Wilkinson, 2018, pp. 167-174). Mathematical language comes with specialised vocabulary, such as irrational numbers, specialised ways of presenting arguments, such as justifications and reaching a consensus, and dense noun phrases, such as area under a curve. Hence, learners need to learn a mathematical language that is precise and technical by integrating the language of the mathematical knowledge with mathematical procedures (Wilkinson, 2018). In the same vein, Sarabi and Gafoor (2017) argue that learners develop an attitude of hating mathematics if they do not succeed with the pragmatics of the mathematics language. The authors further state that mathematics language is characterised by terminology, symbols and syntactic principles that are essential components of the learning and teaching of mathematics. For learners to be proficient in mathematics, there is a need for them to be proficient in mathematical vocabulary, so that they are able to understand algebraic word problems and also develop relevant strategy to solve them.

Leshem and Markovits (2013) define mathematics as a language of science that is also used every day to communicate and describe everyday situations. Mathematical language also means explaining to make sense of problems in mathematics. Mathematical language is the language of using symbols notations and numbers as in words and sentences that form a mathematical story, which is the mathematical grammar that requires solutions. Similarly, mathematical language can be defined as a language of symbols, concepts, definitions and theorems (Ilany & Margolin, 2010). In word problem-solving, the learner is faced with two languages that are mixed together when finding the solution to a mathematical problem, all accompanied by text; the two languages being the natural language and mathematical language. Mathematical language requires the learners to be aware of the mathematical components while natural language requires the learner to have a literacy approach to the whole textual unit (Ilany & Margolin, 2010). Ilany and Margolin (2010) further contend that mathematical language is more precise and less flexible as it brings in a deep structure of

statements that are unambiguous, such as fractions and numbers, and equality and inequality signs.

Jamison (2000) shares a very interesting observation on the language of mathematics, namely that it is non-temporal, implying that there is no past, present or future. However, mathematical language makes use of learning tools that are the same as those in any language class, such as writing, speaking, listening, memorising models and learning the history and culture. In mathematics language, one starts by learning the definitions in the communication discourse, then a definition–theorem–proof format where learners apply procedures, and claim, justify and reach consensus (Jamison, 2000). In the same vein, Lager (2006) defines mathematical language as contextualised by interactions that learners work on, related to algebraic word problems. The mathematics register language acquisition framework requires learners to translate problem situations from everyday language into algebraic expressions. Mathematical language is linked to the rational constructivist mathematics learning framework where the learner integrates new information into existing cognition to acquire higher-order concepts. The rational constructivists believe that, while some mathematical knowledge can be transmitted, sometimes the abstraction of a concept still has to be achieved by the learner (Lager, 2006).

Mathematical language requires learners to ‘be able to read long words included in the mathematical vocabulary (such as equation) and also to interpret grammatical constructions representing mathematical relations (double the amount of money), even if they are more unusual in colloquial language’ (Bergqvist et al., 2018, pp. 41-55). The authors highlight the importance of reading ability as it is always needed to solve a written task. The students need to be able to read the language in the mathematics to solve algebraic word problems (Bergqvist et al., 2018). Galligan (2016) recommends features of mathematical language as the mathematics register, language in the classroom and technical communication. The mathematics register is the etymology of the words, phrases, syntax, semantics and associated meanings used to express

mathematical ideas. The language in the classroom is the mathematical language that is used to communicate ideas and dialogue that learners use to communicate when developing mathematical proficiency, particularly when solving algebraic word problems; while technical communication is the oral and written form of the accepted standard language and symbols used to communicate the ideas (Galligan, 2016). Since mathematical concepts are often encapsulated in words (language), an adequate grasp of the correct terminology in mathematics is crucial. Accordingly, Galligan (2016) asserts that it is important to have the correct word for a concept because it provides immense assistance in the understanding of that concept. Furthermore, teachers need to use the correct mathematical vocabulary when developing learners' mathematical proficiency, particularly when solving algebraic word problems in schools located in rural and semi-rural areas. This could elicit learners' interest in doing mathematics in their future studies.

Hornburg et al. (2018) established that mathematical language is critical for development of numeracy skills, which are important for later academic achievement. These authors define mathematical language from a numeracy perspective; learners' understanding of key words in mathematics that include quantitative words such as 'more' and 'less'; ascending and descending order; increasing and decreasing with their associate mathematical operations; and spatial words such as 'before' and 'after' (Hornburg et al., 2018). They further state that mathematical language mediates the relationship between general language and early numeracy skills. In word problem-solving, learners must first comprehend the problem before they select the correct operation to execute the calculation. Hence, mathematical language influences the learners' performance in algebraic word problem-solving. For learners' comprehension of the word problem, they need to think about the question that the word problem is asking, find and understand the relevant/irrelevant information in the algebraic word problem and collaborate with other students to find problem-solving solutions (Orosco & Abdulrahim, 2018).

In algebraic word problem-solving, a language of the discipline, which is what mathematical language is, has been regarded as a gateway to developing learners' mathematical proficiency during the instructional process (Kunene, 2019). The linguistic, disciplinary content, mathematical procedures, the definition–theorem–proof format and mathematical register are all within the rational constructivist mathematics learning framework. When learners construct their knowledge as they learn during algebraic word problem-solving, all the above listed constructs become the gist of the text and the talk that is involved in the process. The above collection of constructs confirms the necessity for mathematical language to drive the discourse of developing learners' mathematical proficiency towards solving algebraic word problems in lower grades within the mathematics discipline.

2.10 Use of Formative Assessment

The DBE (2011) define assessment as a continuous planned process of identifying, gathering and interpreting information about the performance of learners using various forms of assessment. Hill and Barber (2014) define assessment as the process to establish what learners know and can do. Assessment is generally classified into two broad categories, namely assessment designed to support teaching and learning in classrooms and assessment programmes for public reporting, certification, for selection and as system accountability. Furthermore, Lubisi (1999) states that assessment entails making sense of a learner's knowledge and skills. In addition, assessment should be both informal or formative (assessment for learning) and formal or summative (assessment of learning). Evans et al. (2014) alluded to the notion that assessment can be formative, enabling a learner or teacher to determine whether the responses meet specific criteria; it can be diagnostic, enabling at least an initial identification of strengths and potential areas of learning difficulty; and it can be used to provide guidance provide effective feedback. Assessment can also motivate learners by way of positive feedback from the teacher, which can help learners to develop mathematical proficiency towards solving algebraic word

problems (Verschaffel et al., 2020). In both cases, regular feedback should be provided to learners so as to enhance their learning experiences in the process of teaching and learning mathematics particularly when learners are solving algebraic word problems (DBE, 2011).

Formative assessment entails daily monitoring of learners' progress (DBE, 2011). This is done using homework, classwork, tests and experiments. Formative assessment should be structured in such a way that it develops learners' mathematical proficiency towards solving algebraic word problems. The Curriculum and Assessment Policy Statement (CAPS) document emphasises the use of formative assessment to provide feedback to learners and to inform teacher planning. Teachers and learners can mark these formative assessment tasks during the instructional practice.

Both self-assessment and peer assessment are important as they actively encourage learners to learn from each other as they engage in meaningful mathematics and reflect on their own performance (Ratminingsih et al., 2017). Self-assessment affords learners an opportunity to develop the crucial skill for learning and for future employment. It encourages learners to take responsibility for their own improvement and it serves as a gateway for the learning of mathematics by learners. Learners should be motivated by a desire to be proficient in mathematics, explore, develop and to improve themselves, but not through fear of failure (Henry et al., 2019).

Formal assessment tasks, on the other hand, are marked and officially recorded by the teacher. All formal tasks are subject to moderation for the purposes of quality assurance and to ensure that appropriate standards are maintained. Formal assessment provides teachers with a systematic way of evaluating how well learners progress. In addition, formal assessment is viewed as a dependent of mathematical content coverage (Shute & Rahimi, 2017). Summative (or formal) assessment provides a grade, which contributes to the final mark used for promotion purposes and to determine whether learners are ready to go to the

next grade. To address variety in the different forms of assessment, the DBE (2011) stipulates that application exercises that include problem-solving activities should be done at all cognitive levels, in all knowledge areas and on all mathematics concepts.

Teachers should assess the full range of cognitive abilities of learners by way of exercises and formative assessment that address learners' attainment of mathematical proficiency. During instructional practice, teachers should provide learners with at least three algebraic word problem activities on a frequent basis, such that learners' attainment of mathematical proficiency can be guaranteed (DBE, 2011). The Norms and Standards for Teacher Education, which set out the design and delivery of teacher education in South Africa, emphasise the need for teachers to effectively and efficiently perform the role of assessor (DBE, 2011). The terms of this role requires teachers to understand that assessment serves as an essential feature of the teaching and learning process. Furthermore, this understanding should be demonstrated by integrating formative assessment into teaching and learning as a matter of course. The purpose of formative assessment in the current study was to help teachers to plan the lesson and support learners' development of mathematical proficiency towards solving algebraic word problems.

2.11 Theoretical Framework

Kilpatrick et al. (2001) theory of mathematical proficiency guided this study. There are five intertwined strands of mathematical proficiency, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. For learners to be successful in mathematics, they are expected to develop all the strands of mathematical proficiency and be able to apply them, even when solving algebraic word problems (DBE, 2018). Figure 5 shows intertwined strands of mathematical proficiency.

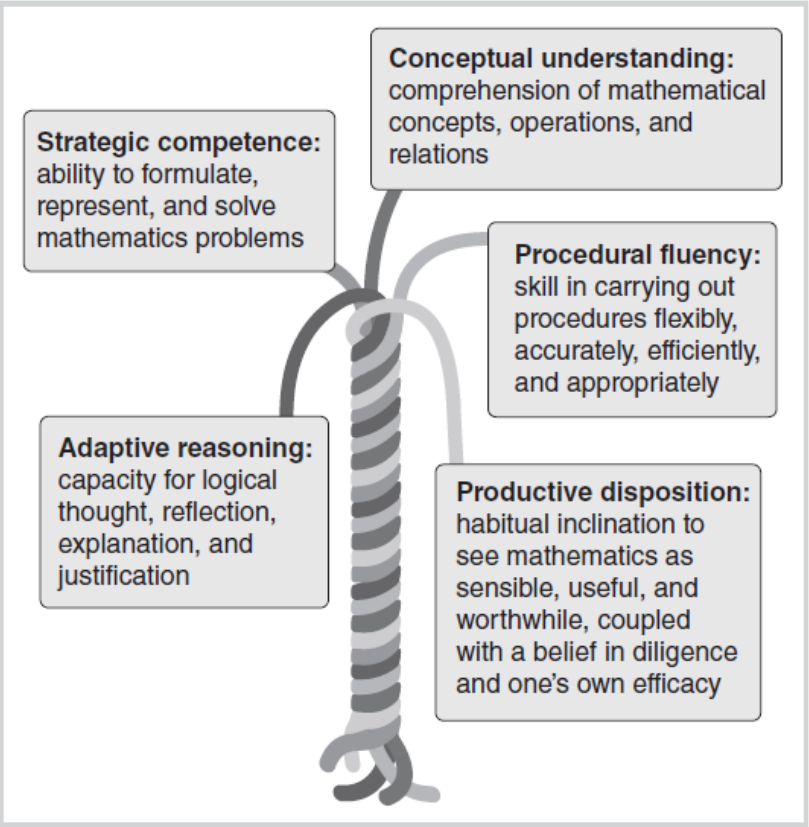


Figure 5: Strands of mathematical proficiency (Kilpatrick et al., 2001, p. 117)

Conceptual understanding is described as the comprehension of mathematical concepts, operations and relations (Kilpatrick et al., 2001). A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. To find one's way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar and how they are different. The degree of learners' conceptual understanding is related to the richness and extent of the connections they have made. Teachers should help learners to comprehend their ideas and relate them from what they known to unknow.

Procedural fluency is described as the skill of being able to carrying out procedures flexibly, accurately, efficiently and appropriately. In the domain of numbers, procedural fluency is especially needed to support conceptual understanding of place value and the meanings of rational numbers. It also

supports the analysis of similarities and differences between methods of calculating. These methods include, in addition to written procedures, mental methods for finding certain sums, differences, products or quotients, and methods that use calculators, computers or manipulative materials such as blocks, counters or beads. Learners need to be efficient and accurate in performing basic computations with whole numbers, without always having to refer to tables or other aids. They also need to know reasonably efficient and accurate ways to add, subtract, multiply and divide multidigit numbers, both mentally and with pencil and paper. After teachers have taught learners to make sense of the algebraic word problems, learners should be expected to carry out the procedures of solving algebraic word problems appropriately.

Strategic competence is described as the ability to formulate, represent and solve word problems. This strand is similar to what has been called problem-solving and problem formulation in the literature of mathematics education and cognitive science, and mathematical problem solving, in particular, has been studied extensively. Although in school, learners are often presented with clearly specified problems to solve outside of school, where they encounter situations in which part of the difficulty is to figure out exactly what the problem is. Then they need to formulate algebraic word problems so that they can use mathematics to solve the problems encountered. Consequently, they are likely to need experience and practice in problem formulating and in problem-solving. They should know a variety of solution strategies and which strategies might be useful for solving a specific problem. Teachers should help learners to know multiple way and strategies in which to solve algebraic word problems. In addition, learners also need to know the relevant strategies to be used in solving a particular algebraic word problems (Zubainur, 2020).

Adaptive reasoning is realised as a capacity for logical thought, reflection, explanation and justification. Such reasoning is correct and valid, stems from careful consideration of alternatives and includes knowledge of how to justify the conclusions. In mathematics, adaptive reasoning is the glue that holds everything

together; the lodestar that guides learning. One uses adaptive reasoning to navigate the many facts, procedures, concepts and solution methods, and to see that they all fit together in a way that they make sense. In mathematics, deductive reasoning is used to settle disputes and disagreements. Answers are correct because they follow from some agreed upon assumptions by way of series of logical steps. Learners who disagree with a mathematical answer need not rely on checking this with the teacher, collecting opinions from their classmates or gathering data from outside the classroom. In principle, they need only check that their reasoning is valid. Teachers should develop learners' adaptive reasoning so that, after the learners have solved a given algebraic word problem, they should be able to defend their solution.

Productive disposition is observed as a habitual inclination; as such, teachers should encourage learners to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's efficacy. If learners are to develop conceptual understanding, procedural fluency, strategic competence and adaptive reasoning abilities, they must believe that mathematics is understandable, not arbitrary; that, with diligent effort, it can be learned and used; and that they can figure it out. Developing a productive disposition requires frequent opportunities to make sense of mathematics, to recognise the benefits of perseverance and to experience the rewards of sense making in mathematics. Kilpatrick et al. (2011) recommend that teachers should help learners to develop strands of mathematical proficiency and use them simultaneously, since they are intertwined, that is, the five strands do not work in isolation. In this study, Kilpatrick et al.'s theory will be used as a lens to guide how Grade 4 mathematics teachers develop learners' mathematical proficiency for solving algebraic word problems.

CHAPTER 3: RESEARCH METHODOLOGY

In this chapter I present the research methodologies that can be used in social sciences research. The methodology used for this study is discussed. Additionally, this chapter deals with the research paradigm that guided data collection. Thereafter, I describe the research design, sampling, data collection methods and how the data were analysed. Lastly, the quality criteria and ethical considerations which were considered when the study was conducted are clearly outlined.

This study has followed a qualitative research approach. The qualitative research approach is the approach that is framed this study in terms of using words to engage learners in discourse-based mathematics instruction that could help learners develop mathematical proficiency towards solving algebraic word problems. Qualitative methods are explanatory and textual, and include passive observation, participant observation and open-ended interviews or an analysis of participants (in this case, a Grade 4 mathematics teacher and his learners) (Risjord et al., 2001). A qualitative research approach was suitable since the study considered the thoughts, feelings, experiences, meaning of language and processes of individuals and groups within the classroom settings (Lopez & Whitehead, 2013). The study aimed to explore how Grade 4 mathematics teacher develop learners' mathematical proficiency when solving algebraic word problems. As a result, the qualitative research approach was appropriate because this study explored the proposed research title in depth (Carlsen & Glenton, 2011).

Kuhn (1962) first used the word paradigm to mean a philosophical way of thinking. The word has its aetiology in Greek where it means pattern. In educational research the term paradigm is used to describe a researcher's 'worldview' (Mackenzie & Knipe, 2006). This worldview is the perspective, thinking, school of thought or set of shared beliefs that inform the meaning or interpretation of research data. As Lather (1986) explains, a research paradigm inherently reflects the researcher's beliefs about the world that they live in and

want to live in. It constitutes the abstract beliefs and principles that shape how a researcher sees the world and how they interpret and acts within that world. Kivunja and Kuyini (2017) stated that 'When we say a paradigm defines the researcher's worldview, we mean that a it constitutes the abstract beliefs and principles that shape how a researcher sees the world and how they interpret and acts within that world' (pp. 26-41). It is the lens through which a researcher looks at the world. As a result, it is of paramount importance for the researcher to locate their school of thought when conducting a research Guba and Lincoln (1994) define a paradigm as a basic set of beliefs or worldviews that guide research action or an investigation. Similarly, Denzin et al. (2006) define paradigms as human constructions that deal with first principles, or the ultimate indication of where the researcher is coming from, so as to construct meaning embedded in data. Paradigms are thus important because they provide beliefs and dictates that, for scholars in a particular discipline, influence what should be studied, how it should be studied and how the results of the study should be interpreted. The paradigm defines a researcher's philosophical orientation.

Merriam's perspective that orientates qualitative case study is constructivism. She maintains that 'the key philosophical assumption upon which all types of qualitative research are based is the view that reality is constructed by individuals interacting with their social worlds' (Merriam, 1998, p. 6). Similarly, she comments 'that reality is not an objective entity; rather, there are multiple interpretations of reality' (Merriam, 1998, p. 22). Therefore, espousing this philosophical assumption, the primary interest of qualitative researchers is to understand the meaning or knowledge constructed by people. In other words, what really intrigues qualitative researchers is the way people make sense of their world and their experiences in this world. The current study's epistemological point of view is aligned with Merriams' perspective. I share these same sentiments with this author. For learners to be able to develop mathematical proficiency towards solving algebraic word problems, they should be able to construct knowledge and make sense of the world around them.

3.1 Research Design

This study adopted a qualitative case study design, as proposed by Merriam (1998). She highlighted the fact that a case can be a person, a programme, a group of people (teachers), a specific policy and so on, which represents more comprehensive data. This study followed the case of a teacher and his respective learners to come up with a rich comprehensive data. Furthermore, Merriam also emphasises the unique distinctive attributes that make it suitable for a case study. For example, she talks of a) particularistic, which means focuses on particular situation, event, programme or phenomenon being researched; b) descriptive, which yields a rich, thick description of the phenomenon under the current study; and c) heuristic, which illuminates the reader's understanding of phenomenon under study (Merriam, 1998). The unique distinctive attribute that was suitable for this study was descriptive, which refers to the case of a Grade 4 mathematics teacher developing learners' mathematical proficiency when solving algebraic word problems to generate a rich description of the phenomenon research on (Merriam, 1998).

3.2 Sampling

Convenience sampling is non-probability sampling that is often used in qualitative research (Pace, 2021). This sampling technique often selects participants who are available with regard to location (such as a school in educational research) (Stratton, 2021). Merriam's (1998) case study approach supports non-probability sampling. Non-probability sampling is well suited for exploratory research intended to generate new ideas that could be systematically tested later (Alvi, 2016). This study used convenience sampling because it allowed for selecting a few participants who are conveniently available with regard to access and location (Lopez & Whitehead, 2013). The study sampled one Grade 4 mathematics teacher and 167 Grade 4 mathematics learners. Of the 167 learners, only 79 were permitted by their parents to take part on the study. The teacher and the learners were conveniently selected with regard to access and

location. As a result, convenience sampling is not costly and neither is it as time consuming as other sampling strategies (Stratton, 2021). The teacher and the learners were from the school where I was employed. As a result, they were easily accessible without any cost and without consuming time. Convenience sampling was appropriate for this study because I was able to access the participants after school hours (sports time). The classes were constituted as follows: class A consisted of 41 learners; while class B consisted of 38 learners. All learners were from a Sepedi-speaking background. This study was located at Mapudithomo primary school, in Kgakatlou Circuit of the Capricorn South District in the Limpopo province of South Africa.

3.3 Data Collection

Data were collected by means of observations, written task and interviews. The three instrument that were used in this study were developed in such a way that could help the teacher to develop learners mathematical proficiency. Section 8.1, 8.2, 8.3, and 8.4 under appendices outline in detailed how the instrument were developed. Before the data collection process commenced, I sat down with the teacher and discussed the learning episodes that were designed by the researcher to help the teacher to develop learners' mathematical proficiency towards solving algebraic word problems. Input from the teacher was welcomed and changes were made to accommodate the thoughts of the teacher. The teacher was also given the opportunity to read about strands of mathematical proficiency, The Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding and discourse-based mathematics instruction, so that he became fully aware of what he needed to do when developing learners' mathematical proficiency. After the reading was done by the teacher, we sat down again and conceptualise what the teacher had read. Thereafter, I gave the teacher the two learning episodes that we had discussed together to teach in the classroom. The teacher was told to engage learners with following question during the process teaching Grade 4 learners to develop mathematical proficiency: 'Explain how you got your answer and which strategy

did you use to get the answer'. The Grade 4 mathematics teacher was expected to develop learners' mathematical proficiency towards solving algebraic word problems while teaching learning episodes 1 and 2.

There were 167 learners given consent form to go and give their parents at home, so that they can give their children permission to participate on the study. Out of 167 consent form that were distributed to learners, only 79 consent form were singe by parents of the learners that permitted them to participate on the study. The learners were divided into two classes. Class A consisted of 41 learners, while class B consisted of 38 learners. Since number of learners was reduced to 79, data collection took place after school hours to allow learners with a signed consent form to attend the lesson. Those whose parents did not sign their consent form did not attend the lesson, they attended sports, since the lesson took place during sports time. Each day, the teacher was able to teach one class during sports time, running for one hour. The whole arrangement gave the teacher an opportunity to reflect on the lesson that he presented before going on to the next lesson. The learners in each class were arranged in groups. Each group had between five and seven group members and all groups were of mixed gender and mixed ability (Mokwana, 2017). Class A had seven groups, while class B had six groups. Altogether, there were thirteen groups of learners. This arrangement afforded learners an opportunity to learn from their peers.

The learning episodes were prepared to target the content area of numbers, operations and relationships. The reason for choosing this content area was that the weight (marks) had been increased to 50 per cent in grades 4, 5 and 6 to ensure that learners acquire adequate numeracy skills before they proceed to the Senior Phase (DBE, 2011). Within this content area, the topic that was covered was on whole numbers, with a particular focus on i) addition and subtraction of four-digit numbers; ii) multiplication of two-digit by two-digit numbers; and iii) division of three-digit numbers by a one-digit number (DBE, 2011). The first learning episode targeted addition and subtraction within the context of finance. The second learning episode targeted addition, subtraction, multiplication and division within a variety of contexts. After the presentation of each learning

episode, learners were given written tasks to respond to. These were used to crosscheck whether learning had taken place. Subsequently, learners' responses to the given tasks were collected and marked, with the intention of later analysing them to identify the challenges they encountered. The marked scripts were given back to the teacher so that he could go through them before giving learners effective feedback. After the feedback was given to the learners, the scripts were given back to me so that I could analyse them. The interview process took place immediately after the completion of the learning episode and the written task. The theory of mathematical proficiency was used to guide the presentation by the teacher so that he would remain within the scope of the study of developing learners' mathematical proficiency towards solving algebraic word problems.

3.3.1 Observations

Lopez and Whitehead (2013) define observation as the process of watching the daily life and the behaviour of the participants in their natural setting in order to record aspects such as social position and function, or actions and interactions.

In this study, I played a role of a participant as an observer. I was acting as both participant and observer. This openness affords researchers an opportunity to create a productive relationship that strives to develop learners' mathematical proficiency towards solving algebraic word problems with the teacher (participant). The teacher allowed me to step in and out of the lesson, as he presented it (Lopez & Whitehead, 2013). In addition, I observed how the teacher facilitated learning; how he interacted with his learners while developing their mathematical proficiency towards solving algebraic word problems involving non-verbal interaction; and, lastly, how he encouraged interactions among the learners. As a participant observer, I occupied multiple positions, which afford me opportunities to move to different locations in the classroom and view events that were occurring inside the classroom from different angles/perspectives (Ciesielska et al., 2018). Observation was relevant to this study as the researcher gained a first-hand picture of what was happening in classroom when the teacher

was developing learners' mathematical proficiency towards solving algebraic word problems (Barrett & Twycross, 2018). The observation was videotaped so that the recording would be available and could be transcribed verbatim. The theory of mathematical proficiency was used to check whether the teacher afforded learners an opportunity to develop conceptual understanding, procedural fluency, strategic competency, adaptive reasoning and productive disposition (Karatas, 2022).

3.3.2 Learners' written task

Grade 4 learners were taught by their teacher during two learning episodes, after each lesson, learners were given a written task to complete. In learning episode 1, learners were given an activity drawn from the context of finance. The activity consisted of three questions. Question 1 had eight sub-questions, that is, question letter A to H. Learners responded to all the questions. However, analysis of results were drawn from question 1 sub-question G and question 2. The assumption was that the two selected questions could yield rich data. Similarly for learning episode 2, there were seven questions, A to G. Questions C and D were used for data analysis with the hope that they would yield rich data. Responses to the tasks given during classroom interactions were collected and marked with the intention of later analysing them to identify the challenges encountered by the learners. The marked scripts were given back to the teacher so that he could give effective feedback to the learners. In addition, learners' written work was relevant for this study because it was used to identify learners' challenges when solving algebraic word problems (Mokwebu, 2013).

3.3.3 Interviews

Jentoft and Olsen (2019) describe an interview as a widely used method for collecting data in social sciences, where the purpose is to reveal other peoples' (participants') views, descriptions and perspectives on the themes that are addressed. While Hockey and Forsey (2020) state that interviews provide a particularly sound basis for gaining insights into peoples' experiences, thoughts and feelings. Interviews provide the most direct and straightforward approach to gathering detailed and rich data regarding a particular phenomenon being studied

(Barrett & Twycross, 2018). The phenomenon explored in this study was how a Grade 4 teacher develops learners' mathematical proficiency towards solving algebraic word problems. A common approach in qualitative research is the semi-structured interview, where core elements of the phenomenon being studied are explicitly asked about by the interviewer (DeJonckheere & Vaughn, 2019). A well-designed semi-structured interview ensures that data are captured in key areas, while still allowing flexibility for participants to bring their own personality and perspectives to the discussion. As a result, this study used semi-structured interviews. The teacher and learners were interviewed separately. The teacher was interviewed to allow him freedom and flexibility when responding to interview questions. In addition, the interview allowed him to bring his personality and perspectives to the fore (Barrett & Twycross, 2018). Learners were interviewed in groups to find out how they experienced the lessons, and whether the lessons were delivered and received as planned. The interviews process took place immediately after the completion of each learning episode and the written task. The duration of an interview session was approximately 30 to 60 minutes. This depended on the teacher and learners' responses to the interview questions (Rabiee, 2004). The theory of mathematical proficiency was used to guide interviews questions. The questions were asked in such a way that they developed the learners' strands of mathematical proficiency. The interviews were audiotaped and later be transcribed verbatim (Merriam, 1998).

3.4 Data Analysis

Merriam (1998) defines data analysis as the process of making sense of data. The proposed steps used when making sense of data involve consolidating, reducing and interpreting data. Using content analysis as an analytic strategy, data collected from observations, learners' written tasks and interviews were initially analysed separately, guided by the strands of mathematical proficiency indicators. Table 2 represents the indicators for each strand of mathematical proficiency that were used to draw conclusions about learners' development of mathematical proficiency, and also to analyse data and, thereafter, reconcile the

analysis using methodological triangulation. The work of Corrêa (2021) differs from Table 2 on the indicators for conceptual understanding. Corrêa (2021) indicators for conceptual understanding are: connect mathematical content; Retrieve mathematical content and understand mathematical content, which is different from the one in Table 2. The indicators used in Table 2 for conceptual understanding are relevant for this study to conclude learners' development of mathematical proficiency.

Table 2: Strands of mathematical proficiency Adapted from Corrêa (2021, p. 36)

Strands of mathematical proficiency	**Data analysis Indicators for the strands of mathematical proficiency**
Conceptual understanding	Read algebraic word problems and understand mathematical concepts
	Represent algebraic word problems mathematically (graphically, pictorially and symbolically)
	Connect prior knowledge and new knowledge
	Understand mathematical operations and which operations to use when solving any given algebraic word problems
	Translate algebraic word problems
	Analyse the question. Check what is given to you and write down the important information that will help you to solve algebraic word problems
	Draw a conclusion
Procedural fluency	Ability to choose the correct procedure
	Ability to carry out the procedure of solving algebraic word problems appropriately
Strategic competency	Ability to formulate algebraic word problems so that they can use mathematics to solve it
	Ability to use the correct strategy to solve algebraic word problems
	Ability to use multiples ways and strategies to solve algebraic word problems
Adaptive reasoning	Ability to:
	explain algebraic word problems
	justify their solution
	draw conclusions

Productive disposition	Does learners see mathematics as:
	sensible
	useful
	worthwhile
	Effective feedback

3.4.1 Analysis of data collected through observations

The videotape was listened to on several occasions to make sense of the proceedings. Thereafter, data were transcribed verbatim. The transcriptions were read and reflected on to attach meaning to what happened in the classroom. Direct quotes were used, where necessary, to confirm the claims made, particularly in cases where the teacher attempted to develop learners' mathematical proficiency when solving algebraic word problems. Thus, the emerging themes, patterns and processes were categorised, guided by the five strands of mathematical proficiency (Kilpatrick et al., 2001).

3.4.2 Analysis of data collected through learners' written task

Learners' written responses were collected and marked by the researcher. The main intention was to identify challenges encountered by learners when they were solving algebraic word problems. Challenges faced by learners when solving algebraic word problems were identified, and later analysed and categorised, guided by the five strands of mathematical proficiency.

3.4.3 Analysis of data collected through interviews

The audiotape was listened to on several occasions to make sense of the teacher's and learners' responses. Thereafter, data were transcribed verbatim, followed by consolidating, reducing and interpreting the transcriptions, guided by the five strands of mathematical proficiency (Merriam, 1998). The emerging themes, patterns and processes was compared to, or related to, those identified when analysing data from the observations. Thereafter, methodological triangulation was used to ensure or validate the data that were collected and analysed from observation, learners written task and interviews.

3.5 Quality Criteria

According to Lincoln and Guba (1985), there are four principles used to ensure the trustworthiness of a qualitative research, namely credibility, transferability, dependability and confirmability.

3.5.1 Credibility

The credibility of this study was ensured by prolonged engagement, persistent observation and methodological triangulation, as proposed by Lincoln and Guba (1985). In respect of prolonged engagement, I observed the lessons throughout. I gave the teacher enough time to complete his lessons and to ensure that learners completed the given tasks. During classroom interactions, care was taken not to interfere much and, in that way, I persistently observed the proceedings. I remained active throughout the process of gathering data. Methodological triangulation was ensured with the use of different methods of data collection (observations, written task and interviews) to increase credibility of the study (Lincoln & Guba, 1985).

3.5.2 Transferability

Transferability is concerned with the extent to which the findings of one study can be applied to other situations (Guba & Lincoln, 1985; Merriam, 1998). In positivist work, the concern often lies in demonstrating that the results of the work at hand can be applied to a wider population. Guba and Lincoln (1985) further allude to the notion that the findings of a qualitative project are specific to a small number of particular environments and individuals. As a result, it is impossible for the researcher to specify the findings. Therefore, only the reader can demonstrate whether the findings and conclusions are applicable to other situations and populations (Guba & Lincoln, 1985). Furthermore, transferability can be achieved when the research accounts for a thick description of the study. Thick description refers to giving a thorough account of the participants' views, intents, circumstances, motives, meanings and understandings (Younas et al., 2023).Thick description enables judgement of how well a research context fits in other contexts (Anney, 2014). For the purposes of this study, transferability was

ensured by the convenience sampling of a Grade 4 mathematics teacher and Grade 4 mathematics learners as participants in this study (Sihlangu, 2022). The participants were sampled from one primary school and all learners were from a Sepedi-speaking background. No generalisations were made when the findings were interpreted (Schloemer & Schröder-Bäck, 2018).

3.5.3 Dependability

Lishner (2015) alluded to the notion that dependability is used to measure or demonstrate the consistency and reliability of the result of the study. The author maintained dependability by tracking the precise methods used for data collection, analysis and interpretation, and by providing adequate contextual information about each piece collected of data . As a result, if this study is replicated by other researchers, consistent results should be generated (Lishner, 2015). In addressing the issue of reliability, Shenton (2004) stated that the positivist employs techniques to show that, if the work were to be repeated in the same context, with the same methods and with the same participants, similar results would be obtained. However, Shenton (2004) acknowledges that the changing nature of the phenomena scrutinised by qualitative researchers renders such provisions problematic in their work. This means that it is unlikely for the participants to provide similar responses, as their understanding develops as a result of the repetition of the same research (Carcary, 2009). In this study, the teacher used the same learning episode to teach learners in two (class A and B) different Grade 4 classes. Learners were given the same questions to develop mathematical proficiency. As a result, similar findings were expected.

3.5.4 Confirmability

Methodological triangulation was used to ensure confirmability of the study, since three data collection methods were used, namely observation, written task and interviews (Bekhet & Zauszniewski, 2012). Analysis and interpretation of data were initially separated and later reconciled using methodological tribulation to ensure confirmability (Anney, 2014). In this study, bias was reduced through an audit trail to confirm that the findings are derived from the collected data (Tobin & Begley, 2004).

3.6 Ethical Considerations

According to Adams and Schvaneveldt (2011), if research results are obtained without following proper ethical considerations, then those results are deemed invalid. The following ethical considerations were considered in this study.

3.6.1 Voluntary participation

The ethical rules stipulate that participation should be voluntary and that the participants have to be fully informed about the study (Arifin, 2018). In addition, the ethical principles of respect and dignity of the study were ensured by the recognition of the participants' rights and the capacity to make their own decisions. Furthermore, participants were empowered to make decisions freely and were given all the information needed to make an informed decision (Rivera et al., 2004). Since participation was voluntary, the teacher and his learners were free to withdraw themselves from the study at any time, without giving a reason or without any cost.

3.6.2 Informed consent

A request letter addressed to the school principal for permission to conduct the study was written and submitted to the principal to sign. A consent form for teacher to complete before he participated in the study and a consent form asking parents to allow their children to participate in the study were drafted and signed. Lastly, an assent form for minors was provided to the learners for them to sign. Learners were given a consent form to give to their parents to sign. The consent form for parents and assent form for the learners were written in simple basic English (Ruiz-Casares & Thompson, 2016). The consent form for parents was translated into Sepedi so that parents could understand what is written in it and also what they are getting their children into. In addition, parents were given a provision to call the researcher in case they needed to seek clarity. The consent form had the section in it where the teacher as participant, parent/legal guardian of the learner and the researcher could sign. Only after the parents/legal guardians of the learners had signed were learners requested to complete and

sign the assent form (Pyle & Danniels, 2016). Before the learners signed, the information provided on the assent form were translated into the language (Sepedi) that they better understood (Bravo-Sotelo, 2020). This was done prior to the data collection process. A participant's agreement to participate in this study was obtained only after a thorough explanation of the research process (Arifin, 2018). Seventy-nine learners were allowed by their parents to participate in the study.

I applied for a clearance certificate from the Turfloop Research Ethics Committee (TREC) and also applied for ethical clearance from Limpopo provincial Department of Education before conducting the research.

3.6.3 Anonymity

Anonymity of the participants was achieved by not revealing the names and identities of the participants (the Grade 4 mathematics teacher and the Grade 4 mathematics learners) during data collection, analysis and reporting of the study findings (Arifin, 2018). Privacy of the interview environment was managed carefully during interview sessions. The interview sessions were recorded and saved in a protected folder. The identities of the Grade 4 mathematics learners were labelled as the number of group in which they participated (for example, learners in group A), which made it easy to not reveal their names. The Grade 4 mathematics teacher was labelled as a teacher since he was the only mathematics teacher sampled.

3.6.4 Confidentiality

Privacy and confidentiality of the observation videotaped and interviews audiotaped were managed carefully by keeping the records secure by using password-protected files (Surmiak, 2020). The transcriptions of the interviews, observations and learners' responses were kept safe in a locked cardboard. Since the teacher and learners were responding an interview schedule, codes were used to hide their identities.

CHAPTER 4: FINDINGS AND DISCUSSION

In this chapter I present results and the data analysis. Data were collected from observations, learners written tasks and interviews administered in response to the problems posed in Chapter 1 of this study. Two fundamental research questions drove the collection of the data and the subsequent data analysis. The questions that were used to guide data collection and data analysis were: 'What are Grade 4 learners' challenges when solving algebraic word problems?' and 'How does a Grade 4 mathematics teacher develop learners' mathematical proficiency towards solving algebraic word problems?' Responses were transcribed verbatim and analysed separately. The analysis of the study data was guided by the theory of strands of mathematical proficiency, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Moreover, each strand was guided by the data analysis indicators that were discussed in Chapter 3 dealing with data analysis. The data for analysis for learning episode 1 was drawn from two questions, question 1 sub-question letter G and question 2. Similarly, for learning episode 2, data for analysis were also drawn from two questions, question letter C and D. These questions were selected because they played a substantial role in the process of developing learners' mathematical proficiency. Excerpts were used to give a snapshot of transcriptions of data that was captured under section 8.5 and 8.6, while number of excerpts were used to support the arguments within the analysis.

4.1 Learning Episode 1: Finance as Context

The teacher attended two classes during learning episode 1, class A and class B, separately. However, the analysis was done collectively.

4.1.1 Data gathered from observation

❖ **Description of data gathered from observation**

In this learning episode, Grade 4 mathematics learners were given a task to work on in groups ranging from five to seven members (Mokwana, 2017). Before they

started to respond to the questions in learning episode 1, the teacher introduced the lesson by giving learners a question that was used to determine learners' prior knowledge in class A and class B respectively. Transcription of data was captured in Appendix E, item number 8.5.1. After the teacher introduced his lesson in class A and class B, he gave learners an activity related to learning episode 1. The teacher gave learners an opportunity to read the scenario presented in the learning episode 1. Learners read the scenario several times to familiarise themselves with the scenario. After the learners had read the scenario, the teacher translated the scenario into Sepedi to familiarise learners with what has been written in the scenario. While the teacher was translating the scenario, he was engaging learners at the same time (Bravo-Sotelo, 2020; Setati et al., 2002). Thereafter, the teacher told learners to start working on the questions given to them and also, informed them to follow the procedure they used in the example he gave before the lesson. As learners were on the task, the teacher moved around the classroom, monitoring the learners' progress.

- **Analysis of data gathered from observation**

In this section, I present the analysis of data from my observations that were recorded from class A and B separately. The analysis was guided by the strands of mathematical proficiency. The teacher in class A did not allow learners to write their solutions on the piece of paper, since learners in class A did not struggle to get the answer. However, in class B, the teacher allowed two groups of learners to write what they were saying on the piece of paper. The intention of the teacher was to determine whether learners could write out what they are saying mathematically or symbolically.

Excerpt 1

1.1 Teacher: you want to buy each one of you my size [300 ml] cold drink. How many are you in the classroom
1.2 Learners: 41
1.3 Teacher: how much is the price of my size [300 ml] cold drink
1.4 Learners: R8
1.5 Teacher: if we want to buy 41 learners my size [300 ml] cold drink, how much can we spend all together?.
1.6 Learner A: we are going to multiply 41 by 8 and get 328
1.7 Teacher: how and which strategy did you use to get your answer?
1.8 Learner A: I used the calculator to get the answer.

1.9 Teacher: Ok, indeed we multiplied 41 by 8. Do you still remember column method?
1.10 Learners: Yes
1.11 Teacher: we are going to use column method to calculate the answer. Next time do not use the calculator right.
1.12 Learners: Ok sir

❖ Conceptual understanding

The teacher was expected to develop learners' conceptual understanding. However, when the teacher introduced the lesson, he gave learners a simple question, which resulted in the learners getting to the answer straight away. The response of learner A, when she said 'we multiply 41 by 8 to get the answer' (1.6), indicated that the learner had never applied indicators that develop conceptual understanding. Those indicators are read algebraic word problems and understand mathematical concepts; represent algebraic word problems mathematically (graphically, visually and symbolically); connect prior knowledge whit the new knowledge; understand mathematical operations and which operations to use when solving any given algebraic word problems; translate algebraic word problems; check what is given to you and write down the important information that will help you to solve algebraic word problems and draw a conclusion and analyse the question (Corrêa, 2021). For this particular question, it was clear that the learner managed to represent algebraic word problems symbolically. However, this seems not sufficient to develop the learners' conceptual understanding. When the teacher demonstrated the solution on the white board, he wrote down the number of learners and the price of a 'my size [300 ml]' cold drink. According to the indicators for conceptual understanding, the teacher identified the important information that could be used to find the solution. Taking out the important information from the scenario has become the most important key factor for developing learners' conceptual understanding. However, while the teacher was taking out the important information, based on my observation, he did not involve learners in the process of extracting important information on scenario. In my view, learners missed that important information.

Excerpt 2

2.1 Teacher: your mathematics teacher wants to buy all of you ice cream. A plain ice cream cost R7,00. While plain ice cream with crunch chocolate cost R9,00. How many of you would like to eat plain ice cream? May you please indicate by raising your hands.

2.2 Learners: 15 learners raised their hands
2.3 Teacher: how much money will I spent to buy all of you ice cream? Discuss with your group mates before answering the question
2.4 Learner: we are going to multiply 38 by 7 and the answer is 266. Meaning you will spend R266.
2.5 Teacher: how and which strategy did you use to get your answer?
2.6 Group J: we used column method to calculate the answer
2.8 Teacher: write down the strategy that you used to find your answer on the piece of paper
2.9 Group J: ok sir, 38 multiply by 7 gave us 299
2.10 Teacher: not exactly
2.11 Group J: why sir course number of learners is 38 and the ice cream cost 7?
2.12 Teacher: remember there are 15 learners amongst you who prefer to eat a plain ice cream, while the remaining learners prefer a plain ice cream with crunch chocolate. So how much money will I spend all together.
2.13 Group L: ohooooo
2.14 Teacher: since you understand what the question is required can you solve the question.
2.15 Group L: this means we are going to multiply 15 by 7 and multiply the answer with 9 to get the total price. Like 15 by 7 gave us 105. Then 105 by 9 gave us 945. Which means you will spend R945.

The teacher in class B introduced the lesson by giving learners a difficult question to solve compared to the question given to class A. This indicates that the teacher reflected on the lesson he presented to class A (Schwartsm & Karsenty, 2020). He did not give the learners a question that allowed them to get the solution straight away (Gourdeau, 2019). According to the question, learners had to calculate the total amount of money that the teacher would have spent when buying them ice creams. In the process of solving the solution, learners were supposed to calculate the number of learners who wanted plain ice cream and number of learners who wanted plain ice cream with crunched chocolate, separately, and later add the total price together. The first attempt that was done by group J a shown in Figure 33 in class B, under Appendix E item 8.5.1, which indicates that the learners multiplied 38 by 7. According to my view, learners did not read the algebraic word problems with understanding. Learners in group J did not analyse the question correctly. This could be because of their lacking mathematical vocabulary to understand the question (Sibanda, 2017). They could have realised that the question needed them to calculate the price separately and later add them together to get the total price. Therefore, learners in this group did not show indicators that develop learners' conceptual understanding.

The teacher realised that the learners were lost. He took a step and interacted with the learners by reminding them that ‘there are 15 learners amongst you who prefer to eat a plain ice cream, while the remaining learners prefer a plain ice cream with crunch chocolate (Garde & Brodie, 2022). So how much money will I spend all together’ (2.12). Group L, as shown in Figure 34 in Appendix E, item 8.5.1, indicate that learners multiplied 15 by 7 and got the answer. They then multiplied the answer they got by 9. In their minds, the learners were hoping that they could get the total price. However, they had made an incorrect analysis of the question (Pape & Wang, 2003). The teacher tried to improve his practice from class A to B. In class B, the teacher incorporated discourse-based mathematics instruction by allowing learners to discuss the question with their group mates (Yimam & Dagnew Kelkay, 2022). Unfortunately, learners struggled to analyse the question with understanding. As a result, conceptual understanding was not fully developed since one indicator emerged.

- **Procedural fluency**

In class A, the teacher introduced the lesson by giving learners a simple question in order to determine learners’ prior knowledge. Learners were asked how much they spent to buy 41 learners a ‘my size [300 ml]’ cold drink that cost R8. Their response was ‘multiply 41 by 8 to get the answer’ (1.6). The response of learners indicates that learners were successful in choosing the correct procedure to solve the algebraic word problem. The teacher did not allow learners to write the answer down on the piece of paper. According to the observation that I made, I have realised that the reason why the teacher did not allow the learners to write down the answer on the piece of paper was that the teacher was satisfied with the response that the learners gave. Since the learners did not write the response down on the piece of paper, it was difficult to conclude whether the learners’ procedure was well developed in class A, since I was not sure whether they had the ability to carry out the procedure of solving algebraic word problems appropriately. Figure 35 shows the demonstration of the teacher on the white board, as illustrated in Appendix E 8.5.1. While the teacher was busy demonstrating, he was also interacting with his learners.

The teacher in class B gave learners a difficult question, one that required learners to think deeply before they could solve the algebraic word problem (Ulandari et al., 2019). The procedure that learners used to solve the algebraic word problem, calculated by group J and group L as shown in Figure 33 and Figure 34 respectively, indicates that the learners' chosen procedure was incorrect. This could be because of the complexity of the question that was given to learners (Das,2019). The example that the teacher used indicates that, when they are given difficult algebraic word problems, learners found it difficult to solve the question. This could be because the learners lack a conceptual understanding, which results in them struggling to choose the correct procedure.

- **Strategic competency**

The teacher introduced the lesson in class A. He gave learners a question that required the learners to calculate the total price that they would spent when buying 41 learners a 'my size [300 ml]' cold drink that cost R8. The response of learners, as mentioned on Excerpt 1 (1.6), indicates that learners got to the correct answer. The teacher asked learners on how they got their answer (1.7). Learner A responded by saying that 'she used the calculator to get the answer' (1.8). In my view, even when learners got to the correct answer to the introductory question, they did not have the ability to formulate algebraic word problems to solve the question (Kafle, 2019). Learner A, who responded to the question, used a calculator to get to the answer. The use of a calculator does not show all the steps that could afford learners an opportunity to understand algebraic word problems. Therefore, relying on a calculator could disadvantage learners as they proceed through their grades. As a result, during the introduction of the lesson, learners in class A did not seems to have a strategy to solve algebraic word problems.

In class B, the teacher introduced the lesson by giving the learners a difficult question that required them to think critically. Learners' responses were recorded on pieces of paper. The response of learners in group J, as shown in Figure 33

in Appendix E, item 8.5.1, indicates that learners struggled to use the correct strategy to solve the algebraic word problem. Therefore, the teacher realised that learners were confused. The teacher reminded learners that 'there are 15 learners amongst you who prefer to eat a plain ice cream, while the remaining learners prefer a plain ice cream with crunch chocolate. So how much money could you spend all together' (2.12). In my view, this was an instance where the teacher was trying to afford learners an opportunity to the develop strategic competency of solving algebraic word problems, since the teacher identified important information that the learners were supposed to write down in order to solve the algebraic word problems. Learners' responses after the teacher had made the contribution indicate that the learners struggled to formulate algebraic word problems so that they could use mathematics to solve the question.

- **Adaptive reasoning**

During the lesson he presented to class A and class B, the teacher asked learners a question on how they got to their answer and which method they used to get to their answer (1.7 & 2.5). This was an instance where the teacher afforded learners an opportunity to develop adaptive reasoning. The response of learner A in class A was that she got the answer by multiplying 41 by 8 (1.6). Moreover, the method used by learner A to get to the answer was to use a calculator. The indicators for adaptive reasoning are the ability to explain algebraic word problems; justify the solution; and draw a conclusion. The response of the learners indicates that the learner A was justifying her solution to how she got to her answer.

Learners in class B they were frustrated by the questions. The response of the learners in group J as shown in Figure 33 was '38 multiplied by 7' which gave them 266 (2.9). The teacher told the learners that the answer was incorrect (2.10). Learners asked the teacher 'why their answer was incorrect because they took number of learners in the class and multiply that by 7' (2.11). The reasoning of the learner indicates that the learner did not understand the question. As a result, it was difficult to draw a conclusion. The teacher reminded the learners that 15

learners preferred plain ice cream that cost R7, while the remaining learners out of 38 learners preferred ice cream with crunch chocolate, which cost R9 (2.12). However, group reasoning was recorded as ‘this means we are going to multiply 15 by 7 and multiply the answer with 9 to get the total price. Like 15 by 7 gave us 105. Then 105 by 9 gave us 945. Which means you will spend R945’. Group L as shown in Figure 34 indicated that they were able to reason and were able to draw a conclusion. However, the computation was incorrect, which led to the incorrect answer.

❖ **Productive disposition**

While he was presenting his lesson in class A and class B, the teacher demonstrated on the white board, as illustrated in Figure 32 and Figure 35, respectively, in Appendix E item 8.5.1. The teacher draws a conclusion in each class. In class A, the teacher said they would spend R328 when they bought a ‘my size [300 ml]’ cold drink for each learner in the classroom. Learners used gestures to indicate agreement with teacher in class A. This indicates that learners see mathematics as sensible, useful and worthwhile. They see the importance of mathematics in real-life situations.

The teacher gave learners feedback in class B, after learners had made several attempts to respond the question. Learners from groups J and L realised the mistakes that they had committed when solving the algebraic word problem. Appendix E item 8.5.1 represents the mistakes committed by learners groups J and L as shown in Figure 33 and Figure 34 respectively when solving algebraic word problems. The teacher drew a conclusion by adding the price of learners who prefer plain ice cream to the price of the learners who prefer plain ice cream with crunched chocolate in order to get the total price. As the teacher was demonstrating, learners were showing interest in what the teacher was doing. This indicates that learners were understanding and appreciating mathematics.

4.1.2 Data gathered from learners’ written task

❖ **Description of data gathered from learners’ written task**

As mentioned in the introduction to this chapter, learners were given a task during learning episode 1. Question 1 sub-question letter G and question 2 were used to analyse data. After the task was completed, scripts were collected and marked. Learners' challenges were identified

❖ **Analysis of data gathered from learners' written task**

Data from learners' written task were collected from two classes, class A and class B. Learners were arranged in groups ranging from five to seven members. The number of groups in class A was seven and in class B the number of groups was six. In total, there were thirteen groups of learners ranging from five to seven members in both classes. All groups were of mixed gender and mixed ability (Mokwana, 2017). All learners in each group wrote the activity from learning episode 1. The presentation of learning episode 1 took place in the afternoon during the sports period. Learners were asked to respond all the questions. However, for data analysis purposes, question 1 sub-question letter G and question 2 were used. The teacher was able to teach one class per day, during the sports period. The questions for data analysis purposes are captured in Table 3.

Table 3: Questions for data analysis on learning episode 1

Unemployment rate in South Africa is extremely high. Mr Mamabolo decided to open a school wear market to make a living. He is selling the school uniforms: short sleeve shirt for R90,00; Long sleeve shirt for R100,00; School Trouser R110,00; School Shorts R80,00; School Skirt R60,00; Pullover/Jersey R140,00, school tie at R25,00; school socks R20,00; school bag R150,00 and school shoes R290 g) Your mother buy 3 school trouser and 3 school shorts, how much did she pay. Show how you got your answer 2. Your mother bought all the items mentioned in 1. How much did she pay altogether? Show how you got your answer.

Before responding to the questions, learners were advised to read the scenario captured in Table 3 to help them to respond question 1 letter G and question 2, the questions that were used for data analysis. In respect of the responses of learners looking at question 1 letter G, all 13 of groups of learners wrote a response to the activity, seven groups of learners managed to get the correct answer, five groups of learners got an incorrect answer and one group of learners

got a correct and incorrect answer. For question number 2, 11 groups of learners got an incorrect answer and one group got a correct answer, as indicated in Table 4 in Appendix E item 8.5.2. The remaining group did not write anything down. Let us now look at how the learners responded to the questions captured in Table 3 as per the strands of mathematical proficiency (Reano, 2016).

❖ Conceptual understanding

The teacher gave the learners questions during learning episode 1. Learners responded all the questions. However, the focus was on the question mentioned in Table 3. The response of learners from group A is represented in Figure 6.

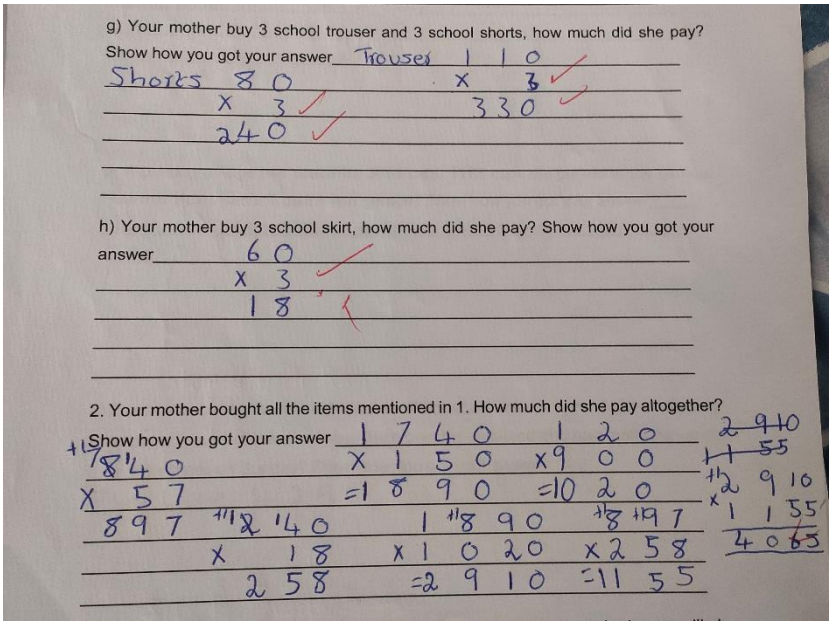


Figure 6: Response of learners from group A

The response of learners illustrated in Figure 6 indicates that learners were able read algebraic word problems and understand the mathematical concepts that were used in question letter G. learners were able to analyse the question and write down the important information that helped them solve the question. However, when looking at this question according to the response of the learners, it seems as though the learners failed to complete answering the question. This could be as a result of learners being unable to draw a conclusion.

Figure 7 represents the response of learners from group B. Learners from group B did not get to the correct answer. In this question, learners were supposed to multiply 110 by 3 and 80 by 3 separately and, later, add the totals together. However, learners multiplied 110 by 6 and 80 by 6 separately. I tried to make sense of where does 6 came from. It looks as if learners added 3 school trousers to 3 school shorts. Learners failed to understand that school trousers and school shorts are two different items. As a result, the two cannot be treated as one item. This indicates that learners from group B fail to read algebraic word problems with understanding, resulting in them getting to an incorrect answer. They also failed to analyse the question.

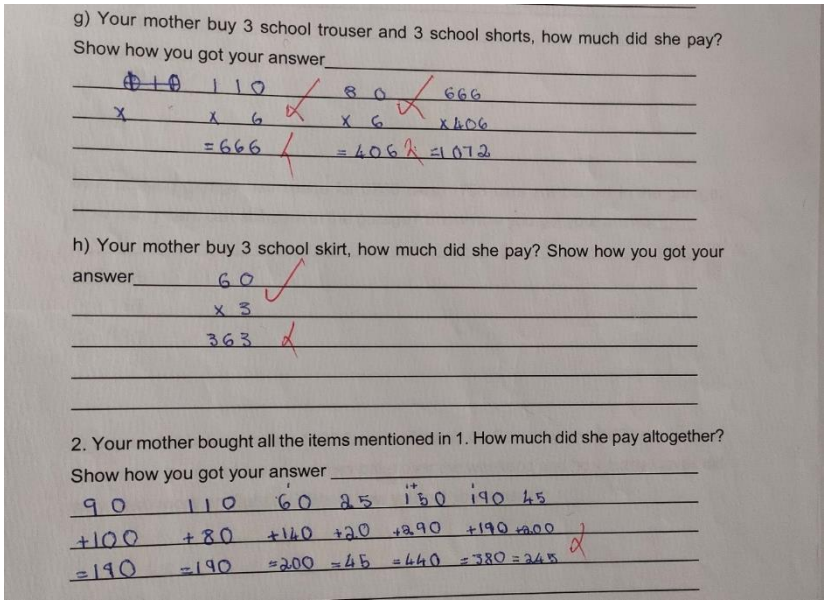


Figure 7: Response of learners from group B

Figure 8 represent the response of learners from group C. Learners in group C they multiplied 190 by 6 to get to their answer. I tried to figure out where they got 190 and 6 from. It looks as though learners took the price of school trousers, R110, and the price of the school shorts, R80, to get 190, and that they obtained 6 by adding 3 numbers of school trousers and 3 numbers of school shorts. In my view, I think that the reason behind learners adding the price of school trousers and the price of school shorts was prompted by the question ‘how much did she pay altogether’, because the question was looking for the total amount paid. Learners thought that they should add everything to get to the answer. According

to this interpretation, it looks as if the challenges learners face could be as a result of a lack of mathematical vocabulary. Moreover, learners failed to read the question with understanding.

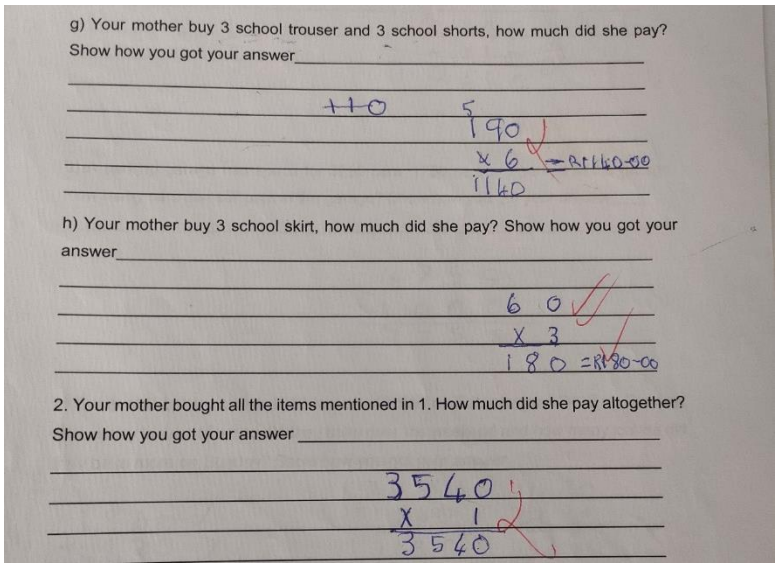


Figure 8: Response of learners from group C

In question 2, learners were expected to add the totals of each product they had calculated in question 1 sub-question letter A-H. Group A-K got to an incorrect answer. When I doublechecked the reason why learners did not get to the correct answer for question 2, I realised that majority of learners did not get to the correct answer on question 1 sub-question letter A-H, which led them to get to an incorrect answer, as shown in Table 4 in Appendix E item 8.5.2. Figures 6 and Figure 8 indicate that learners failed to understand mathematical operations and which operation they should have used. Learners were supposed to use addition. These figures indicate that learners used multiplication. I tried to make sense of what could have happened and decided that this could be because learners failed to understand algebraic word problems (Bravo-Sotelo, 2020). Furthermore, this misunderstanding could be as a result of a lack of mathematical vocabulary.

Figure 9 represents group L. Learners in group L did not write anything down. They left an empty space, as shown in Figure 9. This empty space could be

because learners did not know what to do. Learners lack the confidence to solving algebraic word problems and lack motivation to perform calculations.

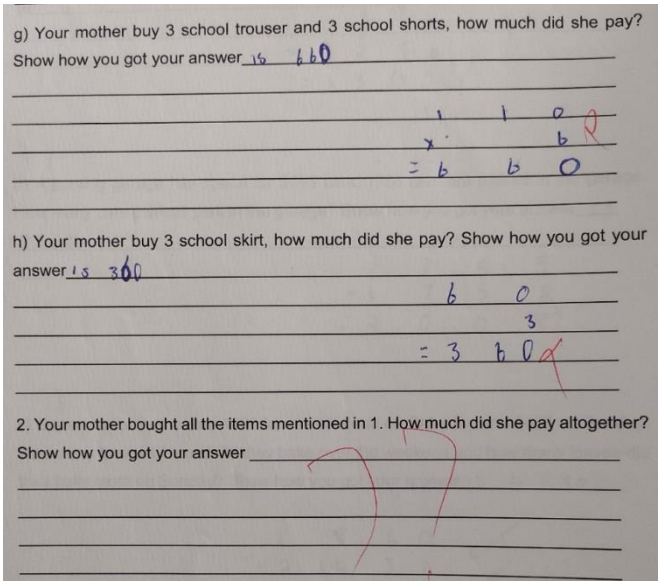


Figure 9: Response of learners from group L

Figure 10 represents the response of learners from group M. Learners in group M got to the correct answer for question letter A-H, which made it easy for them to get the correct answer. The computation by group M learners indicates that they had read algebraic word problem with understanding. They were able to analyse the question and write down the important information that helped them to get to the correct answer.

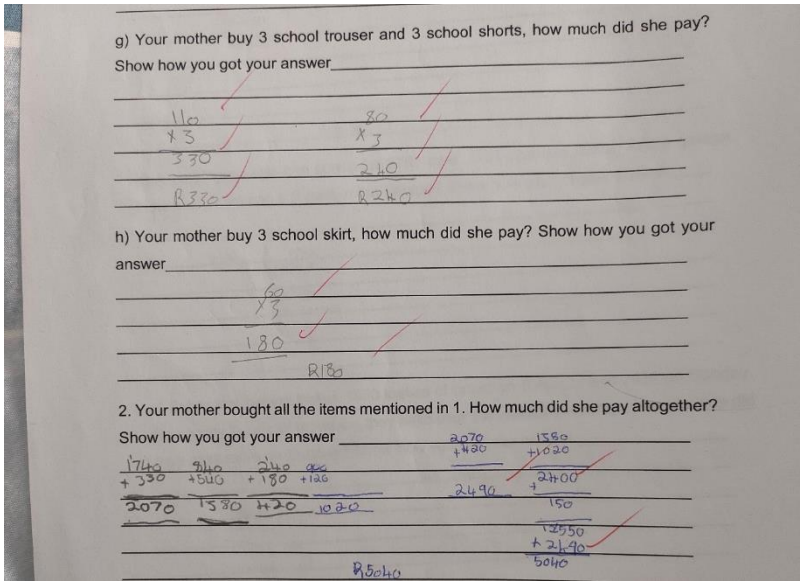


Figure 10: Response of learners from group M

❖ Procedural fluency

The procedure that Grade 4 learners were supposed to follow when responding question letter G and question 2 was they were supposed to multiply 110 by 3, to get 330, and 80 by 3, to get 240, and, later, add the totals together to get to the total price that their mother would have spent. In question 2, learners were supposed to add the total cost of all products mentioned in question letter A-H together to get to the total amount. However, looking at the response of learners in group A-M to question letter G. six groups of learners got to the correct answer, five groups got to an incorrect answer and the remaining group got to both a correct and incorrect answer. I tried to look at the response of the learners per group and realised that learners committed mistakes when multiplying. Groups C and G as shown in Figure 8 and Figure 13 respectively added the price of school trousers and the price of school shorts together and multiply by 6. They got 6 by adding the quantity of school trousers and shorts together to get 6. The response of learners, as illustrated in Figure 7, indicates that learners had a sense of what was required from them. However, the procedure that they used was incorrect and the values that they used were also incorrect. Learners added number of items to get to a total of children (6). Therefore they multiplied 110 by 6 and 80 by 6 which caused them to get to an incorrect answer. These mistakes could have been made because the learners lacked conceptual understanding. Furthermore,

learners struggled to choose the correct procedure because of a lack of conceptual understanding. Strands of mathematical proficiency develop one another, they do not operate in isolation (Kilpatrick et al., 2001).

❖ Strategic competency

Grade 4 mathematics learners were expected to formulate algebraic word problems and use mathematics to solve them; use the correct strategy to solve algebraic word problems; and, lastly, use different/multiple strategies to solve algebraic word problems. In question 1 sub-question letter G, seven groups of learners managed to formulate, represent and solve the algebraic word problems. Of these seven groups, six groups of learners used the same strategy, which was $110 \times 3 = 330$ and $80 \times 3 = 240$ and, later, they added the total of each item together to get the total amount spent, which was $330 + 240 = 570, \therefore = R570$.

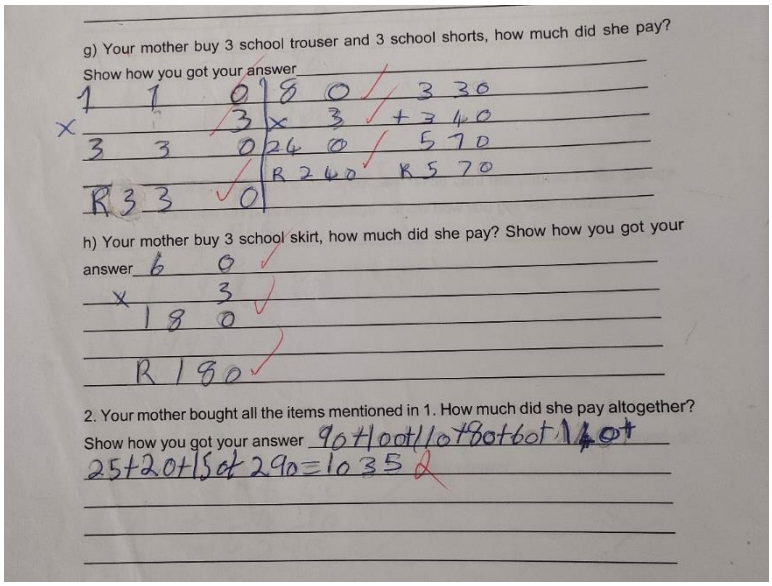


Figure 11: Representative responses of learners from group I

The remaining groups out of seven groups of learners that got to the correct answers used a different strategy.

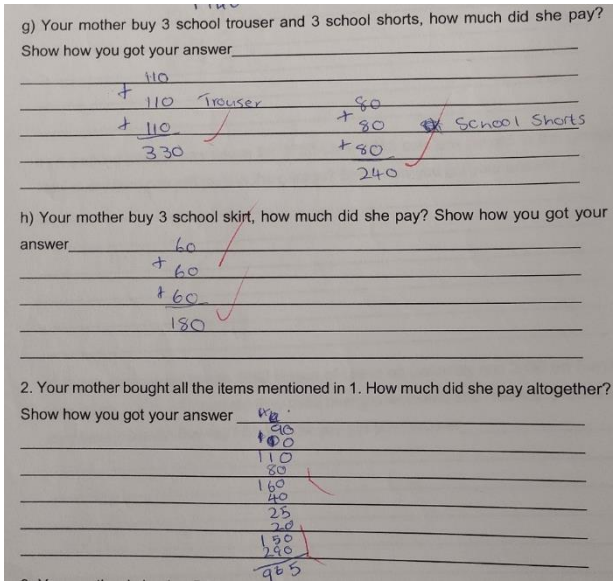


Figure 12: Representative strategy used by learners from group J

Learners in group J as shown in Figure 12 indicated that they understood the algebraic word problems that were presented to them because they were able to use a different strategy to get to the answer. Learners knew that there were 3 items that needed to be bought. Instead of using 110 multiply by 3, they decided to write 110 + 110 + 110, which gave them 330, which meant that three school trousers cost R330, and 80 + 80 + 80, which gave them 240, which meant that three school shorts cost R240. However, when I looked at the strategies they employed, I discovered that it worked for this type of a question. For future purposes, however, I would not advise them to use that strategy because they could struggle when given large numbers to multiply. The six groups of learners had insights into how they should solve the algebraic word problems; however, their level of understanding of the question was poor, which resulted in them getting to an incorrect answer.

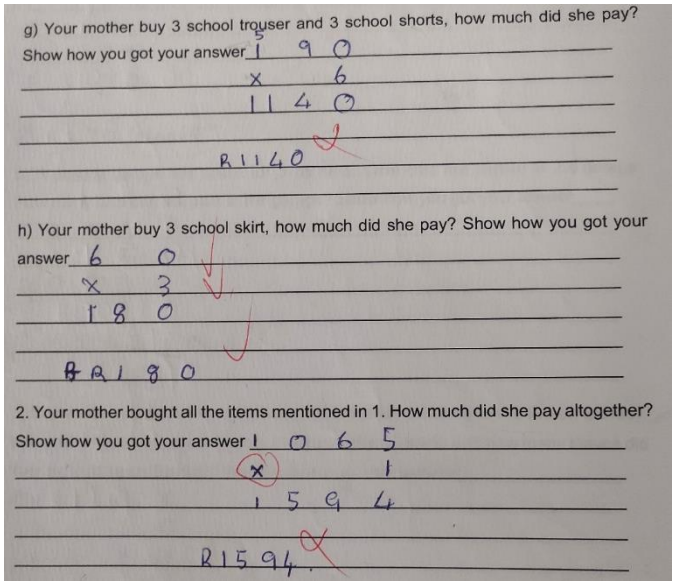


Figure 13: Response of learners from group G

Figure 13 indicates that learners added the prices (110 + 80 = 190) together and multiply the total by the total number of items (3 + 3 = 6) to get to the answer. Looking at this question and the approach learners used, it shows that they understood what they were doing, unfortunately the questioning style that was used in this question was one that led learners to an incorrect answer. These questions should have been asked separately.

problem. According to my view, the learners managed to read and understand algebraic word problems. As a result, they had the ability to formulate algebraic word problems and choose the correct strategy to solve algebraic word problems.

Figure 14 represents the response of learners in group D, which indicates that learners did not have a clue about what they were doing. Figure 14 shows that learners did not have the strategy to solve the problem. It looks as though learners were simply responding to question 2 without making sense of it.

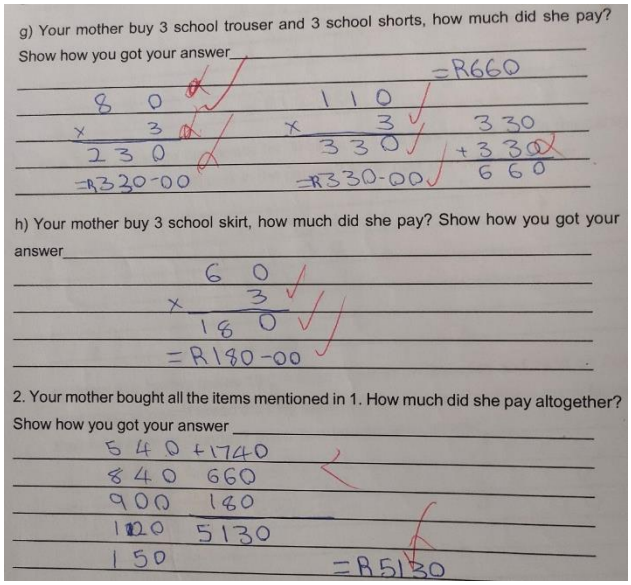


Figure 14: Response of learners from group D

❖ Adaptive reasoning

In this section, the Grade 4 learners were expected to justify their solution logically and sensibly, and also be able to draw a conclusion. Of seven groups of learners who got to the correct answer for question 1 sub-question G, two groups managed to reach a conclusion about their response. Learners in group I, represented in Figure 11, and group H, represented in Figure 15, were able to calculate the total price for buying three school trousers and three school shorts. This could be as a result of learners being able to analyse the question and know what had been given to them, and what was required (Carotenuto et al., 2021). As a result, learners were able to reason logically and sensibly. The remaining five groups of learners who got to the correct answer calculated the answers the answer for school trousers and school shorts separately. They did not add the total of three school trousers that cost R110 and the total of school three school shorts that cost R80. They failed to write down the total amount of money that the mother used when buying these items. As a result, the learners failed to draw a conclusion.

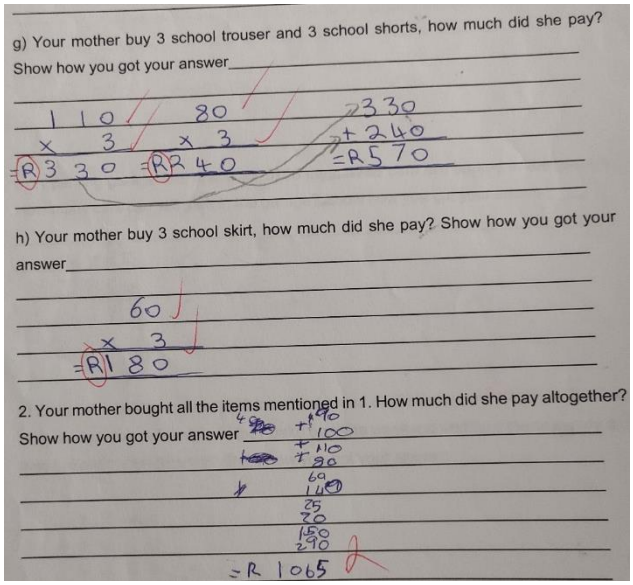


Figure 15: Learners response from group H

For question 2, group M as shown in Figure 10 manage to explain the algebraic word problems that were presented to them and they were able to justify their solution by saying ‘they have added the total price of question 1 A-H, to get the total money that their mother has spent’. Group M managed to draw the conclusion and said ‘their mother spends R5 040 for the whole items she bought under question 1’. Out of 13 groups one group (M) managed to get the correct answer as mentioned, while the remaining groups (12) struggled to explain and justify their solution. As a result, they could not draw a conclusion for their writing.

❖ Productive disposition

The researcher marked the learners’ scripts and gave the scripts to the teacher. The teacher was given a time to go through the marked scripts. Immediately after the teacher was satisfied with the marking, he went back to the class to give feedback to the learners. The learners were given their scripts and the teacher gave them feedback. The feedback from the teacher caused learners to realise the mistakes they have committed. Feedback played an important role since it made learners to see mathematics as sensible, useful and worthwhile. Feedback made learners realised that mathematics is important to them, since it can helped them to calculate the amount of money their mothers could spend when buying their school uniform.

4.1.3 Data gathered from the teacher's interviews

The researcher conducted interviews with the teacher guided by the interviews schedule that can be found in section 8.3. The interview schedule was guided by the theory of mathematical proficiency. The teacher was expected to account for how he developed learners' mathematical proficiency towards solving algebraic word problems. The audiotape was listened to on several occasions to make sense of the responses of the teacher. Thereafter, data were transcribed verbatim. A snapshot of the transcription was captured on excerpt 3. The whole transcription of data was captured in Appendix E item 8.5.3.

Excerpt 3

3.1 Researcher: How do you develop Grade 4 learners' mathematical proficiency, particularly when solving algebraic word problems? Explain

3.2 Teacher: I put learners in groups ranging from five to seven and give them an activity that include algebraic word problems, from there I allow learners to read the scenario with understanding and ask them to pick up important information. Once they have picked up an important information, I encourage them to represent the information mathematically. From there I ask them what is the question, after they tell me the question, I ask them which mathematical operation is suitable to solve the question, once they respond, I allowed them to follow the same procedure for other questions.

3.3 Researcher: What are the teaching strategies or methodologies that work for you to develop learners' mathematical proficiency? Explain in reference to solving algebraic word problems.

3.4 Teacher: I use learning-centred classroom incorporated with discourse-based mathematics instruction to create an environment that afford learners an opportunity to develop mathematical proficiency. This strategy afford learners an opportunity to: discuss and share their mathematical ideas; explain their reasoning; compare and justify problem-solving strategies; communicate their ideas; challenge each other's reasoning through questioning, reflect on and clarify their thinking.

3.5 Researcher: What challenges did you notice, faced by learners when solving algebraic word problems? Outline them

3.6 Teacher: Learners lack conceptual understanding of mathematical operations; they confuse addition with multiplication. For example, when they multiply 2 x 2 they get 4 and when they add 2 + 2 they also get 4, therefore they think 3 x 3 is equal to 6 since 3 + 3 = 6. However, in the question that was given to learners, I have realised that some learners did not understand the question. Learners were multiplying R110(price of school trouser) by 6, instead of 3 and R80,00 by 6 instead of 3 and others were adding the price of school trouser and school shorts which was R110 + R80 = R190, and that R190 was multiplied by 6, which was the total price of the learners. Therefore, this indicate that some learners lack conceptual understanding.

3.7 Researcher: What would you regard as key actions to be taken care of by the teachers when assisting learners to develop mathematical proficiency when solving algebraic word problems? Account why you regard them as key actions?

3.8 Teacher: Teachers should use learning-centred classroom incorporated with discourse-based mathematics instruction to create an environment that afford learners an opportunity to develop mathematical proficiency. help learners to read with understanding, so that it can be easy for learners pick up important information from the scenario of the algebraic word problems. they should help learners be able to represent algebraic word problems mathematical, and they should also help learners to be able to choose or formulate the best strategy to use when solving algebraic word problems and

lasty help learners to defend their writing and draw conclusion. In addition to that effective feedback could encourage learners to see mathematics as useful, sensible and worthwhile.

❖ Analysis of data gathered from teacher interviews

Conceptual understanding

The teacher used a learning-centred classroom incorporated with discourse-based mathematics instruction to develop learners' mathematical proficiency. A learning-centred classroom is characterised by a culture of interaction between teachers and learners in the process of 'doing mathematics'. The teacher plays an important role in establishing this culture and in designing learning experiences to help learners learn mathematics, using whatever teaching and learning strategies s/he thinks are most suitable (DBE, 2018). Discourse-based mathematics instruction is viewed as a 'learning medium' through which learners' construct or develop mathematical understanding and knowledge by explaining how mathematical tasks can be accomplished or how procedures work (Bradford, 2007). In addition, it affords learners an opportunity to engage in social interaction and communication with others by challenging each other's ideas, comparing different solution strategies, sharing ideas and reasoning about mathematical concepts and relationships (Bradford, 2007).

After introducing his lesson and in order to determine the learners' prior knowledge, the teacher arranged learners into groups that ranged from five to seven members. All groups were of mixed gender and mixed ability, such that learners could learn from their peers (Mokwana, 2017). This indicates that the teacher was creating an environment that could afforded learners an opportunity to interact among themselves and also interact with the teacher. When he was determining the learners' prior knowledge, I realised that the teacher was trying to bring learners' prior knowledge close to the new knowledge that they had learned inside the classroom. The teacher gave learners an opportunity to read algebraic word problems with understanding and, later, allowed them to analyse the scenario so that they were be able to identify important information that could help them to respond the question (3.2). After that, the teacher asked learners a question to challenge their thinking (Hiebert & Wearne, 1993). The question

afforded the learners an opportunity to engage in social interaction and communication, which was informed by the implementation of discourse-based mathematics instruction by the teacher as a strategy to develop learners' mathematical proficiency (Steeley, 2017). There are four indicators for developing learners conceptual understanding, which were identified in Table 2, namely read algebraic word problems and understand mathematics concepts; connect prior knowledge with new knowledge; understand mathematical operations and which operations to use when solving any given algebraic word problems; translate algebraic word problems; check what is given to you and write down the important information that will help you to solve algebraic word problems and draw a conclusion and analyse the question. This indicates that the teacher was developing learners' conceptual understanding of solving algebraic word problems.

- **Procedural fluency**

The last sentence of the response of the teacher on question 1 states 'I allowed them to follow the same procedure for other questions' (3.2). I tried to make sense of what procedure the teacher was referring to. However, when I go back to the proceedings, I realised that the teacher said that learners should read algebraic word problems with understanding, analyse then, understand mathematical operations and connect prior knowledge to new knowledge (3.2). In my view, this could be the procedure the teacher he was referring to. However, when I look at the response of the teacher to question 3 (3.5), he started by outlining common mistakes learners make when dealing with addition and multiplication (3.6). The response of the teacher indicates that learners were unable to choose the correct procedure for solving algebraic word problems flexibly, accurately, efficiently and appropriately (3.6).

- **Strategic competency**

The teacher told the learners to read algebraic word problems with understanding and analyse the problem. In the process of analysing algebraic word problems, learners were told to identify important information that could help them solve

algebraic word problems (3.2). In my view, the teacher understood that if learners were able to analyse the question and write down the important information, it would be easy for them to formulate algebraic word problems. When the teacher asked a learner the question ‘what are you asked to find?’, in my view, the teacher was trying to develop the learners’ ability to think about the correct strategy that they could use to solve algebraic word problems.

- **Adaptive reasoning**

When responding to question 2 of the interviews, the teacher said that ‘I use learning-centred classroom incorporated with discourse-based mathematics instruction to create an environment that afford learners an opportunity to develop mathematical proficiency. This strategy affords learners an opportunity to: discuss and share their mathematical ideas; explain their reasoning; compare and justify problem-solving strategies; communicate their ideas; challenge each other’s reasoning through questioning, reflect on and clarify their thinking’ (3.4). The response of the teacher indicates that he was creating an environment that would affords learners an opportunity engage in social interaction and communication with others so that the learners could improve their mathematical reasoning and proof. In addition, the teacher was also advising learners to be in a position to draw a conclusion.

- **Productive disposition**

When responding question 5 of the interview, the teacher indicated that ‘teachers should use learning-centred classroom incorporated with discourse-based mathematics instruction to create an environment that afford learners an opportunity to develop mathematical proficiency. help learners to read with understanding, so that it can be easy for learners pick up important information from the scenario of the algebraic word problems. They should help learners be able to represent algebraic word problems mathematical, and they should also help learners to be able to choose or formulate the best strategy to use when solving algebraic word problems and lasty help learners to defend their writing

and draw conclusion. In addition to that effective feedback could encourage learners to see mathematics as useful, sensible and worthwhile' (3.8).

The last sentence of the paragraph above indicates that if teachers use a learning-centred classroom incorporated with discourse-based mathematics instruction, this could afford learners an opportunity to develop mathematical proficiency. As a result, effective feedback could result in learners being interested in doing mathematics, and could help them to begin to appreciate mathematics and see it as useful, sensible and worthwhile.

4.1.4 Data gathered from learners' interviews

Thirteen groups of learners (groups A–M) were interviewed, guided by the interview guideline captured in section 8.4. The audiotape of the interview proceedings was listened to on several occasions to make sense of the responses of the learners. Thereafter, data were transcribed verbatim. A snapshot of the transcription was captured in excerpt 4. The whole transcription of data is captured in Appendix E item 8.5.4. However, follow up questions and probing questions were not captured in the transcription of the data.

Excerpt 4

4.1 Researcher: What is your understanding about solving algebraic word problems? Explain
4.2 learners: We must read and understand the question before we respond to it
4.3 can you restate the algebraic word problems in your own words? Explain
4.4 learners: Your mother buys 3 school trouser and 3 school shorts. So, they want use to find how much did she pay
4.5 Researcher: What are you asked to find? Explain
4.6 Group F: We are asked to find how did our mother paid for 3 school trouser and 3 school shorts
4.7 Researcher: Is there enough information to help you to find the answers? Explain
4.8 Group D: Yes, they told us how much the school trouser cost and how much the school shorts cost. There were 3 items each that need to be bought"
4.9 Researcher: Explain how did you get to your answer?
4.10 Group A: By multiplying 110 x 3 and we get 330, and also multiplying 80 x 3 and we get 240

❖ **Analysis of data gathered from learners' interviews**

Conceptual understanding

The response of learners to question 1 of interviews was similar. Learners were asked what their understanding of solving algebraic word problems was (4.1). The response of learners was 'We must read and understand the question before

we respond to it' (4.2). This clearly indicates that the teacher emphasised that, when they are presented with algebraic word problems and attempt to respond to the question, they should first read and understand the question, before answering it. The responses to question 2 by the majority of learners read as follows 'your mother buys 3 school trouser and 3 school shorts. So, they want use to find how much did she pay'. The responses of learners to question 2 shows that the majority of learners did not manage to restate algebraic word problems in their own words, they simply respond by reading the question as is. This could be as a result of learners lacking the mathematical vocabulary necessary to enhance their ability to restate algebraic word problems in their own words.

Group F learners' responses to question 3 stated that 'we are asked to find how did our mother paid for 3 school trouser and 3 school shorts'. This response indicates that learners were aware of what was required of them. The response of group F learners is illustrated in Figure 16

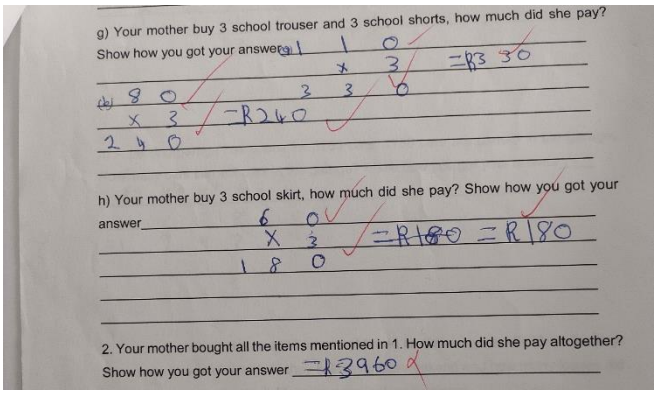


Figure 16: Learners response from group F

Figure 16 indicates that group F learners partially understand the question and what was required from them to do it. However, learners did not complete question letter G. They only calculated the price of 3 school trousers and the price of school shorts. They did not calculate the total amount that their mother would spend when buying 3 school trousers and 3 school shorts altogether. This could be as a result of learners not analysing the question fully. After they had read the first part of the question, they rushed to respond to the question and forget to

complete the question. Table 6 indicate the transcription of learners' interviews from learning episode 1 in Appendix E item 8.5.4. Table 6 show that learners in group E and group L did not respond the question 3 of the interview. Figure 17 represents the response of group E.

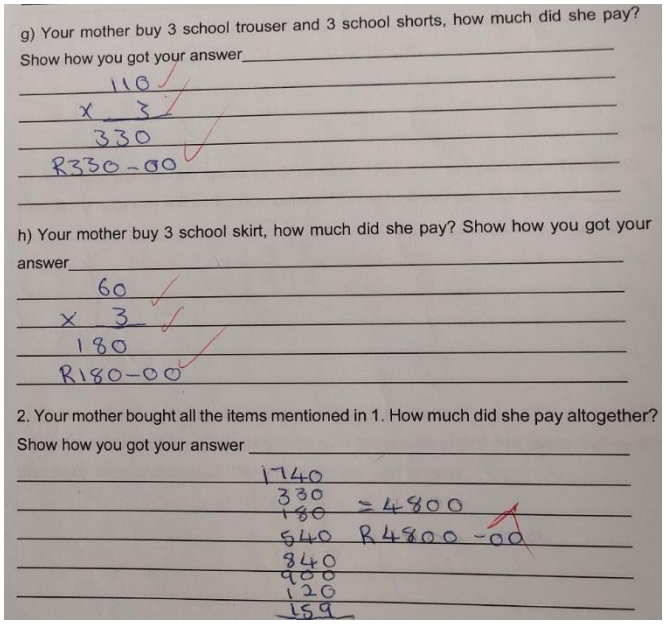


Figure 17: Response of learners from group E

Figure 17 indicates that the learners were reluctant to respond to the interview question, however, on their script they had indicated that they had the potential to respond to the question. Group L learners also did not respond to the interview question. I looked at their scripts and realised that they had multiplied 110 by 6, as shown in Figure 9. However, what they wrote was incorrect, which indicates that learners did not have an idea of what they were doing. Hence, they remained silent in response to the interview question. Their silence could be as a result of not understanding the question.

❖ Procedural fluency

The question that was used to determine the procedure that learners used to solve algebraic was 'What procedure/method did you used to solve algebraic word problems?' Learners responded by saying that they used the column method to get to the answer. I took my time to go through the learners' scripts. I

realised that, of the 13 groups of learners, seven groups managed to choose the correct procedure of multiplying 110 by 3 for school trousers and multiplying 80 by 3 for school shorts. Learners got to the correct answer by following the above mentioned procedure to get to the correct answer. Five groups of learners got to an incorrect answer as a result of using an incorrect procedure. Some groups followed the procedure of multiplying 110 by 6 and 80 by 6, as indicated in Figure 7, while other groups used the procedure of multiplying 190 by 6, as indicated in Figure 8. On the question for school shorts, learners from the remaining group, which was group D represented in Figure 14, presented the correct procedure. However, their answer was incorrect. They multiplied 80 by 3 and got 230, which was incorrect. I tried to understand what led them to get 230 and I think that the learners here made error when computing the answer.

- **Strategic competency**

Grade 4 learners were expected to formulate algebraic word problems so that they could use the correct strategy to solve the problem. There were 13 groups of learners and, out of 13 groups, all groups managed to formulate algebraic word problems and choose the correct strategy to solve them. However, all of them did not get to the correct answer because these groups used incorrect values. Learners were supposed to multiply 110 by 3 and 80 by 3 to calculate the price of school trousers and the price of school shorts, separately, and, later, add the prices to get the total price that the mother would have to spend. Groups C and G, see Figure 8 and Figure 13 respectively, used 190 by 6, while groups B and L used 110 by 6, as shown in Figure 7 and Figure 9, respectively. Group B, C, G and L represented by Figure 7, Figure 8, Figure 13 and Figure 9 respectively used the column method to calculate, however, they got to an incorrect answer because they used incorrect values. These challenges could be as a result of learners failing to analyse the algebraic word problems properly, which resulted in them getting to an incorrect answer. Group J, as shown on Figure 12 used a different strategy. They said 110+110+110 = 330 and 80+80+80 = 240. The answer was correct. This indicates that learners in this group were able to

formulate algebraic word problems and use a multiples strategy to solve the problem.

❖ **Adaptive reasoning**

In respect of adaptive reasoning, learners were expected to justify their solution in a logical and sensible manner, and draw a conclusion. Questions 4 and 5 read as follows: ‘Is there enough information to help you to find the answers?’ (4.7) and ‘Explain how did you get to your answer?’ (4.9). These questions were used to determine the reasoning and proof of learners. The response to question 4 by group D was captured as follow: “Yes, they told us how much the school trouser cost and how much the school shorts cost. There were 3 items each that need to be bought’. The response of group D indicates that the learners’ reasoning improved as they were able to pick up the important information. The response to question 5 by learners in group A was captured as follow: ‘By multiplying 110 x 3 and we get 330, and also multiplying 80 x 3 and we get 240’. The response of learners to question 5 indicated that learners understood the question and, as a result, they managed to reason logically and draw a conclusion on how much the mother spent on each item.

❖ **Productive disposition**

As the researcher was interviewing learners, he was interacting with them, Making learners feel comfortable enough to respond to the questions that were asked. Where learners struggled to understand the question, the researcher repeated the question so that the learners were able to understand the question and make sense of it before they attempted to respond to it. This type of engagement made learners to appreciate mathematics and made them to see mathematics as sensible, useful and worthwhile.

4.2 Learning episode 2: Different contexts

In learning episode 2, the teacher decided to combine the two classes. The teacher realised that he could manage both classes. There were classes A and

B. Class A had 41 learners in it, while class B had 38 learners in it. The total number of learners was 79. Since one classroom could not accommodate 79 learners, the teacher decided to use the school hall, which accommodates more than 100 learners. The teacher used the same groups of learners that were created in the separate classes. There were still thirteen groups of learners, ranging from five to seven members. Learners were seated according to their groups. The only change that was made was to combine the two classes. Learners were given an activity to write down.

4.2.1 Data gathered from observation

❖ **Description of data gathered from observation**

Before they started to respond to the questions in learning episode 2, the teacher introduced the lesson by giving learners a question that was used to determine the learners' prior knowledge. The question that was used to determine the learners' prior knowledge was: 'A parking garage at Paledi Mall has a space of 3 765 cars. 1 758 cars are parked in the garage. How many cars can still park in the garage? Show how you got your answer. After a while 1 398 cars arrived at the parking. How many cars can still park in the garage? Show how you got your answer.'

A transcription of the data is captured in Appendix letter F item number 8.6.1. After the teacher introduced the lesson. He gave learners an activity from learning episode 2. The teacher gave learners an opportunity to read the scenario presented in learning episode 2. Data were drawn from question letter C and D from learning episode 2 for analysis.

Excerpt 5

5.1 Teacher: now that you have read the scenario, how many cars can still park in the garage?
5.2 Learners: 2007
5.3 Teacher: How did you get the answer?
5.4 Learner: 3765 minus 1758
5.5 Teacher: ok. Is the question complete?
5.6 Learners: No
5.7 Teacher: why are you saying No?
5.8 Learner B: Because they said 1398 cars arrived in the parking, meaning they have increase numbers of cars in the parking

Teacher: ok, I am giving you time to complete the task, after one member from the group should come and demonstrate their solution on the bord.

5.9 Learners: ok sir

5.10 The teacher gave learners a chance to complete their task, as learners were working on the task, the teachers was moving around checking if learners are responding to the question given to them. after a while, one learner from one of

❖ Analysis of data gathered from observation

Conceptual understanding

The introduction of the lesson by the teacher demonstrated that the teacher had reflected on the previous lesson. The teacher started the lesson with a difficult question, which demonstrated that the teacher was developing learners' conceptual understanding. The role of the teacher in this lesson was to facilitate the lesson. Learners were in control of their learning. The solution that the learners came to indicates that the learners understood the question that was given to them, even though they did not have the courage to explain their solution in front of other the learners. The responses of learners, indicated in Figure 18 and Figure 19, show that learners were able to choose the correct mathematical operation to solve the algebraic word problem. Figure 19 shows that learners used a different approach to solving algebraic word problems, which indicates that learners understood the concept.

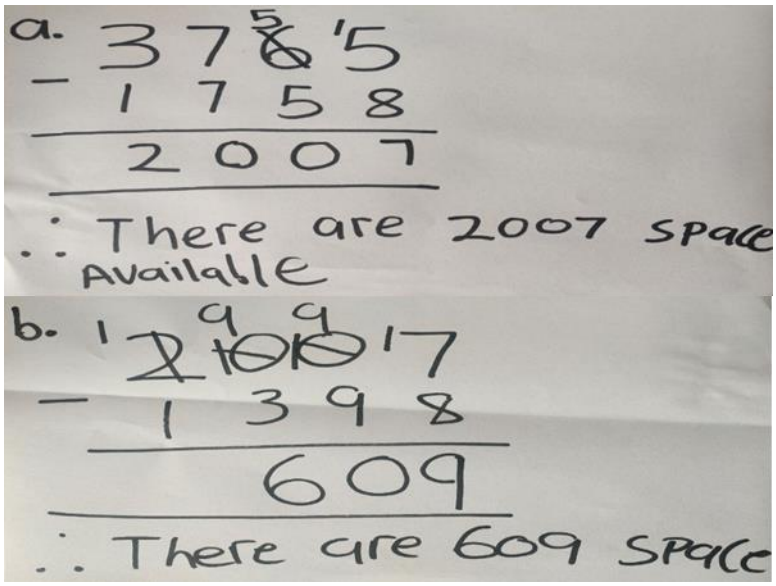


Figure 18: Response of learners from group D

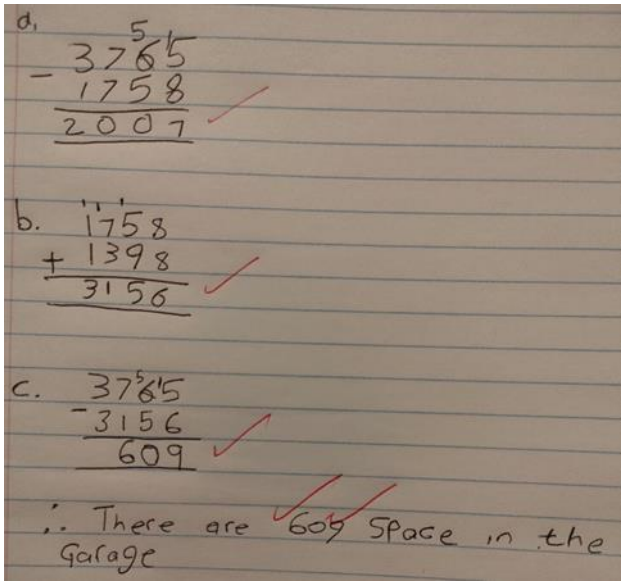


Figure 19: Response of learners from group J

❖ **Procedural fluency**

Figures 18 and 19 indicate that learners were able to choose the correct procedure to solve the algebraic word problems presented to them. Figure 18 shows that learners used subtraction to get to their answer, which was the correct procedure to use. On the other hand, Figure 19 shows that the learners used subtraction to find the total space left, after they had used addition to add the number of cars that were parked and numbers of cars that had arrived at the parking garage. They took the total and subtracted the total space. The solution was the same as the solution presented in Figure 18 and, as a result, the learners' procedure in this case was developed.

❖ **Strategic competence**

Figures 18 and 19 indicate that learners were able to formulate an algebraic word and use mathematics to solve the problem. The strategy of using subtraction that learners used, as depicted in Figure 19, was correct as learners found the initial space that was available. After the second set of cars arrived at the parking garage, learners simply subtracted the total space remaining from the total numbers of the cars that had arrived at the parking garage. Figure 19 shows that the learners used a different approach to get the correct answer. This indicates that conceptual understanding and procedural fluency were developed, since

learners were able to formulate a different strategy to solve the algebraic word problem.

❖ **Adaptive reasoning**

The learner who wrote the solution on the white board did not explain her solution. I think that this was because she was shy and she lacked the confidence to explain herself in front of her peers. However, one learner came up and explained the solution. Her explanation was correct (5.8). This indicates that the adaptive reasoning was partially developed because it is not easy for learners to stand up and explain their solution to the whole class.

❖ **Productive disposition**

The teacher gave learners an example of a real-life situation to make learners aware that the mathematics that they are learning in classroom can be applied in a real-life context. The facial expressions of learners was good when the teacher talked about Paledi mall (a local shopping mall). This indicates that learners appreciate it when the teacher use that which they are familiar with.

4.2.2 Data gathered from learners' written task

❖ **Description of data gathered from learners written task**

Transcription of data gathered from learners written task is presented in section 8.6 item 8.6.2. Learners were given a task to complete during learning episode 2. This episode consisted of seven questions (A to G). The shaded questions C and D are shown in Table 7 in Appendix F item 8.6.2.

❖ **Analysis of data gathered from learners written task**

Conceptual understanding

The approach to question C by majority of learners indicates that they did not understand the question. There were two questions in one question and the learners attempted to respond to one of the two questions. Some of learners subtracted a larger number from a smaller number, which gave them an incorrect answer. What led the learners to get to the incorrect answer was that learners

subtracted a larger number from a smaller number which a negative answer. Figure 20 illustrates group F learners' lack of conceptual understanding.

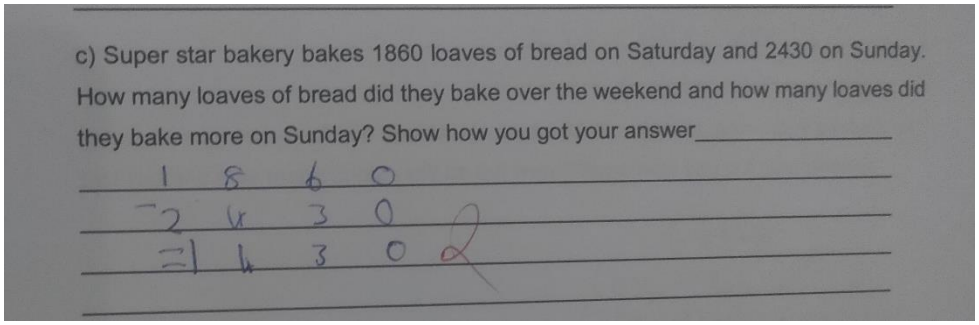


Figure 20: Response of learners from group F

According to the curriculum, Grade 4 learners are not allowed to subtract a larger number from a smaller number because the solution will be negative. Figure 20 shows that the learners' solution was positive, which also indicates that learners did not understand that when they subtract a larger number from a smaller number the answer should be negative. Another aspect that Figure 20 indicates is the fact that that learners did not respond to the first part of the question, which was about the total bread that was baked on Saturday and Sunday. They simply responded to the second part of the question, which also indicates a lack of conceptual understanding of the problem.

Figure 21 indicates that learners understood that the question required two solutions, however, their representation of their solution was incorrect. In the first part of their solution, they took 2 430 loaves and multiply it by 2. I think that the reason for them to use 2 was because the Super Star Bakery baked loaves of bread on Saturday and Sunday, therefore they concluded that they could use 2 to get to the total number of bread loaves that was baked over the weekend. Looking at their response, their thinking would have been correct if the number of breads baked was 1 860 only over the weekend or 2 430. Figure 21 clearly indicates that the learners did not struggle with mathematics, they struggled with language proficiency, which made it difficult for them to develop a conceptual understanding.

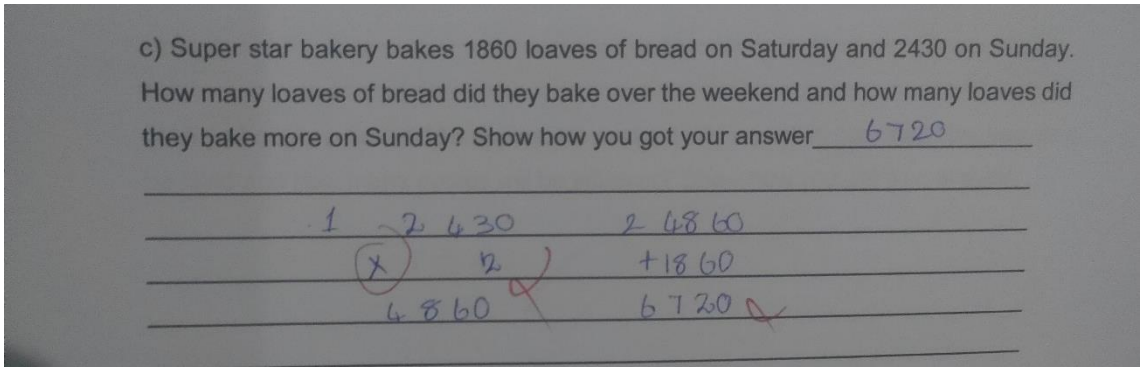


Figure 21: Response of learners from group K

In respect of question D, of the 13 groups of learners, seven groups did not get to the correct answer. The challenge that learners faced when solving question D might be a lack of conceptual understanding. Figure 22 demonstrates that learners had an insight into what the question required, however, instead of using division sign, the learners decided to use multiplication. Looking at Figure 22, it is clear that the learners' first attempts to solve the question were discarded. This could be as a result of learners lacking confidence in themselves. The values that learners wrote down, as illustrated in Figure 22, shows that the learners were unable to represent the text correctly. Learners wrote 28 multiplied by 6 (28 x 6), which raises questions about their thinking capacity. What they wrote down did not represent the question. The use of 6 might be taken from the learning episode 1, where the mother was taking care of 6 children. However, in this question, this was not applicable. In this question, there were 10 children and 40 sweets, and the learners were supposed to share the 40 sweets between the children. Figure 22 reveals that learners did not understand the question and, as a result, they ended up writing values down that were did not form part of the question. Figure 22 indicates that the learners responded to the activity without making sense of it. In addition, learners failed to represent algebraic word problems mathematically. Figure 22 also reveals that the learners struggled with mathematical language and, as a result, were not able to write down the value of 10.

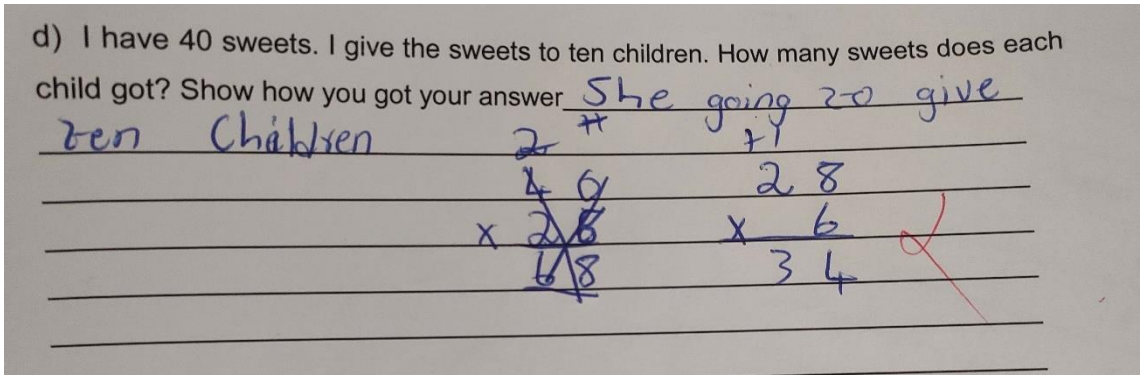


Figure 22: Response of learners from group A

Figure 23 reveals that the learners in group B also did not understand question D. The learners wrote down multiplication and division at the same time. This indicates that learners lacked conceptual understanding, which resulted in them being confused. They were not sure about which sign they should use. Lastly, the learners used addition, which indicates that these learners did not understand the concept that was taught to them.

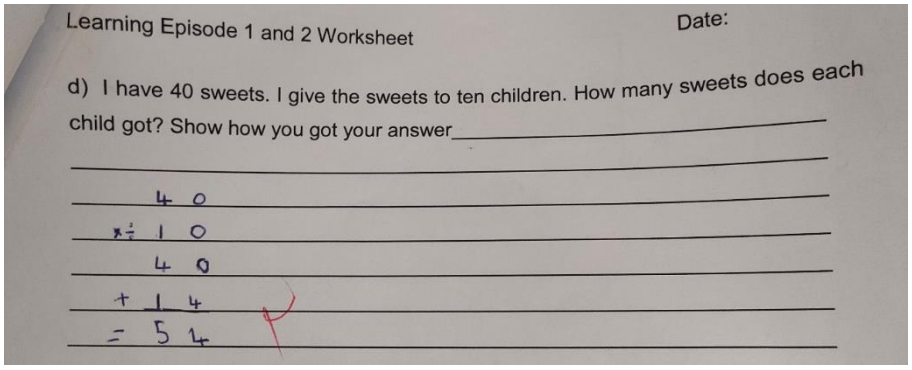


Figure 23: Response of learners from group B

Figure 24 illustrates that learners did not understand the mathematical operation that they were supposed to use. Instead of using division to find number of sweets each child should get, they decided to use subtraction, which led them to an incorrect answer. The response of the learners implies that one child is going to get 30 sweets, which means other children might not get any sweets.

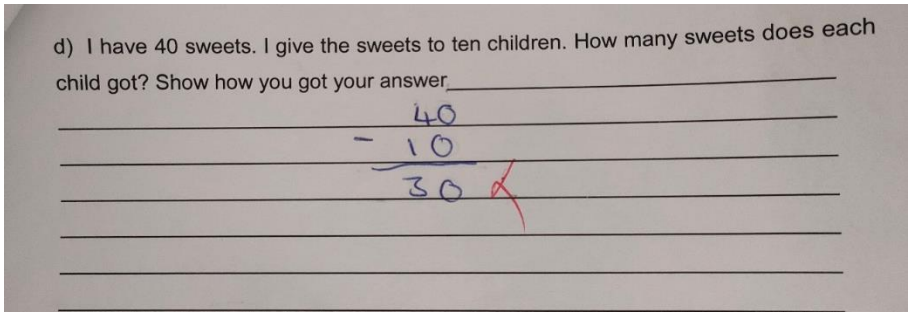


Figure 24: Response of learners from group E

Figure 25 indicates that learners' conceptual understanding was developed, since learners were able to use the correct mathematical operation. Learners divided 40 by 10, which gave them 4, and they wrote a conclusion to say that each child got 4 sweets.

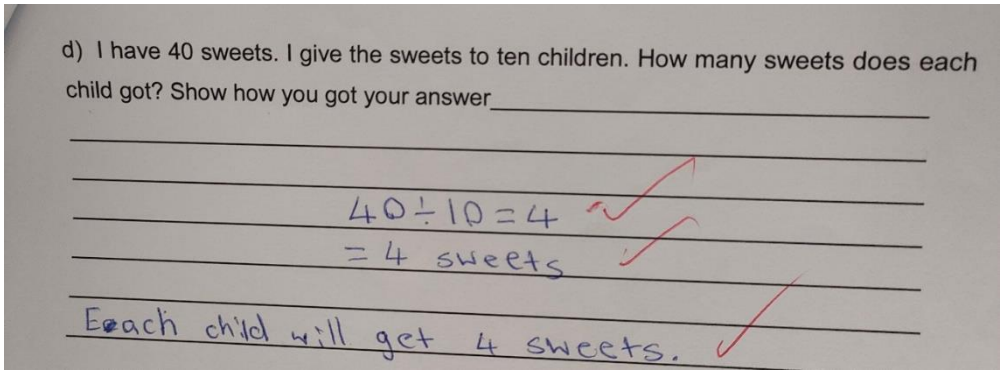


Figure 25: Response of learners from group D

❖ **Procedural fluency**

The response of learners to question letter C, as shown in figure 20, indicates that learners in Grade 4 are not familiar with the standard procedure of subtracting numbers. The procedure that learners were supposed to follow, was to subtract a smaller number from a larger number for them to get to a positive answer. However, the figure shows that learners lacked procedural fluency in respect of subtracting numbers. The response of the learners to question letter D indicates that the learners were unable to choose the correct procedure to solve algebraic word problems. Figure 26 represent the learners' inability to choose the correct procedure.

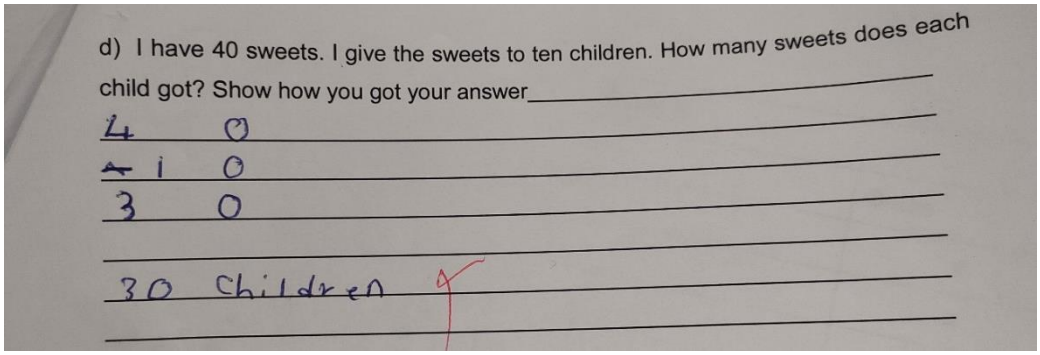


Figure 26: Response of learners from group G

The procedure that learners were supposed to follow was to divide the number of sweets by the number of children to get to how many sweets each child should get. However, as illustrated in Figure 26, the learners used the procedure of subtracting the number of sweets from number of children. As a result, the learners failed to carry out the correct procedure for solving algebraic word problems appropriately. This activity shows that learners' conceptual understanding was not developed.

❖ **Strategic competency**

Figure 20 indicates that the majority of learners had an insight into the strategy to use when solving algebraic word problems. However, the solution they got to was incorrect because they subtracted a larger number from a smaller number. Moreover, the question they responded to was not the first part of the question in question letter C, it was the second part of the question in question letter C. The first part of the question was looking for the number of bread loaves that were baked over the weekend and learners were supposed to use addition to get to the total number of bread loaves that were baked over the weekend. For the second part of the question, the strategy and the mathematical operation used by learners was correct. However, the manner in which used to carry out their strategy was incorrect as they subtracted the larger number from the smaller number.

In question letter D, the learners were asked to determine how many sweets each of 10 children would get from 40 sweets. Figure 27 shows how learners were supposed carry out the long division method.

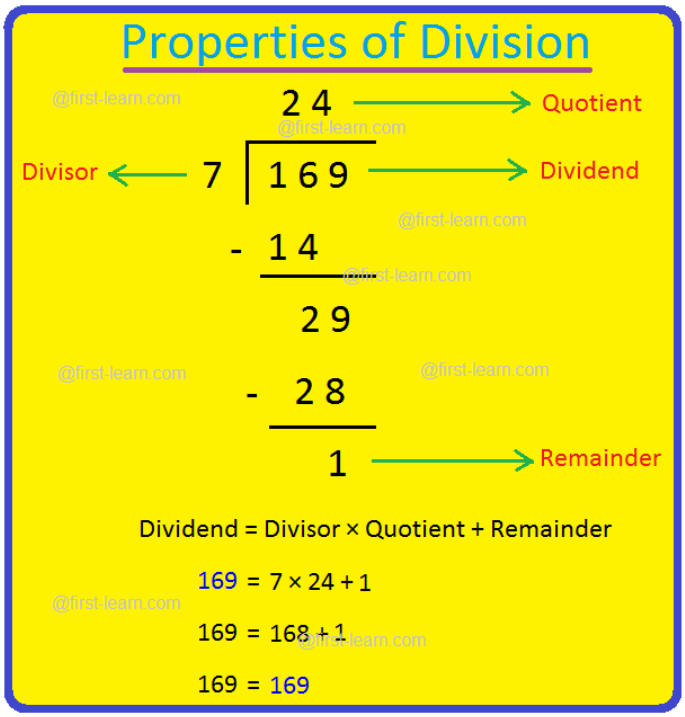


Figure 27: Properties of division (adapted from @first-learn.com)

Figure 28 represents group C learners' response to a question in a mathematics classroom.

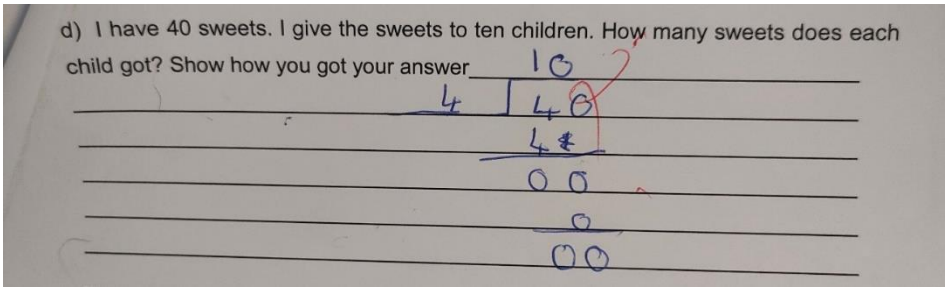


Figure 28: Response of learners from group C

Figure 28 shows how learners used long division to calculate the algebraic word problem. Figure 28 shows that learners had the ability to use the correct strategy to solve the problem. However, how they carried out the strategy was incorrect. Figure 28 shows that learners managed to put the dividend by in the correct place,

as shown in Figure 28, and put the divisor in the place of quotient and the quotient in the place of a divisor. The strategy that they used was correct. However, they misplace the divisor and the quotient. The challenge was the manner in which they carried out the strategy. Figure 25 shows that learners had the ability to formulate algebraic word problems in order to solve the problem mathematically. Figure 25 that represent the response of learners from group D indicates that learners could use different strategies to solve the algebraic word problems. Learners carried out the strategy efficiently in other to get the answer. They divided 40 by 10 to get 4 as the correct answer

- **Adaptive reasoning**

Figure 29 shows that learners were expected to respond two questions. However, this figure indicates that learners responded to only one question. The majority of learners did not respond the second question. This indicates that the learners lacked conceptual understanding and, as a result, they failed to show their reasoning capacity in their writing.

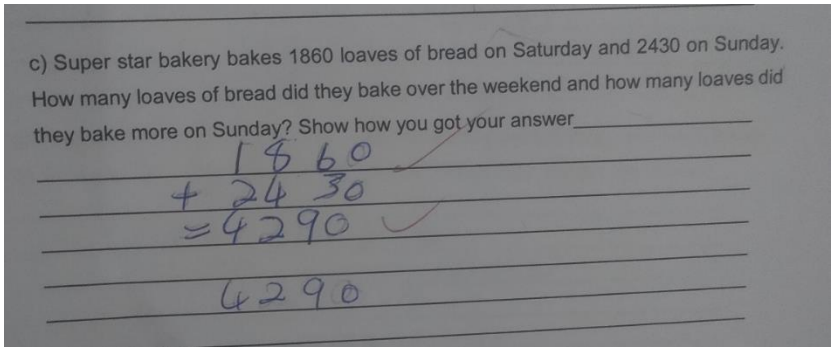


Figure 29: Response of learners from group I

Figure 25, which represents response of learners from group D, indicates that the learners' reasoning was well developed. This figure shows that the learners managed to calculate the algebraic word problem and write down their conclusion that 'each child got four sweets'. Their workings show that the learners were able to justify their solution, since they managed to draw a conclusion from their response to the question.

❖ **Productive reasoning**

The use of loaves of bread caused learners to appreciate mathematics as they are familiar with loaves of bread. The teacher also used the examples that learners come across in everyday life situation which encourages learners to see mathematics as sensible and worthwhile.

4.2.3 Data gathered from the teacher's interviews

The procedure in item 4.1.3 was followed. The researcher conducted interviews with the teacher, guided by the interview schedule in section 8.3. However, a snapshot of the transcription was captured and presented as excerpt 6. A transcription of all the data captured can be found in Appendix F item 8.6.3.

Excerpt 6

6.1 What challenges did you notice, faced by learners when solving algebraic word problems? Outline them

6.2 The challenges I have identified when learners were solving the question of Super star bakery is that learners were able to solve only one solution, without responding to the second solution of the question. I also realised that the questioning style was the one that made learners to have challenges, if the question were to be separated, learners were going to be able to respond all of them. Lastly, I have noticed that learners a facing a challenge of knowing the standard procure of subtracting numbers, whereby they supposed to subtract a bigger number with a smaller at their level.

❖ **Analysis of data gathered from teacher interview**

Conceptual understanding

The response of the teacher indicates that, when developing learners' conceptual understanding, it is necessary to afford learners an opportunity to read the scenario several times to make sense of it (3.2). In the process of reading, learners are expected to write down and represent important aspects of the scenario mathematically.

❖ **Procedural fluency**

The teacher gave learners the procedure that they should use when encountering algebraic word problems. The procedure was that learners should read an algebraic word problem several times in order to conceptualise it. In terms of the process of reading, the teacher also stated that learners should be in a position to write down/represent the important aspects of algebraic word problems

mathematically (3.2). Finally, the learners should be able to choose the correct procedure to solve algebraic word problems flexibly, accurately, efficiently and appropriately (3.4).

❖ **Strategic competency**

The teacher was asked what teaching strategy he would use to develop strategic competency of the learners (3.3). His response was: 'I use learning-centred classroom incorporated with discourse-based mathematics instruction to create an environment that afford learners an opportunity to develop mathematical proficiency. This strategy affords learners an opportunity to: discuss and share their mathematical ideas to formulating algebraic word problems and choose the correct strategy to solve it; explain their reasoning; compare and justify problem-solving strategies; communicate their ideas; challenge each other's reasoning through questioning, reflect on and clarify their thinking.' The teacher stated that the two strategies that are mentioned in excerpt 3 item 3.4 afforded learners an opportunity to discuss and share their mathematical ideas in order to formulate algebraic word problems and to choose the correct strategy to solve these problems. This indicates that a learning-centred classroom incorporated with discourse-based mathematics instruction creates an environment where learners can interact with their peers in the process of doing mathematics. As a result, learners are able to formulate algebraic word problems and have the ability to choose the correct strategy to solve algebraic word problems.

❖ **Adaptive reasoning**

The teacher stated, when learners are engaged in a learning-centred classroom, they become active participants in doing mathematics. As a result, learners develop confidence to solving algebraic word problems and to justify their answer. The teacher incorporates discourse-based mathematics instruction to afford learners an opportunity to engage in mathematical talk. Mathematical talk improve learners' reasoning. In addition, learners develop the ability to draw conclusions when solving algebraic word problems

❖ **Productive disposition**

When he was introducing his lesson, the teacher gave learners an example of a parking lot at Paledi mall. Parking lots are things that learners come across in everyday life situations, which made learners appreciate mathematics and see it as sensible, useful and worthwhile.

4.2.4 Data gathered from learners' interviews

The similar procedure was followed from item 4.1.4. The snapshot of transcription of data were captured on Excerpt 7. However, the whole transcription of data was captured on Appendix F under item 8.6.4. Moreover, follow up questions and probing questions were not captured on the transcription of the data

Excerpt 7

7.1 Researcher: What is your understanding about solving algebraic word problems? Explain
7.2 Group D: Read and understand the question before we respond to it
7.3 Researcher: Can you restate the algebraic word problems in your own words? Explain
7.4 Group D: They want to know how many bread of loaf did they bake over the weekend
7.5 researcher: What are you asked to find? Explain
7.6 Group B: How many loaves of bread did they bake over the weekend and how many loaves did they bake more on Sunday? Show how you got your answer
7.7 Researcher: Is there enough information to help you to find the answers? Explain
7.8 Group E: Yes, because super star bakery baked 1860 loaf of bread on Saturday and 2430 of bread on Sunday
7.9 Teacher: Explain how did you get to your answer?
7.10 We took 1860 – 2430 and the answer was 1430

❖ **Analysis of data gathered from learners' interview**

Conceptual understanding

Learners' responses to question 1 indicate that the learners were told that, when they encounter algebraic word problems, they should read the algebraic word problems and understand them before they can be solved (7.2). The way learners' responded question 1 indicates that learners repeated what their teacher told them to do. They did not show the posture of understanding the algebraic word problem, particularly in their response to question 1. In question 2, group D learners managed to restate the algebraic word problem without changing its meaning (7.4). However, some learners simply agreed that they could restate the algebraic word problem in their own words without changing the

meaning, only to find out that they did not restate the problem. This indicates that some learners still face the challenge of understanding the concepts.

The responses to question 3 by group B indicate that learners had an insight into what was required from them, however they failed to solve the algebraic word problem accurately (7.6). Figure 30 indicates that learners were lost, they even failed to write down the mathematical operation. However, looking at their working shows that the learners used subtraction, even though they did not show this in their writing. This indicates that learners still struggle to decide which mathematical operation to use when solving algebraic word problems.

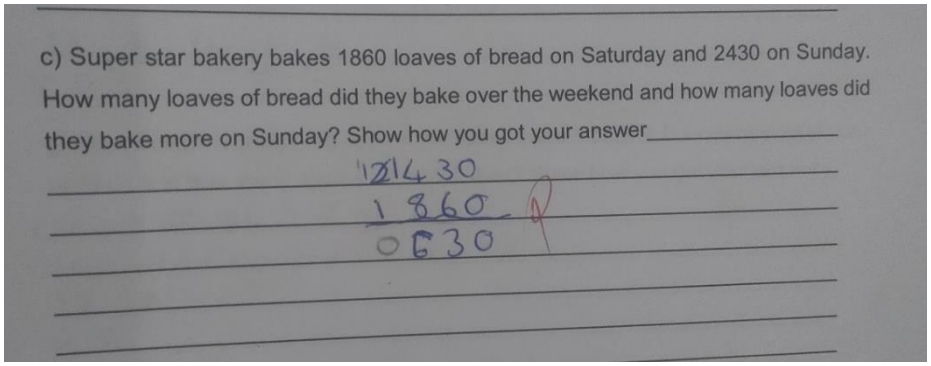


Figure 30: Response of learners from group E

❖ **Procedural fluency**

The response by group E to question 4 indicates that the majority of learners understood that they had sufficient information to carry out the procedure flexibly, accurately and efficiently (7.8). However, Figure 31 indicates that the learners failed to carry out the procedure accurately (producing a correct answer).

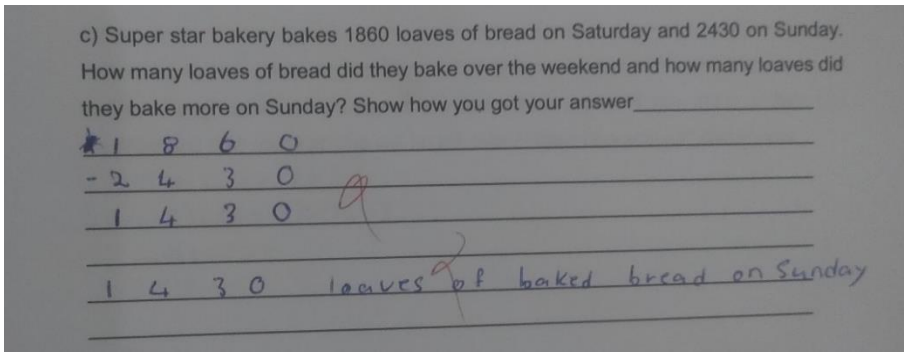


Figure 31: Response of learners from group G

Figure 31 shows that learners faced challenges when solving the algebraic word problem. The first challenge was that the learners did not solve the first part of the question, which was to find the numbers of loaves of bread that were baked over the weekend. The second challenge was that, when solving the second part of the question, the learners did not follow the standard procedure of subtracting numbers. In Grade 4 learners were expected to subtract the smaller number from the larger number. However, as illustrated in Figure 31, the learners did the opposite, resulting in them getting to an incorrect answer. Hence, Figure 31 shows that the learners' procedural fluency was poorly developed.

❖ **Strategic competency**

The responses of learners to questions 5 and 6 of the interviews indicated that the learners used the column method to add and/or subtract. Figure 30 and 31 indicate that, indeed, learners used the column method. However, the writing down of learners shown in figures 30 and 31, respectively, shows that the majority of learners did not understand the question. As a result, the majority of learners got to an incorrect answer and they did not respond to the second part of the question, which required them to find how many more loaves they baked on Sunday. This indicates that the learners' strategy was correct, although their computation was incorrect, as learners used an incorrect mathematical operation or did not follow the standard procure of subtracting a smaller numbers from a larger numbers.

❖ **Adaptive reasoning**

A response by group B to question 5 of the interviews stated that: 'We took 1 860 – 2 430 and the answer was 1 430' (7.10), indicating that the learners did not follow the standard procedure for subtracting numbers in Grade 4, where learners are supposed to subtract a small number from a large number. When I interviewed the learners and asked why did they a larger number from a smaller number, their response was 'we wanted to get the number of breads that were baked more on Sunday'.

The response of learners in group K is shown in Figure 21 and indicates that learners in group K used multiplication instead of addition to solve the problem. Learners were supposed to add 1 860 and 2 430, instead they multiplied 2 430 by 2 to get 4 860. When I asked them why they used multiplication, their response was 'because the question wanted us to calculate the number of breads baked over the weekend, since the weekend is having 2 days, we decided to multiply by 2'. The reasoning of the learners indicates that they did not understand the question clearly because the number of bread loaves that were baked on Saturday and Sunday was given. Again, in question 2, learners took the total they got in number 1 and added it to the number of bread loaves that were baked on Saturday. The second part of the question also shows that learners did not have an idea of what they were doing. In question 2 learners were expected to calculate how many more loaves they baked on Sunday, which required learners to subtract the number of breads baked on Saturday from the number of breads baked on Sunday. This indicates that learners still struggle with conceptual understanding, which results in learners not being able to reason appropriately.

❖ **Productive disposition**

The example that the teacher used to introduce the lesson involved the Paledi mall parking lot, which the majority of learners were familiar with. The use of the Paledi mall parking lot meant that the learners realised the importance of mathematics in a real-life situation. In addition, the activity that was used in learning episode 2 used the loaves of bread. This meant that the learners could

see mathematics as sensible and useful since they buy bread almost every day of their lives.

4.3 Conclusion

In this chapter, findings from two learning episodes were reported. The findings were based on the responses of a Grade 4 mathematics teacher and Grade 4 mathematics learners, focussing on Kilpatrick et al.'s (2001) strands of mathematical proficiency, namely conceptual understanding, procedural fluency, strategic competency, adaptive reasoning and productive disposition. The responses of the Grade 4 mathematics teacher and Grade 4 mathematics learners were separated. Furthermore, the response of the teachers in respect of learning episodes 1 and 2 were summarised, as were the responses of the learners in respect of learning episodes 1 and 2.

4.3.1 Mathematics teacher's response.

❖ **Conceptual understanding**

The approach of the teacher in learning episode 1 in class A did not clearly indicate that the teacher was developing the learners' conceptual understanding because the teacher did not fully engage the learners. The teacher gave the learners instructions that they should use when solving algebraic word problems. According to the teacher, learners were expected to follow instructions. The teacher solved the algebraic word problems. He did not give learners the platform to solve the algebraic word problems on the board. He provided a solution to the learners. The example that the teacher used to introduce the lesson was a simple question. The question did not challenge learners to think. As a result, the introduction of the lesson to class A in learning episode 1 did not clearly develop the learners' conceptual understanding. Learners memorised the question and provided the solution. However, the introduction of the lesson to class B indicates that the teacher reflected on the lesson he presented to class A. The teacher gave the learners a difficult question to deal with (Van Harpen & Presmeg, 2013). Learners did not get to the answer immediately (Polya, 2004). They struggled to get to the correct answer. However, the introduction of lesson to class B indicates

that the teacher created a conducive environment for developing conceptual understanding by allowing the learners to take part in their learning and engaging the learners in classroom discourse, were the learners share their ideas and communicate with their peers in the process of doing mathematics.

The approach to learning episode 2 indicates that the teacher, indeed, reflected on the class A lesson in learning episode 1. When introducing learning episode 2, the teacher wrote the algebraic word problem out on top of the white board and allowed the learners to read the question several times. The algebraic word problem that the teacher used was a challenging question, which challenges the learners' thinking. He then posed a question to the learners. When learners responded to the question, the teacher asked one learner to come up and write the solution down on the board. The learner came up to the board and the teacher asked another learner to come up and explain what the other learner had written. One learner stood up and explained the solution; in that way the teacher engaged the learners in a learning-centred classroom that incorporated discourse-based mathematics instruction to allow learners to participate actively in their learning. Learners were able to interact with one another, connecting topics and concepts, and speaking mathematics by participating in a learning-centred classroom incorporated with discourse-based mathematics instruction as the method employed by the teacher to develop learners conceptual understanding (DBE, 2018).

- **Procedural fluency**

In learning episode 1, the teacher emphasised the procedure that learners should follow whenever they are presented with algebraic word problems. However, the results of the learners participating in learning episode 1 revealed that the learners had memorised the procedure and that they were able to carry out the procedure accurately without understanding it. The procedure that learners were supposed to follow in learning episode 1 was to multiply 110 by 3 and 80 by 3, separately, and, later, add the results together. Out of the 13 groups of learners, eight groups of learners managed to carry out the procedure accurately, while

five groups of learners did not managed to carry out the procedure accurately. This indicates that majority of learners could carry out the procedure of solving algebraic word problems flexibly, accurately, efficiently and appropriately. However, a few learners struggled to choose the correct procedure as a result of a lack of conceptual understanding.

- **Strategic competency**

The teacher afforded learners an opportunity to read an algebraic word problem for several time, such that learners can comprehend the algebraic word problems and make connections with concepts. Consequently, learners were able to formulate algebraic word problems and choose the strategy to solve the algebraic word problems under learning episode 1 question one letter G. However, majority of learners under learning episode 1 question 2 were unable to choose the correct strategy to solve the algebraic word problems. Out of thirteen groups, only one group emerge and use the correct strategy to solve an algebraic problem.

Under learning episode 2, when the teacher introduced the lesson, the teacher created an environment where learners were collaborating when doing mathematics which minimise learners' errors as they were working together on the example given to them. Learners' response was satisfactory on the example that was given to them. However, learners under learning episode 2 question letter C. Out of thirteen groups of learners, 6 groups of learners managed to use the correct strategy to solve an algebraic word problem. Two groups of learners partially used the correct strategy; hence they did not get all the marks. Lastly, 5 groups of learners were not successful in choosing the correct strategy to solve the algebraic word problems. As a results, errors were committed by learners as they were solving the algebraic word problems.

- **Adaptive reasoning**

In learning episode 1 class A, the teacher did not encourage learners to reason or justify their solution. The teacher was in control of the lesson; the teacher did not use a learning-centred classroom incorporated with discourse-based

mathematics instruction to develop learners' adaptive reasoning. However, in the case of class B in learning episode 1 and in learning episode 2, the teacher used a learning-centred classroom incorporated with discourse-based mathematics instruction to develop the learners' adaptive reasoning. Initially, learners were reluctant to speak in front of their peers. The teacher intervene by creating an environment that afford the learners an opportunity to communicate with their peers and to justify their reasoning.

❖ **Productive disposition**

In both learning episodes, the teacher used examples that the learners would come across in their everyday life situations. In class A learning episode 1, the teacher used the example of buying a 'my size [300 ml]' cold drink and in class B the teacher used ice cream as the example. When learners go to a shopping mall, they come across ice cream and 'my size [300 ml]' cold drinks, which made them to appreciate mathematics. The teacher gave the learners effective feedback, which made them realise the mistakes that they have committed while solving algebraic word problems. Effective feedback led to the learners appreciating mathematics and see mathematics as sensible, useful and worthwhile.

4.3.2 Learners' responses

❖ **Conceptual understanding**

Learners' conceptual understanding in learning episode 1 indicates that the learners partially understood algebraic word problems because the teacher gave learners an opportunity to read the algebraic word problem scenario, after which, the teacher translated the algebraic word problems into Sepedi, which helped learners understand the problem (Bravo-Sotelo, 2020; Setati et al., 2002). In response to question 1 sub-question letter G, of the 13 groups of learners, seven groups managed to get to the correct answers, which indicates that learners understood the algebraic word problem. In contrast, five groups of learners did not get to the correct answer, indicating that they still lacked a conceptual understanding, which can be informed by a lack of mathematical vocabulary. The remaining group got one mark for using correct values but failed to carry out the

correct procedure to solve algebraic word problems. In response to question 2, 11 groups of learners did not get to the correct answer, one group left a blank space and the last group managed to get to the correct answer.

In learning episode 2, the teacher introduced the lesson and used a challenging question, allowing the learners to tackle the question as a group. Immediately after the learners responded to the question, the teacher allowed them represent their solution on the whiteboard, which meant that the learners were engaged in the lesson. The learners indicated a level of understanding of the introduction to the lesson, since the teachers modelled the lesson.. The data for analysis were drawn from question letters C and D. In respect of question letter C, of the 13 groups of learners, six groups of learners did well completely, two groups did well partially because they received only one mark, which indicates that they did not understand the question, and, lastly, five groups of learners did not do well at all. The six groups of learners indicated that they understood the mathematical concept that was taught in the classroom, could represent the algebraic word problem mathematically and, lastly, could choose the correct mathematical operation to solve the algebraic word problem. The seven groups of learners showed little knowledge and understating of solving the algebraic word problems.

The result for learning episodes 1 and 2 indicate that there was an equal opportunity for the learners to develop a conceptual understanding as half the number of groups got to the correct answer. The learners indicated that they lacked arithmetic knowledge. This lack of arithmetic knowledge was informed by a lack of mathematical vocabulary in learners. As a result, their conceptual understanding was only partially developed.

- **Procedural fluency**

The outcomes of learning episodes 1 and 2 show that learners had the ability to choose the correct procedure. Of the thirteen groups of learners, eight groups of learners were able to carry out the procedure to solve the algebraic word problem flexibly, accurately, efficiently and appropriately. In contrast, five groups of

learners struggled to carry out the procedure to solve the algebraic word problem accurately. As a result, this shows that the learners' procedural fluency was developed since all the learners were able to choose the correct procedure required to solve the algebraic word problem, despite the fact that some learners did not carry the procedure out accurately.

- **Strategic competency**

According to the findings from learning episode 1 question 1 sub-question letter G, of the 13 groups of learners, eight groups of learners managed to use the correct strategy to solve the algebraic word problem correctly. The remaining five groups of learners managed to choose the correct strategy, however, they failed to use the correct values to solve the algebraic word problem, and, as a result, their computation was incorrect and they got to an incorrect answer. Learners were expected to multiply 110 by 3 and 80 by 3 to get the price of three school trousers and the price of three school shorts separately and, later, add them together to find the total price of school trousers and school shorts. The five groups used 110 by 6 and 80 by 6, which led them to get to the incorrect answer. In question 2, the majority of learners were unable to choose the correct strategy to solve algebraic word problems, while only one group managed to use the correct strategy to solve question letter C.

Learners were given the activity in learning episode 2 to respond to. The activity consisted of seven questions, question letters A to G. The learners were expected to respond to all the questions. However, question letters C and D were used for analyses purposes. The findings in respect of question letter C indicated that six groups of learners got to the correct answer. This indicates that the learners were able to formulate algebraic word problems and were able to choose the correct strategy to solve the problem. However, five groups of learners did not use the correct strategy, hence, they got to an incorrect answer. The remaining two groups of learners did not get all the marks. Data from question letter D reveals that the majority of learners struggled to formulate and choose the correct strategy to solve algebraic word problems. Of the thirteen groups of learners,

seven groups of learners did not get the correct answers and the reimaging groups of learners got the correct answers. Considering number of learners who got to the correct answer and those who got to an incorrect answer in learning episodes 1 and 2, and equal number of learners got to correct answer as learners who got to an incorrect answer. As a result, learners' strategic competency was equally developed.

- **Adaptive reasoning**

The responses to learning episodes 1 and 2 revealed that the learners were reluctant to explain their answer when they were interviewed. Learners were expected to explain algebraic word problems, justify their solutions and draw conclusions. During the interviews session, most of learners were silent. They did not respond to interviews questions. However, in terms of their writing, some learners were able to justify and draw conclusions about their solutions.

- **Productive disposition**

In learning episodes 1 and 2, learners were given the feedback by their teacher. The feedback afforded the learners an opportunity to reflect on their work and allowed them to see the mistakes that they had committed while solving algebraic word problems. This enabled the learners to see mathematics as sensible, useful and worthwhile. In addition, the teacher used examples in real-life situations that made the learners appreciate mathematics.

CHAPTER 5: RESPONSE TO RESEARCH QUESTIONS

The purpose of the study was to explore a Grade 4 mathematics teacher's development of his learners' mathematical proficiency towards solving algebraic word problems. This chapter deals with the response to the research questions that were posed in Chapter 1 of the study. The questions were: 'What are Grade 4 learners' challenges when solving algebraic word problems?' and 'How does a Grade 4 mathematics teacher develop learners' mathematical proficiency towards solving algebraic word problems?' Findings in Chapter 4 assisted in answering the research questions.

5.1 Learners' Challenges When Solving Algebraic Word Problems

Chapter 4 of the study presented the findings and a discussion. In the discussion it was found that learners face several challenges when solving algebraic word problems. The challenges faced by learners were identified in learning episodes 1 and 2. The findings revealed that learners lack conceptual understanding, strategic competency, adaptive reasoning, mathematical vocabulary, arithmetic skills and reflective practice.

5.1.1 Mathematical vocabulary

The findings discussed in chapter 4 revealed that the challenges learners face when solving algebraic word problems are as a result of them not understanding the language used in algebraic word problems (Lee & Goerss, 2007). Learners struggled to understand the word 'restate', as mentioned in section 8.4 bullet point 2. As I was the person who interviewed the learners', I had to code switch from English language as a medium of to the learners' mother tongue language (Sepedi) (Bravo-Sotelo, 2020). After translating the word 'restate' into their mother tongue, learners were able to respond to the interview question. Furthermore, learners also struggled to understand the word 'operation', as mentioned in section 8.4, bullet point 7. As a result, teachers need to be cautious when operating with entities and not separate operations from their mathematical terms (Sihlangu, 2022).

The findings in section 4.1.2 revealed that learners in group C and G as shown in Figure 8 and Figure 13 respectively, lacked a conceptual understanding because they added the price of school trousers and the price of school shorts, which was recorded as R110 + R80 = R190 (the price of school trousers and school shorts added together), and then they multiplied the result by 6 to get the answer. Learners were supposed to multiply the result by the number 3, however, they used the number 6 because the statement said that their mother took care of 6 children. This statement indicates that learners struggled to analyse algebraic word problems with understanding. This struggle is informed by a lack of mathematical vocabulary. Mathematical vocabulary plays a critical role in developing learners' mathematical proficiency towards solving algebraic word problems. Sibanda (2017) stated that Grade 4 learners face a challenge in solving mathematical word problems because they lack mathematical vocabulary, which is informed by the transition from Grade 3 to Grade 4. In South African schools, learners in Grade 3 are taught mathematics in their mother tongue, while, in Grade 4, learners are expected to use English as the medium of instruction, which makes it difficult for them to learn mathematics (Sibanda, 2017).

5.1.2 Conceptual understanding

The response to a question in learning episode 2 by group E learners, shown in Figure 30, revealed that the majority of learners did not understand that the question required two solutions. The learners attempted to respond to only one solution. This indicated that the learners struggled with a conceptual understanding of algebraic word problems. They could not tell whether the question needed one or two solutions. Figure 21 in item 4.2.2 revealed that learners struggle to conceptualise algebraic word problems. Figure 21 showed that the learners used multiplication instead of using addition to get the total number of bread loaves that were baked over the weekend. The learners' response illustrated in Figure 22 shows that they did not understand the question, since they wrote 28 x 6 and got 34. Their workings show that they lacked a conceptual understanding because what they wrote did not form part of the question. Learners write without attaching meaning to what they are writing. The

usage of multiplication, as illustrated in Figure 22, shows that the learners did not grasp the concept taught and, as a result, they failed to choose the correct operation to solve the algebraic word problem. Figure 24 also illustrate that learners lack a conceptual understanding of solving algebraic word problems because they wrote 40 - 10 = 30. This indicates that learners did not understand the question, hence they used subtraction instead of using division. In addition, challenge of conceptual understanding faced by the learners could also be influenced by learners' lack of mathematical vocabulary

5.1.3 Adaptive reasoning

The response of the learners to interviews relating to learning episodes 1 and 2 revealed that the learners struggled to justify their solution. The questions in both learning episodes 1 and 2 had two solutions. In addition, learners were expected to draw a conclusion. However, the learners only solved one problem and, of the 13 groups of learners, only one group manage to draw a conclusion in learning episodes 1 question 2. Group M as shown in Figure 10 manage to add the answer from question 1 letters A to G to respond to question 2. Question 2 sought the total price that their mother spent on the items mentioned in question 1. During the interviews concerning learning episode 2, I asked the learners to justify the solution they came up with for question letter C. The majority of the learners did not respond to my question. When I double check the reason behind their silence, I found that the learners did not understand the question. Moreover, the questioning style of the algebraic word problems was also problematic for the learners because they could not tell whether the question was looking for two solutions or not.

5.1.4 Arithmetic skills

According to the Grade 4 curriculum, learners are expected to add or subtract numbers up to four digits in length and multiply numbers up to three digits in length by one-digit numbers. The learners' responses in learning episode 1 revealed that they made careless mistakes. Learners were expected to multiply R110 by 3 to get to the answer for the number of school trousers and R80 by 3 to get to answer for the number of school shorts. In contrast, some of the groups

of learners added the price together (R110 + R80 = R190) and the number of children together (3 + 3 = 6), and then they multiplied R190 by 6 to get to the total price that their mother had paid. This challenge was also informed by a lack of language proficiency. The response of the learners from group G, shown in Figure 31, indicated that learners were subtracting a larger number from a smaller number. The curriculum in Grade 4 does not allow learners to subtract a larger number from a smaller number. The response from the learners in group B, shown Figure 23 indicated that learners wrote down two mathematical operations (multiplication and division). These are some of the arithmetic skills that learners lack when solving the algebraic word problems (Pongsakdi et al., 2020).

5.1.6 Reflective practice

Learning episode 2 revealed that the learners did not reflect on what they had learned from the introduction to the lesson. When the teacher introduced the lesson, he asked the learners a question and learners responded. He asked learners whether the question was complete and learners said ‘No’. However, when learners were given the activity from learning episode 2, they could not reflect on what they had learned from the introduction to the lesson. The knowledge they gained from the introduction of the lesson was preparing them to respond to similar algebraic word problems type questions.

5.2 Development of Learners’ Mathematical Proficiency

5.2.1 Pedagogical content knowledge

A Teacher’s pedagogical content knowledge (PCK) plays an important role in developing learners’ mathematical proficiency (Lee et al., 2018). Furthermore, subject matter knowledge (SMK) the helped the teacher to prepare for the lesson. While the teacher prepared the lesson, he anticipated the misconceptions that the learners would bring to the classroom. Therefore, it was easy for the teacher to address these misconceptions. The teacher’s PCK made the lesson more interesting for the learners. The teacher used different examples, which included real-life situations that were related to the content. PCK affords teachers an

opportunity to implement learning-centred classrooms incorporated with discourse-based mathematics instruction in the process of developing learners' mathematical proficiency. A teacher who lacks PCK cannot implement learning-centred classrooms incorporated with discourse-based mathematics instruction strategies in the process of developing learners' mathematical proficiency.

5.2.2 Reflective practice

After teaching learning episode 1 in class A, the teacher set down and reflected on the lesson. He realised that he had used lower order, simple question to introduce the lesson and did not allow learners to attempt the example by themselves. He wrote for the learners. During learning episode 1 class B and learning episode 2, the teacher used a challenging question to introduce lessons, he allowed learners to work on the question and then he gave them an opportunity to write their solution on the whiteboard. The reflective practice of the teacher plays a significance role in developing learners' mathematical proficiency towards solving algebraic word problems (Raoano, 2016). After each lesson a teacher needs to sit down and reflect on their lesson, determine what worked and what did not work, and come up with strategy to improve teaching and learning, particularly when developing learners' mathematical proficiency. The strategies that the teacher used to develop learners mathematical proficiency include learning-centred classrooms incorporated with discourse-based mathematics instruction.

5.2.3 Learning-centred classroom

A learning-centred classroom is an environment that allows meaningful learning and teaching to take place (DBE, 2018). It is the responsibility of the teacher to create such environment so that strands of mathematical proficiency towards solving algebraic word problems can be developed. The way in which a teacher conducts a classroom depends on the way in which they view mathematics. A teacher who sees mathematics as a body of knowledge that has to imparted to learners will mostly tell learners what to do and how to do it. On the other hand, a teacher who sees mathematics as a body of knowledge that learners should actively explore and engage with, creates a learning environment where learners

can make sense of mathematics. Furthermore, learners are afforded an opportunity to express their ideas and ask questions to engage in a meaningful learning of mathematics, particularly when solving algebraic word problems (DBE, 2018).

A learning-centred classroom affords learners an opportunity to make sense of mathematics, particularly when learners are given examples that they come across in real-life situations. Learners who are engaged in learning-centred classroom can speak the mathematics language and they are able to make connections between the topics and concepts they come across when learning mathematics, particularly when solving algebraic word problems. Furthermore, teachers need to incorporate learning-centred classrooms into their instructional practice when developing learners' mathematical proficiency to solve algebraic word problems (DBE, 2018).

5.2.4 Discourse-based mathematics instruction

The central idea of discourse-based mathematics instruction is that learning mathematics occurs by way of participation in mathematical discourse practices orchestrated by the teachers, by choosing mathematical tasks into which the content to be learned is embedded (Kazemi, 2008; National Council of Teachers of Mathematics, 2010; Sfard, 2008). Discourse-based mathematics instruction affords teachers an opportunity to create an atmosphere in the mathematics classroom that includes constructing arguments, questioning, explaining, exemplifying, agreeing, disagreeing, justifying, discussing and comparing (Moschkovich, 2007; Stein, 2007). As a result, these discourse-based mathematics instruction practices serve as instructional strategies that enhance the learners' development of mathematical proficiency towards solving algebraic word problems (Bennett, 2018). Furthermore, it affords learners an opportunity to discuss and share their mathematical ideas, to explain their reasoning, to compare and justify problem-solving strategies, to communicate their ideas, to challenge each other's reasoning through questioning, to reflect on and clarify their thinking, and to listen to other's points of view (Steeley, 2017). Engaging

learners in discourse-based mathematics instruction practices develops learners' mathematical vocabulary that enhances learners' conceptual understanding of solving algebraic word problems (Kazemi, 2008; Van de Walle et al., 2015).

5.2.5 Problem posing

Problem posing is an effective mathematical activity for improving learners' mathematical proficiency. The manner in which the teacher poses the question to their learners determines how learners' mathematical proficiency can be developed. In learning episode 1, the teacher used a simple question to introduce the lesson. The manner in which the teacher posed the question in learning episode 1 in class A meant that the learners were involved in rote learning rather than deep learning. The learners memorised the procedure that the teacher used to get the solution without understanding the process that they should undergo in order to solve algebraic word problems. However, in class B and in learning episode 2, the teacher used challenging questions to enforce deep learning, where the solution to the question was not immediate (Polya, 2004).

The process of problem posing affords learners opportunities to understand the complexities of mathematical concepts (Priest, 2009). In addition, problem posing transfers learners' understanding to their concrete knowledge (Bernardo, 2001), and they can apply this knowledge to the algebraic word problems posed to them (Mestre, 2002). The literature indicates that the development of concrete, comprehensive mathematical knowledge by means of problem posing can influence development of mathematical proficiency in learners towards solving algebraic word problems (Van Harpen & Presmeg, 2013). Consequently, teachers should take care of problem posing when developing learners' mathematical proficiency towards solving algebraic word problems.

5.3 Summary

This chapter presented the responses to the two research questions that guided the exploration how Grade 4 mathematics teacher's development of learners' mathematical proficiency towards solving algebraic word problems. The

responses revealed that learners faced challenges with mathematical vocabulary and that they were also lacked a conceptual understanding, adaptive reasoning, arithmetic skills and reflective practice. From the findings of the study, it is evident that a lack of mathematical vocabulary affects learners' conceptual understanding. It was also found that a learning-centred classroom incorporated with discourse-based mathematics instruction improve learners' mathematical proficiency towards solving algebraic word problems.

CHAPTER 6: CONCLUSION, LIMITATION, RECOMMENDATIONS AND IMPLICATIONS

6.1 Introduction

This chapter presents the conclusion, recommendations and the implications of this study. The conclusion summarises the key findings of the study, which are related to the objectives of the study (Faryadi, 2019). It also summarises the responses to the research questions. The research questions for the study were: 'What are Grade 4 learners' challenges when solving algebraic word problems?' and 'And how does a Grade 4 mathematics teachers develop learners' mathematical proficiency towards solving algebraic word problems?' Lastly, the conclusion deals with how the study contributes to the existing body of literature. This chapter also outlines the recommendations, implications and opportunities of the future research.

6.2 Conclusion

This study reported on a Grade 4 mathematics teacher's development of learners' mathematical proficiency towards solving algebraic word problems. The objectives that guided the study were: to identify Grade 4 learners' challenges when solving algebraic word problems; and to document how a Grade 4 mathematics teacher develops learners' mathematical proficiency towards solving algebraic word problems. To achieve the objectives of the study, two learning episodes were presented to Grade 4 mathematics learners by a Grade 4 mathematics teacher. Data were collected through observation, learners' written task and interviews. The theory of mathematical proficiency was used to guide the analysis of the study. Findings of the study reveal that learning-centred classroom incorporated with discourse-based mathematics instruction has improved learners' mathematical proficiency when solving algebraic word problems. The study also revealed that learners are still facing a great challenge of solving algebraic word problems because of lack of mathematical vocabulary, conceptual understanding, adaptive reasoning, arithmetic skills and reflective practice (Reano, 2016). As a result, there is a need for teachers to intensify

discourse-based mathematics instruction within learning-centred classroom to develop learners' mathematical vocabulary that could result in learners' development of conceptual understanding as corner stone of mathematical proficiency

6.3 Limitation

The qualitative nature of this research study and the convenient sampling of one mathematics teacher and 79 learners from one school, limit the findings and the scope of the research (Le Roux, 2020). Furthermore, this study was conducted in a classroom where learners were coming from Sepedi-speaking backgrounds. As a result, the teacher was struggling to engage learners fully in discourse-based mathematics instruction as a practice to develop learners' mathematical proficiency towards solving algebraic word problems. The teacher was teaching learners during sports time when other learners were playing outside, while others were attending, this could have limited the findings of the study as learners' concentration was disturbed by the noise outside the classroom. The time that was allocated during sports time limited the teacher. As a result, the teacher was unable to interact with all the groups in the classroom. Apart from that, the teacher was also inadequately experienced with discourse-based mathematics instruction, as he was struggling to implement it in a large size classroom after classes A and B were merged in the school hall (Bradford, 2007).

6.4 Recommendations

This study was conducted in a classroom where the researcher was observing how the teacher develops learners' mathematical proficiency towards solving algebraic word problems. The participants of the study were conveniently sampled, since they were available with regard to access, location, time and willingness (Lopez & Whitehead, 2013). This study sampled a Grade 4 mathematics teacher and 79 Grade 4 mathematics learners. It is recommended that similar studies should be conducted using a sample of more than one mathematics teacher to gain insights into how different teachers develop their learners' mathematical proficiency towards solving algebraic word problems

using learners from different primary schools to identify other challenges that learners may face when solving algebraic word problems.

6.5 Implications

The findings of this study have implications for developing Grade 4 learners' mathematical proficiency towards solving algebraic word problems. Teachers need to keep their minds open to new ideas that seek to improve their instructional practice. Algebraic word problem activities should not only be given to learners as classwork or homework, but teachers should also strive to equip learners with the necessary skills that could help them to solve algebraic word problems.

Teachers should always encourage learners to open their minds to ideas that could improve their ability to solve algebraic word problems. Learners should be encouraged to read algebraic word problems more often and to make sense of them in order to improve their mathematical vocabulary and their conceptual understanding. This could help learners to develop procedural fluency, strategic competency, adaptive reasoning and productive disposition.

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8. APPENDICES

8.1 Appendix A: Learning Episodes

8.1.1 Learning episode 1: Finance as context

Solving money problems using addition and subtraction learners use money in their daily lives and in Grade 4 they solve money problems using their addition and subtraction skills. They are solving context free problems as well as problems in a real context, and they are working with whole numbers only, that have 4 digit numbers (National Education Collaboration Trust, 2015).

Unemployment rate in South Africa is extremely high. Mr Mamabolo decided to open a school wear market to make a living. He is selling the school uniforms: Short sleeve shirt for R90,00; Long sleeve shirt for R100,00; School Trouser R110,00; School Shorts R80,00; School Skirt R60,00; Pullover/Jersey R140,00, school tie at R25,00; school socks R20,00; school bag R150,00 and school shoes R290

Read the above scenario before answering the following questions.

1. Your mother is looking after 6 children at your family, including yourself, 3 boys and 3 girls.

a) Your mother buy 6 short sleeve shirt, how much did she pay. Show how you got your answer

b) Your mother buy 6 school jersey, how much did she pay. Show how you got your answer

c) Your mother buy 6 school Bags, how much did she pay. Show how you got your answer

d) Your mother buy 6 pair of socks, how much did she pay. Show how you got your answer

e) Your mother buy 6 school tie, how much did she pay. Show how you got your answer

f) Your mother buy 6 pair of school shoes, how much did she pay. Show how you got your answer

g) Your mother buy 3 school trouser and 3 school shorts, how much did she pay. Show how you got your answer

h) Your mother buy 3 school skirt, how much did she pay. Show how you got your answer

2. Your mother bought all the items mentioned in 1. How much did she pay altogether? Show how you got your answer

3. Your mother is having R6000 after paying for all items, how much change will she receive. Show how you got your answer.

8.1.2 Learning episode 2: Different contexts

a) A farmer has to plant 2 560 mealies. He has already planted 1 290. How many more mealies must he still plant? Show how you got your answer

b) A parking garage has space for 3 755 cars. 1 758 cars are parked in the garage. How many cars can still park in the garage? Show how you got your answer

c) Super star bakery bakes 1860 loaves of bread on Saturday and 2 430 on Sunday. How many loaves of bread did they bake over the weekend and how many loaves did they bake more on Sunday? Show how you got your answer

d) I have 40 sweets. I give the sweets to ten children. How many sweets does each child got? Show how you got your answer

e) 256 children go on a school outing. 4 buses transport the children. How many children were there on each bus? Show how you got your answer

f) There are 156 Grade 4 learners at athletics. Only 9 learners can run at a time. How many races will be there? Show how you got your answer

g) Vuyo bought a box of 156 apples. He packs 8 into a packet, how many packets can be filled and how many apples will be left over? Show how you got your answer

8.2 Appendix B: Observation Schedule Guide

The following questions will guide what to observe as the teacher facilitate learning:

- How does the teacher introduce the lesson?
- How does the teacher facilitate learning, particularly at the early stages?
- Does the teacher offer learners opportunity to make sense of the given tasks?
- Are learners allowed to interact with each other in an attempt to make sense of the activity?
- Do learners ask questions when they are stuck?
- Are the learners free to respond to teachers' questions?
- How does the teacher deal with challenges experienced by learners when solving the given questions?
- Are there any non-verbal interactions? If yes, at what stage did they occur/happen?
- Was the teacher targeting achievement of any strand of mathematical proficiency?
- At what stage did the teacher target achievement of any of the strands of mathematical proficiency?
- How does the teacher use questions to promote achievement of any strands of mathematical proficiency?
- Did the teacher consciously/unconsciously ask questions that assisted the learners to develop learners' mathematical proficiency for solving algebraic word problems?

8.3 Appendix C: Interview Schedule for the Teacher

- How do you develop Grade 4 learners' mathematical proficiency, particularly when solving algebraic word problems? Explain
- What are the teaching strategies or methodologies that work for you to develop learners' mathematical proficiency? Explain in reference to solving algebraic word problems.
- What challenges did you notice, faced by learners when solving algebraic word problems? Outline them
- What are the assessment strategies that seem to be working when you assist learners to develop learners' mathematical proficiency? Explain
- What would you regard as key actions to be taken care of by the teachers when assisting learners to develop mathematical proficiency when solving algebraic word problems? Account why you regard them as key actions?

8.4 Appendix D: Interview Schedule for the Learners

- What is your understanding about solving algebraic word problems? Explain
- Can you restate the algebraic word problems in your own words? Explain
- What are you asked to find? Explain
- Is there enough information to help you to find the answers? Explain
- Explain how did you get to your answer?
- Explain which method/procedure did you use to get the answer
- Describe why did you choose the operation you used to get the answer
- What are the challenges you faced when solving algebraic word problems? Explain

8.5 Appendix E: Transcription of Data Gathered from Learning Episode 1

8.5.1 Transcription of data gathered from observation

Introduction of the Lesson:

Class A

Teacher: you want to buy each one of you my size [300 ml] cold drink. How many are you in the classroom

Learners: 41

Teacher: how much is the price of my size [300 ml] cold drink

Learners: R8

Teacher: if we want to buy 41 learners my size [300 ml] cold drink, how much can we spend all together.

Learner A: we are going to multiply 41 by 8 and get 328

Teacher: Which strategy did you use to get your answer?

Learner A: I used the calculator to get the answer.

Teacher: Ok, indeed we multiplied 41 by 8. Do you still remember column method?

Learners: Yes

Teacher: we are going to use column method to calculate the answer. Next time do not use the calculator right.

Learners: Ok sir

The teacher demonstrates the answer to learners on the white board.

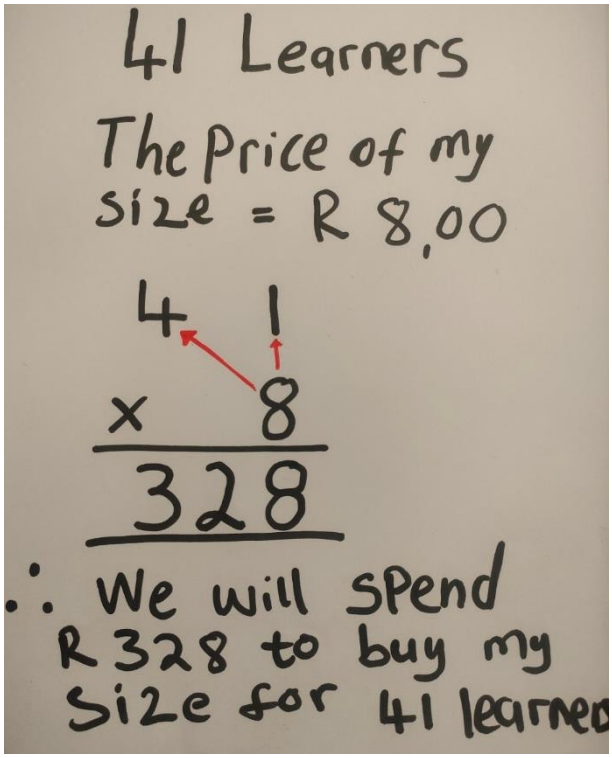

Figure 32: Demonstration by the teacher in class A learning episode 1

Class B

Introduction of the lesson:

Teacher: your mathematics teacher wants to buy all of you ice cream. A plain ice cream cost R7,00. While plain ice cream with crunch chocolate cost R9,00. How many of you would like to eat plain ice cream? May you please indicate by raising your hands.

Learners: 15 learners raised their hands

Teacher: how much money will I spent to buy all of you ice cream? Discuss with your group mates before answering the question

Group J: we are going to multiply 38 by 7 and the answer is 266. Meaning you will spend R266.

Teacher: which strategy did you use to get your answer?

Group J: we used column method to calculate the answer

Teacher: write down the strategy that you used to find your answer on the piece of paper

Group J: ok sir, 38 multiply by 7 gave us 299

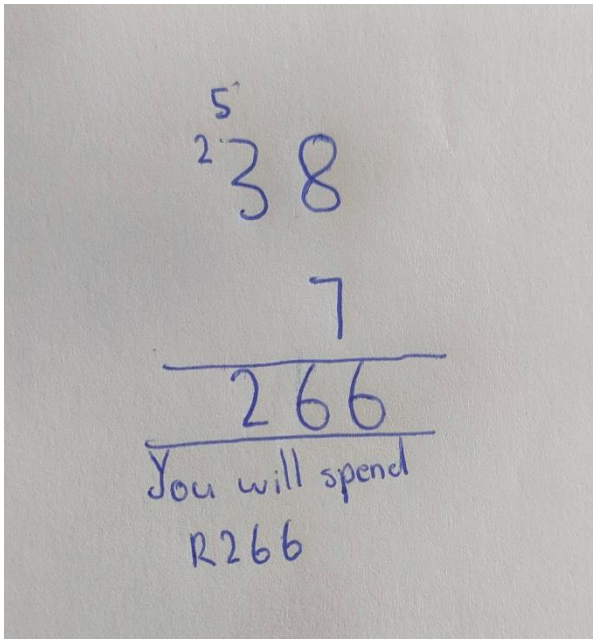

Figure 33: Learners response from group J

Teacher: not exactly

Group J: why sir course number of learners is 38 and the ice cream cost 7?

Teacher: remember there are 15 learners amongst you who prefer to eat a plain ice cream, while the remaining learners prefer a plain ice cream with crunch chocolate. So how much money will I spend all together.

Group J: ohooooo (agreed)

Teacher: since you understand what the question is required can you solve the question.

Group L: this means we are going to multiply 15 by 7 and multiply the answer with 9 to get the total price. Like 15 by 7 gave us 105. Then 105 by 9 gave us 945. Which means you will spend R945.

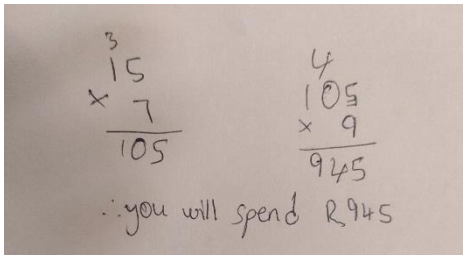

Figure 34: Learners response from group L

Teacher: now you went too far, the approach that you used earlier on. You may use it but using 15 by 7 and 23 by 9. Then you add them together.

Group L: ok sir

Teacher: the teacher demonstrates the solution on the white board.

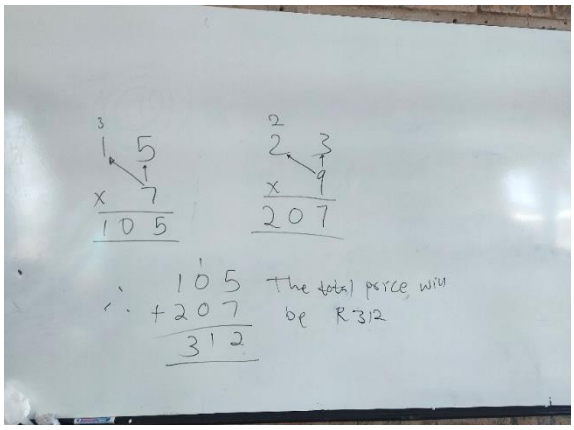

Figure 35: Demonstration by the teacher in class B learning episode 1

8.5.2 Transcription of data gathered from learners' written task

Summary of learners' results were recorded. The shaded questions are the one that were used to analyse data

keys	1-3	A-M	√	x	√/x	Empty Space
Description	Number questions	Number of groups represented by letters	Correct answer	Incorrect answer	Correct/incorrect	No answer

Table 4: Summary of learners' results to questions in learning episode 1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Represesantation of learners' response from learning episode 1													
2	Number of questions	A	B	C	D	E	F	G	H	I	J	K	L	M
3	1. A	x	√/x	√	√	√	√	√	√/x	√	√	√	√/x	√
4	B	√	√/x	√	√	√	√	√/x	√/x		√	√	√/x	√
5	C	√	√/x	√	√	√	√	√	√	√	√	√	√	√
6	D	√	√/x	√	√	√	√	√	√	√	√	√	√	√
7	E	√	√/x	√	√	√	√	√	√	√	√	√	√/x	√
8	F	√	√/x	√	√	√	√/x	√	√	√	√	√	√/x	√
9	G	√	x	x	√/x	√	√	x	√	√	√	x	x	√
10	H	x	√/x	√	√	√	√	√	√	√	√	√	x	√
11	2.	x	x	x	x	x	x	x	x	x	x	x		√
12	3.	x	x	x	x	x	x	x	x	x	x	x		x

8.5.3 Transcription of data gathered from the teachers' interviews

Table 5: Transcription of the teacher interview from learning episode 1

Questions	Response of the teacher
1. How do you develop Grade 4 learners' mathematical proficiency, particularly when solving algebraic word problems? Explain	I put learners in groups ranging from five to seven and give them an activity that include algebraic word problems, from there I allow learners to read the scenario with understanding and ask them to pick up important information. Once they have picked up an important information, I encourage them to represent the information mathematically. From there I ask them what is the question, after they tell me the question, I ask them which mathematical operation is suitable to solve the question, once they respond, I allowed them to follow the same procedure for other questions.
2. What are the teaching strategies or methodologies that work for you to develop learners' mathematical proficiency? Explain in reference to solving algebraic word problems.	I use learning-centred classroom incorporated with discourse-based mathematics instruction to create an environment that afford learners an opportunity to develop mathematical proficiency. This strategy afford learners an opportunity to: discuss and share their mathematical ideas; explain their reasoning; compare and justify problem-solving strategies; communicate their ideas; challenge each other's reasoning through questioning, reflect on and clarify their thinking.
3. What challenges did you notice, faced by learners when solving algebraic word problems? Outline them	Learners lack conceptual understanding of mathematical operations; they confuse addition with multiplication. For example, when they multiply 2 x 2 they get 4 and when they add 2 + 2 they also get 4, therefore they think 3 x 3 is equal to 6 since 3 + 3 = 6. However, in the question that was given to learners, I have realised that some learners did not understand the question. Learners were multiplying R110(price of school trouser) by 6, instead of 3 and R80,00 by 6 instead of 3 and others were adding

	the price of school trouser and school shorts which was R110 + R80 = R190, and that R190 was multiplied by 6, which was the total price of the learners. Therefore, this indicate that some learners lack conceptual understanding.
4. What are the assessment strategies that seem to be working when you assist learners to develop learners' mathematical proficiency? Explain	I use class activities and learners respond to them in groups. After the activity, learners are given the effective feedback
5. What would you regard as key actions to be taken care of by the teachers when assisting learners to develop mathematical proficiency when solving algebraic word problems? Account why you regard them as key actions?	Teachers should use learning-centred classroom incorporated with discourse-based mathematics instruction to create an environment that afford learners an opportunity to develop mathematical proficiency. help learners to read with understanding, so that it can be easy for learners pick up important information from the scenario of the algebraic word problems. they should help learners be able to represent algebraic word problems mathematical, and they should also help learners to be able to choose or formulate the best strategy to use when solving algebraic word problems and lastly help learners to defend their writing and draw conclusion. In addition to that effective feedback could encourage learners to see mathematics as useful, sensible and worthwhile.

8.5.4 Transcription of data gathered from learners' interviews

Table 6: Transcription of learners interviews from learning episode 1

	Group A	Group B	Group C	Group D	Group E	Group F	Group G
1. What is your understanding about solving algebraic word problems? Explain	You must read the question and understand before you respond to the question	You have to read and understand the question before you answer the question	You must read the question and understand before answering it	Read the question and understand it	Read the story and understand, before you answer it	We must read and understand the question before we can answer it	We read first and understand the question, then we give answer
2. can you restate the algebraic word problems in your own words? Explain	Your mother buys 3 school trouser and 3 school shorts. So, they want use to find how much did she pay	Yes, because we know how much is the school trouser and school shorts	Yes, you buy 3 school trouser and 3 school shorts	yes	Yes, they bought school trouser and school shorts to the children	We understand that the mother bought 3 school trouser for their children	School trouser was 110 and school trouser was 80
3. What are you asked to find? Explain	We are asked to find How much did she pay?	Because I know that trouser Is R110 and school shorts is R80	How much they must pay	How much did she pay		We are asked to find how did our mother paid for 3 school trouser and 3 school shorts	How much your mother was going to pay
4. Is there enough information to help you to find the answers? Explain	Yes, we say 110 times 3 and get the answer	We must say 110 times 3 and get the answer, we also write 80 times 6	Yes, school trouser is R110 and school shorts is R80	Yes, they told us how much the school trouser cost and how much the school	Yes, she bought 3 school trouser and 3 school shorts	School trouser and school shorts	School trouser R110 and school shorts is R80

	Group A	Group B	Group C	Group D	Group E	Group F	Group G
				shorts cost. There were 3 items each that need to be bought			
5. Explain how did you get to your answer?	We calculated 110 times 3 also 80 times 3 to get the answer	Multiplication	We add 110 and 80 to get 190 and 3 school trouser and 3 school shorts to get 6, from there we have multiply 190 by 6 and get the answer	We used column method and multiplication	We got the answer by writing 110 multiply by 3 and get 330 by column method	We have multiply 80 x 3 = 240 and 110 x 3 = 330	We said 190 x 6 = 1140
6. Explain which method/procedure did you use to get the answer	multiplication	Multiplication	Multiplication	Multiplication and column method	We used column method to get the answer	We used multiplication	Column method
7. Describe why did you choose the operation you used to get the answer	Because it was easy to use multiplication	Because multiplication is what we know	Because it is easy to get the answer	110 x 3 = 330	Because we must multiply 110 by 3 to get answer	Because it is easy to get the answer	Because we thought it will give us the correct answer
8. What are the challenges you faced when solving algebraic word problems? Explain	No challenges	The challenge we had is that we wrote 6 instead of 3	We did not understand the question and we ended up making mistake by adding 110 and 80 to get total price and	We could not calculate the question of 3 School shorts	Yes, because we were not aware and we did not understand the question well.	No. we did not encounter any challenges	Challenges is that, we have multiplied 190 by 6, instead we should have multiplied

	Group A	Group B	Group C	Group D	Group E	Group F	Group G
			add 3 school trouser and 3 school shorts to get total of school uniform.				110 by 3 and 80 by 3 and get the correct answer.

	Group H	Group I	Group J	Group K	Group L	Group M
1. What is your understanding about solving algebraic word problems? Explain	You read and understand before you respond the question.	We must read and understand the question before we respond to it	Read and understand the question before you answer it	We have to understand the word problem in order to respond it	You read the question before you respond to it.	You read the question before you respond to it
2. can you restate the algebraic word problems in your own words? Explain	They say we must multiply 110 by 3	Your mother buy 3 school trouser and 3 school shorts, how much did she pay?	yes	How we got our answer	I can say to my friend that you should multiply each of the school trouser by 3 and school shorts	I can say to my friend that you should multiply each of the school trouser by 3 and school shorts
3. What are you asked to find? Explain	How much did she pay	We are asked to find the money she spend	How much did they pay	The price of the school uniform items		I am ask to find the answer of total of 3 school trouser and school shorts
4. Is there enough information to help you to find the answers? Explain	Yes, 110 x 3 = 330, 80 x 3 = 240	Yes, they told us much the price of school trouser and school shorts is.	3 school trouser and 3 school shorts	Yes, the price of the school uniform	110 x 3 = I got 330 School shorts 80 x 3 = 240	Yes, we are given the price of school trouser and school shorts

	Group H	Group I	Group J	Group K	Group L	Group M
5. Explain how did you get to your answer?	By multiplying 110 x 3 and we get 330, and also multiplying 80 x 3 and we get 240	We multiplied the price of the school trouser and the price of the school shorts	We got the answer by adding 110 + 110 + 110 = 330	We said 110 x 6 and get the answer and also 80 x 6 to get the answer. We also add the final answers to get the total money she paid altogether	we used multiplication	110 x 3 =330 80 x 3 = 240
6. Explain which method/procedure did you use to get the answer	Addition and multiplication	To multiply we used column method	We used addition	Column method		We used multiplication
7. Describe why did you choose the operation you used to get the answer	Is the one that make it easy for us to get answer	We wanted to get the correct answer	We used addition because we understand it better	Multiplication was best to get the answer	It is easier to use	It is easy to use
8. What are the challenges you faced when solving algebraic word problems? Explain	No, Challenges	No, Challenges	No, Challenges	We did not understand the question, and we used 6, because the statement says your mother is looking after 6 children.	No challenges	No challenges

8.6 Appendix F: Transcription of Data Gathered from Learning Episode 2

8.6.1 Transcription of data gathered from observation

Introduction of the lesson:

Teacher: The teacher introduced the lesson by asking learners' if they still recall subtraction and column method.

Learners: yes sir

Teacher: A parking garage at Paledi Mall has a space of 3765 cars. 1758 cars are parked in the garage. How many cars can still park in the garage? Show how you got your answer. After a while 1398 cars arrived at the parking. How many cars can still park in the garage? Show how you got your answer

Before answering this question can you all read the question on the bord and try to make sense out of it

Learners: yes, sir we can read.

Learners read the question three times

Teacher: now that you have read, how many cars can still park in the garage?

Learner: 2007

Teacher: How did you get the answer?

Learner: 3765 minus 1758

Teacher: ok. Is the question complete?

Learners: No

Teacher: why are you saying No?

Learner: one learner from the group said "because they said 1398 cars arrived in the parking, meaning they have increase numbers of cars in the parking

Teacher: ok, I am giving you time to complete the task, after one member from the group should come and demonstrate their solution on the bord.

Learners: ok sir

The teacher gave learners a chance to complete their task, as learners were working on the task, the teachers was moving around checking if learners are responding to the question given to them. after a while, one learner from one of the groups came and represent the solution of their group on the bord. Figure 36 represent the solution of the group D of learners.

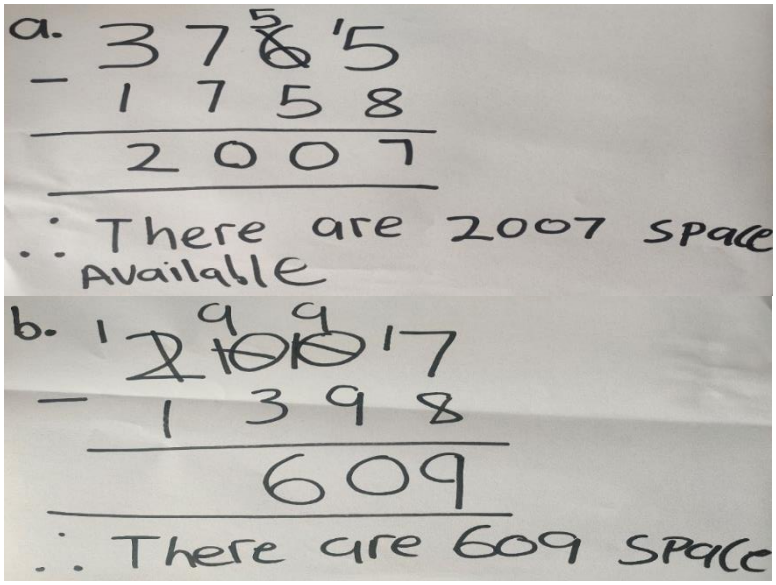

Figure 36: Learners respond from group D

After the learner representing their solution on the white bord, the teacher asks the learners a question

Teacher: is it correct

Learners: yes

The teacher asks the learners to explain their solution. The learner smiled and did not explain the solution. The teacher asked the class if there is anyone who can come in front and explain the solution. One learner came and explain.

Learners: we took 3765 and subtract 1758 cars to get the answer of 2007. After a while 1398 cars arrived at the parking and we took 2007 and subtract it with 1398 and we got 609 space of the cars that was remaining.

As the learner was explaining the teacher was moving around to check what other groups have done. The teacher came across the learners working represented on figure 37

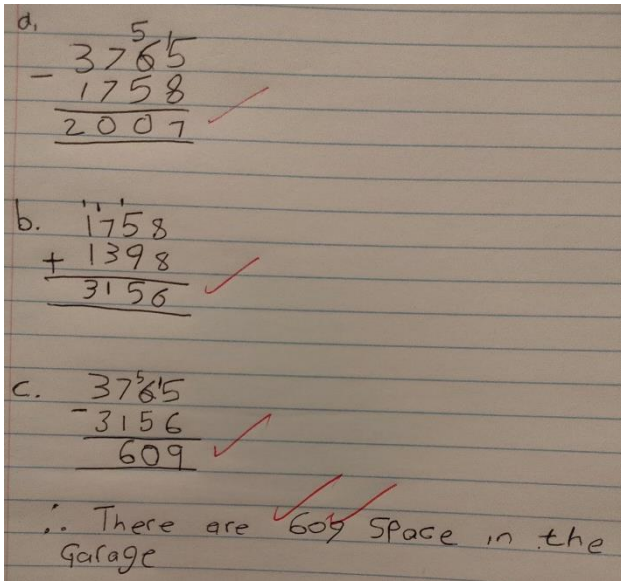

Figure 37: Learners response from group J

After the teacher allow that learner to sit down. From there learners were given a worksheet to complete the activity of learning episode 2

8.6.2 Transcription of data gathered from learners' written task

keys	1-3	A-Y	√	x	√/x	Empty space
Description	Number questions	Number of groups	Correct answer	Incorrect answer	Correct/incorrect	No answer

Table 7: Summary of learners' results to questions in learning episode 2

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Represesantation of learners' response from learning episode 2													
2	Number of questions	A	B	C	D	E	F	G	H	I	J	K	L	M
3	A	√/x	√	x	√	√	√	√	√	√	x	√	x	x
4	B	x	√	x	x	√	√	x	√	√	√	√	x	x
5	C	√	x	√/x	√/x	x	x	x	√	√	√	x	√	√
6	D	x	x	x	√	x	x	x	√	x	√	√	√	√
7	E	√	√	√	√	√	√	x	x	x	√	√	x	√
8	F	x	x	x	√/x	x	x	x	x	x	x	√	x	x
9	G	x	√	x	x	√	√	x	√	√	√	√	x	x

8.6.3 Transcription of data gathered from the teachers' interviews

Table 8: Transcription of the teacher interview from learning episode 2

Questions	Response of the teacher
1. How do you develop Grade 4 learners' mathematical proficiency, particularly when solving algebraic word problems? Explain	When developing learners' mathematical proficiency, what is important, is to make learners understand the algebraic word problems, by allowing them to read the algebraic word problems several times and interact with each other. In the process of reading, learners should take out the important aspect from the algebraic word problems so that they can be able to write them aside. Consequently, it become easy for learners to predict the strategy, procedure and mathematical operation to use, to solve algebraic word problems. Steps that were outline above are to be followed by learners to respond to any algebraic word problems the come across.
2. What are the teaching strategies or methodologies that work for you to develop learners' mathematical proficiency? Explain in reference to solving algebraic word problems.	I use learning-centred classroom incorporated with discourse-based mathematics instruction to create an environment that afford learners an opportunity to develop mathematical proficiency. This strategy afford learners an opportunity to: discuss and share their mathematical ideas in formulating algebraic word problems and choose the correct strategy to solve it; explain their reasoning; compare and justify problem-solving strategies; communicate their ideas; challenge each other's reasoning through questioning, reflect on and clarify their thinking.
3. What challenges did you notice, faced by learners when solving algebraic word problems? Outline them	The challenges I have identified when learners were solving the question of Super star bakery is that learners were able to solve only one solution, without responding to the second solution of the question. I also realised that the questioning style was the one that made learners to have challenges, if the question were to be separated, learners were going to be able to respond all of them. Lastly, I have noticed that learners a facing a challenge of knowing the standard procure of subtracting numbers, whereby they supposed to subtract a bigger number with a smaller at their level.
4. What are the assessment strategies that seem to be working when you assist learners to develop learners' mathematical proficiency? Explain	I gave learners an activity to work in groups, after they complete the activity, we discuss feedback together
5. What would you regard as key actions to be taken care of by the teachers when assisting learners to develop mathematical proficiency when	I focus much on the instruction given to the learners, taking out the most important part of the instruction, from there I present the key concept to the learners so that they can understand and be able to select the correct mathematical operation to be used to solve the problem. Give learners a chance to work the algebraic word problems on their own, and facilitate their learning. Allow them to be in control of their learning, let them take decision about their learning.

Questions	Response of the teacher
solving algebraic word problems? Account why you regard them as key actions?	

8.6.4 Transcription of data gathered from learners' interviews

Table 9: Transcription of learners interviews from learning episode 2

	Group A	Group B	Group C	Group D	Group E	Group F	Group G
1. What is your understanding about solving algebraic word problems? Explain	You read the question and understand the question before you answer the question.	Solving algebraic word problems, you must read the question and understand and show how you respond	You must read and understand the question	Read and understand the question before you respond it	Read and answer the question with understanding	You have to read and understand the question before you respond to it.	We must understand before we respond the question
2. can you restate the algebraic word problems in your own words? Explain	No	Yes, we must find the number of loaf of bread that were baked over the weekend	They have baked 1860 loaf of bread on Saturday and 2430 loaf of bread on Sunday	They want to know how many bread of loaf did they bake over the weekend	How many loaves of bread did they bake	Yes, they wanted to know how many loaves of bread did they bake over the weekend and we must multiply and get the correct answer	Yes, they want us to find number of bread they baked over the weekend
3. What are you asked to find? Explain	How many loaves of bread did they bake over the weekend and how many loaves did they bake more on	How many loaves of bread did they bake over the weekend and how many	We are asked to find the total of bread they have baked over the weekend	How many loaves of bread did they bake over the weekend and	How many loaves of bread did they bake over the weekend	How many loaves of bread did they bake over the weekend	We are asked to find how many bread did they baked over

	Group A	Group B	Group C	Group D	Group E	Group F	Group G
	Sunday? Show how you got your answer	loaves did they bake more on Sunday? Show how you got your answer		how many loaves did they bake more on Sunday? Show how you got your answer			the weekend
4. Is there enough information to help you to find the answers? Explain	yes	Super star bakery baked loafs of bread	You have to add 1860 and 2430 and we said 1860 + 2430 give us 4890	Yes, because they told us the bakery have baked 1860 bread on Saturday and 2430 bread on Sunday	Yes, because super star bakery baked 1860 loaf of bread on Saturday and 2430 of bread on Sunday	Yes, there are 1860 loaf of bread baked on Saturday and 2430 on Sunday.	yes
5. Explain how did you get to your answer?	Addition	We took 1860 – 2430 and the answer was 1430	We add 1860 + 2430 = 4290		We wrote 2430 – 1860, 3 minus 6 is not impossible. 4 minus 8 is not impossible	subtract 1860 - 2430	1860 – 2430 = 1430 loaf of bread
6. Explain which method/procedure did you use to get the answer	Because it is easy to find the answer	Minus	We used addition	We used addition	Column method	We used subtraction	Column method
7. Describe why did you choose the operation you used to get the answer	We did not understand the question	Is because we said minus	Because addition is so easy, it gives us answer easy	It is easy to use	Because they want us to find how many loaves of bread did, they baked	It was easy for us to get answer	We did not understand

	Group A	Group B	Group C	Group D	Group E	Group F	Group G
					over the weekend		
8. What are the challenges you faced when solving algebraic word problems? Explain	We did not understand the question clearly.	We had a challenge because, instead of adding we subtract	Here we do not have to say 2430 – 1860 = 1480, we have to say 2430 + 1860	We used the wrong method to calculate	The challenge we had is that, we did not understand the question clearly	The challenge is that we did not subtract correctly	Challenges is that we have been arguing, and we end up getting incorrect answer

	Group H	Group I	Group J	Group K	Group L	Group M
1. What is your understanding about solving algebraic word problems? Explain	By reading and understanding the words before we answer the question	We must read and understand before we respond to the question	Read and understand before answering the question	Read and understand the question	You must read and understand before you give the answer	We need to read the question, understand the question before we respond to it.
2. can you restate the algebraic word problems in your own words? Explain	Yes, they said we must plus 1860 + 2430	How many loaves did they bake over the weekend	yes	Yes, they want us to find number of bread baked over the weekend	yes, on Saturday the super star bakery baked 1860 bread and on Sunday baked 2430 bread	Yes, here they want us to add the loaves of bread that the super star bakery baked over the weekend
3. What are you asked to find? Explain	How many loaves of bread	How many loaves of bread did they bake	How many loaves did they bake on Saturday	We are asked to find the number of bread the superstar bakery has baked over the weekend	How many loaves of bread did they bake over the weekend	How many loaves of bread did they bake over the weekend

	Group H	Group I	Group J	Group K	Group L	Group M
4. Is there enough information to help you to find the answers? Explain	Yes, 1860 + 2430 = 4290	Yes, they told us how many loaves did they bake on Saturday and Sunday	Yes , 1860 loaves of bread on Saturday day and 2430 loaves of bread on Sunday	1860 bread + 2430	Yes, they gave us the number of bread that the superstar bakery baked on Saturday and Sunday.	Yes, there is enough information to help us to respond the question. We are given number of bread baked on Saturday and Sunday.
5. Explain how did you get to your answer?	We said 1860 + 2430 = 4290	We add the number of bread they bake on Saturday and on Sunday	1860 + 2430	2430 multiply by 2 (2430 x 2)	We calculated 1860 + 2430 and we got 4290	We adding 1860 + 2430 = 4290
6. Explain which method/procedure did you use to get the answer	Addition	Addition	plus	Column method	We used column method to add	Addition
7. Describe why did you choose the operation you used to get the answer	It was easy to get the answer	Because it was easy for us to get the answer	Because it easy for us to respond the question	It was easy for us to get the answer	Because it is easy to get the answer	Addition
8. What are the challenges you faced when solving algebraic word problems? Explain	We did not know that the question needs two solutions, this means we did not understand the question	We had the challenge because we did not understand the second question	We thought the question was looking for one solution not 2 solutions.	We did not understand the question correctly	We were not aware that the that the question need two solutions	It was difficult because we were not aware about the question

8.7 Appendix G: A Letter to the School Principal

P.O. BOX 968
Marble hall
0450
Date

Name of School
P.O. BOX …….
SOVENGA
0727

Dear Sir/Madam

RE: REQUEST FOR PERMISSION TO DO RESEARCH AT (Name of School) PRIMARY SCHOOL

I hereby apply for permission to conduct a study at your School. My research project will involve Grade 4 mathematics teachers. The title of my research is "investigating how Grade 4 mathematics teachers develop learners' Mathematical Proficiency when solving algebraic word problems". Data will be collected using observations, learners' written tasks and interviews.

I will be participating with the teachers and request that I videotape observations and audiotape interviews. The information obtained will be treated with the strictest confidentiality and will be used solely for this research purpose. It is my presumption that the research findings will make a creditable contribution towards identifying different strategies, techniques and methods of teaching for Mathematical Proficiency for Grade 4 Mathematical classroom.

I hope my request will be permitted

Yours sincerely

Mashaba G

_____ Date:_____

10.5 APPENDIX E: A LETTER TO THE SCHOOL PRINCIPAL

P.O.BOX 968
Marble hall
0450
22 August 2022

Mapudithomo Primary School
P.O.BOX 2890
SOVENGA
0727

Dear Sir/Madam

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT MAPUDITHOMO PRIMARY SCHOOL

I hereby apply for permission to conduct a study at your school. My research project will involve Grade 4 mathematics teacher and his/her learners. The title of my research is "Grade 4 mathematics teachers' development of learners' mathematical proficiency towards solving algebraic word problems". Data will be collected using observations, learners' written tasks, and interviews.

I will be participating with the teacher and request that I videotape observations and audiotape interviews. The information obtained will be treated with the strictest confidentiality and will be used solely for this research purpose. It is my presumption that the research findings will make a creditable contribution towards identifying different strategies, techniques, and methods of teaching for mathematical proficiency for Grade 4 mathematical classroom.

I hope my request will be permitted

Yours sincerely

Mashaba G

Date: 22 August 2022

School Principal

DEPARTMENT OF EDUCATION
MAPUDITHOMO PRIMARY SCHOOL
EMIS No.: 923242279
P.O. BOX 2890 SOVENGA 0727
2022 -08- 22
CELL: 082 933 3214
PRINCIPAL
LIMPOPO PROVINCE

Date:

8.8 Appendix H: Consent Form

Title of study: 'Grade 4 mathematics teachers' development of learners' mathematical proficiency towards solving algebraic word problems'

Researchers' details		Supervisor
Name	Mashaba G	Prof. Maoto RS
Department	Education	Education
Physical Address	Stand No. 2471 Moremadi Park, 0727	3rd Floor, New K Block Building. University of Limpopo, 0727
Contact no.	0737816041/0728003064	0769049032
Email	Mashaba.glenzitoo@gmail.com	Satsope.maoto@ul.ac.za

PURPOSE OF THE CONSENT FORM

You are invited to take part in a research study. Before you decide to participate in this study, it is important that you understand why the research is being done and what it will involve. Please read the following information carefully. Please ask the researcher if there is anything that is not clear or if you need more information.

Purpose of the study: The purpose of the study is to explore how Grade 4 mathematics teachers develop learners' Mathematical Proficiency when solving algebraic word problems.

Procedure

During data collection process, you will be requested to subject learners to learning episodes, three in total. Data will be collected using different techniques that include Observations, learners' written responses of given tasks and interviews. Learners will be allowed to work in groups to respond to the questions of the planned learning episodes. The observations will be videotaped so that the recordings will be available and be transcribed verbatim. Similarly, interviews will be audiotaped and later be transcribed verbatim (Merriam, 1998).

I have read and I understand the provided information and have had the opportunity to ask questions. I understand that my participation is voluntary and that I am free to withdraw at any time, without giving a reason and without cost. I understand that I will be given a copy of this consent form. I voluntarily agree to take part in this study.

Participant's signature: _____ Date _____

Researcher's signature: _____Date _____

Principal Signature:_____Date _____

8.9 Appendix I: University of Limpopo Ethical Clearance Certificate

University of Limpopo
Department of Research Administration and Development
Private Bag X1106, Sovenga, 0727, South Africa
Tel: (015) 268 3935, Fax: (015) 268 2306, Email: anastasia.ngobe@ul.ac.za

TURFLOOP RESEARCH ETHICS COMMITTEE

ETHICS CLEARANCE CERTIFICATE

MEETING: **26 July 2022**

PROJECT NUMBER: **TREC/323/2022: PG**

PROJECT:

Title:	Grade 4 mathematics teachers' development of learners' Mathematical proficiency towards solving algebraic word problems.
Researcher:	G Mashaba
Supervisor:	Prof. RS Maoto
Co-Supervisor/s:	N/A
School:	Education
Degree:	Master of Education (Mathematics Education)

PROF D MAPOSA
CHAIRPERSON: TURFLOOP RESEARCH ETHICS COMMITTEE

The Turfloop Research Ethics Committee (TREC) is registered with the National Health Research Ethics Council, Registration Number: **REC-0310111-031**

Note:

i) **This Ethics Clearance Certificate will be valid for one (1) year, as from the abovementioned date. Application for annual renewal (or annual review) need to be received by TREC one month before lapse of this period.**

ii) **Should any departure be contemplated from the research procedure as approved, the researcher(s) must re-submit the protocol to the committee, together with the Application for Amendment form.**

iii) **PLEASE QUOTE THE PROTOCOL NUMBER IN ALL ENQUIRIES.**

Finding solutions for Africa

8.10 Appendix J: Permission to Conduct Research from the Limpopo Department of Education

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

CONFIDENTIAL

Ref: 2/2/2 Enq: Makola MC Tel No: 015 290 9448 E-mail:MakolaMC@edu.limpopo.gov.za

Mashaba G
Private Bag x 1106
Sovenga
0727

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.
2. The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"GRADE 4 MATHEMATICS TEACHERS DEVELOPMENT OF LEARNER MATHEMATICAL PROFICIENCY TOWARDS SOLVING ALGEBRAIC WORD PROBLEM "**
3. The following conditions should be considered:

3.1 The research should not have any financial implications for Limpopo Department of Education.

3.2 Arrangements should be made with the Circuit Office and the School concerned.

3.3 The conduct of research should not in anyhow disrupt the academic programs at the schools.

3.4 The research should not be conducted during the time of Examinations especially the fourth term.

3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).

3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.

Cnr 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X 9489, Polokwane, 0700
Tel:015 290 7600/ 7702 Fax 086 218 0560

4 Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.

5 The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes.

19/08/2022

Mashaba KM
DDG: CORPORATE SERVICES

Date

Cnr 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X 9489, Polokwane, 0700
Tel:015 290 7600/ 7702 Fax 086 218 0560

8.11 Appendix K: Affidavit Letter from the Language Editor

Andrew Scholtz Editing

Associate Member of:
Professional EDITORS' Guild 30 1993–2023
Promoting excellence in editing

Membership number: SCH039
Membership year: March 2023 to February 2024
084 602 1938
atjscholtz@gmail.com
www.editors.org.za

224, Raptors View, Hoedspruit, 1380
PO Box 1172, Hoedspruit, 1380

084 602 1938 • 076 079 0214 • atjscholtz@gmail.com

Affidavit

Date: 8 January 2024

To Whom it May Concern

I hereby confirm that I have undertaken a copy-edit of the Master of Education dissertation titled *Grade 4 Mathematics Teachers' Development of Learners' Mathematical Proficiency Towards Solving Algebraic Word Problems* written by Glen Matji. I have suggested a number of changes that the author may or may not accept, at their discretion.

Each of us has our own unique voice as far as both spoken and written language is concerned. In my role as editor, I try not to let my own 'written voice' overshadow the voice of the author, while at the same time attempting to ensure a readable document.

Please refer any queries to me.

Andrew Scholtz

Qualifications:

- MA (Digital Media in Education) – University of Kwazulu-Natal (2006)
- Accreditation of Assessors in Higher Education (Short Course) – Rhodes University (2007)
- Postgraduate Diploma in Dispute Settlement – University of Stellenbosch Business School (2013)
- SLP Family Law (Short Course) – North West University (2013)
- Strengthening Postgraduate Supervision (Short Course) – Rhodes University (2019)
- UCT Copy-editing Online Short Course – University of Cape Town (2020)
- Approved freelance editor and proofreader for Juta & Company (Pty) Ltd

Evidence of qualifications are available on request.