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ORIGINAL RESEARCH

Opportunities for Grade 11 Students to Learn Euclidean Geometry in Some South African Schools

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Abstract

This research paper investigated the opportunities for Grade 11 students to learn Euclidean geometry in some South African schools. The study aimed to examine the Euclidean geometry curriculum covered in these schools and the instructional time used in teaching this content. The research was conducted within a single education district in the Gauteng province and involved six secondary schools. Data collection relied on teaching and learning materials. Results revealed that the depth of content coverage varied across schools, with two schools notably lacking in comprehensive instruction. Furthermore, concerning instructional time, three schools fell short of the recommended duration for teaching the content. These findings suggest that students may not have received adequate learning opportunities on the topic. The paper discussed the implications of these findings and proposed recommendations for addressing the observed shortcomings.

Keywords: Content coverage; Euclidean geometry; Instructional time; Opportunity to learn; Time on task

Introduction

Geometry forms a significant part of the school mathematics curriculum in most education systems, including South Africa. The value of geometry possibly lies in its ability to help foster students' problem-solving skills (Armah et al., 2017; Narh-Kert & Sabtiwu, 2022). However, students' performance in most geometry examinations does not give an impressive picture of their learning, especially in Euclidean geometry.

While national examinations and international competency tests are used as a measure of how students perform in mathematics, these do not seem to consider the Opportunities to Learn (OTL) made available to students at various schools. Most of the assessment and accountability systems in place, such as the National Senior Certificate (NSC) examination, the Trends in International Mathematics and Science Study (TIMSS), Olympiads, and other tests, appear to assume that all students receive sufficient opportunity to learn the curriculum as expected. However, it is not necessarily true that students are provided with equal opportunities to learn in their various classes and schools. The diversity of schools in South Africa (and in many other sub-Saharan countries), with its varying levels of socioeconomic status, implies a diversity in student learning opportunities. Consequently, the quality of education and student learning achievement are affected. It has been noted in many examination reports and studies (Department of Basic Education [DBE], 2023; Ngirishi & Bansilal, 2019; Tachie, 2020) that students tend to perform below expectations in mathematics, particularly in Euclidean geometry. To address this issue, there is a need to understand and address the student OTL mathematics in schools. Against this backdrop, this study investigated the opportunities for Grade 11 students to learn Euclidean geometry in some South African schools. This was done by investigating the coverage of the Euclidean geometry content, and the instructional time used in teaching this content in the schools. The research questions addressed are: (1) how does the coverage of Euclidean geometry content in Grade 11 compare across schools? (2) how is instructional time allocated for teaching Euclidean geometry in Grade 11 classrooms utilised across schools?

The research was a qualitative case study. It applied a content analysis method to teaching and learning artefacts to examine the OTL a critical mathematics curriculum content made available for students in some schools in South Africa. It sheds light on the quality of mathematics education students are exposed to in the schools.

Opportunities to Learn (OTL)

Burstein (2014) notes that the International Association for the Evaluation of Educational Achievement initially employed OTL to assist in understanding students' academic performance in international evaluation studies, thus allowing for a valid comparison of students' achievement. It encapsulates the idea that every student should have equitable access to the resources, materials, and experiences necessary for meaningful learning to take place.

OTL includes the conditions and resources necessary for students to learn the specified contents and skills for their age and grade level (Ogbonnaya, 2021). These conditions and resources include access to textbooks, qualified teachers, subject content, enough instructional time, and a safe and conducive learning environment, amongst others (Mtshali et al., 2023). OTL is crucial for student learning because, without adequate access to conducive conditions and adequate resources for learning, students may be hindered in their ability to learn effectively and achieve success (Kurz et al., 2020).

OTL may be conceptualised using various indicators, depending on which one is important to the person, for example, content taught, instructional time, time on task, types of questions, and quality of instruction. However, Kurz's (2011) conceptualisation encapsulates the indices of OTL in three dimensions: content, quality (quality of instruction and cognitive demands of instructional task), and time. The focus of this study was content and time, hence these two are discussed further.

Content coverage includes the topics and sub-topics that are covered in an educational programme. According to Stephen (2013), content coverage refers to whether students have been exposed to the core curriculum and if there is alignment between the content taught in the curriculum and the content assessed in tests or assessments. In other words, the extent to which what was taught in class overlaps with that which is required from students during assessments. The coverage of content is essential because it determines what knowledge and skills learners acquire. The in-depth coverage of curriculum content in mathematics has been found to positively relate to student achievement (Charles-Ogan & George, 2019; Engel et al., 2016; Schmidt et al., 2011; Shikuku, 2012). Kurz et al. (2014) observed that for students to have the opportunity to learn effectively, teachers must ensure the comprehensive coverage of content outlined in the intended curriculum. This involves employing suitable pedagogical approaches to deliver the content effectively.

Some researchers have investigated OTL concerning the content addressed during instructional periods. Mtshali et al. (2023) investigated the content coverage of algebra in Grade 9 in some secondary schools in Gauteng, South Africa using data from teaching and learning artefacts. The researchers found that the students were not afforded ample opportunity to learn the content due to non-coverage of the curriculum content in these schools. In a study on mathematics content coverage by secondary school mathematics teachers in Nigeria, Aduwa (2020) found that the teachers did not cover all the curriculum content in their classes. Stols (2013) investigated the curriculum content coverage of mathematics in Grade 12 in 18 secondary schools in Gauteng, South Africa. He discovered that there was limited coverage of the curriculum in 16 of the 18 schools in the study. In an earlier study, Taylor (2008) noted that a low level of curriculum coverage is a major hindrance to student mathematics learning opportunities in South Africa.

As one of the indices of OTL (Kurz et al., 2014), the concept of time refers to the time dedicated to teaching and learning. This indicator is classified into three categories: allocated time (the time designated for instruction of the subject), instructional time (the portion of allocated time dedicated to instruction) and engaged time (the proportion of instructional time during which learners actively participate in learning activities) (Elliott & Bartlett, 2016). In this study, time refers to the instructional time used in teaching the contents of the topic to students. Teaching time refers to the duration during which students are actively involved in learning activities under the guidance of the teacher. Like content coverage, numerous researchers concur that instructional time positively influences student learning (Cattaneo et al., 2017; Lavy, 2015; Rivkin & Schiman, 2015). Therefore, to effectively provide students with learning opportunities, teachers need to dedicate instructional time to cover the prescribed content.

Research Methodology

This study was a case study of six randomly sampled schools in one district in Pretoria, South Africa. These schools were located in low socioeconomic status communities. A case study design enabled an in-depth study of the learners' OTL in Euclidean geometry in these schools following a qualitative research approach. Document analysis was used as the method of data collection. As such, the learners' notebooks on Euclidean geometry were analysed. In each of the selected schools, the notebooks of three top-performing learners (whom the teacher believed always attended class, wrote notes, and did classwork and assignments) were selected and used in this study. The selection of top-performing learners was made following a suggestion by Stols (2013) that such learners are likely to capture all notes on the topic discussed during teaching and learning. The mathematics teachers in all the participating schools had a minimum experience of four years in teaching mathematics at the secondary school level.

The data analysis involved a content analysis of the learners' notebooks. A checklist containing all the contents of Euclidean geometry, as stated in the curriculum, was used to indicate the content taught in each school. Moreover, the dates on which the content was taught in these schools (indicated in the learners' notebooks) and the lesson periods (indicated in the lesson timetables) were used to calculate the instructional time. The limitation is that the scheduled time according to the timetable might not reflect the actual time spent on teaching and learning in the classroom due to delays in starting classes, transitions between lessons, and disruptions. To ensure the trustworthiness of the process, three experienced teachers who work with the Department of Basic Education were asked to independently analyse the learners' notebooks using the same instrument.

Findings

Content coverage

The Euclidean geometry content expected to be taught in Grade 11 is in three parts as follows.

Part 1: "Accept results established in earlier grades as axioms and also that a tangent to a circle is perpendicular to the radius, drawn to the point of contact" (DBE, 2011, p34).

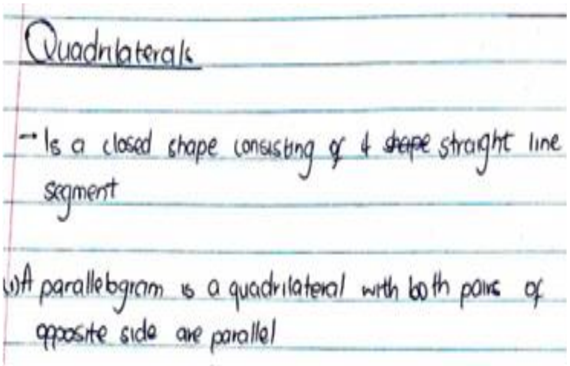
The statement, "Accept results established in earlier grades as axioms" requires an understanding of Euclidean geometry as an axiom system. It is within this system that a collection of ideas, definitions and axioms are the departure point for proving theorems, some of which are based on previously proven theorems. Moreover, an axiom "is a statement the truth of which is to be accepted without argument or logical evidence, because it is thought worthy as a starting point for further logical argument" (Movshovitz-Hadar, 2001, p2). In terms of the Grade 11 curriculum, axioms refer to the ideas, definitions, and theorems established in earlier grades.

The content covered in earlier grades includes: (1) "Basic results regarding [properties of] lines, angles and triangles, especially the similarity and congruence of triangles", (2) "Investigate line segments joining the midpoints of two sides of a triangle," (3) "Define the following special quadrilaterals: the kite, parallelogram, rectangle, rhombus, square and trapezium," and (4) "Investigate and make conjectures about the properties of the sides, angles, diagonals, and areas of these quadrilaterals. Prove these conjectures" (DBE, 2011, p25).

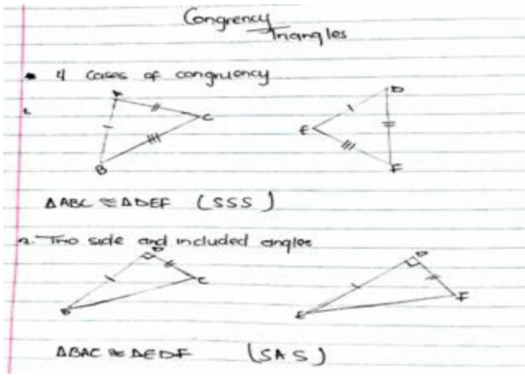
The teachers were not necessarily expected to 're-teach' the contents covered in earlier grades but rather to use them in their teaching to investigate and prove theorems of the geometry of circles (part 2) and to solve riders (part 3). However, the use of axioms in Grade 11 classes might demand the revision of the ideas, definitions and axioms established in earlier grades in the class. Hence, in this study, we checked the revisions and/ or use of these ideas, definitions and axioms, including “a tangent to a circle is perpendicular to the radius, drawn to the point of contact” as an indication of the content coverage of part 1 of the curriculum.

Revision of some of the content from earlier grades was covered in various ways across the six schools. In School A, there was evidence of the revision of properties of lines, angles, triangles, congruent triangles, and quadrilaterals. Figure 1 presents samples of the revision of some of the content from earlier grades, which was carried out in School A. Similar triangles, and line segments joining the midpoints of two sides of a triangle were not revised or used in teaching Euclidean geometry to learners in School A.

Figure 1: Samples of earlier grades’ content that were revised in School A



(a) Revision of quadrilateral

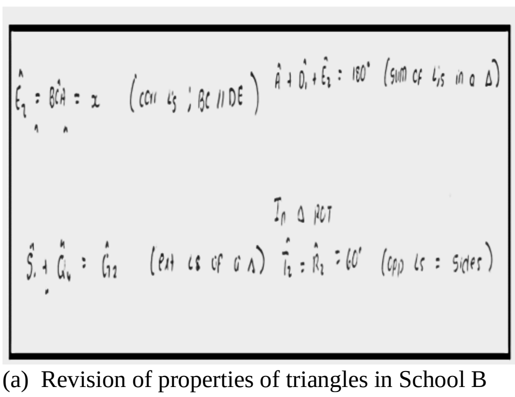


(b) Revision of similarity and congruence of triangles

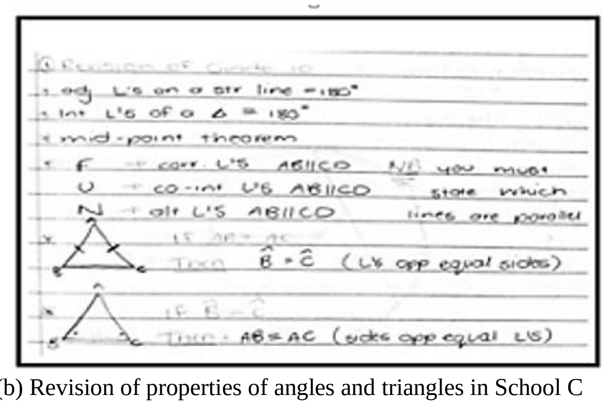
In School B, it was found that some content from past grade levels (the properties of parallel lines, the sum angle of a triangle and the exterior angle of a triangle) was revised and used in solving problems. The sum angles of a triangle as supplementary, the exterior angle of a triangle as equal to the sum on the two opposite interior angles, as well as isosceles triangles with the concept of angles at the base of opposite sides being equal were revised. Similar and congruent triangles, quadrilaterals, and the midpoint theorem were not revised at this school.

In School C, there was evidence of revision of the properties of angles, lines, and triangles. In addition, the midpoint theorem was revised, as seen in Figure 2(b). Congruent triangles and similar triangles were also revised. The definitions and properties of quadrilaterals were not revised in School C.

Figure 2: Samples of earlier grades’ content that were revised in Schools B and C



(a) Revision of properties of triangles in School B



(b) Revision of properties of angles and triangles in School C

In School D, the properties of lines, congruent and similar triangles, and the midpoint theorem were not revised. However, the properties of angles, isosceles triangles, and the properties and definitions of quadrilaterals were revised. In School E, it was found that most of the content established in earlier grades was not revised. However, some angle properties (the sum of angles in a triangle, and the sum of angles in a straight line) were revised. Congruency, the properties and definitions of quadrilaterals, types of triangles, and the midpoint theorem were not revised.

All content covered in earlier grades (properties of lines, angles and triangles, similar and congruent triangles, midpoints theorem, and quadrilateral) was revised in School F. The axiom, "A tangent to a circle is perpendicular to the radius, drawn to the point of contact" was taught in all the participating schools. Figure 3 shows evidence of the teaching of this axiom in Schools B and C.

Figure 3: Evidence of the teaching of "a tangent to a circle is perpendicular to the radius drawn to the point of contact" axiom in Schools B and C

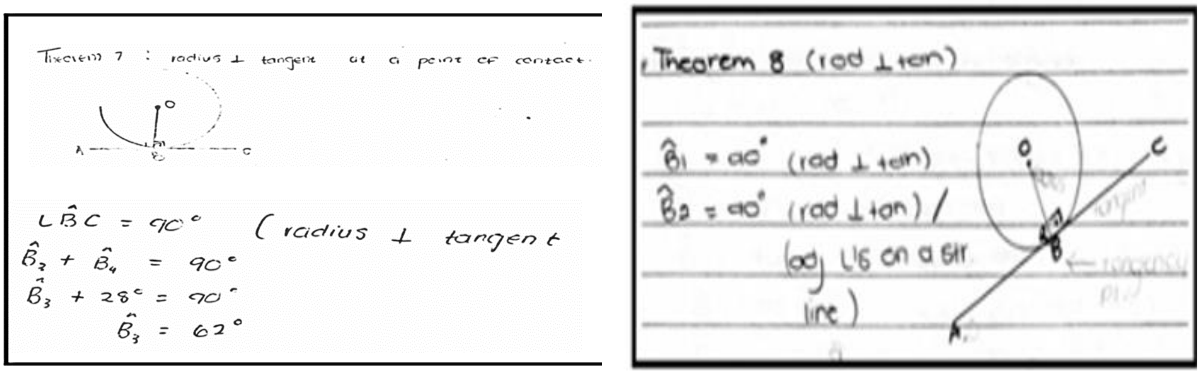


Table 1: Summary of the coverage of the part 1 content in the participating schools

Earlier grades Contents revised/taken as axioms		School					
		A	*B*	*C*	*D*	*E*	*F*
(1) Basic results regarding properties of	lines	✓	✓	✓	-	-	✓
	angles	✓	✓	✓	✓	✓	✓
	triangles	✓	✓	✓	✓	✓	✓
	congruence	✓	-	✓	-	-	✓
	similarity	-	-	✓	-	-	✓
(2) Line segments joining the midpoints of two sides of a triangle		-	-	✓	-	-	✓
(3) Special quadrilaterals	Kite	✓	-	-	✓	-	✓
	Parallelogram	✓	-	-	✓	-	✓
	Rectangle	✓	-	-	✓	-	✓
	Rhombus	✓	-	-	✓	-	✓
	Square	✓	-	-	✓	-	✓
	Trapezium	✓	-	-	✓	-	✓

Part 2: Investigate and prove the theorems of the geometry of circles.

This part focuses on proving some circle theorems. The theorems are statements to be taught along with the use of corollaries and converses to expose learners to a complete system of axioms. The theorems and their associated converses, corollaries, and axioms (where they exist) as stated in the curriculum (DBE, 2011, p43) are as follows:

Theorem 1: "The line drawn from the centre of a circle perpendicular to a chord bisects the chord (perpendicular from centre to chord). Converse: the line drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord" ("line from centre to midpoint chord").

Theorem 2: "The perpendicular bisector of a chord passes through the centre of the circle" (perpendicular bisector of the chord).

Theorem 3: "The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle - on the same side of the chord as the centre (angle at centre is twice the angle at the circumference)". Corollary 1: "Equal arcs subtend equal angles (equal arcs)"; Corollary 2: "Equal chords subtend equal angles on the corresponding arcs of the circle (equal chords)". Corollary 3: "An angle subtended on the circle by a diameter is a right angle (angle in semi-circle)".

Theorem 4: "Angles subtended by a chord of the circle, on the same side of the chord, are equal (angles in same segment)". Converse: "If the line segment joining two points subtends equal angles at two other points on the same side of the line segment, then the four points are con-cyclic" (line segment subtends equal angles on the same side).

Theorem 5: "The opposite angles of a cyclic quadrilateral are supplementary (opposite angles of a cyclic quad)". Corollary 1: "An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle" (exterior angle of a cyclic quad). Corollary 2: "if an exterior angle of a quadrilateral is equal to the interior opposite angle of that quadrilateral, then the quadrilateral is a cyclic quadrilateral (exterior angle equals to interior opposite angle). Converse: if any two opposite angles of a quadrilateral are supplementary, then the quadrilateral is a cyclic quadrilateral" (opposite angles are supplementary).

Theorem 6: "Two tangents drawn to a circle from the same point outside the circle are equal in length" (tangents from same point). Axiom: "a tangent is perpendicular to the radius at the point of contact with the circle" (tangent perpendicular to radius). Corollary: "a line through a point on a circle perpendicular to the radius at that point is a tangent to the circle" (line perpendicular to radius).

Theorem 7: "The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment" (tan-chord theorem). Converse: "the line drawn through the endpoint of a chord of a circle forming an angle with the chord that is equal to the angle subtended by that chord in the alternate segment is a tangent to that circle" (converse tan-chord theorem).

There was evidence of investigation/proving of Theorem 1 in Schools A, C, E, and F, but only Schools A and E went further to investigate/prove the converse of the theorem. There was no investigation/proof of the theorem and its converse in Schools B and D. The investigation/proof of Theorem 2 was only carried out in School A. Furthermore, Theorem 3 was investigated/proven in all the schools, but only Schools C and E explored the converse of the theorem. Figure 4 shows the investigation/proof of some of the theorems in some of the schools.

Theorem 4 was only investigated in Schools B, C, and E, but the converse was not investigated or proven in any of the schools. Schools A, C, E, and F investigated/proved Theorems 5 and 6, but only Schools A and F went further to investigate the converse of Theorem 5; while schools E and F investigated the converse of Theorem 6. Figure 5 shows the investigation/proof of Theorem 5 in School A.

While all the schools investigated/proved Theorem 7, only School A investigated/proved the converse of the theorem. The presentation of this theorem in School F is shown in Figure 6.

Figure 4: Theorem 3 investigated/proven in School E

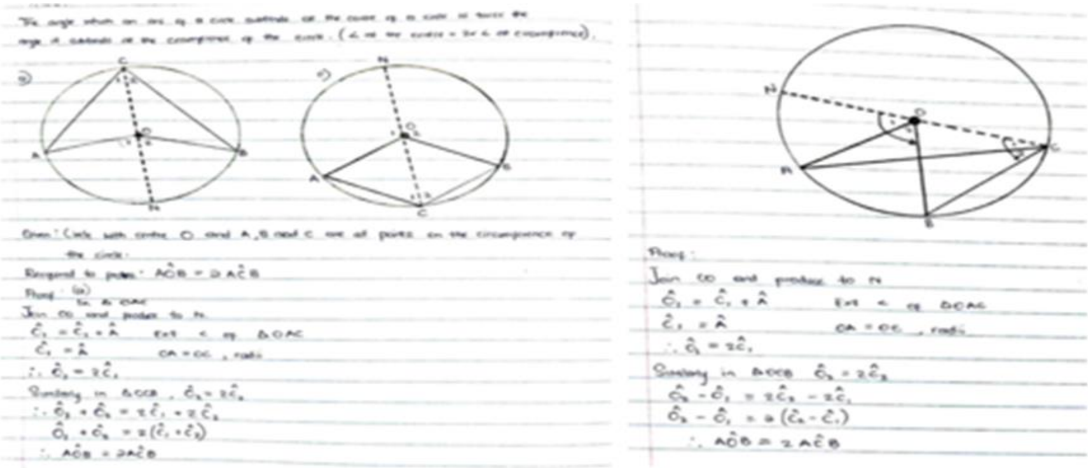


Figure 5: The investigation/proving of Theorem 5, as well as its converse in school A

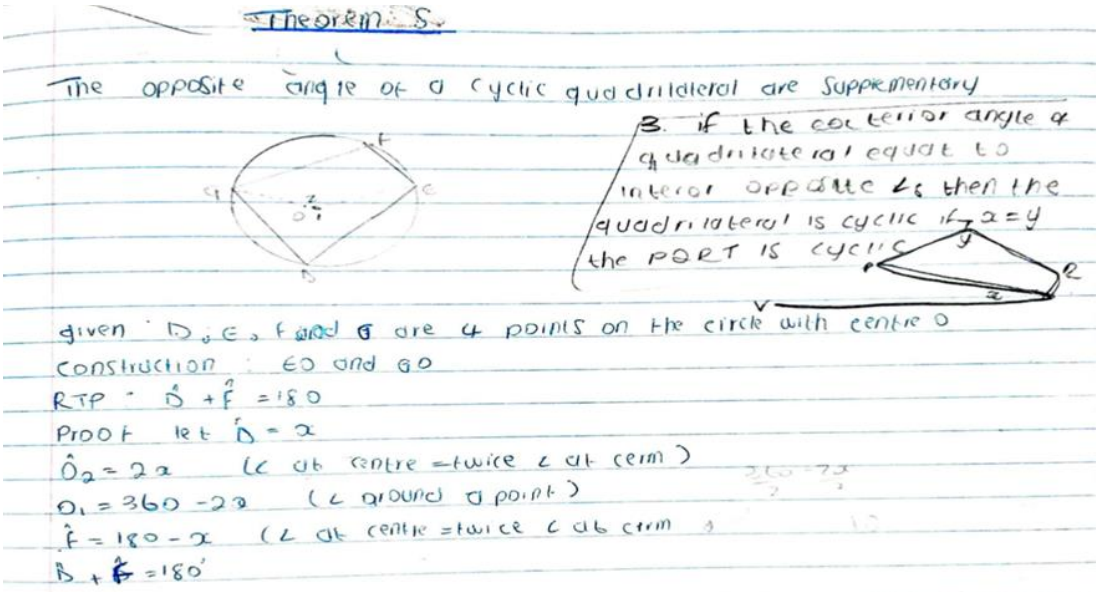


Figure 6: Proof of Theorem 7 in school F

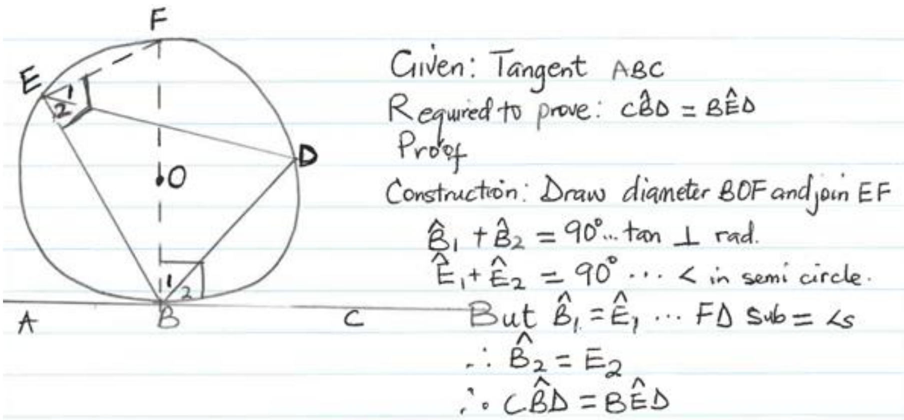


Table 2 presents a summary of the presentations of the theorems and their converses in the schools.

Table 2: The presentations of the theorems and their converses in the participating schools

	School					
Theorem	A	B	C	D	E	F
1. “The line drawn from the centre of a circle perpendicular to a chord bisects the chord”.	✓	-	✓	-	✓	✓
Converse: “The line drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord (line from centre to midpoint chord)”.	✓	-	-	-	✓	-
2. “The perpendicular bisector of a chord passes through the centre of the circle”.	✓	-	-	-	-	-
3. “The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)”	✓	✓	✓	✓	✓	✓
Converse.	-	-	✓	-	✓	-
4. “Angles subtended by a chord of the circle, on the same side of the chord, are equal”	-	✓	✓	-	✓	-
Converse.		-	-	-	-	-
5. “The opposite angles of a cyclic quadrilateral are supplementary;	✓	-	✓	-	✓	✓
Converse”.	✓	-	-	-	-	✓
6. “Two tangents drawn to a circle from the same point outside the circle are equal in length”	✓	-	✓	-	✓	✓
Converse.	-	-	-	-	✓	✓
7. “The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment”	✓	✓	✓	✓	✓	✓
Converse.	✓	-	-	-	-	-

Part 3: Use of theorems and their converses, where they exist, to solve riders.

A rider is a geometry problem to be solved using axioms or theorems. To assess the use of the theorems and their converses to solve riders, we investigated if problems were solved in class or given to the learners as assignments (or homework) on each theorem and the converse (where a converse exits). It was found that even in some cases where the theorems were not investigated, some teachers gave students assignments requiring the use of the theorems. Table 3 indicates the total number of questions used to test the theorems, their corollaries, and converses in the schools.

None of the schools assessed problems requiring the use of all of the theorems and their converses. All the schools were found to have solved problems requiring the use of Theorems 1, 3, 5, and 7. Questions requiring the use of Theorems 4 and 6 were assessed in five of the selected six schools. Lastly, questions requiring the use of Theorem 2 were the least used, with only three out of the five schools assessing it during the teaching of the topic.

Regarding the use of converses, Schools A and F solved problems or asked questions that required the use of the converses of Theorems 1, 3, 4, 5, and 7. In School B, questions that involved the use of the converses of Theorems 1 and 3 were solved or assessed. School C’s examples/assessments involved the use of the converses of Theorems 1 and 5, and School D only used the converse of Theorem 5. School E used the converses of Theorems 1, 3, and 4. While some of the theorems were not taught (investigated or proven) in some of the schools, as shown in Table 2, Table 3 shows that the learners were given tasks (assignments/homework) on the theorems.

Table 3: Number of questions used to test the theorems in the schools

	School					
	A	B	C	D	E	F
Theorem 1: "The line drawn from the centre of a circle perpendicular to a chord bisects the chord (perpendicular from centre to chord)".	8	5	5	3	6	5
Converse: "The line drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord (line from centre to midpoint chord)".	8	1	0	0	6	0
Theorem 2: "The perpendicular bisector of a chord passes through the centre of the circle (perpendicular bisector of the chord)".	4	1	0	0	0	1
Theorem 3: "The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle - on the same side of the chord as the centre (angle at centre is twice angle at the circumference)".	6	10	6	12	6	6
Corollary 1: "Equal arcs subtend equal angles (equal arcs)".	0	0	0	0	6	0
Corollary 2: "Equal chords subtend equal angles on the corresponding arcs of the circle (equal chords)".	0	0	0	0	6	0
Corollary 3: "An angle subtended on the circle by a diameter is a right angle (angle in semi-circle)".	1	4	6	3	6	6
Theorem 4: "Angles subtended by a chord of the circle, on the same side of the chord, are equal (angles in same segment)".	1	0	5	13	2	8
Converse: "If the line segment joining two points subtends equal angles at two other points on the same side of the line segment, then the four points are con-cyclic (line segment subtends equal angles on the same side)".	0	0	0	0	2	0
Theorem 5: "The opposite angles of a cyclic quadrilateral are supplementary (opposite angles of a cyclic quad)".	5	5	6	4	4	5
Corollary 1: "An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle (exterior angle of a cyclic quad)".	0	0	0	1	0	0
Corollary 2: "If an exterior angle of a quadrilateral is equal to the interior opposite angle of that quadrilateral, then the quadrilateral is a cyclic quadrilateral (exterior angle equals to interior opposite angle)".	0	0	0	0	0	0
Converse: "If any two opposite angles of a quadrilateral are supplementary, then the quadrilateral is a cyclic quadrilateral (opposite angles are supplementary)".	0	5	0	0	0	5

	School					
	A	B	C	D	E	F
Theorem 6: "Two tangents drawn to a circle from the same point outside the circle are equal in length (tangents from same point)".	2	3	6	2	0	4
Axiom: "A tangent is perpendicular to the radius at the point of contact with the circle (tangent perpendicular to radius)".	0	1	0	0	1	0
Corollary: "A line through a point on a circle perpendicular to the radius at that point is a tangent to the circle (line perpendicular to radius)".	0	0	0	0	0	0
Theorem 7: "The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment (tan-chord theorem)".	4	10	13	6	11	7
Converse: "The line drawn through the endpoint of a chord of a circle forming an angle with the chord that is equal to the angle subtended by that chord in the alternate segment is a tangent to that circle (converse tan-chord theorem)".	1	0	0	4	0	3

Instructional time

The curriculum provides guidelines for the number of hours and days that may be spent on teaching each subject topic in all grades. Mathematics is allocated 4.5 hours per week and Euclidean geometry is allocated 15 days (three school weeks), which amounts to 13.5 hours (DBE, 2011, p7). With the schools allocating 30 minutes per period, they had four double (1 hour) periods, and a single (30 minutes) period for mathematics a week.

The instructional time on the topic differed among the schools. In School A, 16 days (13 double periods and three single periods) amounting to 14.5 hours were spent teaching the topic. This implies one day more than the recommended 15 days, and 1 hour more than the recommended 13.5 hours for the topic. Thirteen days were spent on teaching the topic in School B. The school fell two days short in teaching the topic. In the thirteen days used to teach the topic, the school had 11 double periods and two single periods. This indicates that the total hours the school spent on teaching the topic was 12 hours. Fourteen days, consisting of 12 double periods and two single periods, were spent teaching the topic in School C. In School D, only three days of double periods, amounting to three hours, were spent on teaching the topic. This shows a large shortfall in the instructional time for this topic in School D.

Eight days (six double periods and two single periods amounting to seven hours) were spent teaching the topic in School E. Hence, the school taught the topic for seven days less than the recommendation. In School F, nine days (seven double periods and two single periods amounting to eight hours) were spent teaching the topic. The school taught the topic for six days less than the 15 days recommended by the Department of Education. These schools' instructional time devoted to Euclidean geometry in terms of number of days, periods and hours is presented in Table 4.

School A used a total of 16 days, which is equivalent to 14.5 hours according to the school timetable, to teach the topic. This surpasses the number of days recommended by the Department of Education by one day. Schools B and C used 13 and 14 days, respectively, to teach the topic, which amounts to 12 and 13 hours respectively according to their schools' timetable. Schools E and F followed with eight and nine days. This indicates that School E taught the topic for seven days less than the number of days recommended by the Department of Education, while School F taught it for six days less. School D was found to have spent far less time teaching the topic. Only three of the 15 days allocated for the topic were used in this school to teach the entire topic.

Table 4: Instructional time used for Euclidean geometry in schools.

School	No of Days	Periods		Hours
		Double	Single	
A	16	13	3	14.5
B	13	11	2	12
C	14	12	2	13
D	3	3	0	3
E	8	6	2	7
F	9	7	2	8

Double period = 1 hour, Single period = 30 minutes

Hence, not all the schools spent an adequate amount of time teaching on the topic. One school spent an appropriate amount of time, while two schools (B and C) can be said to have spent an acceptable amount of time teaching the topic as they fell short by half an hour and one and a half hours respectively. Therefore, it can be said that three schools (A, B, and C) spent approximately the allocated time on the topic; however, three schools (D, E, and F) spent far less than the allocated time teaching the topic.

Discussion

The findings show that the curriculum content for Euclidean geometry was not fully covered in all of the schools and, in particular, the content was poorly covered in two of the schools. The first part of the content ("accept the concepts established in earlier grades as axioms, and that a tangent to a circle is perpendicular to the radius, drawn to the point of contact") was neglected in three schools. This content was meant to be used to lay the foundation for the Euclidean geometry proofs and the solving of riders. So, the fact that these were poorly covered implies that many, if not most, of the students in those schools would struggle to grasp Euclidean geometry proofs and the solving of riders, which consequently would lead to poor learning of the topic. Previous studies (Charles-Ogan & George, 2019; Engel et al., 2016; Shikuku, 2012) have shown that where teachers fail to cover the curriculum content, student achievement is limited. Approximately a decade ago, Stols (2013) uncovered the poor coverage of the mathematics curriculum in some schools in South Africa. Today, the situation seems to be much the same. One may infer that students' persistently poor achievement in Euclidean geometry in school certificate examinations could be linked to poor coverage of the curriculum content in some schools.

Most of the schools in this study did not spend appropriate time teaching the topic. Three schools used less than 60% of the allocated instruction time for this topic. This could account for the poor coverage of the content in the schools. If inadequate time is spent teaching a topic, it is likely that the teacher may not pay attention to detail or give individual attention to students where needed, especially in an overcrowded classroom (Ogbonnaya et al., 2016). To provide students with sufficient OTL teachers must use the optimum instructional time to teach the content (Stephen, 2013).

The coverage of curriculum content in the classroom is the responsibility of the teacher. Teachers are expected to use their subject knowledge expertise and the relevant pedagogical approaches to ensure that the topics are fully covered using the time allocated for the teaching of each topic. Teachers' failure to achieve content coverage could suggest, among other things, that they may be lacking in the subject knowledge or relevant pedagogical approaches required to teach the topic. Teachers who have a strong grasp of mathematical concepts are more likely to teach the entire content of the curriculum. In contrast, teachers who do not have a solid understanding of mathematical concepts may avoid teaching certain topics, thereby denying learners the opportunity to learn the full curriculum. Teachers with strong pedagogical content knowledge can choose and implement appropriate instructional strategies, such as using manipulatives, visual aids, or technology, to cover the curriculum efficiently (Grossman, 1990; Shulman, 1987). These teachers can identify the most critical aspects of a topic and allocate time effectively. They can anticipate common misconceptions and address them proactively (Grossman, 1990; Kultsum, 2017; Shulman, 1987) thereby reducing the time spent on re-teaching and increasing the focus on advancing through the curriculum. This may not be the case with teachers who lack strong pedagogical content knowledge, as they may struggle to employ the most appropriate instructional strategies needed to deliver the complete curriculum content within the instructional timeframe. Therefore, there may be a need for topic-specific professional development training for teachers on topics that they are not teaching as expected. We also encourage collaborative lesson planning for mathematics teachers to work together to identify critical aspects of each topic and develop strategies for teaching them.

Teachers' inability to use the allocated instructional time in teaching the topic could be due to their inability to manage instructional time effectively. Sometimes teachers use more time than necessary to teach some topics, which eventually leads to time constraints in teaching other topics. Another factor could be large class sizes. Most classrooms in low socioeconomic environments are often overcrowded, requiring teachers to spend more time on classroom management. This situation reduces instructional time and curriculum coverage (Blatchford et al., 2011). Thus, we recommend that teacher professional development training should not be limited to curriculum topic contents or pedagogical approaches but should also include how to manage large classrooms and instructional time. As noted by Shava and Heystek (2018), teachers can benefit from training on how to effectively manage instructional time to improve their instructional practices. In addition, we recommend that teachers integrate technology in teaching Euclidean geometry to provide interactive and engaging learning experiences. This can help them cover the topic more efficiently and enhance learners' understanding of the concepts.

Conclusion

The content of Euclidean geometry as stated in the curriculum was not completely covered in the participating schools. The foundational contents of the topic (lower grades topics), upon which the students needed to build their investigation and prove the theorems of the geometry of circles, were not well revised in the schools. Furthermore, none of the schools covered all the Euclidean geometry theorems, their converses, corollaries, and axioms as stated in the curriculum. In addition, most of the schools used less than the allocated instruction time for teaching the topic. It can, therefore, be concluded that the Grade 11 students in the selected schools for this study were not given sufficient opportunity to learn Euclidean geometry.

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