

**Investigating Mathematics Teachers' Beliefs about the Nature of Mathematics
and their Impact on Classroom Practices**

by

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DECLARATION

I declare that the mini-dissertation hereby submitted to the University of Limpopo, for the of degree of Master of Education has not previously been submitted by me for a degree at this or any other university; that it is my work in design and in execution, and that all material contained herein has been duly acknowledged.

MAPHUTHA B. K (Mrs)

Date

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ABSTRACT

This study investigated Mathematics teachers' beliefs about the nature of Mathematics and their impact on classroom practices. It was conducted in a public semi-urban school in the Capricorn District-Limpopo Province. It was a case study targeting two FET teachers with teaching experiences of 15 years or more.

The central research questions addressed in this study are, namely: What are Mathematics teachers' beliefs about the nature of Mathematics? And, what is the relationship between teachers' beliefs and their classroom practices?

Data were collected through pre-observation interviews, classroom observation and through post observation interviews. Pre-observation interviews were conducted once before the participants were observed. I was a complete observer during my colleagues' lessons. Interviews and observations data were analysed using categorisation and interpretation of data in terms of common themes and synthesis into an overall portrait of the case. Each case study teacher's data were analysed individually (that is within-case analysis) first and thereafter cross-case analysis was done in order to compare the two case studies.

It was found that in one of the case teachers, professed beliefs were at odds with her beliefs in practice. The teacher's enacted beliefs influenced how Mathematics was taught in the classroom. She stated that she adopted the absolutist way of teaching because she was introducing topics. In the other case teacher the findings indicated that there is a strong relationship between the teachers' professed beliefs and his enacted beliefs. In this case, it was found that the teaching and learning in the classroom was influenced by what the teacher believed in. The teacher indicated that the learners he taught were in Grade 12; hence they should be examination oriented and learn to follow exactly what the teacher was teaching so that they would pass at the end of the year. Although the two case teachers differed in the relationship between their professed and enacted beliefs, all their classroom practices were shaped by their beliefs on how Mathematics should be taught.

LIST OF TABLES

Table 1: The Differences between Absolutist and Fallibilist Teaching Styles.....	22
Table 2: Description of the Participants.....	36
Table 3: Summary of Thabiso’s Beliefs.....	54
Table 4: Summary of Lerato’s Beliefs.....	67
Table 5: Teachers’ Beliefs about the Nature of Mathematics and the Teaching and Learning of Mathematics.....	68
Table 6: Mathematics Teachers’ Beliefs about the Nature of Mathematics.....	69
Table 7: Mathematics Teachers’ Beliefs about the Teaching of Mathematics...71	
Table 8: Mathematics Teachers’ Beliefs about the Learning of Mathematics...72	

TABLE OF CONTENTS

DECLARATION	i
ACKNOWLEDGEMENTS	ii
ABSTRACT	iii
LIST OF TABLES	iv
TABLE OF CONTENTS	v
CHAPTER 1: INTRODUCTION	1
1.1. Introduction and background	1
1.2. Purpose of the study and research questions	4
CHAPTER 2: LITERATURE REVIEW	5
2.1. Introduction	5
2.2. What are teachers' beliefs?.....	6
2.3. Mathematics teachers' beliefs about the nature of Mathematics and the teaching and learning of Mathematics.....	8
2.4. Mathematics teachers' beliefs and classroom practices in curriculum reform	11
2.5. Mathematics teachers' beliefs studies conducted in South Africa	15
2.6. Concluding remarks	20
2.7. Theoretical framework.....	21
CHAPTER 3: METHODOLOGY	24
3.1. Research design	25
3.2. Participants and sampling	26
3.3. Data collection	27
3.3.1. Pre-observation interviews	28
3.3.2. Classroom observations.....	29
3.3.3. Post-observation interviews.....	30
3.4. Data analysis	30
3.4.1. Within-case analysis	32
3.4.2. Cross case analysis.....	32
3.5. Ethical considerations.....	32

3.6. Quality criteria	33
3.6.1. Validity.....	33
3.6.2. Multimethod strategy	34
3.6.3. Mechanically recorded data.....	35
CHAPTER 4: FINDINGS AND INTERPRETATIONS.....	36
4.1. Within-case analysis	37
4.1.1. Case 1: Thabiso	37
4.1.1.1. Professed beliefs	37
4.1.1.2. Classroom practices	44
4.1.1.3. Relationship between beliefs and practice	52
4.1.1.4. Concluding remarks	53
4.1.2. Case 2: Lerato	54
4.1.2.1. Professed beliefs	55
4.1.2.2. Classroom practices	60
4.1.2.3. Relationship between beliefs and practices	66
4.1.2.4. Concluding remarks	66
4.2. Cross-case analysis	67
4.2.1. Beliefs about the nature of Mathematics	69
4.2.2. Beliefs about the teaching of Mathematics	70
4.2.3. Beliefs about the learning of Mathematics	71
4.2.4. Concluding remarks.....	73
CHAPTER 5: CONCLUSION AND INTERPRETATIONS, LIMITATIONS AND RECOMMENDATIONS	74
5.1. Introduction	74
5.2. Conclusion and interpretation	74
5.3. Limitations of the study	78
5.4. Recommendations	78
REFERENCES	80
APPENDICES.....	86
APPENDIX A: Interview protocol	86

APPENDIX B: Post-observation interviews for Lerato	88
APPENDIX C: Post-observation interviews for Thabiso.....	89
APPENDIX D: Transcription of Thabiso’s lesson no 1	90
APPENDIX E: Transcription of Thabiso’s lesson no 2	102
APPENDIX F: Transcription of Lerato’s Lesson no 1	109
APPENDIX G: Transcription of Lerato’s lesson no 2.....	115
APPENDIX H: The letter written to the school to ask for a permission to conduct a research.....	120
APPENDIX I: Approval letter from the school	121
APPENDIX J: The letter requesting teachers to be participants in the study	122
APPENDIX K: Acceptance letter from the participant (Lerato)	123
APPENDIX L: Acceptance letter from the participant (Thabiso).....	124

CHAPTER 1: INTRODUCTION

1.1. Introduction and background

The South Africa's process of curriculum change has been undergoing continual review. Driven by Outcomes-Based Education (OBE) philosophy, it was introduced in 1994 as Curriculum 2005 (C2005). In 2002, it was considered a policy with the new name for the General Education and Training band (Grades R-9) as the Revised National Curriculum Statement (RNCS), which later became the National Curriculum Statement (NCS). For the Further Education and Training (FET) band (Grades 10-12) it was introduced right away as the National Curriculum Statement in 2006. The underlying principles, as put by Department of Education, included: social transformation; high knowledge and high skills; integration and applied competence; progression; articulation and portability; human rights, inclusivity, environmental and social justice; valuing Indigenous Knowledge Systems and credibility, quality and efficiency and Outcomes-Based Education (DoE, 2003).

Recently the National Curriculum statement had been amended. The amended policy, which is called Curriculum and Assessment Policy Statement (CAPS), will come into effect in January 2012. Unlike NCS, articulation and portability, integration and critical learning and OBE are no longer principles of CAPS. However, these three principles are replaced by Active and Critical Learning that encourages an active and critical approach to learning rather than rote learning and uncritical learning of given truths (DoBE, 2011).

The present study focuses on teachers as they were struggling with the implementation process of curriculum changes, currently NCS, which is implemented in the FET band. Although NCS is on the verge of being replaced by CAPS, it will still be implemented in Grades 11 and 12 in 2012 and 2013, respectively. Therefore, the implementation of CAPS in the FET will run concurrently with NCS until 2014 in Grade 12. In NCS, Outcomes-Based Education was regarded as the foundation (DoE, 2003). At the core of Outcomes-Based Education are learner-centeredness

and activity-based approach (DoE, 2003). Teachers became challenged to revisit or revise the way they used to teach. To cater for learner-centeredness, they had to talk less and listen more. Mathematics teaching was no longer to be seen as 'step-by-step' explanations of procedures while learners learn by listening and practising (Swars, 2005).

Teachers are expected to facilitate learning through giving activities in which learners will be actively involved. For the realisation of engaging learners in active learning, teachers need to employ activity-based approach. Activity-based approach is defined as the learning approach in which activities form the core of a learning cycle whereby new understandings depend on and arise out of activity (LDoE, 2006). Learners in activity-based learning are expected to do hands-on activities and hence be active participants in their learning process, as such, rote learning is discouraged.

Activity-based learning gives learners an opportunity to engage in a task that is unfamiliar to them, in such a way that they can learn by reflecting on that engagement (LDoE, 2006). In activity-based learning, teachers, as designers of the learning programme, should design the learning activities that best suit the learners' level of understanding (*ibid*). Therefore, this calls for teaching strategies which can direct learners towards the active learning role. The strategies are: cooperative learning, simulations and gaming, case studies, problem-based learning and self-learning modules (*ibid*).

The teaching strategies point out to the seven teacher roles outlined in the norms and standards for educators. The roles of the teacher are defined, namely: mediators of learning; interpreters and designers of learning programmes and materials; leaders, administrators and managers; scholars, researchers and lifelong learners; community members, citizens and pastors; and assessors and subject specialists (DoE, 2003).

With all this pedagogy outlined, it is the teachers' beliefs that are at the centre of how they teach because their planning and their practices impact on learning (Hoadley & Jansen, 2009). Therefore, these changes call for teachers to align their practices

with that of the aspirations of the new curriculum. However, all the changes needed, rely on the teacher's system of beliefs (Lazim & Osman, 2009).

Successful curriculum change depends heavily on teachers' beliefs (Handal & Herrington, 2003). If teachers' beliefs are not taken into consideration, teachers as immediate curriculum implementers can pose a challenge to the curriculum reform. This may be caused by the fact that change is difficult (Nelson, 2009). On the other hand (Chen, 2010), stated that the success of [every] reform is largely dependent on the implementation of the reform-oriented curriculum, and teachers' beliefs are one of the key factors influencing the implementation.

Numerous studies about Mathematics teachers' beliefs have been conducted (Karaagac & Threlfall, 2004; Nisbet & Warren, 2000; Villena-Diaz, 2003; and Vistro-Yu, 2001). These studies have consistently confirmed one major idea that Mathematics teachers' beliefs and conceptions, particularly about the nature of Mathematics and about the teaching and learning of Mathematics, have an impact on the type of Mathematics lessons they deliver in the classroom (Vistro-Yu, 2001). From these studies it is clear that teachers' beliefs influence how the lessons are delivered (teaching methods) and also influence the teachers' roles during the lessons. The way Mathematics is taught is essential for learners' success in Mathematics.

In South Africa, studies about teachers' beliefs and curriculum reform had been conducted, however, being limited number as already indicated. In these limited studies, participants were either pre-service teachers or novice teachers; focus on some of the studies was on Mathematics teachers' beliefs after the implementation of C2005. In this new curriculum (NCS), the main focus on studies conducted was also on either pre-service or novice teachers and again on Mathematical Literacy not Mathematics. Since the main focus in South African studies were either on novice or on Mathematical Literacy, it is not clear as to which beliefs about Mathematics and its teaching and learning are long serving Mathematics teachers holding.

One can wonder as to whether these teachers' beliefs are consistent with the aspirations of NCS, or as beliefs are robust (Leatham, 2002), resistant to change and can either facilitate or inhibit curriculum reform (Koehler & Grouws, 1992), are they still holding the beliefs that they had before curriculum change? This indicates that despite the curriculum change that took place in South Africa, Mathematics teachers' beliefs were not well studied. It is against this backdrop that this research is set out to investigate teachers' beliefs about the nature of Mathematics and their impact on classroom practice.

1.2. Purpose of the study and research questions

The purpose of this study is to investigate Mathematics teachers' beliefs about the nature of Mathematics and their impact on classroom practice in this new curriculum.

The following questions guided my investigation:

- *What are Mathematics teachers' beliefs about the nature of Mathematics?*
- *What is the relationship between teachers' beliefs and their classroom practices?*

CHAPTER 2: LITERATURE REVIEW

The previous chapter discussed the introduction and background for the study. It further provided the purpose of the study and the research questions that guided the study. This chapter is organised in six sections. I start by giving a brief introduction about some research on Mathematics' teachers' beliefs. This is followed by a focus: on what are teachers' beliefs, on Mathematics teachers' beliefs about the nature of Mathematics and the teaching and learning of Mathematics, on Mathematics teachers' beliefs and classroom practices in a curriculum reform and studies on Mathematics teachers beliefs conducted in South Africa. Thereafter, I give concluding remarks based on lessons drawn from the readings. Lastly, I capture theoretical framework that guided this study.

2.1. Introduction

Research on Mathematics teachers' beliefs has been established as far back as the 1970s (Lester, 2007). However, this field started to receive more attention in the 1990s when researchers were starting to focus on many aspects of teaching and learning Mathematics. In most cases, the main focus of the researchers was teachers' beliefs about the nature of Mathematics and its teaching and learning. Other emergent areas of focus on Mathematics teachers' beliefs, such as inconsistency between the beliefs, beliefs about curriculum, technology and also gender, started to receive more attention. In these cases, the researchers were interested in how teachers' beliefs are changed. There is still a need for more studies on Mathematics teachers' beliefs despite the vast literature on this field. This should be carried out because a study on teachers' beliefs has the potential to provide significant and profound insight into many aspects of teachers' professional world (Murphy, 2000).

Research in the field of beliefs provided assumptions that Mathematics teachers' beliefs affect what they do in their classrooms. It further indicated that there is a strong relationship between Mathematics teachers' beliefs about the nature of Mathematics and their classroom practices (Yates, 2005; and Beswick, 2007) and that beliefs influence practice (Vistru-Yu, 2001). Uumasiki and Nason (2004)

confirmed that what goes on in the Mathematics classroom may be directly related to the beliefs teachers hold about the nature of Mathematics. According to Cooney and Shealy (as cited in Beswick, 2005), teachers' beliefs also have long been regarded as critical to the reform of Mathematics education.

There is a precaution for the researchers who would like to pursue this field of beliefs. The precaution is that the researchers need to first define beliefs according to how these beliefs will be observed in their studies. Leatham (2002) highlighted this issue when stating that the defining of beliefs is of utmost importance. Pajares (1992) explicitly cautions that it will not be possible for researchers to come to grips with teachers' beliefs without first outlining clearly what they mean by beliefs, and how that meaning will differ from that of similar constructs. Pajares further concluded that the difficulty in studying teachers' beliefs has been caused by definitional problems, poor conceptualizations, and differing understandings of beliefs and belief structures.

2.2. What are teachers' beliefs?

Many researchers are at odds about the definition of beliefs. Beswick (2005) mentioned that the construct of beliefs lacks a commonly agreed-upon definition despite being a very popular element of research in the current decades. This was substantiated by Yook (2010) when stating that the term has acquired a rather non-specific, indistinct usage due to the lack of consensus on what the construct of beliefs refers to.

The main difficulty in defining beliefs seemed to be caused by the researchers' inability to distinguish between beliefs and knowledge (Pehkonen, 2004). Pehkonen and Pietilä (2003) highlighted another difficulty as being a distinction between beliefs and other affective elements such as emotions, attitudes, and values. These affective components and beliefs are strictly connected (Moscucci, 2007) and, moreover, they affect beliefs, whereas knowledge is not dependent on them (Boz, 2008).

Beliefs are said to be held in clusters which are continuously affecting each other (Pehkonen, 2004). Beswick (2010) stated that:

It has long been acknowledged that an individual's beliefs are not held in isolation from one another, but rather they are related in complex ways that make their relationships, particularly with behaviour, difficult to unravel.

Hence beliefs form a belief system, which is used as a metaphor to represent how the individual's beliefs are structured (Pehkonen, 2004).

Teachers hold beliefs which they use in their teaching practice. These beliefs are said to play an important role in many aspects of teaching, as well as in life (Borg, 2001). Most importantly, these teachers' beliefs could be used to understand teachers' thought processes and instructional practices (Zheng, 2009). Every teacher has his or her own teaching and learning beliefs, hence Khonamri and Salimi (2010) stated that teachers have beliefs in all aspects of their teaching.

Teachers hold beliefs about the subjects they are teaching and how those subjects should be taught. Perry, Howard and Conroy (1996) acknowledged that all Mathematics teachers hold certain beliefs about Mathematics and its teaching and learning. Raymond (1997) and Speer (2005) suggest that there are two types of beliefs that are normally held by Mathematics teachers. These are the professed beliefs and the attributed beliefs. The professed beliefs are defined as the beliefs that are manifested during interviews or what teachers stated during interviews. The attributed beliefs are defined as beliefs that researchers infer based on observational data or what reflected in teachers' practices (Raymond, 1997; and Speer, 2005). Those beliefs are mostly said to be either inconsistent or consistent to each other.

Furinghetti and Pehkonen (cited in Pehkonen & Pietilä, 2003) gave some recommendations to take into considerations when dealing with beliefs and related terms (such as knowledge, attitudes). They recommended that, when dealing with beliefs, one should:

- consider two types of knowledge (objective knowledge and subjective knowledge);

- consider beliefs as belonging to subjective knowledge;
- include affective factors in the belief systems, and distinguish affective and cognitive beliefs, if needed;
- consider degrees of stability, and to acknowledge that beliefs are open to change; and
- take care of the context (e.g., population, subject, etc) and the research goal within which beliefs are considered.

From these recommendations, Pehkonen and Pietilä (2003) defined and understood beliefs to be “an individual’s subjective knowledge and emotions concerning objects and their relationships, and they are based usually on his/her personal experience”. It is their definition and understanding of beliefs that I adopted in this study.

2.3. Mathematics teachers’ beliefs about the nature of Mathematics and the teaching and learning of Mathematics

Most studies conducted on teachers’ beliefs about the nature of Mathematics and how that influenced classroom practice were skewed on pre-service teachers and/or novice teachers. Little has been done to follow up experienced in-service teachers’ beliefs about how they viewed the nature of Mathematics and how that relates to their classroom practices. Jeon (1999) investigated an expert teacher’s belief about the nature of Mathematics and its manifestation in teaching in a high school Mathematics classroom. The study employed case study design in which data was collected through an in-depth interview and field observations. It was found that the teacher’s belief about Mathematics showed a generalistic approach where Mathematics is approachable to everybody. On the manifestation of belief in the teaching, it was found that the teacher act as an assistant in students’ learning by helping students during her lessons. This indicates that what teachers believe is manifested on their teaching practice.

Sometimes inconsistency occurs between teachers’ beliefs and practices. Raymond (1997) investigated relationships between a beginning elementary school teacher’s beliefs and Mathematics teaching practice. In his study, a proposed model of

relationship between belief and practice provided a conceptual framework for examination of the fact that influence belief, practice and the level of inconsistency between them. Data were gathered over ten months through audio-taped interviews, observation, document analysis and a belief survey.

Analysis included the categorisation and comparison of belief and practice and the development of a revised model of relationship between belief and practice. Findings indicated that the teacher's beliefs and practice were not totally consistent. To some extent, her practice was more closely related to her beliefs about Mathematics content than to her Mathematics pedagogy. Her beliefs about Mathematics content were highly influenced by her own experience as a student and her beliefs about Mathematics pedagogy were primarily influenced by her own teaching practice. It appeared that factors such as experience and teaching practice have the influence on beliefs.

Inconsistency between teachers' beliefs and practices could be caused by the non approval of teaching methods. In their study, Karaagac and Threlfall (2004) investigated a teacher's beliefs and their actual practices. It was part of an ongoing project on teachers' beliefs and practices in State Schools (SS) and in privately owned examination Preparation Schools (PS) in Turkey. It was a case study of one Mathematics teacher with 11 years experience of teaching in different PS. The teacher under the study has been chosen because he was aware of the discrepancy between his beliefs and his practice and he was also able to reconcile the two. Data were collected through interviews and classroom observations.

It was found that the teacher's practice has been mediated by the Numerical Value Technique (NVT), which is the method that allows students to solve problems in a very short time and usually without any recourse to the theoretical knowledge supposedly required by the question. Yet, the teacher's beliefs about NVT were negative. This created tension that might have motivated change in that teacher's belief system, but it did not. Thus, another factor for inconsistency might be non-approval of the teaching methods. The participant teacher used that method just

because it was prescribed, although he preferred the traditional method of teaching Mathematics.

In the teaching and learning of Mathematics, teachers could be classified according to two contrasting Mathematical beliefs. Nisbet and Warren (2000) in their study examined primary teachers' beliefs on teaching, learning and assessing Mathematics. In particular, it considered the nature of the beliefs of primary school teachers with regard to Mathematics as a subject and teaching Mathematics.

With regard to beliefs about teaching Mathematics, the results of this study suggest that teachers' are either traditionalists with a transmission approach, or they have been convinced of the merits of the constructivist approach. Although support for the constructivist view was found to be higher than that of the traditional view, it was found that in some classrooms, the teaching of Mathematics occurs in a sterile environment with little use of hands-on experiences and limited attention given to relating Mathematics to interesting ideas in the world. Although teachers may hold certain beliefs this may be in contrast with what they do in their classrooms.

Contrary to the above, teachers could be classified into two contrasting beliefs and their professed beliefs may be found to be in line with their attributed beliefs. Villena-Diaz (2003) investigated the beliefs and practices of elementary Mathematics teachers in High (HPS) and Low Performing Schools (LPS) in Metro Manila. Teachers' beliefs and practices were categorised using the two contrasting beliefs in Mathematics education: the School Mathematics Tradition (SMT), which involves classroom routines and discourses that are usually rigidly controlled by the teacher, and the Inquiry Mathematics Tradition (IMT), which stresses more active learning on the part of the students, by exploration, problem-solving and collaboration, etc. About the nature of Mathematics, it was found that the HPS teachers' beliefs lean towards IMT, while the LPS teachers' beliefs lean towards SMT. And, in practice, it was found that HPS teachers' beliefs lean towards IMT, whereas LPS teachers' beliefs lean towards SMT.

The study recommended in-service training and staff development that will provide opportunities for teachers to re-examine their beliefs and practices and also included a course in the pre-service and in-service programmes that will deal with philosophical aspects of Mathematics education.

Boz (2008) investigated Turkish pre-service Mathematics teachers' beliefs about Mathematics teaching. The study involved 46 pre-service Mathematics teachers. Data were collected through an open-ended questionnaire which consisted of one open ended question with many sub questions. Data analysis involved categorisation. Findings indicated that most of the participants in the study held non-traditional beliefs about Mathematics teaching.

Established here is that teachers' beliefs can be categorised according to the beliefs in Mathematics education. The beliefs that teachers are holding could either be consistent or not consistent with their practice. These may also occur in the curriculum reform because teachers' beliefs play important role in the classroom.

2.4. Mathematics teachers' beliefs and classroom practices in curriculum reform

In most of the studies conducted to investigate beliefs and curriculum change, inconsistency between the beliefs and the aspirations of that curriculum had been found. Frykholm (1995) conducted an ethnographic study which investigated Mathematics teachers' beliefs of 44 pre-service Mathematics teachers. The study was set out to determine whether teachers were in line with the reforms proposed in the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989).

The findings indicated that, although most teachers stated that they were implementing the principles as stipulated in the standards, most of the participants were unable to implement those principles. The cause was due to their perceived lack of training in the principles underpinning the reform. Other causes that hindered the implementation were that teachers felt that the standards were "not as practical as they were made out to be, especially in dealing with the structure of most schools

— short periods, no collaboration, no team teaching,” as well as rigid departmental policies, lack of support from cooperating teachers, and textbooks (Frykholm, 1995).

Due to the climate of reform in Mathematics education, Babbette (2000) conducted the study that investigated the factors that contribute to secondary Mathematics teachers’ choices during teaching. She found it to be particularly important to explore which influences and experiences are important as beginning teachers begin developing their practices. Two case studies were conducted with beginning (first 3 years) secondary Mathematics teachers.

Data were collected through interviews and observations. Results suggested that teachers’ beliefs about Mathematics and its teaching and learning, knowledge, and context form an interconnected, complex reality that critically influences what happens in practice. For example, one of the participants’ observed practices involved spending time on tasks not centred on intended mathematical curriculum concepts. This study raised important questions concerning how to best prepare teachers to teach in ways that support ideals of current Mathematics education reform.

Norton, McRobbie and Cooper (2002) conducted a study in which they were investigating teachers’ goals and practices following an introduction of a curriculum called Investigative Mathematics Syllabus in Queensland. It was a case study of nine Mathematics teachers from two schools. Data were collected through teacher questionnaire, classroom observations using field notes, interviews using audio-recordings and from students’ examinations and mark allocations. A questionnaire was used to collect data on demographics and beliefs about Mathematics and teaching of Mathematics.

The findings indicated that teachers were still using teacher-centred approach in their classes. The teachers’ choices of this teaching approach were influenced by the syllabus documents. Most teachers had calculation-based goals for less able students and conceptual goals for more able students, which were encouraged by the curriculum; however, their teaching strategies were at odds with the aspirations

of the curriculum in that most teachers were using show and tell method of teaching. This indicated that there is a distinct relationship between teachers' goals; beliefs and practices. Teachers tend to use the teaching strategies that they feel comfortable with.

Noor (2003) conducted the study that focused at discovering differences between teachers' assumptions about teaching and learning, particularly related to problem solving in Mathematics, as compared to those espoused by the curriculum. Data were collected through interviews, classroom observations and content analysis.

Despite the constructivist nature of the Malaysian Mathematics curriculum, the findings indicated that majority of teachers beliefs were still traditional in nature because teachers still used teacher-centred approach, providing no room for the students to explore, discuss and make conjectures on problem solving tasks given to them. These teachers were still creating learning environment which was not conducive in promoting learning envisaged by the curriculum despite various attempts by the Curriculum Development Centre of the Ministry of Education to change teachers' beliefs about teaching and learning.

In their study, Ziegler and Chapman (2004) had investigated Mathematics teachers' perspective of what is necessary to teach Application-Oriented high school Mathematics with a focus on projects. It was a case study with two experienced high school teachers. They were selected because they were teaching the curriculum since its inception. Data were collected through semi-structured interviews and written accounts of lessons based on projects. Analysis involved scrutinising the interview transcripts and organising them into themes.

The results showed that in implementing the curriculum, the teachers observed found the 'stand and deliver' approach to be counter-productive and as such the curriculum to them was not different to the traditional one and after a few semester of trying alternative ways, they shifted their perspective to a non-traditional way.

Yates (2005) used a survey to investigate relationships between teachers' beliefs about Mathematics and its teaching and learning of Mathematics, their pedagogical practices in Mathematics and their experiences of curriculum reforms in Mathematics. The study involved 127 experienced teachers with 10 or more years of experience in the teaching of Mathematics and had experienced a number of curriculum reforms in Mathematics during their teaching career, with the most recent reform involving a constructivist approach to the teaching and learning of Mathematics which was introduced in 2001. Data were analysed using SPSS.

It was found that teachers' espoused beliefs about Mathematics were unrelated to their beliefs about Mathematics teaching and learning. Furthermore, teachers differed in their beliefs, with those with stronger beliefs making greater use of some constructivist teaching practices in their classrooms. Teachers who experienced a high number of reforms reported utilising computers and the Internet more often in Mathematics lessons and sought constructive information about student Mathematics learning more frequently. However, the study also found a significant relationship between teachers' experiences of reform and the use of technology and some assessment practice.

Another study on teachers' beliefs and Mathematics curriculum reform was conducted by Chen (2007) in Chongqing, China. This was part of a larger ongoing research project to investigate the consistency between Mathematics teachers' beliefs and the underlying philosophy of the new curriculum at the junior secondary level in Chongqing. Data were collected qualitatively through document analysis and quantitatively through a questionnaire.

The findings indicated that the underlying philosophy of the reform-oriented Mathematics curriculum in mainland China was congruent to constructivist ideas to a great extent, and more importantly, the beliefs about Mathematics and Mathematics instruction held by a large proportion of the teachers were, mostly, consistent with the underlying philosophy of the reform-oriented curriculum, although some inconsistencies existed.

In her study, Vistru-Yu (2001) investigated the implications of teachers' beliefs about Mathematics for classroom and teacher education reform. This study involved 57 secondary school Mathematics teachers. Data were collected through survey and questionnaire. It was found that teachers hold strong beliefs about Mathematics that are consistent with the problem solving view.

Mathematics teachers may hold beliefs that are consistent with the aspirations of the curriculum. On the other hand, there may be teachers whose beliefs about Mathematics may not be consistent with the expectations of the curriculum. This may be caused by teachers being comfortable with the methods they were using prior the reform or the contextual factors, such as short periods, no collaboration, no team teaching as well as rigid departmental policies, lack of support from cooperating teachers, and textbooks that teachers found themselves faced with. As South Africa is implementing the new curriculum the question is: are the teachers' beliefs consistent or not consistent with the aspiration of the new curriculum?

2.5. Mathematics teachers' beliefs studies conducted in South Africa

In South Africa studies of teachers' beliefs and curriculum reform had been conducted. For these studies, I had also included the study that focused on Mathematical Literacy as this subject is an alternative to "pure" Mathematics. In some of these studies, also inconsistencies between Mathematics teachers' beliefs and curriculum reform had been established.

A South African ethnographic study which described teachers' current viewpoint on school Mathematics and classroom teaching in relation to the new curriculum requirements was conducted by Roussouw (1999). This study involved 8 Grade 3 Mathematics teachers where data were collected through direct observation and in-depth interviews of those teachers. It was found that the views of the teachers involved in the study about Mathematics and mathematical activities were in direct conflict with the pedagogical practice articulated in Curriculum 2005 (C2005), which offers learners opportunities to engage in problem solving, logical thinking,

recognising patterns and implementing a pedagogy that focuses on conjecture, conceptual exploration, and reflective, critical discussion.

Through the process of systematic observation of classroom interactions and interviews, it was possible to identify teaching styles that do not accord with the expectations of C2005. The predominant views of Mathematics and Mathematics teaching among the subjects of this study are that of a system of algorithm transmitted by teachers to be committed to memory by their students.

MALATI (Mathematics Learning and Teaching Initiative) has conducted research to determine its impact on learners and teachers in order to formulate a 'workable' model of teacher and curriculum development. This was reported by Newstead (1999). They were researching on whether or not the current model that is being trialled in 7 Western Cape schools has any significant effect on teachers' beliefs and classroom practice (and, in an additional study, on learners' achievement). To monitor the process of change in teachers, information on the practice and beliefs of Grades 6 to 9 teachers in four of their project schools, data have been collected as follows:

- Each teacher completed an extensive teacher beliefs questionnaire in November 1997 and also completed the very same questionnaire again in November 1998.
- The MALATI project worker co-ordinating the teacher development at each school collected extensive field notes on reflections, discussions and critical incidents concerning each teacher during the year.
- At three points during the school year, namely February, July and October/November, two MALATI project workers visited the teachers for two consecutive Mathematics lessons. During these visits, the Mathematics lessons were videotaped. Immediately after the second Mathematics lesson, the teacher was interviewed about his beliefs and practice, both general and specific to the two lessons observed. However, this reported the findings from a case study of one teacher.

It was found that, despite intensive teacher development and support, the changes that took place in the classroom probably did not lead to a significant improvement in learning. The changes included the teacher stopping to take responsibility for the learners' learning, of which within the MALATI framework was a positive step because they did not believe in the teacher to passing on his ready-made knowledge to the learners. Although it showed that the teacher's beliefs aligned with the philosophy of MALATI during the interview, but that had not manifested in the classroom.

Another study that found inconsistency between beliefs and practice was conducted by Ramakumba (2010). She conducted a study with the aim of examining strategies that teachers employ in their classrooms in response to their beliefs about OBE. The study was conducted in Gauteng Province, South Africa. The researcher adopted a qualitative exploratory design. The method of choice for this study was a combination of elements of phenomenology and ethnography. Nineteen teachers were interviewed and observed. The sample was drawn from two former Model C schools and three township schools. Data were analysed qualitatively. The findings confirmed that there are multiple beliefs that constitute a personal epistemology. In the absence of certainty about OBE and faced with a multitude of classroom challenges, teachers relied on their experience to make decisions regarding what was important to know. They drew on their own personal teaching theories more than what they thought about OBE to make judgments of learning processes.

This study concludes that the link between teachers' beliefs, conceptualisation of OBE and teaching practice is weak. Teachers' beliefs about the nature of Mathematics, knowledge, teaching and learning Mathematics had stronger connections with, and represented the basis for teachers' pedagogical purpose behind their preferred teaching practice.

Teachers' beliefs can affect the implementation of the new curriculum. Sidiropoulus (2008) investigated how teachers' beliefs and understandings of the curriculum affect the implementation pathway of a Mathematics reform intended for ALL. It was an explorative in-depth case study of two teachers working in Johannesburg, Gauteng

Province. Data were collected through interviews, documents, classroom observations and a researcher's journal.

It was found that the two educators had a superficial understanding of the intentions of the curriculum, both in terms of the required pedagogy and purpose of the reform. This understanding caused them not to implement the curriculum to its full course. This happened because policy and planning did not provide sufficient strategies or support to them. For both educators, the teaching of Mathematics in context was outside their paradigm of understanding as was their limited grasp of 'spirit' of this new reform. It was further revealed that educators teaching Mathematical Literacy felt and expressed an overwhelming threat to the status of their professional teaching identity.

Teachers may take the roles in which they think are in line with curriculum expectations; however, they find themselves acting parallel with the curriculum. A case study of a pre-service secondary Mathematics teacher which focused on the teacher's beliefs about his role as Mathematics teacher was conducted by Lloyd (2004). It was a qualitative study and data were collected over the final five months of the teacher's university teacher education programme. Data were collected through interviews, written course assignments, and observations of student-teaching.

Findings indicated that the teacher valued classroom roles in which students, rather than the teacher, explained traditional Mathematics content. As his student-teaching internship progressed, the teacher began to develop new roles and engaged students in mathematical processes. These results emphasize the need for pre-service teachers to recognize how teacher and student roles impact interrelationships between understanding and mathematical activity, and illustrate the nature of teacher learning that can occur during an internship. This teacher indicated to be knowledgeable about learner-centred approach; however he was applying it in the incorrect way.

In all these studies, inconsistency between curriculum and teachers' beliefs was established; however, what emerges is that beliefs may depend on a particular context. Webb and Webb (2008) investigated the beliefs and practices of a novice teacher of the Nelson Mandela Metropolitan University through the lens of changing contexts and situations. They used both quantitative method, through a questionnaire, and qualitative method, through interviews.

It was found that the teacher involved, demonstrated an absolutist viewpoint and fallibilist viewpoint at different occasions during the same lesson. This teacher emphasised mastery of skills at one stage and a problem solving approach at another stage of the lesson. She acted as both instructor and facilitator. She demonstrated both traditional "chalk-and-talk" style as well as a more innovative contextual, problem solving approach.

They concluded that inconsistency between the beliefs and practices may be an observer's perspective that is not necessarily shared by the teacher because the participant's classroom practices depended on how her beliefs were situated in a particular context. They also indicated that context and complexity of classroom interactions have become increasingly more demanding on novice teachers. And that this understanding has implications when trying to change, measure and understand teachers' beliefs and practices during periods of curriculum reform, such as is currently demanded by the NCS.

Teachers may hold beliefs that are incompatible with the intentions of the curriculum. This may pose a challenge to the curriculum because of how teachers understand the intentions of the curriculum (policy and planning) and personal teaching theories and purposes of teaching. With the knowledge of the reform, teachers may display two parallel beliefs in the same lesson. They may demonstrate the belief compatible with the reform and the one that is inconsistent with the reform.

2.6. Concluding remarks

In conclusion, the studies reviewed in this section raised several issues and limitations that need to be addressed and also provided help into organising the theoretical framework of this study. Before raising these issues, I found it necessary to summarise the studies reviewed under this chapter so that it will be easier to notice those issues.

Firstly, although the study of beliefs in Mathematics education has begun to receive more attention from Mathematics researchers and educators, many of these studies were conducted in foreign countries. Most of these studies involved large samples in which the researchers generalised their findings. These studies confirmed that teachers' beliefs about Mathematics and the teaching and learning of Mathematics are generally consistent with their actual practice. One may ask as to whether or not the results of these studies could be generalized even to South African Mathematics teachers.

Secondly, in the studies that used case study methodology, which involved small samples where participants were either pre-service or novice teachers, the findings mostly indicated that the beliefs are influenced by how the teachers were taught as students. Again, the other question could be what about the long-serving Mathematics teachers. What are their beliefs? Again, what influences their beliefs?

Thirdly, in South Africa, focus on curriculum reform in most of the studies was on Mathematics teachers' beliefs in the C2005. In this NCS, the main focus of studies conducted was on OBE as an approach or on Mathematical Literacy teachers or on pre-service teachers, but not on veteran Mathematics teachers. As such, the above question arose again, what about veteran Mathematics teachers?

Lastly, other studies reviewed under curriculum reform indicated that, although teachers are aware of what the reform needs from them, in most cases, they do not comply with the curricula needs (Frykholm, 1995; Babbete, 2000; Noor, 2003; and Norton *et al.*, 2002).

Newstead (1999) stated that, in many cases, teachers express beliefs about the learning and teaching of Mathematics that do not translate into their classroom practice. This corroborates with Sidiropoulos (2008) when stating that what is evident is that instructional practice in classrooms, especially Mathematics class remains unchanged despite the willingness of teachers to embrace reform change. Is what these researchers found, applicable in each and every Mathematics classroom? Definitely, until one takes the tools and go to the research field, these questions will not be answered.

2.7. Theoretical framework

Fetherston (1998) stated that the importance of theoretical framework in research is that it can assist the researcher with the classification and identification of patterns in the data. He further indicated that a good framework can explicitly articulate the underlying approach to the analysis and interpretation of the collected data, and this will enable the readers to know or understand where the researcher is coming from. This coincides with Aanestad's (2006) postulation when stating that theory provides the ways in which data can be collected and analysed.

Teachers' beliefs of Mathematics and Mathematics teaching and learning are always interpreted in the light of a philosophy or several philosophies of Mathematics because these constitute their views, attitudes, preferences about the nature of Mathematics as well as the teaching and learning of Mathematics (Vistru-Yu, 2001). In relation to the nature of Mathematics, teaching and learning of Mathematics, Ernest (1996) identified teachers' beliefs about the nature of Mathematics in two ways, the absolutist and the fallibilist viewpoints. Absolutism could be associated with the traditional way of seeing teaching and learning in Mathematics. In absolutism, Mathematics is seen as an objective, absolute, certain and incorrigible body of knowledge (*ibid*). For absolutists, Mathematics teaching and learning is characterised by giving learners mainly unrelated routine tasks that involve the application of learnt procedures, and stressing that every task has a right answer (*ibid*).

Ernest further illustrated that in fallibilism, Mathematics is experienced as warm, human, personal, intuitive, active, collaborative, creative, investigational, cultural living, related to human situations, enjoyable, full of joy, wonder, and beauty. This is in line with the aspirations of many countries' curriculum reforms. In the recent curriculum reforms of many countries, South Africa included, Mathematics is presented as a web of related concepts with different ways of representing and solving problems. Furthermore, Mathematics can be explored, contested, justified, and communicated, and the current reform (NCS) Mathematics teaching develops conceptual depth, procedural flexibility and reasoning among learners (Molefe & Brodie, 2010). Mathematical knowledge can be seen as a social construct and is therefore fallible meaning it can be revised and corrected (Ernest, 1996).

Threlfall (1996) extended the viewpoints by giving the distinction between absolutist and fallibilist teaching styles. According to Threlfall, a teacher holding absolutist beliefs will portray teaching styles that differ with the fallibilist teaching styles. The following table outlines the differences.

Table 1: *The Differences between Absolutist and Fallibilist Teaching Styles*

<i>Absolutist teaching</i>	<i>Fallibilist teaching</i>
Behaviourist approach	Constructivist approach
Clear and coherent presentation	Self discovery
Pupil practice and exercises	Real world examples and problems
Emphasis on content	Emphasis on process
Discouragement of discussion	Encouragement of discussion

The current reforms in Mathematics curriculum need teachers to move from behaviourist approach and adopt a constructivist approach, which is the fallibilist teaching approach. The classroom that embraces the reform will always be different from the traditional classroom.

This model of differentiating between the two ways of describing teachers' beliefs about the nature of Mathematics and its teaching and learning has been described in many ways by the researchers. Perry, Howard and Tracey (1999) noticed that the duality of factors is not new and has been described by many authors in various ways. They have derived a model of teacher beliefs from their research and from various Mathematics education reform statements at that time. The model is based on two factors that describe what teachers believe about Mathematics, Mathematics teaching, and Mathematics learning. The two factors were called transmission and child-centredness. These factors are defined in the following ways:

- *Transmission*: the traditional view of Mathematics as a static discipline that is taught and learned through the transmission of mathematical skills and knowledge from the teacher to the learner and where Mathematics is seen as a rigid system of externally dictated rules governed by standards of accuracy, speed and memory.
- *Child-centredness*: students are actively involved with Mathematics through constructing their own meaning as they are confronted with learning experiences that build on and challenge existing knowledge.

This model concurs with Ernest's two ways of identifying teachers' beliefs. Transmission is an absolutist way, whereas child-centredness is a fallibilist way. Ernest (1989) highlighted that the key components of Mathematics teacher are the teacher's view of the nature of Mathematics, view of the nature of Mathematics teaching and the view of the process of learning Mathematics. Therefore, for the categorisation of teachers' beliefs as derived from interview and observation data, I have looked at their views about the nature of Mathematics, their roles in the teaching of Mathematics and their views about the learning of Mathematics. An interpretation of these categories provided me with an opportunity to locate which of the Ernest's ways a teacher falls in, which might be either fallibilist or absolutist.

CHAPTER 3: METHODOLOGY

The previous chapter is organised into six sections. I started by giving a brief introduction about some research on Mathematics' teachers' beliefs. This was followed by a focus on what are teachers' beliefs, on Mathematics teachers' beliefs about the nature of Mathematics and the teaching and learning of Mathematics, on Mathematics teachers' beliefs and classroom practices in a curriculum reform and studies on Mathematics teachers beliefs conducted in South Africa. Thereafter, I gave concluding remarks based on lessons drawn from the readings. Lastly, I captured theoretical framework that guided this study.

This chapter is structured in the following order: I begin by presenting why I chose a qualitative research. This is followed by the research design. The participants and sampling procedures are also discussed after the research design. The chapter also outlines the methods that were used for data collection and how data were analysed. Lastly, issues of ethical considerations and quality assurance criteria are outlined.

This study is intended at investigating Mathematics teachers' beliefs about the nature of Mathematics and their impact on classroom practices. For the realisations of these intentions, I chose a qualitative approach. According to Creswell (1994), qualitative research is defined as "an inquiry process of understanding a social or human problem based on building a complex, holistic picture, formed with words, reporting detailed views of informants, and conducted in a natural setting". Qualitative research provides perspective rather than truth; empirical assessment of local decision makers' theories of action rather than generation and verification of universal theories; and context-bound explorations rather than generalizations (Patton, 1990). Qualitative research was found to suit this study because I was interested in gaining a deeper understanding of teachers' beliefs and their practices in their everyday working situations.

Leedy and Ormrod (2005) gave a clear distinction between the characteristics of quantitative and qualitative research. Among other characteristics that authenticated the suitability of qualitative research for this study was that: firstly, it is often

exploratory in nature and its purpose is to explore and interpret. Secondly, the qualitative process is more holistic and “emergent”, with the specific focus, design and interpretations developing and possibly changing on the way and normally theories help to explain the phenomenon under study.

One other reason for the choice of qualitative research is that beliefs and practices could be easily identified. Thus, de Vos (2001: 243) explicitly stated that qualitative research involves identifying the participant’s beliefs and values that underlie the phenomena. In qualitative research, there is no step-by-step plan or a fixed recipe to follow (de Vos, 2001), meaning there is flexibility in this type of research.

3.1. Research design

According to Huysamen (cited in de Vos 2001) a research design is a “plan or a blueprint according to which data are collected to investigate the research hypothesis or question in the most economical way”. It is the overall plan for conducting the whole research study (de Vos, 2001). The plan that was found suitable for this study was a case study. A case study is defined as a type of a qualitative research in which in-depth data are gathered relative to a single individual, programme, or event, for the purpose of learning more about an unknown or poorly understood situation (Leedy & Ormrod, 2005). Using case study, therefore, provided me an opportunity to gain in depth information about each participant teacher’s beliefs and how they impact on classroom practices.

There are two types of case studies: single and multiple-case studies (Leedy & Ormrod, 2005). Creswell (2007: 74) defined a single case study as the case study where “the researcher focuses on an issue or concern, and then selects one bounded case to illustrate this issue”. In multiple case studies, the one issue or concern is selected; however, multiple case studies to illustrate the issue are also selected. In this study, I had first focused on single case study because my interest is in particular people and their activities as individuals. Secondly, I focused on the participants as multiple case studies because I wanted to make comparisons

(Creswell, 2007). There are several categories of case studies, namely: exploratory, explanatory, descriptive and interpretive (Creswell, 2007).

According to Aanestad (2006), case studies are common in interpretive tradition, focus being on human interpretation and meaning of which are human behaviour and the reasons behind it. This study employed interpretive category. "Interpretive refers to the fact that the aim of the qualitative research is not to explain human behaviour in terms of universally valid laws or generalisation, but rather to understand and interpret the meanings and intentions that underlie everyday human action" (Mouton in de Vos, 2001: 240). This was in line with the purpose of this study because the main focus of this study was on understanding teachers' beliefs and their relationship with what they are doing in their classrooms.

3.2. Participants and sampling

de Vos (2001) stated that "Qualitative research requires that data to be collected should be rich in description of people and places" (p. 253). de Vos further stated that the idea of a qualitative research is to purposefully select informants or documents that will best answer the research question. Focusing on the nature of this study, nonprobability sampling was found to be appropriate. In nonprobability sampling, the researcher has no way of forecasting or guaranteeing that each element of the population will be represented in the sample (Leedy & Ormrod, 2005).

Sampling is the act, process, or technique of selecting a suitable sample, or a representative part of a population for the purpose of determining parameters or characteristics of the whole population (Patton, 1990). In order to gain a deeper understanding and insight of teachers' actions, I found nonprobability purposive sampling suitable. In purposive sampling, people or other units are chosen for a particular purpose (Leedy & Ormrod, 2005).

In order to gain a deep, rich data for this study, I wanted teachers who had 15 or more years teaching Mathematics from Grades 10-12, because I wanted teachers who have experienced many reforms. These teachers should also have had a four

year diploma or a degree in Mathematics. This procedure was based on the assumption that these teachers are more knowledgeable on Mathematics and its teaching and learning. Therefore, sampling procedure in this study was criterion purposeful (Patton, 1990).

This study was conducted in a public semi-urban school in the Capricorn District-Limpopo Province. Initially I intended in using four participants who will match the intention of study. However, only two participants, Lerato and Thabiso (pseudonyms) participated in this study. The participants were those with at least four years teaching diploma or a degree in Mathematics education and fifteen years or more experience in teaching Mathematics. Out of nine educators teaching Mathematics in the FET band in this school, five teachers, including Lerato and Thabiso, were the only ones who met the criteria. However, the abovementioned two were the only ones who showed willingness in the study.

3.3. Data collection

This study was conducted within the interpretive paradigm, as it focused on human interpretation and meaning (human behaviour and reasons behind it) (Walsham, 1995). An interpretive case study was used to collect data from the participant teachers. Interpretive approaches rely heavily on naturalistic methods which are, namely, interviews and observations (Walsham, 1995).

According to Pajares (1992) beliefs cannot be directly observed or measured but must be inferred. Inferences about beliefs require assessment of what individuals say, intent and do (Mapolelo, 2003). Due to the above reasons, data collection in this study occurred in three phases, namely: pre-observation interviews, classroom observations and, lastly, post-observation interview. This resulted in six data collection episodes with the two teachers. Pre-observation interviews were conducted to gather information of teachers' beliefs about the nature of Mathematics and their beliefs about the teaching and learning of Mathematics. Interviews are usually conducted when one needs a detailed understanding of an issue (Boyce & Neale, 2006). All two participants were interviewed once before their first classroom

observations using interview protocol. For interview protocol, see appendix A. They were also interviewed once after all their classroom observations. Data in this study were collected in the third term of 2010 in which the curriculum was still NCS.

3.3.1. Pre-observation interviews

Berry (1999) noted that one of the advantages of using interviews to collect data is that they provide us with a chance to access what is inside a person's head, and thus it makes it possible to measure what a person knows (that is, knowledge and information); what a person likes or dislikes (values and preferences), and what a person thinks (attitudes and beliefs). This is in agreement with Lim (1999) when remarking that one of the advantages of interview is the possibility of accommodating spontaneity as well as preconceived and more tightly structured aspects. This is because interviews give an opportunity to clarify questions in the way that they will be clearly understood by participants.

For pre-observation interviews, the standardised open-ended interviews were used. According to Creswell (2007), in this type of interviews, the interviewers stick to a strict script in which there is no flexibility in the wording or order of questions. Teachers were asked the same questions in the same order, because I wanted data that are complete. The advantage of this type of interview is that it facilitates organisation and analysis of the data (Creswell, 2007), although this method provides less flexibility for questions (Berry, 1999).

Open-ended questions (e.g., what would you consider to be a good Mathematics lesson?) and sequenced questions, which refers to using a special kind of questioning technique called funnelling (Berry, 1999), were used in designing the interview protocol. This funnelling technique means asking from the general to the specific; from broad to narrow (e.g., do you feel there is a best way to teach Mathematics? What is it? How did you come to embrace this as a best practice?). Asking truly open-ended questions does not pre-determine the answers and allows room for the informants to respond in their own terms (Patton, 1990).

3.3.2. Classroom observations

According to Cohen, Manion and Morrison (2000) “whatever the problem or the approach, at the heart of every case study lies a method of observation” (pg, 185). Ziegler and Chapman (2004) remarked that it is important to learn from teachers what they do and how they make sense of what they do in their classrooms because teachers are a determining factor of how Mathematics curriculum is interpreted and taught. Observation is the foundation of a qualitative research and it is a way for the researcher to see and hear what is occurring naturally in the research site (McMillan & Schumacher, 2010).

Classroom observations were conducted for each participant. Each teacher was observed for a maximum of two lessons in one class of their choice. Each lesson-observation took a period of one hour because of the time allocated for a lesson at the participants’ school. Classroom observations were used because they help with the provision of data that reach beyond self-report and triangulation of data gathered through interviews and, thus adding credibility to the study (Berry, 1999). Observations were conducted in order to check the following four things: firstly, teachers’ teaching styles and methods used during lessons. Secondly, the classroom interactions; thirdly, to establish whether there is consistency between the interview response and the classroom practices and, lastly, to check as to whether or not the teachers’ practices were aligning with which type of the Mathematics teaching style (absolutist or fallibilist).

In interpretive studies the researcher is directly involved in the process of data collection and analysis (Creswell, 2007). During classroom observations, I was a complete observer. I chose that form of observation so that I would be able to monitor the class interactions closely. All classroom observations were scheduled in advance with the teachers’ consent. These observations were done a week after the interviews were conducted. A video camera was used to capture all Thabiso’s lesson interactions. One of Lerato’s lesson interactions was video recorded whereas the other was tape recorded. Transcriptions of Thabiso’s lessons are provided in

appendix D and E, whilst the transcriptions of Lerato's lessons are provided in appendix F and G.

3.3.3. Post-observation interviews

Post-observation interviews were unstructured because I was interested in seeking clarity on some of the issues raised in the class during observations. These interviews depended on what I had observed in the first lesson and found it recurring in the second lesson for each teacher. The questions were based on the part of the lessons where I needed some clarity on certain phenomenon that happened in the classroom. For post-observation interviews questions for Lerato and Thabiso see appendix B and C respectively.

3.4. Data analysis

Analysis is a reasoning strategy with the objective of taking a complex and resolving it into its parts (de Vos, 1998). According to Miles and Huberman (1994), data analysis in qualitative research consists of three linked subprocesses: data reduction, data display, and conclusion drawing or verification. These subprocesses are further outlined below:

Data reduction refers to the process whereby data can be reduced and transformed through such means as selection, summary, paraphrasing, or through being subsumed in a larger pattern so that the data will be understandable. Data display means taking the reduced data and displaying it in an organised, compressed way that permits conclusion drawing. Lastly, conclusion drawing and verification involves making interpretations and drawing meaning from the displayed data (de Vos, 1998: 340).

Data analysis in a case study, according to Creswell (2007), involves the following steps:

- Organisation of details about the case
- categorisation of data;
- interpretation of single instances;
- identification of patterns; and

- Lastly, synthesis and generalisation.

Data in qualitative studies are normally in written words. Data in this study are in the form of transcribed interviews, and written descriptions of lesson observations from video recorded lessons. The process of data analysis in this study has taken place in this fashion. Details of each participant were first given and summarised on Table 3 in Chapter 4 and thereafter outlined in detail in the within-case analysis of each participant.

Data were first reduced through the sifting of some extra talks that the participants used. Since data were in the form of words, extra talk included words such as “ehh”, “actually” and “especially” which the participants used when still thinking about the responses to the questions during interviews. Data were coded using words such as beliefs about Mathematics, beliefs about teaching, and beliefs about learning and colour was used in each code to facilitate pattern seeking and categorisation and paving the way for easy analysis. The following themes from theory under which data were categorised were used:

- Teacher’s beliefs about the nature of Mathematics;
- Teacher’s beliefs about the teaching of Mathematics; and
- Teacher’s beliefs about the learning of Mathematics.

Aanestad (2006) stated that in interpretive case studies data analysis can be data-driven or theory-driven. When a case study is (re)interpreted according to a theoretical framework it is said to be theory-driven whereas data-driven is when interpretations are generated by the data itself.

Data analysis in this study occurred in two phases. In the first phase, within-case was done then followed by cross case analysis. In all these phases, analysis was theory-driven in that the cases were interpreted according to the theoretical framework.

3.4.1. Within-case analysis

Within-case analysis means each case study was analysed individually. I used theory to analyse the teachers' beliefs. This gave me an in-depth understanding of each individual's belief about Mathematics and practice. According to Miles and Huberman (1994), within-case analysis means comparing data against the given theory. The process of data analysis started immediately after the pre-observation interviews in order to identify teachers' beliefs so that this will give me a chance to look for the relationship of these beliefs with the practice in the observation. I used a table to display and summarise the findings from within-case analysis for each case. The participants' beliefs about Mathematics and its teaching and learning were interpreted according to Ernest (1996) and Threlfall (1996) as either absolutist or fallibilist.

If you conducted more than one case (multiple-case study), then you are compelled to do cross case analysis (Miles & Huberman, 1994). Therefore, in the second phase, as already mentioned above, cross-case analysis of the two case studies was done.

3.4.2. Cross case analysis

In cross-case analysis data in one case are compared to data in the other cases (Miles & Huberman, 1994). I had done cross case analysis in order to illustrate similarities and differences across the two participants' beliefs. I got back to the categories developed during the within-case analysis and used them in the comparison of the two cases. I used the table to display and summarise the findings from cross case analysis. This helped me in facilitating conclusion.

3.5. Ethical considerations

Whenever human beings are the focus of investigation, one must look closely at the ethical implications of what is being proposed to be done. Education focuses primarily on human beings (McMillan & Schumacher, 2010). This study focused on

teachers and their practices. Leedy and Ormrod (2005) stated that whenever human subjects are involved, the following ethics should be observed

- Informed consent;
- Right of respondents privacy;
- Protection from harm; and
- Honesty with professional colleagues.

With the view of the above categories for ethical considerations, the following had been taken into consideration:

- The letter to inform and ask for permission from the management of the school where my study had taken place was written. For the letter asking for permission and the response letter see appendix H and appendix I respectively. FET Mathematics teachers were informed about the study and its purpose. They were also written the letter requesting them to be participants in the study. For the letter requesting teachers to be participants in this study see appendix J. For the response letters see appendix K and L for Thabiso and Lerato respectively.
- All the participants were assured of the confidentiality of the research and the results. In order to ensure confidentiality, anonymity of the participants were taken into consideration, hence pseudonyms (Lerato and Thabiso) were used. Pseudonyms were also used for learners whom the two participants interacted with them individually during the lessons.
- All participants were assured that all the collected data in the form of notes and discs would be destroyed immediately after analyses; and, lastly,
- All participants were assured that the findings of this study would be reported in a complete and honest way, without fabrication of data.

3.6. Quality criteria

3.6.1. Validity

According to McMillan and Schumacher (2010), validity in qualitative research, refers to the degree of similarity between the explanations of the phenomena and the

actualities of the world. It is the degree to which the interpretations have common meanings between the participants and the researcher. McMillan and Schumacher further indicated that claims of validity rest on data collection and analysis techniques, and they provided ten possible strategies of which a qualitative researcher can use in order to enhance validity. The strategies included:

- prolonged and persistent fieldwork,
- multimethod strategies,
- participant language,
- verbatim accounts,
- low-inference descriptors,
- multiple researchers,
- mechanically recorded data,
- participant researcher,
- member checking,
- participant review, and
- Negative or discrepant data.

In this study, two strategies to enhance validity were employed. Those strategies were multimethod strategy and mechanically recorded data.

3.6.2. Multimethod strategy

This is the use of many methods of collecting data. McMillan and Schumacher (2010) describe this strategy as where the most qualitative researchers employ several data collection techniques in a study but, usually, select one as the central method, either observation or interviews. Observations and interviews were used to collect data in this study. Interviews were selected as the central method. These methods permitted triangulation and thus increased the credibility of the findings. Triangulation is more appropriate when a researcher is conducting a case study (Cohen, Manion & Morrison, 2000).

3.6.3. Mechanically recorded data

To enhance validity, I had intended to use video camera for all lessons of Lerato's class; however, I found it impacting negatively on the learning environment during the first observation as learners were distracted from the lesson. Some learners were trying to hide from the video camera, whereas some were focusing and paying more attention to it than to the lesson. I then decided to use a voice recorder in the second lesson so that learners will feel free. The voice recorder was put on the front desk that was used as a resource desk by the teacher; however, this compelled me to even write down the activities given so that they will help me in transcription of the tape and also in data analysis. I found this helpful as learners did not notice what was happening, and I felt that the atmosphere was the one they were used to. All the lessons of Thabiso were video recorded. All the mechanically recorded data were destroyed immediately after data analysis.

CHAPTER 4: FINDINGS AND INTERPRETATIONS

The previous chapter indicated why I chose a qualitative research. This was followed by the research design. The participants and sampling procedures were also outlined after the research design. The chapter also discussed the methods that were used for data collection and how data was going to be analysed. Lastly, issues of ethical considerations and quality assurance criteria were outlined.

This chapter presents the findings and the interpretations of the data collected in this study. It starts with the provision of descriptions of the participants in Table 2 and a brief description of each case before it could be analysed. This is followed by within-case analysis and, thereafter, a cross-case analysis of the two participants. Lastly it gives the summary of the findings.

Table 2: Description of the Participants

Participant's name	Gender	Age	Qualification	Grade and classes teaching	Teaching experience	In-service training attended
Lerato	Female	Early 50's	Primary Teachers Diploma (PTD), Advanced Certificate (ACE) in Education (FET Mathematics)	Grade 10, Grade 11C and Grade 11E	26 years	Attended in-service training at Mastec in 2008
Thabiso	Male	Late 40's	Education Diploma in Mathematics and Science, BSc Mathematics and Statistics education and BSc(Hons) Mathematics	Grade 12 A, B, C, D	20 years	Attending Mathematics workshops at the circuit offices

The interview data were transcribed and coded by looking for similar patterns and categorised using theory. From theory, three themes were generated. The themes are: beliefs about the nature of Mathematics, beliefs about the teaching of Mathematics and beliefs about the learning of Mathematics.

4.1. Within-case analysis

4.1.1. Case 1: Thabiso

Thabiso is a male Zimbabwean teacher at his early forty's. He has completed his education diploma in Mathematics and Science education, BSc in Mathematics and Statistics and also BSc (Hons) in Mathematics education. He has been teaching for the past 20 years: 8 years in Zimbabwe and 12 years in South Africa. At the time of this study, he was teaching Mathematics in Grade 12. At the school where he was teaching, there were six Grade 12 classes and he was teaching only four classes: 12A, 12B, 12C and 12D. The teacher used to attend the Mathematics workshops at the circuit offices.

4.1.1.1. Professed beliefs

Teacher's beliefs about the nature of Mathematics

Thabiso viewed Mathematics as a school subject. This view was portrayed during the interviews. The way he presented his responses indicated this view. He viewed Mathematics in terms of the subject he is teaching and how he is teaching it. From his responses it could be inferred that he views Mathematics as numbers, as a procedural subject and also as an integrated subject.

Mathematics as numbers

Thabiso believes Mathematics to be numbers and application of numbers. When asked to define Mathematics, he responded by saying that: "Mathematics is about dealing with numbers and it is about the application of information in terms of numbers". He stressed this issue by saying that: "Mathematics is formed by the arithmetic analysis of numbers. In other words, you will be actually dealing with numbers, addition, subtraction, multiplication and division, etc".

The teacher obviously believes that Mathematics is all about numbers and hence when doing Mathematics you will be dealing with numbers.

Mathematics as a procedural subject

Thabiso indicated that Mathematics is all about procedures. He stressed that in Mathematics steps are very important. This was indicated when saying that: “You have to follow all steps which are necessary to get to solve the problem”. He also mentioned that:

Mathematics uses some procedures. Yes there are many ways to kill a cat, there are many ways of doing a problem, but it must be procedural, it must be coherent, it must be so chronological. You cannot just kill a cow by starting to stab it in the stomach. You just have to follow the procedures.

This also appeared when showing Mathematics as a clear and coherent subject which need step by step when working out some problems. He openly stated that: “Mathematics is very clear, you can actually show a chronological way of presentation, very coherent; right from step number one up to the last stage, especially when you are carrying out arithmetic processes”.

The responses clearly indicated the transmission view or rather the absolutist view of Mathematics where procedure is more stressed and presentation is clear and coherent.

Mathematics as an integrated subject

Thabiso showed another view of Mathematics. He indicated that Mathematics is found in all other learning areas. He showed that Mathematics is everywhere and it can be used in all spheres of life. He is of the idea that without Mathematics other areas will be difficult. He stated that:

Mathematics is a philosophy which permeates at through all other learning areas. Without Mathematics the other areas becomes very difficult like Physical Sciences, like Geography where we are doing map reading, map interpretation, calculation of distances, so that is a philosophy which permeates through all other areas.

When asked about where mathematical knowledge comes from he repeatedly stated that: "Mathematical knowledge comes from as I told you that is a philosophy that permeates through other learning areas, it means every sphere, everywhere, there is Mathematics, finance, geography, whatever just mention a few".

This view also appeared again when asked to say anything about his Mathematics teaching. He responded by saying that:

There is a need for us as actually teachers to encourage these learners actually opt to do Maths [Mathematics] as their route through the FET until tertiary level. Is a philosophy which I mentioned earlier which will help our country at large, because most governments they rely on Maths [Mathematics] and Science, because Mathematics and Science those are very quite specific areas which can help us to manoeuvre all through these new technologies, now we have new technologies in the communication industries like cellular phones. All this need Maths [Mathematics]. Even accountants, we cannot have accountants who do not know maths. So I encourage Maths [Mathematics] to be done at lower level and more bit by bit.

These utterances reveal that Thabiso clearly acknowledges that Mathematics is an integrated subject which is needed and could be applied everywhere and in every sphere of life. He recognizes Mathematics to be applicable and important in our real life citing examples of communication, industries, government and in accounting.

Teacher's beliefs about the teaching of Mathematics

The teacher is holding the belief that he is an imparter of knowledge through explaining concepts. He proudly stated that when he was interviewed. Although this seemed to give an impression that Thabiso dominates his lessons he further indicated that he is accommodative and provides remedial work. This teacher also highlighted that he uses technology during his teaching of Mathematics and he strongly believes in summative assessment, where learners will be assessed at the end of the topic.

The teacher should be an explainer and illustrator

Explanations, illustrations or demonstrations seemed to be the core of teaching Mathematics for Thabiso. The teacher indicated that his role is that of the explainer during the Mathematics lesson. Thabiso thought that for a Mathematics teacher to be successful in the teaching of Mathematics s/he should be able to explain clearly. According to Thabiso, explanations help the learners to understand the concepts well. Again, the teacher indicated clearly that illustrations or demonstrations help learners to follow what the teacher had done during the lesson. He said:

When you are introducing a new concept, you have to demonstrate the concept clearly and give examples, and then now you administer the questions starting from simple ones to more complex, and learners should be following exactly what you have actually done during the lesson.

He reiterated that:

When I introduce a topic, I will explain why the topic is very important and give practical or environmental application of the particular topic. Then I will start to illustrate or demonstrate the concept in detail, then I expect my learners to follow suit.

When asked about his strengths and how he is using them in his classroom, he showed that: “My strength is on the aspect of explaining. I am very proud that I am very audible, and I normally explain quite clearly”.

The utterances above give an idea that Thabiso is proud about his ability to transmit knowledge to the learners and the learners following exactly what he had done.

To be accommodative and provide remedial

The teacher also indicated that he is accommodative. The teacher indicated that, as a Mathematics teacher, he is able to accommodate different learning styles and provides remedial to those who need it in his classroom. He indicated that this does not enable slow learners to catch up with what is being taught only, but also to catch up or even overtake the so called fast learners. He showed that:

We meet different learners in the class. Some are not very capable-they are slow learners. Some are very capable, some are even worse than those two categories. Remedial is very important aspect, remedial where you actually take into cognisance of those slow learners because there are some who are just naturally slow in terms of learning. Once you actually revisit their problem, and try to be slow enough, to keep their pace, they can challenge those ones whom we think they are very fast learners. So you have to be very accommodative. You have to actually accommodate every learner in terms of learning or IQ-intelligence quotient ability.

When asked about his weaknesses, he indicated that: "One of my weaknesses is that since I am a very fanatic of Mathematics, I somehow carried away because I tend to be fast somehow but, however, I am very accommodative".

Technology should be used during the teaching of Mathematics

The teacher indicated that there is a need for technology use during Mathematics lessons.

Yes, there is a best way to teach Mathematics, especially when you are actually bought with the new technology. Maths [Mathematics] should be taught using Computers, and also Mathematics can be practiced that is using some softwares. There are some softwares like Mathematica, Matlab, just mentioning a few, but we do not use them as our schools are not having them.

However, he also showed that the technology they are using in his class is the calculator and the Internet. That is why he mentioned that: "During the lesson, you must actually be able to operate your mini computers- that are your calculators and I encourage my learners to research using internet".

Assessing the learning process

Thabiso believes that assessment should be incorporated in the teaching and learning situation. However, the teacher also believes on summative assessment where assessment takes place in the form of tests and assignments. Thabiso showed that he likes tests than assignments. During the interview, he responded by saying:

'Ya', there is a certain % for example 30% or 40%. I can analyse from how they will be doing their Maths [Mathematics] in class, but I cannot exhaustly say they are

doing well during that particular process until I administer maybe a test or an assessment where I can actually maybe an assessment in terms of a test, especially don't believe in assignment because in an assignment they actually copy each other but a test where one man can stand on its own. 'Akere' we say "indoda ya sibonela", meaning one man for himself. Then I can measure each performance of each individual, and then I can record and actually start to help the individual.

Teacher's beliefs about the learning of Mathematics

Thabiso believed that learners should be fed with all the information that they could be able to reproduce when needed. After all the explanations, illustrations, and demonstrations by their teacher, they have to be in a position to can do what their teacher had done. Consequently, learners are anticipated to be receivers of knowledge passed on from their teacher and act according to it. The teacher expect that learners should listen and give correct answers; learn by following exactly what the teacher had taught and master concepts through doing.

Learners' role is to listen during Mathematics lessons

The teacher highlighted that, during the lesson, learners should listen to the teacher talk and then apply what he had taught. He stated that:

During Mathematics lessons, everyone must have the right asset (stationery) required. Learners should have pencils, mathematical sets, calculator, etc. Listening is a skill in Mathematics, so that you can justify each particular stage and you can follow all steps which are necessary to get to solve your problem. I usually emphasise that you must listen, show all coherence and justify every step.

Thabiso's assertion indicates that learning takes place effectively when learners are listening to what the teacher is saying. Therefore, this indicates that learning depends solely on the teacher and the learners' role is to justify the steps.

Learners should learn by following exactly what the teacher had taught

Thabiso holds certain beliefs about the learners' role during Mathematics lessons. He is of the view that, when he is busy explaining, the learners should take a back seat and listen to what he is saying.

He believes that the learners' role during Mathematics lessons is to be attentive, grasp and apply the subject matter exactly as they are taught. This was reiterated several times during the interview. The statements like "after illustration or demonstrating the concept in detail, and then I expect my learners to follow suit" were uttered several times during the interview. He also indicated that, after following what the teacher had said, they are also expected to give correct answers when asked questions. He said:

A good Mathematics lesson is where there is a very strong interaction between the teacher and the learner, especially when the concept is imparted and learners are following exactly what you [the teacher] have already emphasised and giving the correct answers.

The teacher acknowledges that there should be an interaction between the teacher and the learner. However, the interaction according to him is that the teacher should be the imparter of the information and the learner be the receiver of the information. These learners should also follow exactly what the teacher has said and give correct answers.

Learners should master concepts through practice during Mathematics lessons

Mastering of concepts by learners seemed to be what brings satisfaction to the teacher. The teacher believes that, after he had explained the concepts, learners will master concepts. He clarified that after explaining,

I will now put the problems which are exactly related to the some of the recall back and then moderately...I mean statements will be actually said, until you develop bit by bit until you actually reach a more complex or complicated problem testing the ability to master the concept through doing or through calculating.

In addition, learners should practice regularly in order to master the concepts. He added:

In fact, I believe Mathematics must be done not read. We do not read Mathematics but we do Maths [Mathematics]. So, practice makes perfect. You actually have to practice, once you actually do it practically, you will master a concept, and you can apply this concept across all other problems. So, it is not a theoretical subject, it is a practical subject where you really do hands on.

Thabiso's statements indicated that he recognizes Mathematics to be a practical subject. However, the practicality of the subject is associated with the mastering of concept through practising it.

Summary

Thabiso seemed to be aware that Mathematics is practical and applicable in our real and daily lives; however, his professed beliefs about the nature of Mathematics were consistent with the absolutism. In the teaching of Mathematics, he portrayed the traditional way of teaching that is characterised by stressing procedures, the teacher being the source of information, learners practicing and listening to the teacher hence, discouragement of discussion. This indicates that according to him the process of teaching is not learner-centred. In the learning of Mathematics, Thabiso also indicated the traditional way of learning Mathematics. His belief in the learning of Mathematics indicated that learners' role is to listen to the teacher, master concepts, following exactly what the teacher has said and give correct answers. Therefore, his professed beliefs about the learning of Mathematics and about the teaching of Mathematics were also purely skewed to absolutism.

4.1.1.2. Classroom practices

Thabiso' practices from classroom observations were analysed. These observations were all video recorded. The results were compared so as to see whether or not his professed beliefs have impact on his classroom practices. These are summarised in Table 3 on page 54.

Thabiso is teaching Mathematics in Grade 12. The school has six Grade 12 classes ranging from Grade 12A up to Grade 12 F. The class in which the observations took place was that of Grade 12A. This class was selected by the participant. It was the third period of the day in the first visit, which starts at 09H30 and ends up at 10H30.

In the second visit, the class was observed in the second period that started at 08H30 and ended up at 09H30. The class consisted of about 45 learners. All these learners were seated in their desks facing the chalkboard. Immediately after the teacher greeted them, they started to focus on the teacher in all lessons that were observed. The teacher introduced the topic of the day and some learners took out their books, whereas some just paid attention to the teacher. The teacher started to explain what the lesson is all about.

Teacher's beliefs about the nature of Mathematics

As a procedural subject

Throughout the lesson, procedure was stressed. The belief that Mathematics is a procedural subject influenced the lesson. Learners were often told how to do Mathematics. Here is an example:

Excerpt 1: Extracted from the first lesson

(The lesson was about the revision of application of trigonometry)

Teacher: Which side do you want to calculate first Prince?
Prince: I think it (he stood up going to the chalkboard pointing at side z and ultimately changing his mind pointed at angle X) will be angle X, using angle Y and side y.
Teacher: x and y is it?
Learners: Yes
Teacher: In other words we are calculating this unknown (pointing at angle X) first. You cannot calculate this one (pointing at side z) first. Why is it impossible to calculate this one?
Learners: Two unknowns
Teacher: Two unknowns 'akere'?
Learners: Yes
Teacher: So we have to calculate angle X first. If we are to calculate angle X first, it means why is it that we ought we opted to choose angle X as our first calculation? Because this side (pointing at side x) is given, the opposite to angle x and we relate it to y because Y angle and y side are given, clear?
Learners: Clear

Teacher's beliefs about the teaching of Mathematics

The teacher holds certain beliefs about the teaching of Mathematics and these beliefs manifested in his teaching. Firstly, the teacher believes that during the lesson he should take the part of being an explainer or an illustrator of what to be learnt. This belief had manifested during the teaching as outlined below.

Thabiso's teaching was dominated by explanations. These explanations were done concurrently by talking and writing on the chalkboard. He used the chalkboard mainly to write notes, explain ideas and to write class activities and writing corrections mainly by the teacher and rarely by learners. The teacher spends more time explaining whatever he deems necessary and mainly asked questions that led the learners to respond in a chorus. This was manifested when the learners were responding by saying "hmm" or "yes" many times in the lesson. For example:

Excerpt 2: Extracted from the first lesson

(The lesson was about the revision of application of trigonometry)

Teacher: Let us consider a triangle, $\triangle ABC$, the side opposite to angle A is side a- smaller letter a. The side opposite to B is side b and the side opposite to C is c. We use capital letter for angle and small letter for what?

Learners: *(in a chorus)* Angle.

Learners were given a class activity during the lessons. They were also given an opportunity to write answers on the chalkboard. However, they were usually interrupted by the teacher when they were writing incorrect answers or using the method which the teacher felt it is the long one or which they will not arrive at the correct answer.

Orienting learners about an examination

It was inferred that another teacher's belief about the teaching of Mathematics is to give advice and orient learners about examination. Learners were often given an advice by the teacher where the teacher felt necessary to. Most of the advices were given when the teacher thought that learners are making a mistake or when the teacher stressed something concerning the examination. This appeared many times during the lessons. For example:

Excerpt 3: Extracted from the first lesson

(The lesson was about the revision of application of trigonometry)

Teacher: But in Mathematics, formulas are normally given on the data sheet being twisted 'akere'?

Learners: 'ee'

Teacher: So is the same or you can put it across as (writing the formula on the board $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$). So the two formulas can be given either in this format or in that particular format (pointing the formulas on the board). What is important is for you to identify the right sides, the angles and manipulate their unknown values. You find that there is terminology for example, please take note that when they say solve the triangle, solve the what?

Learners: The triangle

Teacher: What are they saying? They are simply saying find the unknown sides and unknown angles. This means find all unknown sides and unknown angles (writing the statement on the board. So you must be able to apply this particular sine rule. Is there any question up to so far. Any question from the floor? (Learners did not respond). No question means satisfaction.

Teacher: So, these three equations are very important but are we given all this three equations in an examination?

Here some learners said "yes" whereas some said "no".

Teacher: Every year they change, maybe (pointed at number one) this one 2009 and this one (pointing at formula number two) in 2010 and this one (referring to the third one) 2011. So, check as to whether the March one was having which one and the November was having which one. They normally change all of this. They may bring one and most of which is not related to the question which is under consideration. So, be very careful and be able to identify the right formula for the right job.

This belief seemed to have provided the teacher with another role in the teaching of Mathematics because, in almost all the observed lessons, the teacher stressed the need to take care of some basics needed in the examination. During the second lesson, statements like the one which follow were uttered:

Do you understand what I am saying? In an examination do not do this, otherwise you will be penalised. You must show all your calculations thus the examiner is interested in you showing coherence and let us look at the variance. I do not want to calculate it using the formula. I will calculate using a formula but for now I want a table because this table has many marks.

This type of statement shows that learners are constantly reminded about examinations. Another example:

Excerpt 4: Extracted from the second lesson

(The lesson was about the data handling calculation of the mean, standard deviation and variance)

Teacher: We are calculating the variance. The variance = the summation of all the deviations squared divided by n (he wrote the formula on the board). The numerator is this number (pointing at 23598, 92) which we have already calculated. So the **examiner** will come and mark every value. If it has got more marks it means using a table. (he substituted the values on the formula) what is the value now.

Learners: 1966,566667

From this extract, the teacher clearly indicates that he believes that learners should be taught how to answer the questions in an examination. So, they were constantly reminded about the examination.

Teacher's beliefs about the learning of Mathematics

Listening and giving correct answers

According to Thabiso's professed beliefs, the learners' role is to listen and give correct answers. This belief was manifested during the lessons as the teacher was constantly calling attention of the learners by posing questions that led learners to answer as a group hence they were answering as such. The teacher seemed to be satisfied with that type of learning process. Hence the lessons were dominated by explanations and answering of questions in a chorus form.

Following exactly what the teacher has taught

This belief influenced the teacher's classroom practices in that, during the lesson at the point where learners were given an opportunity to solve for the problem on the board, they were usually interrupted by the teacher when doing their work on the board. Here is an example:

Excerpt 5: Extracted from the first lesson

(The lesson was about the revision of application of trigonometry)

Teacher: But angle X we have already calculated. Let us take it as $27^{\circ} + 33^{\circ} + Z = 180^{\circ}$ (writing on the board). And we want angle Z, so angle Z becomes $180^{\circ} - 60^{\circ}$ which is $27^{\circ} + 33^{\circ}$ which is 60° . $180^{\circ} - 60^{\circ}$ which is equal to 120° . Of course that become obtuse which is not normally the case but because maybe is quite obtuse, above 90° . What is left is what?

Learners: The side

Teacher: And the side is small z, "akere"? How do we calculate the small z

Learners: By applying the sine rule

Teacher: Who can apply the sine rule? (he moved around and ultimately gave one learner 3 a chalk to apply the sine rule) let us all observe quick quick

Learner 3: Started to write on the board, $\frac{z}{\sin Z} =$ (interrupted by the teacher)

Teacher: "Hee", a moment, which side is the most applicable which will be the most appropriate? Will you choose the one related to x or the one related to y?

Learners: y

Teacher: y because is the original. I am not discouraging you to use the x one, but I am simply saying if you want to be very accurate. You must use the side which is the original one.

This appeared again in the observed lesson 2. Here, the teacher was indicating clearly what the learner should do when solving mathematical problems. Learners were given an activity whereby they were to calculate the mean, the standard deviation and the variance. They were continually reminded of following exactly what the teacher had done in a given example. Let us look at the interaction below:

Excerpt 6: Extracted from the second lesson

(The lesson was about calculation of the mean, standard deviation and variance in data handling)

Teacher: who can come and calculate the mean, any volunteer? Yes (pointing at one learner). I will give you a chance (telling another learner). This is our number 1 (he wrote mean =... and gave a learner a chalk to finish). The learner wrote $=120+80+$ (interrupted by the teacher). When I said many observations, don't write them all. Just write $120+80+60+$ blah blah blah+ $110+115$. Blah blah blah represent all those calculations. Your calculator will do it. The learner proceeded with writing. The teacher got to the back of the class and started to talk. Do you understand what I am saying, in an examination don't do this, you will be penalised. (The teacher came from the back of the classroom, rubbed off what the learner was doing and started to show them how to do it. He wrote mean = $120+80+60+\dots+110+115/12$, then gave the learner a chalk to proceed and said to the learner "you may go on". The learner proceeded by writing $629/12=52, 4$

From this interaction one can be aware that the teacher's methods were encouraged. Whenever the learners were solving the problem in their own way, they were often interrupted by the teacher if they were not following exactly what the teacher had done.

Assessing the learning process

The teacher believed in assessment at the end of the lesson that is formative assessment. During the lesson, it will be the teacher who is explaining and illustrating some ideas. The learners were passive recipients in that they only participated by punching their calculators or responding to the questions in a chorus. It was only after the explanations where they were given an opportunity to write their responses in their books or one of them on the chalkboard. Here is an example:

Excerpt 7: Extracted from the second lesson

(The lesson was about the data handling calculation of the mean, standard deviation and variance)

Teacher: Example 2. I am going to change the values. Now I am no longer interested in the rainfall pattern but the funeral contribution of the money for the year 2011, for Grade 12A learners (the teacher wrote the values on the board). Calculate:

- (i) The mean
- (ii) The variance
- (iii) The standard deviation

The teacher moved to the back of the class.

Teacher: Just follow exactly what we did in example 1.

(Learners kept on writing in their books and the bell rang, indicating the end of the lesson)

Teacher: Take the activity as homework

Technology use during Mathematics lessons

A Scientific Calculator was the only technology used during the lesson. Learners were often encouraged to use the calculator during the lessons. Here is an example:

Excerpt 8: Extracted from the first lesson

(The lesson was about the revision of application of trigonometry)

Teacher: Now we are simply saying we want... 'hee' I know some learners who are so interested in quickly calculating. The degree of accuracy of your answer is going to be compromised. Your calculator is a minicomputer. It can do everything at one call at the last concept, but I am not encouraging you to do my own way as long as you are able to manipulate. One man's meat is another one's poison. As long as you are able to evaluate correctly, am I clear? So, how I am going to find this one (meaning $5\sin 33^\circ/6$)? If I find $\sin 33^\circ$ I am going to get lot of decimals which are even not easy to handle. So I simply want what? Angle X not $\sin X$. So angle X becomes \arcsin for everything found on the right hand side (meaning $5\sin 33^\circ/6$) which we can find straight away. What do you get?

And urging learners to help the other learner by using a calculator, the teacher stated that:

Teacher: Let us help him. Punch (your calculators) and verify the answer. You have already calculated 9, 54 what?

Learners: centimetres

This also became evident in the second lesson. Learners were encouraged to use their calculators and come up with correct answers. The following excerpt is an example:

Excerpt 9: Extracted from the second lesson

(The lesson was about the data handling calculation of the mean, standard deviation and variance)

Teacher: The columns of the table depend on the quantities that are portrayed in the formula.

Remember our formula has got n . Our formula has got also $x - \bar{x}$. So we tend to say that our data is such a way that our rainfalls which is our x values and went actually to say our mean you know it. What else do we want? We want $x - \bar{x}$ column and we sum it (he drew the table on the board and inserted the values of the rainfall on the columns. All these are our x values and our \bar{x} doesn't change. Let us fill in the column.

(Learners started to call out values and the teacher inserted the values on the table)

Teacher: What do we want? We want to sum up the deviations squared. Then here we must get the total of all this columns (pointing at the deviations column).

(Learners added up the deviations column) one learner (Bristo) called out the answer as 23598, 32

Teacher: Are you all getting this?

4.1.1.3. Relationship between beliefs and practice

In all Thabiso's lessons, there showed consistency between his professed beliefs and his attributed beliefs. His beliefs seemed to be the ones that controlled all his teaching practice. About the nature of Mathematics, his belief was that Mathematics is a procedural subject, hence this was revealed when he was stressing procedure during all the lessons observed. The belief that Mathematics is numbers also manifest during the lessons as the teacher had never gave learners an opportunity to explain their answers. All what they were doing was only to find answers in the form of numbers and whether the answers were correct or not correct, learners were not asked why. About the teaching of Mathematics, Thabiso indicated that he believes in explaining everything to the learners and indeed his lessons were dominated by explanations. He started all the lessons by explaining what the concept to be taught is all about, and followed that by explaining all the procedures needed to solve the

problems within that concept and ultimately explaining what to do when answering questions based on that taught concept in an examination.

His expectations with regard to how learners should learn Mathematics were also shaped up by his beliefs. He believes that learners should follow exactly what he had taught them. Really, during the lessons when learners were given an opportunity to solve the problem on the board, they would be stopped if they were not doing according to how the teacher had showed them. In relation to assessment, his belief that learners should be assessed summatively as individuals was clearly visible upon entering the classrooms as learners were seated individually facing the chalkboard, with no possibility for peer assessment. No walking around and checking of learners' progress was made by the teacher, which indicated his belief of summative assessment.

4.1.1.4. Concluding remarks

These interactions demonstrate that Thabiso is an absolutist teacher who believes Mathematics to be numbers and a procedural subject, although also believing that Mathematics can be found in other subjects. He believes in step by step procedure in solving mathematical problems. He also believed that he should explain, illustrate or demonstrate concepts during the teaching of Mathematics and that he should also orient learners about examination. These are the teaching styles of an absolutist teacher who believes in the transmission of knowledge from the teacher to the learners, and learners mastering concepts by following exactly what the teacher had said. These beliefs were reflected in his teaching practice. The teacher occupied all lessons with explanations in which he needed learners to listen and grasp all the important knowledge imparted. Therefore, the classroom tends to be teacher-centred. The table below summarises his beliefs.

Table 3: Summary of Thabiso's Beliefs

<i>Teacher's belief about</i>	<i>Professed beliefs</i>	<i>Beliefs that manifested during observations</i>	<i>How beliefs impacted on classroom practices</i>
<i>The nature of Mathematics</i>	Mathematics as numbers	Learners were not given an opportunity to explain their answers.	As long as the answers were correct the lesson proceeded.
	Set of procedures	Set of procedures	Procedure was more stressed and hence learners were advised to follow a step-by-step procedures in solving problems
	Philosophy which permeates through all learning areas		
<i>The teaching of Mathematics</i>	Accommodative and provide remedial		
	Explainer and illustrator	Explained and illustrated everything verbally and on the chalkboard	In most cases, learners listened and waited for the slot to answer in a chorus
<i>The learning of Mathematics</i>	Listen and give correct answers		
	Following exactly what the teacher had taught	Learners were often interrupted if they were not using procedure that the teacher has not taught them	Learners were not given a chance to express themselves
	Mastering concepts through practice	Master concepts by following the teacher's methods	Learners were not given a chance to come up with their own ideas

4.1.2. Case 2: Lerato

Lerato is a female teacher who is at her early fifty's. She has completed her Primary Teachers Diploma in Mathematics and Science Education and an Advanced Certificate in Education majoring in FET Mathematics. For

the past twenty-six years, she has been teaching Geography, Sepedi and Mathematics and she is now a master teacher. At the time of this study, she was teaching Mathematics in Grade 11. At the school where she is teaching, there were six Grade 11 classes and she is teaching only two classes: 11A and 11C. This teacher attended the in-service training at MASTEC for a period of six months in 2008.

4.1.2.1. Professed beliefs

Teacher's beliefs about the nature of Mathematics

This teacher viewed Mathematics as science. In the interview she commented that: "Mathematics is a science which sharpens problem solving and logical thinking". She also indicated that: "understanding Mathematics implies doing Mathematics, being initiative and being original using one's own methods to solve mathematical problems which are of a non-routine nature".

This simply implies that the teacher believes that Mathematics enables a person to think logically and be a problem solver. The statement also implies that mathematical problems can be solved in different ways.

Teacher's beliefs about the teaching of Mathematics

Mathematics should be taught using a problem solving approach

Lerato seemed to be aware of the importance of problem solving method in the teaching of Mathematics. During the pre-observation interviews, when asked about the best way to teach Mathematics, she responded by saying:

The best way to teach Mathematics is through problem solving approach. To embrace this best practice is just to learn to move from the traditional way of teaching Mathematics wherein algorithms were taught first and the problem solving skills of the child were overlooked.

When asked to describe her teaching style, she stressed that learners should be taught through problem solving and also emphasized the importance of learner-centred lessons and she responded in this way:

My teaching style, first gives learners a chance to tackle a problem i.e. real life problem in their groups, then allow them time to share the methods they use to

reach to the answer. By so doing, I cater for learners styles. I only provide guidance where necessary but encourage learners to use their own methods.

She further stated that:

A successful Mathematics class is a class in which learners do problems in their groups, share methods of solving problems and learners not sticking to the teachers methods of solving problems she further indicated that it is a class in which all stereotypes and negative attitudes are cleared.

The iterations above, clearly indicates that the teacher is knowledgeable about problem solving strategy and its application in the classroom. These also indicate that Lerato's lessons are not teacher-centred as she encourages learners to use their own methods and discourages the traditional way of teaching.

In the teaching of Mathematics, algorithms should be taught first before an exercise is given

The teacher also showed that she is holding some conflicting beliefs to the ones mentioned above. Although she indicated that she is knowledgeable about problem solving and learner-centred approach, she also indicated that she is holding a traditional method of teaching Mathematics. This belief appeared when asked to describe her past experiences about the teaching of Mathematics and how they may influence how she is teaching now, she responded by saying:

My past experiences in the teaching of Mathematics made me think that Mathematics teaching involved first teaching of algorithms then gave learners an exercise. This kind of habit makes it difficult for me to use problem solving approach as it is difficult to change the status quo.

The above view appeared again, however this time she indicated a conflicting view. When asked how the recent reform in Mathematics has affected her teaching, she responded by saying: "The recent reforms in Mathematics affected my teaching a lot as I find myself in a continuous struggle to move from my comfort zone of traditional way of teaching to a more recent problem solving or problem centred approach".

Lerato indicated that her primary role during Mathematics lessons is to give enough scaffolding which she refers to as guidance. She indicated that:

In introducing a new topic, I give learners just enough scaffolding to equip learners with necessary skills, and then present a real life problem in which learners would apply their skills in logical thinking to solve the problem.

This was reiterated when asked to describe a good Mathematics lesson. She said that:

A good Mathematics lesson is the one where learners are given enough scaffolding (guidance) to enhance their problem solving skills, then a real life problem presented to learners and groups be given a chance to discuss possible ways of solving the problem.

Lerato provided the contradictory statements when interviewed. From the conversations above, it could be inferred that although this teacher is aware that the learners should be active participants in the learning process, she at times tends to use some traditional way of understanding and teaching Mathematics. She stated clearly that her past experiences in the teaching of Mathematics made her think that Mathematics teaching involved first teaching of algorithms then give learners an exercise. She further indicated that this kind of habit makes it difficult for her to use problem-solving approach as it is difficult to change the status quo. However she also indicated that she is struggling to change from traditional way to the recent problem solving meaning she is trying very hard to move towards the fallibilist way of teaching

Teacher's beliefs about the learning of Mathematics

Learners learn best in small groups

The teacher has the belief that a successful Mathematics classroom is a class in which learners do problems in their groups. This was stressed when asked to describe a good lesson in Mathematics. She stated that

A good lesson in Mathematics should allow time for learners to work in their small groups; for learners to share solutions and methods; for individuals to work on their own; and for the teacher to provide just enough guidance where needed.

Learners should listen, be attentive and be focused during Mathematics lessons

The teacher believes that during Mathematics lessons learners should always listen to what the teacher is saying, be attentive and focused during the Mathematics lessons. She stated clearly that:

Discipline is the most important value, so learners must be attentive and focused during class so as not to miss out in understanding basic concepts and algorithms in the teaching of Mathematics.

This view also provided another perspective that Lerato believes that she should teach concepts and algorithms and learners should listen to what she will be saying so that they will not miss out. This indicates that she is the source of information in her class and learning is teacher-centred.

Learners should learn by doing and apply prior knowledge

Lerato viewed learning Mathematics as a process of self discovery that could be unfolded through doing. She explained that:

I encourage my learners to learn Mathematics by doing since experience has taught me that knowledge acquired through doing-i.e., self discovery, has relatively a lasting impact on the mind of the learner. To this effect, practical investigation forms the most essential part of Mathematics activities even now.

She further explained that continuity and the use of previous knowledge in building the new knowledge is vital in the learning of Mathematics. She pointed out that: "I always encourage continuity in learning as experience has taught me that in more advanced grades one has to apply prior knowledge".

The teacher indicates that she encourages logical connections in the application of the previous knowledge in the learners learning.

The statements above indicate that Lerato valued continuity which is brought up by application of prior knowledge. She also valued self-discovery and hence believes that Mathematics is practical and learners could learn it by doing.

Assessing the learning process

Lerato believes in continuous assessment. She believes that learners could assess themselves through peer assessment. She stated that: "After the groups have attempted to resolve the problem, they share their methods with the rest of the class. They also share with other groups the conjectures they have drawn".

Lerato again believes in summative assessment whereby learners will be given tests after being taught. When asked about when generally she knows that her learners are doing well in Mathematics, she stated that: "I generally know that my learners are doing well in Mathematics when more than 50% of the learners pass a standardized test".

Using technology during Mathematics lessons

Lerato believes that Mathematics should be taught using technology. The teacher showed that, during her teaching, she encourages learners to use available technology. She indicated that learners use their calculators as the only available technology during the lessons. She clearly stated that:

At the moment, I have not yet employed relevant PC technologies like Geometer sketchpad or Excels, however, my learners are encouraged to use advanced calculators for compiling tables and graphs while dealing with functions.

Summary

Lerato showed that she is aware of what Mathematics is, and that it is applicable in real life. Her professed beliefs about the nature of Mathematics were consistent with the fallibilism where Mathematics is seen as being related to human situations. In the teaching of Mathematics, she indicated some conflicting views. Although she is aware of problem solving strategy and learner-centred approach she stated clearly that she is still using the traditional way which include learners being attentive and focused during the lesson. In the learning of Mathematics, Lerato also indicated the contradicting views of learning Mathematics. She showed that she is aware of collaborative learning where learners learn in their groups, however, she also

indicated that learners must be attentive and focused during class so as not to miss out in understanding basic concepts and algorithms in the teaching of Mathematics. Although her statements indicated some contradicting views, her professed beliefs about the learning of Mathematics were skewed to learner-centeredness which aligns with fallibilism.

4.1.2.2. Classroom practices

The school had six Grade 11 classes ranging from Grade 11A to Grade 11F. The class observed was that of Grade 11C. This was the class that was taught by the participant. It consisted of 30-35 learners. Upon entering the class, you will find all learners seated in their desks facing the chalkboard. Due to the school time table, the first visit to Grade 11C class was in the second period on a Wednesday, and it was again observed a week after the first observation. In the second lesson, the learners were just seated as they were found during the first lesson.

In all the visits in her classroom, the teacher used whole class teaching. Basically, the teacher will be standing in front, at the chalkboard, telling learners how to solve problems. The learners will be focusing at the teacher explaining concepts and asking questions and, in most cases, leading learners in giving answers where the learners will answer in a chorus.

Teacher's beliefs about the nature of Mathematics

During the lesson observations the teacher stressed the use of procedure explained by her. In the lessons, originality was never encouraged, instead procedure and retrieval of memorised procedure was often stressed. For example,

Excerpt 1: Extracted from the first lesson

(Lesson was about the introduction of Linear Programming)

Lerato: we were saying whenever you are confronted with paragraphs asking you questions in Linear programming, the first thing you should do is to do what?

Before the learners could try to give an answer or state the procedure, she proceeded by saying:

-take the Mathematics out of the paragraph
-identify the variables
-take out your constraints and we said the explicit constraints are....?

Excerpt 2: Extracted from the first lesson

(Lesson was about the introduction of Linear Programming)

Lerato: An objective function is given by only two words, can you name them? Whenever you see cost, profit it is whispering that you are now under the objective function. I even whispered some other day that whenever you see maximum or minimum, but not always but in most cases remember something about objective function. Whenever you are having your objective function, learn to write it in the form $y=mx+c$. Now I want us to represent that data, and I highlighted that I don't want you to forget that I want you to write the objective function in the form $y=mx+c$.

Excerpt 3: Extracted from the second lesson

(The lesson was about reduction of trigonometric identities)

Lerato: Remember I have told you that always strive to use the horizontal reduction so that you will not be confused. Learners, remember that **I have told you** that whenever you are given angles, the first thing to do is to locate the angle, that is, determine in which quadrant does the angle lie. Secondly, check whether the ratio is positive or negative and lastly, whether the ratio is going to change or not.

These extracts indicate that Lerato is holding the belief that Mathematics is a set of procedures that need to be explained to the learners by the teacher. Lerato's explanations of how tasks should be done are all procedural. This was mainly as a result of the preparation of the tests and examinations. She offers explanations of certain procedures without explaining why the procedures are used. From the above

extracts, it appears that Lerato's explanations attempts to provide some important procedures in Linear Programming and in Trigonometry. However, she poses questions that prohibit learners' responses on how to solve problems in their own way instead of following a certain procedure. Her questions are of a procedural nature in that the answers she requires follow a pattern. She strictly states and writes down all the steps to be followed when solving Linear Programming or trigonometric problems.

Lerato also possess another view of Mathematics as a subject that consists of sections which are not connected to each other. This view manifested during lessons observations. The following extract is an example of the situation where the belief manifested itself:

Excerpt 4: Extracted from the second lesson

(The lesson was about reduction of trigonometric identities)

Learner 2: Ma'am, will I be correct if I express $1-\cos^2(240)$ as $\sin^2 240$ and then solve it from that point.

Lerato: You are forward treating identities; here we are still using a reduction formula.

Teacher's beliefs about the teaching of Mathematics

From excerpts 1 and 2, one can assume that, although this teacher knows more about what is expected of her during the lessons; however she is still struggling to adapt to the new teacher roles. This became evident during all lesson observations as the belief that she held before seemed to be the one driving her lessons.

For example, in all the excerpts extracted from her lessons on Linear Programming and Trigonometry, it became evident that the teacher took the role of being an explainer of the idea or procedure. In the first lesson, the teacher was showing the learners how to come up with constraints. Learners were given green and white dice to play.

Excerpt 6: Extracted from the first lesson

(Lesson was about the introduction of Linear Programming)

Lerato: When playing the game, there are rules of the game, not so?

Learners: So

Lerato: We may also call them conditions. These rules limit us, they constrain us. We call them therefore constraints. The rules need to be converted. When looking at the dice we can let the one be x and the other be y . Why do we choose x and y ?

Without even waiting for the learners to respond, she further explained that we would have chosen any other variable, but we choose x and y so that we may be able to represent our information graphically because our graphs consist of x and....?

Learners: y (in a chorus).

From this extract, it is evident that the teacher was explaining what needs to be done when learning Linear Programming. In the very same lesson, Lerato stressed the choosing of variables.

Excerpt 7: Extracted from the first lesson

(Lesson was about the introduction of Linear Programming)

Lerato: When we are given this information in words, we identify the variables as x and y that is the first step okay. When we are looking into that passage full of words remember to look for x and y .

During lesson 1 on Linear Programming, Lerato demonstrated, on the chalkboard, how to graph and shade the feasible region. She again explained the need for procedure to be followed. Excerpt 1 presents what happened in the class during that lesson.

In the very same lesson, procedure was stated several times. For example, in the following extract, procedure was stressed.

Excerpt 8: Extracted from the first lesson

(Lesson was about the introduction of Linear Programming)

Lerato: Class, in every situation, whenever we are confronted with a question in Linear programming, when all has been searched and done, now comes your objective function. Now comes your.....?

Class: (in a chorus) Objective function.

Lerato: An objective function is given by only two words, can you name them? Whenever you see cost, profit it is whispering that you are now under the objective function.

I even whispered some other day that whenever you see maximum or minimum, but not always but in most cases remember something about objective function.

Whenever you are having your objective function learn to write it in the form $y=mx+c$. Now I want us to represent that data, and I highlighted that I don't want you to forget that I want you to write the objective function in the form $y=mx+c$.

The above excerpts are examples of how Lerato carried out her lessons. In these excerpts, it became clear that Lerato's role was that of an explainer. Her explanations dominated the lessons. Learners participated mostly by taking notes and responding to some of her questions in a chorus. The lessons were more dominated by the teacher explaining, without letting the learners have an opportunity to investigate or grapple with the questions. Looking at Lerato's explanations, we see that the most common explanations were that of stressing procedure, telling learners about what to do or what they are expected to do for example, Lerato: "When we are given this information in words, we identify the variables as x and y , that is the first step, okay. When we are looking into that passage full of words remember to look for x and y ". These extracts indicated that Lerato's belief about teaching Mathematics is that of explainer.

Teacher's belief about the learning of Mathematics

Lerato indicated during interview that Mathematics is learned best when learners work in small groups, share methods and solutions and for the teacher to provide just enough guidance where needed. She emphasised a step by step of a procedure to be followed and that learners should always remember the procedure explained by the teacher. In all the visits, learners would be found paying attention

to what the teacher is saying and waiting for the slot to answer questions as group. The teacher always did not tolerate little disorders from those who needed help from other learners. In one of the visits, the teacher portrayed exactly how she wanted her learners to learn Mathematics. For example, in the following extract, the teacher was teaching Trigonometry in Grade 11E. The teacher stressed what she had told the learners to do, thus expecting them to reproduce it in the same way as she had told them.

Excerpt 5: Extracted from the second lesson

(The lesson was about reduction of trigonometric identities)

Lerato: **Remember I have told you** that always strive to use the horizontal reduction so that you will not be confused. Learners, **remember that I have told** you that whenever you are given angles, the first thing to do is to locate the angle. That is, determine in which quadrant does the angle lie. Secondly, check whether the ratio is positive or negative and lastly, whether the ratio is going to change or not.

From there, learners were given the activity to tackle.

The teacher explained that $\sin 150^\circ = \sin (180^\circ - 60^\circ) = \sin 60^\circ$. One learner asked a question and the teacher responded by explaining what is needed.

Learner: Ma'am will I be correct if I use $(90^\circ + 60^\circ)$ for 150° ?

Lerato: **Yes, but I have told** you that always try to use 180° and 360° .

On the very same activity, another learner asked as to whether or not he will be correct if he can express $1 - \cos^2(240^\circ)$ as $\sin^2 240^\circ$ and then solve it from that point.

Lerato: You are forward treating identities, here we are still using a reduction formula.

This indicates that the teacher's methods are always encouraged and the correct procedure is stated by the teacher. Her belief is that learners should always receive knowledge imparted to them by the teacher, and this knowledge should be stored and be retrieved when needed. The teacher seemed to be the authority to decide what is right or what is wrong, or which method to be used during the learning process.

4.1.2.3. Relationship between beliefs and practices

Lerato's professed beliefs were found to be inconsistent with her attributed beliefs. From the way she viewed Mathematics and her views about teaching and learning of Mathematics differed with what happened in the classroom. About the nature of Mathematics, she viewed Mathematics as a problem solving activity of which was not found during her teaching. On the teaching of Mathematics, she indicated that she will group learners and give them an activity to do of which it was never found happening in the classroom. In all her lessons, Lerato portrayed to be an explainer and stressed the procedure to be followed by learners in future when given tasks related to the section she was teaching.

4.1.2.4. Concluding remarks

During the interviews, Lerato portrayed both an absolutist and fallibilist viewpoints at the same time. Lerato's beliefs about the nature of Mathematics and about the learning of Mathematics were that of a fallibilist who views Mathematics as science which is practical and be taught collaboratively. However, for her teaching of Mathematics she indicated an absolutist viewpoint. She stressed the teaching of algorithms first and that learners should be attentive and be focused during Mathematics lessons. However, in practice, Lerato's beliefs could be inferred as that of an absolutist teacher who stresses procedures to solve problems in the learning environment where learners are attentive, focused and following what the teacher had said. As such the class tend to be teacher-centred. Her beliefs are summarised in Table 4 on the next page.

Table 4: Summary of Lerato's Beliefs

<i>Teacher's belief about</i>	<i>Professed beliefs</i>	<i>Beliefs which manifested during observations</i>	<i>How beliefs impacted on classroom practices</i>
<i>The nature of Mathematics</i>	Mathematics as a science that sharpens problem solving and logical thinking.	Procedures were more stressed.	Learners tend to memorise and call out procedures when asked during the lesson.
<i>The teaching of Mathematics</i>	Algorithms should be taught first before an exercise is given.	Telling, explanations and illustrations of how problems should be solved dominated the lessons.	Learners listened to what the teacher was saying and answered mainly by saying yes in a chorus.
<i>The learning of Mathematics</i>	Learners should be taught in small groups.	Whole class teaching was used.	Learning tend to be teacher-centred.
	Learners should be attentive and be focused during Mathematics lessons.	Learners listening and attentively answering questions in a chorus.	Learners listening and attentively answering questions in a chorus.
	Learners should learn by doing.	Following what the teacher had said.	Learners learn by imitating the teacher.

4.2. Cross-case analysis

This study sought out to respond to the following questions: What are Mathematics teachers' beliefs about the nature of Mathematics and the teaching and learning of Mathematics? And what is the relationship between teachers' beliefs and their classroom practices?

I conducted a cross-case analysis across the two cases (Lerato and Thabiso) in order to establish the differences and the similarities of their beliefs as revealed by the interviews. This was again compared to their beliefs as inferred from their classroom practices. The following similarities as discovered from their classroom

practice were recognized. The two cases used whole class teaching in their classroom teaching despite Lerato's indication of using groupwork during the pre-observation interviews. Again, in all their classrooms, procedures were more stressed; explanations and illustrations of how to juggle around mathematical problems played a pivotal role. However, the reasons behind their actions were different. For Lerato, the major reasons behind her actions was that she was introducing new topics and faced with pace setters, which needed her to work under pressure thus causing her to violate some of the principles that she knows are ideal to be applied. For Thabiso, he is not used to using collaborative methods as they are just another form of teaching Mathematics. So he chose the method he found suitable for his learners and, in this case, it was telling method. One other reason was that he was outlining some important basic concepts that needed to be known for an examination.

An analysis across the cases presented the differences between the two cases as established during the interviews. The summary for the differences is presented in Table 5 below. Summaries of the differences according to each theme are also presented in subsequent tables.

Table 5: Teachers' Beliefs about the Nature of Mathematics and the Teaching and Learning of Mathematics

Theme/case	Thabiso	Lerato
Beliefs about the nature of Mathematics	Mathematics is about dealing with numbers, clear and coherent and a procedural subject.	Science which sharpens problem solving skills and a practical subject.
Belief about the teaching of Mathematics	Explainer and illustrator, accommodative and provide remedial.	A facilitator using problem solving skills, teaching algorithms first before giving exercise.
Beliefs about the learning of Mathematics	Listen and give correct answers, following the teacher's methods exactly and mastering concepts through practice.	Learners learn best in small groups, and they should listen, be attentive and be focused during lessons and they should learn through doing.

4.2.1. Beliefs about the nature of Mathematics

Thabiso and Lerato shared different beliefs about the nature of Mathematics. This is manifested during their interviews and how they delivered their lessons. For Thabiso, Mathematics is a clear, coherent, procedural subject. And, above all, it is a philosophy, whereas for Lerato Mathematics is viewed as a practical subject that should be learned by doing. Hence, she viewed it as a science that sharpens problem-solving skills.

The classroom observations shared another perspective of these teachers. Thabiso's practice was consistent with his professed beliefs about the nature of Mathematics. Procedures were more stressed during the lessons and he gave examples from other subjects, such as Geography, whereby data of rainfall pattern of the year were used during one of the observed lessons. In the case of Lerato, her professed beliefs about the nature of Mathematics were found to be inconsistent with her practice. In all her lessons, it was observed that procedure was stressed. There was no evidence of problem solving and Lerato indicated that this was caused by the fact that she was introducing new concepts. The table below summarises the two teachers' beliefs about the nature of Mathematics.

Table 6: Mathematics Teachers' Beliefs about the Nature of Mathematics

Thabiso	Lerato
Mathematics is a philosophy	Mathematics is a science which sharpens problem solving skills and logical thinking
Mathematics is all about procedures	Mathematics is a practical subject
Mathematics is a set of numbers	

4.2.2. Beliefs about the teaching of Mathematics

Table 7 presents the indication of the difference between Thabiso and Lerato with respect to their beliefs about the teaching of Mathematics. Thabiso described his role as that of an explainer and demonstrator. This became evident in the lessons. His teaching was dominated by the teacher's explanations. The teacher explained all concepts and all procedures he felt are needed in the learning of Mathematics. During the post-observation interviews, he gave the reason why explanations played an important role in his teaching. He stated that: "For learners to understand better and clearly or vividly, explain the concept very clear with clear instructions, clear illustrations so that the learner will follow suit".

He also indicated that his role is to be accommodative and provide remedial to the slow learners. However, this role did not clearly become visible during the lesson. What was inferred from his practice was that the teacher was moving with those learners who understand. There was no sign of helping any learner during the lesson. Another belief that was inferred from the practice was that the teacher took another role of giving advice and orienting learners about the examination. The teacher will be telling the learners about how an examiner sets some of the questions and how answers are marked at the Marking Centre. He stated his reason as being that:

They are matriculants, so whenever you are introducing a concept or explaining a concept, or illustrating a concept, it must have something to do with examination because at the end of examination, throughout the entire academic year, you will be emphasising or orienting learners on examination concepts so that these learners would not be surprised when we talk about the real way of answering questions at the end of the year.

Lerato demonstrated a different view from that of Thabiso. During pre-observation interviews, she viewed her role as that of the facilitator who uses a problem-solving approach during Mathematics lessons. During the observations she took the role of being an explainer where procedure was more stressed and retrieval of the memorised procedure was stressed. She clarified the whole process during the post-

observation interview. When asked about why explanations and telling method were the only methods used in her lessons, she responded by saying that:

You know, we are having a challenge, a challenge of time. When we look into our work programmes we are supposed to finish a vast amount of work within a limited time such that this causes us to violate some of the principles which we know are ideal to be applied in Mathematics and therefore for covering a lot of work within a short space of time. One has more often to resort on that kind of method which I do not justify of course. However, after finishing the work then it is then that during the time of revision and reviewing the work that is when I have to go on through the right way.

Lerato further indicated that she used a whole class teaching because she is pressurised by the fact that: “She always works under pressure as she is supposed to finish a stipulated work within a given time”.

Table 7: Mathematics Teachers' Beliefs about the Teaching of Mathematics

<i>Thabiso</i>	<i>Lerato</i>
An explainer or an illustrator	A facilitator using problem solving approach
Be accommodative and provide remedial	Teach algorithms first before giving an exercise

4.2.3. Beliefs about the learning of Mathematics

During the pre-observation interviews, Thabiso regarded the best way to learn Mathematics as to listen, give correct answers, master concepts through doing and follow exactly what the teacher had taught. From what was happening in the classroom, it became evident that the teacher controls the whole learning. However, from the classroom practice, some of these beliefs can be inferred. In the classroom, if a learner could be given a chance to do a problem on the board, s/he would be interrupted by the teacher and asked to follow the example exactly as it was. It became evident that this belief influenced the teaching practice as Thabiso seemed to be satisfied when asking questions and learners responding in a chorus. As long as the answer from the chorus is correct, then the teacher will go further with the problem he was busy with.

Lerato believes that learners learn best in their small groups. This belief was in contrast with her practice. Whole class teaching was used in all her class. When asked why she did not use groupwork, she responded this way:

You happened to come to my lessons, my classes for observations during the time when I was introducing new topics and that was the time when I needed to make enough scaffold so that when they go to their respective groups they will be well equipped with the basics.

This indicates that groupwork is not used when new topics are introduced. This also stresses the idea that the teacher's belief about teaching is to equip learners with the basics that they will need in their groups.

Lerato also believes that the learners' role is to listen, be attentive and be focused during the Mathematics lessons. This belief was also inferred from what happened in the classroom during the observed lessons. The teacher did not tolerate little disorders, even where the learners were asking for assistance from their classmates.

Table 8: Mathematics Teachers' Beliefs about the Learning of Mathematics

<i>Thabiso</i>	<i>Lerato</i>
Listening and giving correct answers	Learners should listen, be attentive and be focused
Following exactly what the teacher had taught	Learn through doing
Master concepts through practice	Learners should learn in small groups

4.2.4. Concluding remarks

Several things can be learned from this cross case analysis. The participants portrayed beliefs about the nature of Mathematics and the teaching and learning of Mathematics. Although analyses of their beliefs were conducted under same themes, these participants portrayed different beliefs especially during interviews as beliefs are held individually. During their lessons that I observed, the two teachers portrayed similar absolutist teaching styles. Although they seemed to be very traditional in their teaching approaches, and taking into consideration that they gave reasons for the methods they used; this was due to the fact that they were introducing new concepts. I cannot assume that all their classes were always like that on daily basis. During some of the lessons, I observed that learners were allowed to solve activities on the board, although it was to replicate the teacher's methods. One can infer that learners were actively involved in the memorisation of procedures provided by their teacher.

CHAPTER 5: CONCLUSION AND INTERPRETATIONS, LIMITATIONS AND RECOMMENDATIONS

5.1. Introduction

The previous chapter presented the results and the interpretations of this study. It started with the provision of descriptions of the participants and a brief description of each case before it could be analysed. This was followed by within-case analysis and, thereafter, a cross-case analysis of the two participants. Lastly, it gave a summary of the findings.

This chapter provides the conclusion, interpretations, limitations of the study and suggestions for the future research.

5.2. Conclusion and interpretation

This study sought to investigate Mathematics teachers' beliefs about the nature of Mathematics and the relationship between their beliefs and the classroom practices. The main research questions that guided this study were:

- What are teachers' beliefs about the nature of Mathematics, and the teaching and learning of Mathematics?
- What is the relationship between the teachers' beliefs and their classroom practices?

The beliefs of teachers under the study were analysed using the following themes:

- Beliefs about the nature of Mathematics;
- Beliefs about the teaching of Mathematics; and
- Beliefs about the learning of Mathematics.

The above themes were used to provide inferences of whether the teacher's professed belief or attributed belief is that of an absolutist or a fallibilist. Relationship between beliefs and practice was used in order to establish consistency or

inconsistency between professed beliefs and classroom practices. This was summarised in concluding remarks.

In order to pursue the above research questions, I chose a case study research design in which I observed two of the participants' lessons in their classrooms and interviewed them on two occasions: pre-observation and post-observation. Theory was used to classify both teachers as either absolutists or fallibilists, and their teaching practice as such. Focusing on teachers who had 15 or more years experience and who had experienced various curriculum reforms, I used purposeful sampling to select participants who matched the above criterion. Collected data from observations and interviews were transcribed and analysed separately for each case in what is referred to as within-case analysis. Cross-case analysis was also done to compare the two cases.

In order to identify teachers' beliefs, I have used Ernest's (1996) viewpoints as outlined in theoretical framework which form part of Chapter 2. Within this theoretical framework, teachers' beliefs about the nature of Mathematics, teaching and learning of Mathematics were classified under absolutism and fallibilism.

Thabiso's case

In pursuing the first research question that was intended to find out what teachers' beliefs about Mathematics are, it became clear that Thabiso holds an absolutist belief. This clearly became evident through the responses of the pre-observation interviews, Thabiso holds a strong belief that Mathematics is procedural, clear and coherent in which you have to follow a chronological order in solving mathematical problems. He strongly believes that learners should always be imitators of the teacher. This was done in order to prepare the learners for examination as they are matriculants.

In pursuing the second question that was intended at finding the relationship between the teachers' beliefs and their classroom practices, it became evident that Thabiso's beliefs impacted on his teaching practice as learners were constantly reminded about how the question might be set in an examination. Learners were also instructed to follow the teacher's methods. It seems that the teacher's actions in

his classroom were consistent with his beliefs. This confirmed the theory that teachers' professed beliefs may be consistent with their attributed beliefs. This emerged during the analysis process of Thabiso's case.

The findings in this study may be the representation of the experienced teacher who applies his own curriculum within his classrooms due to external factors that influence his beliefs. The external factors that influenced Thabiso's beliefs were found to be the grade he was teaching and the examination. Thabiso mentioned that he was orienting learners to examination or it may be that he wanted to finish a certain stipulated section of work in a given time.

There is clear evidence that Thabiso's professed beliefs concurred with his classroom practices. Therefore this suggests that beliefs may be held in such a way that what you believe in will be indicated by your actions, and these will be in such a way that whatever you are doing portray what you think is good to be done.

Lerato's case

The findings in Lerato's case indicated that she is a teacher who believes in problem solving and uses it in her teaching. Maybe she was influenced by her qualification of Advanced Certificate in Education (ACE) in Mathematics. Although Lerato's beliefs were skewed towards a fallibilist view by her understanding of problem solving and collaborative learning where learners should learn in their small groups, she also portrayed a conflicting belief to that of problem solving when stating that in the teaching of Mathematics algorithms should be taught first before an exercise could be given.

In pursuing the second question that was intended at finding the relationship between the teachers' beliefs and their classroom practices, it became evident that the teacher portrayed the traditional way of teaching. For Lerato, teaching Mathematics depends on what part of the content you are teaching. If you are introducing new topics, you have to teach in a traditional way. Her professed beliefs were found to be inconsistent with her teaching practice. She stressed following procedures during all her teaching and seemed to be in control of the whole learning

process. What influenced her teaching was that she was introducing new topics at the time of this study. She also indicated that the work programme as prescribed by Department of Education put her under pressure.

Lerato highlighted that pace setter put more pressure on her and as such influenced her choice of methods to be applied in a classroom. She further indicated that she chose to use whole class teaching as she was introducing new topics during the time of this study. Despite the teacher's preference of learner-centred approach she had found herself using the traditional approach due to external factors such as pace setters.

Comparison between the two cases

The two cases were compared in a cross-case analysis in which the differences and similarities of their beliefs and how they impacted on their classroom practices were revealed. The differences between the two cases were mostly revealed by data from the interviews. Thabiso's professed beliefs and his classroom practices were more of an absolutist's practices. Lerato's professed beliefs were that of a fallibilist whereas her classroom practices were skewed towards absolutist's practices. Similarities emerged from data of their classroom practices. The two teachers used whole class teaching and explanations dominated their lessons. There was also an emphasis on following procedures in both cases.

Factors which caused consistent or inconsistent relationship between professed beliefs and classroom practices found were pace setters, introduction of new topics, Grade level the teacher is teaching and examination. These factors were derived from pre-observation interviews, observations and mainly from post-observation interviews.

The two teachers were not aware of their beliefs until they were interviewed. They were also not aware that their classroom practices were either consistent or not consistent with what they believed in. This study provided them with an opportunity to know their beliefs and reflect on the relationship between those beliefs and their classroom practices.

5.3. Limitations of the study

This study had its limitations. Firstly, it was a coincidence that data were collected when teachers were introducing new concepts. According to the teachers, this is the period when they normally use traditional methods. If prolonged data collection, especially observations, were made, the results of this study could have brought along a further deeper insight into the matter. Also, if intensive post-observation interviews could have been conducted, other factors that hampered the teachers' enactment of the new roles would have been revealed by the results.

Secondly, the sample involved only two participants, which gave this study an in-depth understanding of their beliefs. However, the results may not be generalised as it is the case with case studies. If the study could have involved many participants, this would likely change the results.

Thirdly, the two participants were from one school within which the setting itself might have influenced the results of this study. If the study of this nature could have been conducted in two different schools, or within a circuit or a cluster, maybe this could have produced different results.

5.4. Recommendations

The results presented in this study are of two experienced teachers from one school. The results, as already mentioned, cannot be generalised for all teachers. Future research is suggested among different participant teachers. The suggestion being that this type of research could be conducted on many teachers, new-entrant teachers or full time versus temporary teachers, GET teachers etc. Another research could also be conducted to the issue of whether teacher beliefs and their influence in practice depend on the subject matter which is taught, that is when a teacher is teaching different topics in Mathematics, would the beliefs he/she holds about Mathematics influence his/her teaching in the same or in different ways in that different topics?

Secondly, prolonged data collection could yield different results from the results of this study. Therefore, data collection should not be limited to two observations and two interviews with each teacher.

Thirdly, this type of study could be conducted in two different schools, or a circuit or a circuit cluster. This will maybe produce another dimension of results.

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APPENDICES

APPENDIX A: Interview protocol

1. What were your early experiences like in Mathematics? (Primary school, Secondary school, Tertiary and Family background)
2. What prompted you to become a Mathematics teacher? What were your initial thoughts about becoming a Mathematics teacher?
3. 3 Define Mathematics. How is it formed? Where does Mathematical knowledge come from?
4. How long have you been a teacher?
5. Describe your past experiences with the teaching of Mathematics and how they may influence how you are teaching now?
6. Do you feel there is a best way to teach Mathematics? What is it? How did you come to embrace this as a best practice?
7. Describe what you would consider to be a good Mathematics lesson.
8. Do you think it is necessary for you to encourage or support your learners when they are learning in the class? Why or why not?
9. Do you think it is important to teach Mathematics? Can you briefly describe your general approach of teaching Mathematics? Describe your teaching style and how you accommodate the different learning styles of the learners in your classroom.
10. How do you teach Mathematics? (Please describe more details, i.e. the design, approach or strategy, and the procedure.) What Mathematics teaching method do you prefer? What kind of techniques do you use frequently? Why?
11. What rules do you have for your Mathematics classroom?
12. Do you think rules and procedures should be stressed during Mathematics lessons? Why or why not?
13. What do you consider to be your strengths and weaknesses and how will you use them in your Mathematics classrooms?
14. Which technology do you encourage your learners to use in your classroom? When do they use it?

15. When do you generally know that your learners are doing well in Mathematics (maybe during the lesson)?

16. Do you plan to continue to study Mathematics? Why?

17. Do you have anything else to say about your Mathematics teaching?

Thank you for this conversation

APPENDIX B: Post-observation interviews for Lerato

1. During your initial interviews, group-work was more stressed; however during the lesson observations that did not appeared to be used, why?
2. Telling method seemed to be the most used method in all your lessons, could you please explain why?
3. You have indicated that problem solving strategy is the best strategy you are using during your lessons; however in all lessons observed I found that whole class teaching is used. Could you please elaborate and explain why?

APPENDIX C: Post-observation interviews for Thabiso

1. During your lessons observations, learners were not in groups as you have indicated in the pre-observations interviews. In all the lessons you were using whole class teaching, why?
2. Explanations seemed to play the most important part of your teaching, can you explain why?
3. In all lessons observed, learners were examination oriented; may you please elaborate on this issue?

APPENDIX D: Transcription of Thabiso's lesson no 1

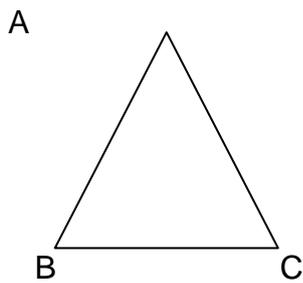
THABISO'S LESSON NO 1

The teacher entered into the class (grade 12A) greeted the learners, and told them why he has been accompanied. He introduced the lesson and it was about the application of trigonometry. He wrote on the board Application of trigonometry, from there he started to teach.

We want to apply trigonometry to a sine rule. I think today we are well acquainted with a sine rule hee?

Learners (in a chorus): yes

Teacher: let us consider a triangle, let us consider triangle ABC. A triangle can actually be drawn (drew a triangle on the board)



It can be any triangle, isosceles triangle, a scalene triangle or an equilateral triangle, with three sides hee?

Learners: yes

Teacher: let us say this is A, this B and this C (naming the apex of the triangle). In other words I am saying let the angle here (pointing at B) be angle B right?

Learners: yes

Teacher: let the angle here (pointing at A) be what?

Learners: A

Teacher: let the angle there (pointing at C) become what?

Learners: C

Teacher: but you should take note that the internal total number of degrees especially for the internal angles of a triangle, they should be what?

Learners: 180°

Teacher: so $A + B + C = 180^\circ$. It means given any two you can actually do your algebra to come up with the third one isn't it?

Learners: ee

Teacher: now the side which is opposite to angle A is what we call side *a* (writing on the board). In other words we are simply saying (illustrating on the board) the length here is *a* units, is that right?

Learners: ee

Teacher: that one is angle B (pointing at B), and the side that is opposite to angle B is what we call what?

Learners: *b*

Teacher: *b* units, like what you see that side which is opposite to angle C is side *c*. Now what are we saying, capital letters for what?

Learners: for angles

Teacher: and small letters for what?

Learners: sides

Teacher: for sides because small letters are representing distances, or the lengths of those sides. Am I clear?

Learners: yes

Teacher: so the sine rule states that, let's state the sine rule (writing on the board) $a/\sin A = b/\sin B = c/\sin C$. This formula is what we call the

Learners: sine rule

Teacher: but in Mathematics formulas are normally given on the data sheet being twisted akere?

Learners: ee.

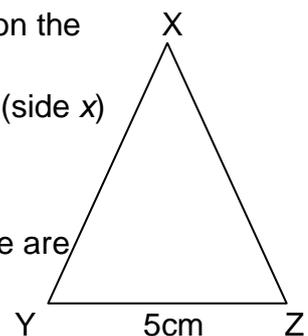
Teacher: so is the same or you can put it across as (writing the formula on the board) $\sin A/a = \sin B/b = \sin C/c$. So the two formulas can be given either in this format or in that particular format (pointing the formulas on the board). What is important is for you to identify the right sides, the angles and manipulate their unknown values. You find that there is terminology for example, please take note that when they say solve the triangle, solve the what?

Learners: the triangle

Teacher: what are they saying? They are simply saying find the unknown sides and unknown angles. This means find all unknown sides and unknown angles (writing the statement on the board. So you must be able to apply this particular sine rule. Is there any question up to so far. Any question from the floor? (Learners did not respond). No question means satisfaction. Let's look at the practical example first. Given a triangle ABC, ah we are not always using ABC we can actually say triangle XYZ so that I can tell whether you understood the concept XYZ, and this triangle XYZ (drawing the triangle on the board) and

suppose this angle (pointing at Y) is 33° (inserting it in angle Y on the board, and this side (pointing side y) is 6cm and that other side (side x) 6cm

then the question say solve the triangle. What are we saying, we are simply saying by solving the triangle, I think I have already



illustrated that this means find all unknown sides and unknown angles. This particular angle (pointing at angle Z, putting a question mark on it) is unknown and this angle (pointing at angle X, also putting a question mark on it) is also unknown. Going to the sides, this side (pointing at side z, also putting a question mark on it) is also unknown. So we have to calculate all those three question marks applying the sine rule. But now how do we go about this sine rule, any idea? Prince?

Prince: I think we are going to use the cosine rule to get the unknown side.

Teacher: which side do you want to calculate first Prince?

Prince: I think it (he stood up going to the chalkboard pointing at side z and ultimately changing his mind pointed at angle X) will be angle X, using angle Y and side y.

Teacher: x and y is it?

Learners: yes

Teacher: in other words we are calculating this unknown (pointing at angle X) first. You cannot calculate this one (pointing at side z) first. Why is it impossible to calculate this one?

Learners: two unknowns

Teacher: two unknowns akere?

Learners: yes

Teacher: so we have to calculate angle X first. If we are to calculate angle X first, it means why is it that we ought we opted to choose angle X as our first calculation? Because this side (pointing at side x) is given, the opposite to angle x and we relate it to y because Y angle and y side are given, clear?

Learners: clear

Teacher: so we are simply saying now, we can actually use the sine rule, because we are simply saying y is given, the Y angle is 33° and the y side (small y) is 6cm. And because of this idea we can apply the sine rule. $\sin Y/y = \sin X/x$ remember small letter x for side. I am applying that one (pointing at the formula on the board), then you can put values. $6/\sin 33^\circ = 5/\sin X$ (writing in the board)

Teacher: what we want is not $\sin X$, we want angle X. But because we cannot go and find x before, we cannot snatch x of course we have to make some arithmetic eh?

Learners: ee

Teacher: now by cross multiplying, cross multiplication means this one multiplied by this one (pointing at the equation on the board). It means $6\sin X = 5\sin 33^\circ$. Satisfied?

Learners: yes sir

Teacher: then you can actually evaluate it means automatically we can leave $\sin X$ on that particular side so that becomes $\sin X = 5\sin 33^\circ/6$. I hope that I am not too fast for you, because sometimes I am carried away and then you complain akere?

Learners: ee

Teacher: now we are simply saying we want ... hee I know some learners who are so interested in quickly calculating. Your degree of accuracy of your answer is going to be compromised. Your calculator is a minicomputer. It can do everything at one call at the last concept, but I am not encouraging you to do my own way as long as you are able to manipulate. One man's meat is another one's poison. As long as you are able to evaluate correctly, am I clear? So, how I am going to find this one (meaning $5\sin 33^\circ/6$)? If I find $\sin 33^\circ$ I am going to get lot of decimals which are even not easy to handle. So I simply want what? Angle X not $\sin X$. So angle X becomes arc sin for everything found on the right hand side (meaning $5\sin 33^\circ/6$) which we can find straight away. What do you get?

Learners: 26, 99°

Teacher: 26,99° and normally we work with degree of accuracy. One decimal place for an angle but two decimal places for any other values as the final answer, but during the calculation more than two decimal places. So if it is one decimal place it becomes 27°. Am I clear?

Learners: yes

Teacher: oh right 26, 99° like this, if we want one decimal place then this one (pointing at the last 9) is going to promote that one (pointing at the second 9). Then that promotes the six so that it becomes 27 to one decimal place. Now that we have calculated this angle (pointing at X) and there is another angle with a question mark, how do we calculate that?

Learner 1(Prince): I just take this one.....(learners started to make noise)

Teacher: one person at a time. Prince you need to give others a chance. Yes (learner 2)

Senong (learner 2): the angles of a triangle add up to 180, so we must say 180 subtract the angles we know to give us that.

Teacher: that is very correct and is not the only option, isn't it? We can also apply the sine rule. Since we now know the other angle but however let us apply. We know that $X + Y + Z$ (writing them on the board) equal to?

Learners: 180°

Teacher: but angle X we have already calculated. Let us take it as $27^\circ + 33^\circ + Z = 180^\circ$ (writing on the board). And we want Z, so Z becomes $180^\circ - 60^\circ$ which is $27^\circ + 33^\circ$ which is 60° . $180^\circ - 60^\circ$ which is equal to 120° . Of course that become obtuse which is not normally the case but because maybe is quite obtuse, above 90. What is left is what?

Learners: the side

Teacher: and the side is small z, akere? How do we calculate the small z

Learners: by applying the sine rule

Teacher: who can apply the sine rule? (moved around and ultimately gave one learner 3 a chalk to apply the sine rule) let us all observe quick quick

Learner 3: started to write on the board, $z / \sin Z =$ (interrupted by the teacher)

Teacher: hee, a moment, which side is the most applicable which will be the most appropriate? Will you choose the one related to x or the one related to y?

Learners: y

Teacher: y because is the original. I am not discouraging you to use the x one, but I am simply saying if you want to be very accurate. You must use the side which is the original one.

Learner 3 proceeded to write on the board, $z = y/\sin Y (\sin Z)$. (he was again interrupted by the teacher)

Teacher: is he correct by the formula wise before the values?

Learners: yes

Then learner 3 proceeded to write and substitute: $z = 6\sin 120^\circ / \sin 33^\circ$. (he was interrupted again before finishing)

Teacher: let us help him. Punch and verify the answer. You have already calculated 9,54 what?

Learners: cm

Teacher: if there were no cm we were just going to say units. If there were mm we were just going to comply with the same measurement portrayed in the question. Any question so far? Anyone who is stuck? So far so good. Yes (pointing at the learner who was asking a question).

Learner 4: is it possible to get 6cm and the other side to get 5m?

Teacher: he is asking is it possible to get 6cm and the other side to get 5m? It is more of a practical. That one will exist in that way. Like my measurement I didn't know that this one is going to be obtuse isn't it? But generally they must add up to 180, so practically it may not be something well but otherwise it must comply so that we apply clearly and correctly but with the same measurement. Like in this case cm throughout or else you can convert m to cm. converting m to cm you divide by 100. So this is the whole idea about the sine rule but on application we don't normally apply only the sine rule. We can also apply the cosine rule. But under what conditions do we normally apply the sine rule?

Learners: 2 sides and an angle

Teacher: 2 sides and an angle, but that angle given must be related to one of the given 2 sides. Am I clear? That is the only rule under which we can apply the sine rule. Let us look at the second concept, the cosine rule. Now when we are looking at the cosine rule, there are conditions under which we can apply the cosine rule.

Before we state it or before we have it on the formula, we must look at under what conditions do we normally apply the cosine rule. Let us look at conditions under which we apply cosine rule (wrote the word conditions on the board). What are the conditions: when we are given what?

Learners: 2 sides and an angle

Teacher: let me start with the number one, given all sides and all angles not given, am I clear?

Learners: ee

Teacher (wrote on the board): 1. all lengths of sides given, no angle given. Here I am simply saying we are given a triangle, the letters let us draw go to the standard one ABC (he drew the triangle on the board). Given side a, b and c but all angles not given. In such a situation you must know that sine rule do not apply but cosine rule is very very important. For we use experience in Mathematics. Number 2?

Learners: 2 sides and an angle

Teacher: 2 sides and an angle?

Learners: and an included angle

Teacher: (the teacher wrote on the board) 2 sides and one angle but the angle must be between the 2 sides. What am I trying to say? I am trying to say given a triangle like this (he drew the triangle on the board). Let us use our standard ABC and we are given 2 sides. Suppose we are given side b and we are given side c, so the angle must be between these two sides. So angle A must be given for us to be able to manoeuvre around isn't it? What else? We can change here (pointing at rule 2). Instead of 2 sides we can say 2 angles and one side. We are just changing these words. It is also applicable. But now let's see with an example. I said it is applicable when we are given 2 angles and 1 side. I am saying for example, let's say (he drew triangle ABC, with $A= 27^\circ$, $B=33^\circ$ on the board) and one side given. I am changing the statement will it be applicable?

Learners: no that one is for sine rule

Teacher: so that one (referring to rule number 3 is not applicable) is not applicable. Given all angles without one side is possible. Let us apply our particular cosine rule in the first place. Who can state the cosine rule? I think is not for the first time you meet the cosine rule.

Learners: a^2 equal (interrupted by the teacher)

Teacher: a^2 . How many sides are left?

Learners: 2 sides

Teacher: there are two which ones? B and c, so must have $b^2 + c^2 - 2bccosA$. the angle related to the side we are calculating. That's formula number 1. Another one is when we are to work with side b. B is related with b^2 and the two sides which are left are $a^2 + c^2 - 2accos$ of which angle?

Learners: B

Teacher: and finally we take that one of c^2 . C^2 equal to?

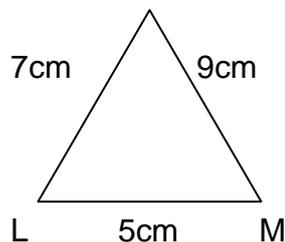
Learners (in a chorus): $a^2 + b^2 - 2abcosC$

Teacher: that becomes equation number 3. So this is 3 equations are very very important but are we given all this 3 equations in an examination?

Here some learners said yes whereas some said no

Teacher: every year they change, maybe (pointed at number one) this one 2009 and this one (pointing at formula number 2) in 2010 and this one (referring to the third one) 2011. So check as to whether the March one was having which one and the November was having which one. They normally change all of this. They may bring one and most of which is not related to the question which is under consideration. So be very careful and be able to identify the right formula for the right job. Let us look at the specific example where we can apply this cosine rule. Hee we are not prophesying, we are getting them from... that is even if you are going to do Mathematics at university level there is no formula which you must not prove. Where is it coming from? What about if you are cheating? So you must know where it is

coming from. Let us look at the specific example. Let us take when all lengths of the sides are given. Solve the triangle. K



Can you solve? Let us all solve quickly each one of you. Solve the triangle and find all angles and you are going on only 2 steps isn't it? 2 steps because one either any of two angles and subtract them from 180° . Remember solve the triangle means find all the unknown angles.

Learners began to solve individually in their book

Teacher: speed and accuracy is needed in Mathematics. Speed and?

Learners: accuracy

Teacher: if you are slow and you are not accurate and if you are fast and not accurate, you are also.... (the teacher moved around checking how learners are doing the activity) some are saying they are getting the error is that true?

Some learners said yes whereas some said no

Teacher: let us be fast

Learners work in their books

Teacher: I think the lesson is over and you are not yet through. Raise up your hands those who are through. Or is a challenge?

One learner said there is an error sir (then the argument started between the learners some saying there is an error whereas some were saying there is no error).

Teacher: is there any error? O'right let us do it. Any way let us calculate. We are going to calculate only one to illustrate then you will finish the rest since there is no ample time. Let us look at angle what?

Learners: K

Teacher: angle K is the one we want and angle K is related to which side? To k. So what is the formula ?actually k^2 equal to?

Learners: $l^2+m^2-2lm\cos K$

Teacher: this one (pointing at k^2) we do have. So $25=81+49-2(9)(7)\cos K$

Learners: no sir

Teacher: $25 = 81 + 49 - 2(9)(7)\cos K$ all of you did you get this one?

Learners: yes

Teacher: o'right let us evaluate $25 = 130-126\cos K$ clear?

Learners: clear

Teacher: I am going to take this $126\cos K$ to that side so that I avoid this negative. So I am having $126\cos K=130-25$

$\cos K=105/126$

angle $K= \arccos(105/126)$ and therefore angle $K=$

this number is perfect, a large number in the denominator means less than one. Then $K=23,6^\circ$ and then we do the same. Let's take for another angle. Which angle?

Learners: L

Teacher: lets us look at L. Angle L is related to which small letter? L

Learner3: sir we can use the sine rule since we have calculated one angle?

Teacher: yes, we can use the sine rule but let us use the cosine rule. $L^2 =$ to what?

Learners: $k^2+m^2-2km\cos L$

Teacher: so what is L

Learner 3: $81= 25 + 49 - 2(5)(7)\cos L$

Teacher: $81 = 74 - 70\cos L$

$$70\cos L = 74 - 81$$

Am I confusing you?

Learners: no

Teacher: $70\cos L = -7$

$$\cos L = -7/70$$

$L = \arccos(-7/70)$ and remember cosine is not affected by minus

$L = 95,7^\circ$. And the last angle what do you do? Since angles $K + M + L = 180^\circ$, then $M = 180^\circ - 95,6 - 33,6^\circ = 50,7^\circ$. So who was saying we will get an error?

Thank very much class.

APPENDIX E: Transcription of Thabiso's lesson no 2

THABISO'S LESSON NO 2

The lesson was about data handling, calculating the mean, the variance and the standard deviation. He wrote data handling on the board. The lesson proceeded in the following manner.

Teacher: we will be doing statistics today, although is an elementary statistics where we just want to look at data handling. Now at data handling I just want to look at calculation of the mean, calculation of the standard deviation and calculation of the variance, am I clear?

Learners: yes sir

Teacher: now you have already seen that there are symbols which we normally use for a population and for a sample. (wrote population on the board) when we look at a population, let me define some of the terms. A population is the entire number of items that are under an observation. The what?

Learners: the entire

Teacher: (wrote the definition on the board). For example, when we look at number of stars on the sky, we cannot count them, but that constitutes a population. Am I clear?

Learners: ee

Teacher: a total number. If I want to study behaviour of people in South Africa, then I have to count all the 49 million people. The 49 million people constitutes to tell a what?

Learners: a population

Teacher: but sometimes if I want to study the behaviour of stars on the sky is impossible for me to calculate. Isn't it?

Learners: hmm

Teacher: sometimes is very difficult to study is very difficult to study the population but once I want to study the population, I will go on to study what is called a sample (wrote the word sample on the board). A sample, which is part of a population. Part of a?

Learners: population

Teacher: here I not looking at a population in Geography where we look at a number of people in a particular given area. I am looking at the number of items. Items can be sweets, chocolates, can be number of stars etc. So we are simply saying sample is a part of population. So should I want to study for example the number of stars in the sky, then I will take a sample of this stars and study, but otherwise when you are choosing part of the population and you want to study the behaviour of the population and you you have chosen a sample, a sample must be a correct representation of the population. For example, if I want to study the people here in Lebowakgomo, their eating habits, I cannot count the entire number of people in Lebowakgomo. I will take a sample. But if I want to study their behaviour in terms of their eating habits I cannot go very far away, maybe let us say there in district. I take a sample ofdistrict, then I analyse that particular sample. Will that sample give me a true picture of the people in Lebowakgomo?

Learners: hmm hmm

Teacher: so it means is a random sample, the sample must be a correct representative of the population which I want to study? Am I clear?

Learners: yes sir

Teacher: when we are looking at some symbols like sigma, this one (sigma, writing it on the board) like of course sigma has got 2 symbols that one for summation (showing it in the air). But now this sigma (pointing at the sigma sign) is for a population. When we are using sigma we are referring to a population, but when we are looking at (s) we are looking at a sample (writing it on the board). In other words sigma is denoting standard deviation for a population. This one (pointing at) is also a standard deviation for a sample. But sometimes these values are the same because you may have such entire number ofso when you look at sigma squared it

will be variance of what? Variance of population. People who are doing Mathematics paper 3 must be very careful about this one otherwise paper 2 they don't penalise for that one. But for paper 3 you must be very clear that am I studying a population or a sample? You must know which symbol to use. Given (a learner) when you are not clear that a particular observation under a study approximates a normal distribution with the two parameters, the mean and the standard deviation or the variance then you must know that my distribution must be in such a way that I am studying a population. Thus why I am looking at this particular situation. For a sample then the variance will become s^2 . But sometimes we can use both since we can study whole of the entire population. Now let us look at an example where we can actually in other words the mean or the what? The average. We add all the observations divide by the number of observations, am I clear? The mean has got a single \bar{x} (writing it on the board). How can we calculate the mean (he wrote on the board) the mean = number total of all the values given in a data set. That is the sum of the total divided by the number of observations. He wrote $\bar{x} = \frac{\text{total of all the observations}}{\text{number of observations}}$. What am I saying, we are simply saying we are summing up all the x -values, all the x_i 's where he wrote ($\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$ the formula). From $i=1$ up to the maximum of observations divided by n (learners started to make noise). I am including notation to cover in both paper 1 and paper 2 because notation is very very important in paper 2 even on contingency table a where we are looking at this i 's and k 's, the mean in rows and columns. Now I am saying x_i (pointing on the board) i starting from 1, look I am simply saying when $i=1$, what happens to this (pointing at x_i) it becomes x number 1, when $i=2$, this become x number 2 and I am summing them up. When $i=3$ it becomes x what?

Learners: 3

Teacher: plus blah blahblah up to the maximum which is what plus x_n , this is what I mean (writing $\bar{x} = \frac{x_1+x_2+x_3+\dots+x_n}{n}$). I am getting all the x_i 's, so don't be worried about the notation like this (wrote the notation on the board). But sometimes it is very tricky because sometimes you may take from $i=0$ in some particular summations, thus why they tend to. Notation gives a very clear picture of what you want to tell us, am I clear? Now that is the formula for the mean. Now if we are to look at the variance, and the variance usually has got a s^2 or σ^2 (he wrote the two formulas on the board)

depending whether is a population or a sample and when we are looking at the variance usually $\sigma^2 = \frac{\sum x_i^2}{n}$. These formulas are provided on the data sheet, am I clear? Now it means the standard deviation which I have already defined as either σ or s is the square root of the what?

Learners: variance

Teacher: is the root of the variance because the variance is the sigma squared (he wrote standard deviation = square root of the variance). So is very easy to find the sigma which is the standard deviation. But now let us look at the specific example so that we can understand because Mathematics is about understanding through worked examples akere?

Learners: ee

Now let us look at the rainfall pattern from January to December. He wrote the activity on the board. The rainfall pattern for the year 2010 has been recorded as follows:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rainfall in mm	120	80	60	20	10	25	2	3	9	75	110	115

And normally which month do normally has the highest rainfall?

Learners: some said January and September

Calculate:

- (i) The mean
- (ii) Variance
- (iii) Standard deviation

How do we calculate the mean?

Learners: (in a chorus) total number of all observation divided by number of observation

Teachers: who can come and calculate the mean, any volunteer? Yes (pointing at one learner). I will give you a chance (telling another learner). This is our number 1 (he wrote mean =... and gave a learner a chalk to finish). The learner wrote $=120+80+$ (interrupted by the teacher). When I said many observations, don't write them all. Just write $120+80+60+$ blah blahblah+ $110+115$. Blah blahblahrepresent all those calculations. Your calculator will do it.

(The learner proceeded writing) the teacher got to the back of the class.

Teacher: do you understand what I am saying, in an examination don't do this you will be penalised. (teacher come from the back, rub what the learner was doing and started to show them how to do it. He wrote mean = $120+80+60+...+110+115/12$, then gave the learner a chalk to proceed and said to the learner you may go on.

The learner proceeded by writing $629/12 = 52, 4$

Teacher: units, newtons, ya what are we measuring? Rainfall in?

Learners: mm

Teacher: o'right, now that becomes we normally summarises by writing like this (he wrote, therefore the mean rainfall or the average for the period is 52,4 mm meaning that each month the average will be 52,4. Let us look at the variance. I don't want to calculate it using the formula. I will calculate using a formula but I want a table. Sometimes this table got many marks akere?

Learners: yes

Teacher: the columns of the table depend on the quantities that are portrayed in the formula. Remember our formula has got n. Our formula has got also \bar{x} . So we tend to say that our data is such a way that our rainfall which is our x values and went actually to say our mean you know it. What else do we want? We want \bar{x} column and we sum it (he dwre the table on the board and inserted the values of the rainfall on the columns. All these are our x values and our \bar{x} doesn't change. Let us fill in the column.

(learners started to call out values and the teacher inserted the values on the table)

Teacher: what do we want? We want to sum up the deviations squared. Then here we must get the total of all this columns (pointing at the deviations column).

(learners added up the deviations column) one learner (molo) called out the answer as 23598,32

Teacher: are you all getting this

Learners: No

Teacher: what are you getting

Learner (Lesty): it is sir, the last digit is comma 92 not 32

Teacher: is it

Learners: yes

Teacher: o right, remember we are on number 2. We are calculating the variance. The variance formula .the variance = the summation of all the deviations squared divided by n (he wrote the formula on the board). The numerator is this number (pointing at 23598,92) which we have already calculated. So the examiner will come and mark every value. If it has got more marks it means using a table. (Substituted the values on the formula) what is the value now.

Learners: 1966,566667

Teacher: so the answer is 966, 58 mm to 2 dp. That is our variance. Now the standard deviation becomes very easy, but examiners normally are tricky, they normally start to say mean here (pointing at the mean) then they take this one first to confuse you (pointing at the standard deviation) then this one will be there (meaning the variance). O'right number 3. So our standard deviation which is our s or sigma = square root of the variance. = square root of 1966, 58 =

Learner called it out: 44,346

Teacher: let us say 44,35 mm to 2 dp. You calculate again 2 but you must do the calculation to show thus why examiners are so interested in you showing coherence. Any question? Should you be given any similar example will you be able to do?

Learners: yes

Teacher: example 2. Im going to change the values. Now I am no longer interested in the rainfall pattern but then funeral contribution of the money for the year 2011, for Grade 12A learners (the teacher wrote the values on the board). Calculate:

- (iv) The mean
- (v) The variance
- (vi) The standard deviation

The teacher moved around

Teacher: just follow exactly what we did in example 1.

(Learners kept on writing in their books)Teacher: Take it as your homework.

APPENDIX F: Transcription of Lerato's Lesson no 1

LERATO'S LESSON NO 1

Lesson was about the introduction of Linear Programming in Grade 11 class. The lesson proceeded in this way:

Lerato: learners I am having two dice the green and the white one to play the game. Are there any two learners who can play the game of dice.

Learners: those two girls at the front ma'am.

Lerato: (she gave the two girls dice to play). When we are playing the game, what is the purpose of the game?

Learner: is to win.

Lerato: But there are rules of the game, not so?

Learners: so

Lerato: I want us to observe the rules of the game. We may also call them conditions. These rules limit us, they constrain us. We call them therefore constraints. These rules need to be converted. When looking at the dice we can let the one be x and the other be y . Why do we choose x and y ? We would have chosen any other variable but we choose x and y so that we may be able to represent our information on a graph/ graphical, because our graphs consist of x and?

Learners: y

Lerato: when we are given the information in words, we have to identify the variables as x and y . That is the first step okay. When we are looking into that passage full of words, remember to look for x and y . We are saying whenever we are confronted with a paragraph asking questions in Linear Programming the first thing you should do is to take Mathematics out of the paragraph. What must you do?

Learners: take Mathematics out of the paragraph.

Lerato: And how do you do it? You first have to find the variables. You first find what?

Learners: the variables.

Lerato: and then from there you need to find the constraints. We are saying we are having explicit constraints and implicit constraints. Who can tell me what the explicit constraints are? (learners struggle to give the answer) they are those that are explained, they are clearly explained, they are clearly given, and then we have just mentioned our explicit constraints. Now I want us to figure out our implicit constraints before we go any further. We are never given our implicit constraints, we have to read the paragraph and then figure them out. Remember the dice game, for example, the implicit constraint, if the statement says the number on each die should be 3 or more, will be $x \geq 3$ and $y \geq 3$. Another example, if the statement says the numbers on the two dice should be not greater than 12, the implicit constraint will be $x + y \leq 12$. We have derived them from the statements. Class, lets proceed. In every situation, whenever we are confronted with a question in Linear Programming when all has been searched and done, now comes our objective function. Now comes your.....?

Learners (in a chorus): objective function.

Lerato: an objective function is given by only two words. Do you know them? Whenever you see cost or profit it is whispering that you are now under the objective function. I even whispered some other day that whenever you see maximum or minimum but not always but in most cases remember something about objective function. Whenever you are having your objective function learn to write it in the form $y = mx + c$. Now I want us to represent that data, and I highlighted that I don't want you to forget that I want you to write the objective function in the form $y = mx + c$. Can we move further?

Learners: yes

(The teacher writes this excercise on the board)

A factory produces two types of wooden tables, the plain table and the luxury table. It costs R250 to produce a plain table and R400 to produce a luxury table. The budget provides a maximum of R4000 a day to cover these costs. To keep up with the orders the factory must produce at least 10 plain tables and at least 1 luxury table a day.

- a) Set up a model for the constraints the factory is working under.
- b) Graph the feasible region.
- c) Write down all the combinations of the tables the factory can produce.
- d) What is the maximum number of tables that can be produced in a day?

Lerato: first and foremost we must find our?

Learners: variables.

Lerato: is that clear?

Learners: yes

Lerato: we are identifying our variables and which are our variables here? We are reading from here (pointing at the statement). Can somebody read for us? (one learner read the problem). We are having a factory here which produces two types of wooden tables and their names are?

Learners: the plain tables and the luxury tables.

Lerato: and then we want to identify our?

Learners: variables.

Lerato: what are they? We are having the plain and the ?

Learners: luxury.

Lerato: ok, now for the purpose of uniformity, I am going to suggest that we say x be the luxury and y be the plain, o'right? Are you there guys?

Learners: yes

Lerato: Okay, R250 is needed to produce a plain and R400 to produce luxury. What does this tells us? It tells us the objective function. It is telling you be aware, open your eyes because there is what?

Learners: objective function.

Lerato: objective function, okay?

Learners: yes

Lerato: and therefore may we construct our objective function, that is $\text{Cost} = 400x + 250y$. I remind you one more time to rewrite your objective function in the form $y = mx + c$. And therefore here we are making y the subject of the formula. So when we make y the subject of the formula we want y to stand alone okay? Are you there?

Learners: yes

Lerato: so we simply try to get rid of this one (pointing at $400x$). And I was just telling you that in Mathematical language, you subtract the same quantity both sides. What you do into this side of the equation you must also do into the other side of the equation, are you there? And please don't be tempted to write this one as $C - 400x$, why?

Learner 1: because we want to write it in the standard form, $y = mx + c$.

Lerato: yes, we do not want it in this form (pointing at $C - 400x$), we want it in the form $y = mx + c$. They are the same thing but we want it in the form $y = mx + c$. Are we happy guys?

Learners: yes ma'am

Lerato: we can simplify here, when we simplify what are we going to have? We do not want the 250, what should we do?

Learners: you should divide by 250

Lerato: yes all over here and then you will have?

Learners: $y = -400x/250 + C/250$

Lerato: are you there guys, can we simplify this one? Then we will have $C = -8x/5 + C/250$. Are you there? And whenever you want to represent your objective function graphically, all you need is? What is it called? The m okay?

Learners: yes

Lerato: $m = -8/5$. What does this mean? Your m is?

Learners: the gradient

Lerato: and what is it made up of? The numerator tells you something, it is the change in y over the change in x , okay? Can we go further with our question? The budget provides a maximum of R4000 a day to cover these costs. If we say maximum, what sign?

Learners: less or equal to

Lerato: less or equal to, it must not go above. In other words it now becomes the constraints. It now becomes a ?

Learners: constraints

Lerato: which kind of constraints is this one? Remember we are having two types.

Learners: explicit

Lerato: explicit because it has been explained. We saying it provide to cover these costs. How much is x and how much is y? x is 400 and y is 250. Therefore we will have $40x + 25y \leq 400$. what is the other constraint, to keep up with the orders, the factory must produce at least 10 tables and 1 luxury table a day. Here they are talking of separate constraints for x and y. if you were to write it in the right form, what will you say about them? We say $x \geq 1$ and $y \geq 10$. Are you aware that you have already identified your objective function and you have set up the model for the constraints? Immediately now I want us to go further. How do we construct our implicit constraints? These are the ones which you should figure them out. And we first look at the nature of data which we are dealing with. Here is tables, we do not have decimal tables “akere”?

Learners: yes

Lerato: so as the results we are dealing with discrete data. Are you there?

Learners: yes

Lerato: and immediately we know we do not have negative number of tables. So your $x; y \geq 0$ which say is either no table or positive number of tables, isn't it?

Learners: yes

Lerato: such that x and y is an element of natural numbers or positive integers or simply whole numbers. Now we want to represent the feasible set. But before you represent your feasible set you first look at your constraints. What is the largest value of x and y, so that you can allocate your spaces evenly. What is the highest value along the y?

Learners: 16

Lerato: 16 ne? And the highest value on the x is?

Learners: 10

(Lerato then drew the feasible set on the board) what kind of tables along the x-axis?

Learners: luxury

Lerato: and along the y?

Learners: plain tables

Lerato: is there anybody who wants to plot? (no learner showed up)

Learners: (plots the lines on the board) where do we shade the less or equal to?

Learners: below

Lerato: who want to finish? (one learner 2 volunteered and learner 1 helped him to finish) also show the arrows according to the inequalities. Which one is our feasible region? Let's talk about the feasible region. By the way we are talking about which type of data?

Learners: discrete data

Lerato: now give me the coordinates of the feasible region. (she then pointed the points and the learners call them out).how do we get the maximum number?

Learners: by adding the combinations of x and y in each coordinates.

Lerato: which one gives us the maximum number?

Learners: $1 + 14 = 15$ (the bell rang marking the end of the period)

Lerato: 15 is our answer because the question needed the maximum number. Remind me to show you how we plot the objective function.

APPENDIX G: Transcription of Lerato's lesson no 2

LERATO'S LESSON NO 2

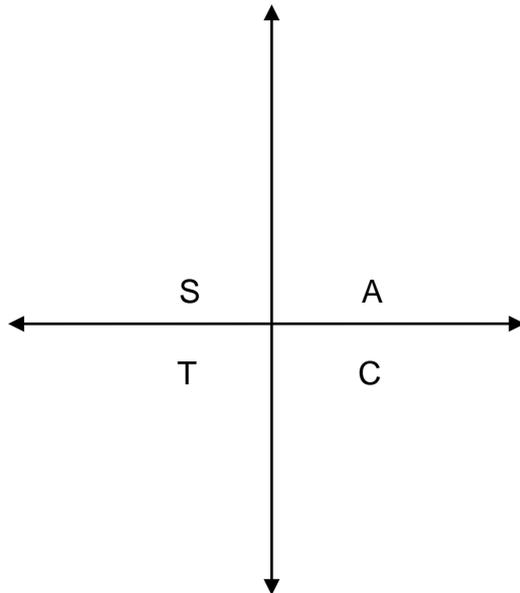
The lesson was about reduction of trigonometric identities normally referred as using the CAST diagram. The lesson proceeded in this way:

Leratowrote the following trigonometric ratios on the board:

1) $\sin (180^\circ + \theta)$ 2) $\sin (180^\circ - \theta)$ 3) $\cos (180^\circ - \theta)$

4) $\tan (360^\circ - \theta)$ 5) $\cos (360^\circ + \theta)$ 6) $\cos (360^\circ - \theta)$

She then draw the CAST diagram on the chalkboard.



Lerato: May you please locate the given ratios into the given Cartesian plane. (She then started to ask questions.

Lerato: number 1, $\sin (180^\circ + \theta)$ is in which quadrant?

Learners (in a chorus): In the third quadrant.

Lerato: number 2, $\sin (180^\circ - \theta)$ is in which quadrant?

Learners (in a chorus): In the second quadrant.

Lerato: number 3, $\cos (180^\circ - \theta)$ is in which quadrant?

Learners (in a chorus): In the second quadrant.

Lerato: number 4, $\tan (360^\circ - \theta)$ is in which quadrant?

Learners (in a chorus): In the fourth quadrant.

Lerato: number 5, $\cos (360^\circ + \theta)$ is in which quadrant?

Learners (in a chorus): In the first quadrant.

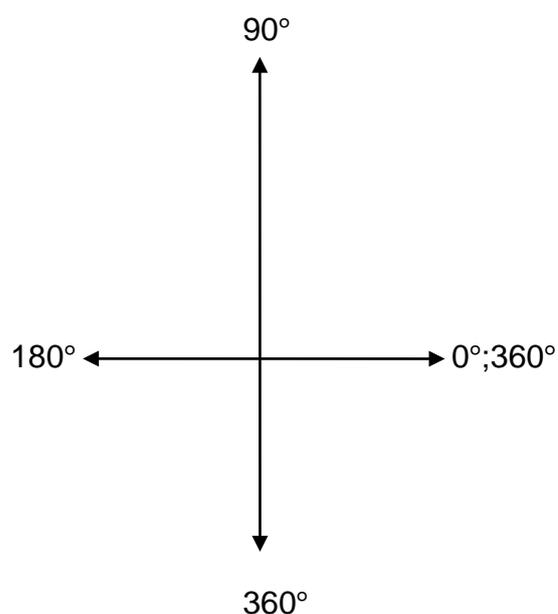
Lerato: number 6, $\cos (360^\circ - \theta)$ is in which quadrant?

Learners (in a chorus): In the fourth quadrant.

Lerato: in which quadrant is 150° ?

Learners (in a chorus): In the second quadrant.

Lerato: (she wrote $\sin 150^\circ = \sin (180^\circ - 30^\circ)$). She then drew another cartesian plane on the board now each arm having an angle.



Lerato: let us write the function values in the quadrants. For example, in this quadrant (pointing in the first quadrant) we write the function value as $(90^\circ - \theta)$ because in Mathematics we are working in an anticlockwise direction. Therefore whenever we are working in a clockwise direction we subtract from the given angle on the arm of a Cartesian plane like I have already given you an example. And again when working in an anticlockwise direction we are adding to the given angle, for example, in this quadrant (pointing in the second quadrant) we are going to have these function values $(180^\circ - \theta)$ and $(90^\circ + \theta)$. Let us proceed, in the third quadrant what will be the function values?

Learners (in a chorus): $(270^\circ - \theta)$ and $(180^\circ + \theta)$

Lerato: what about the fourth quadrant?

Learners (in a chorus): $(270^\circ + \theta)$ and $(360^\circ - \theta)$.

Lerato: we have additional function values. All negative angles are found in the fourth quadrant (writing $-\theta$) in the fourth quadrant. Also $(\theta - 90^\circ)$ is in this quadrant (writing it in the fourth quadrant). She further wrote θ , $(\theta - 360^\circ)$ in the first quadrant, $(\theta - 270^\circ)$ in the second quadrant and $(\theta - 180^\circ)$ in the third quadrant. When applying the reduction formula, we reduce obtuse or reflex angles to their acute angles. **Whenever we are given angles, the first thing to do is to locate the angle, that is, determine in which quadrant the angle lie. Secondly, check whether the ratio is positive or negative and lastly, check whether the ratio is going to change to its co-function or not. And always strive to use 180° and 360° , that is the horizontal reduction.** Let's get to our initial problem, let us reduce the sine of 150° . Yes my girl, give us the answer.

Learner: the angle is in the second quadrant and sine is positive in the second quadrant. Therefore, $\sin 150^\circ = \sin (180^\circ - 30^\circ) = \sin 30^\circ$

Lerato: she is right. What about $\tan 150^\circ$ and $\cos 150^\circ$. $\tan 150^\circ = \tan (180^\circ - 30^\circ) = -\tan 30^\circ$ and $\cos 150^\circ = \cos (180^\circ - 30^\circ) = -\cos 30^\circ$.

Learner: ma'am will I be correct if I use $(90^\circ + 60^\circ)$ for the angle of 150° ?

Lerato: yes, but I have told you that always try to use 180° and 360° . Can we proceed?

Learners: yes

Lerato: (she wrote the following on the board

i. $\cos 210^\circ = \dots$

ii. $\tan 210^\circ = \dots$

iii. $\sin 210^\circ = \dots$

iv. $\sin 300^\circ$

v. $\sin^2 300^\circ$

vi. $1 - \cos^2 240^\circ$

let us do $\cos 210^\circ$ together.

$$\cos 210^\circ = \cos (180^\circ + 30^\circ) = -\cos 30^\circ$$

Lerato: **Remember have told you** that always strive to use the horizontal reduction so that you will not be confused. Learners, **remember that I have told** you that whenever you are given angles, the first thing to do is to locate the angle. That is, determine in which quadrant does the angle lie. Secondly, check whether the ratio is positive or negative and lastly, whether the ratio is going to change or not.

Learners were given time to complete the activity and ultimately the teacher done the corrections on the board talking whilst writing. She wrote

i. $\tan 210^\circ = \tan (180^\circ + 30^\circ) = -\tan 30^\circ$

ii. $\sin 210^\circ = \sin (180^\circ + 30^\circ) = -\sin 30^\circ$

iii. $\sin 300^\circ = \sin (360^\circ - 60^\circ) = -\sin 60^\circ$

iv. $\sin^2 300^\circ = \sin^2 (360^\circ - 60^\circ) = \sin^2 60^\circ$

v. $1 - \cos^2 240^\circ = 1 - \cos^2 (180^\circ + 60^\circ) = 1 - \cos^2 60^\circ$

Learner 2: Ma'am will I be correct if I can express $1 - \cos^2 240^\circ$ as $\sin^2 240^\circ$ and then solve it from that point?

Lerato: you are forward treating the identities, here we are still using reduction formulae.

In wrapping up the lesson the teacher gave this homework, reduce and evaluate without using a calculator

$$1. \frac{\cos^2 330^\circ - 1}{\tan 300^\circ \cdot \sin 300^\circ} .$$

APPENDIX H: The letter written to the school to ask for a permission to
conduct a research

P O Box 424
Lebowakgomo
0737
May 2010

The principal
Secondary School
Lebowakgomo
0737

Dear Sir/ madam

Re: Request for conducting a research at your school

As part of the fulfilment of MED in Mathematics, the University of Limpopo requires in-depth research in a school setup, I therefore request to undergo a research at your school for this title: investigating Mathematics teachers beliefs about the nature of Mathematics and their impact on classroom practice. In order to achieve the purpose of this study, the research will involve two interviews and two observations with the teacher participants. Further be informed that the school and the teachers will not be identified by their names.

Hoping for your positive response

Yours in education

B. K. MAPHUTHA (Researcher)

APPENDIX I: Approval letter from the school

30.07.2010

Enq:

To: Maphutha B.K

Subject: Permission to conduct research.

The above matter bears reference that

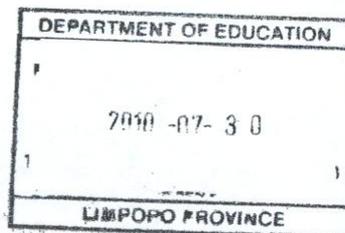
1. We acknowledge receipt of your request for conducting research at this institution.
2. The institution gladly informs you that your request was accepted.

We hope that you will enjoy conducting your study at this institution.

Yours Faithfully



(HOD Maths & Science)



APPENDIX J: The letter requesting teachers to be participants in the study

P O Box 424

Lebowakgomo

0737

03 August 2010

FET Mathematics teacher

Secondary school

Lebowakgomo

0737

Dear colleague

Re: Request for participating in a research

As a student at University of Limpopo, I am required to conduct a research in a school setup as part of the fulfilment of MED in Mathematics. The title of my study is: investigating Mathematics teachers' beliefs about the nature of Mathematics and their impact on classroom practice. In order to achieve the purpose of this study, I therefore humbly request you to be the participant in this research.

The research will involve two interviews and two observations with each teacher participant. Further be informed that you and your school will not be identified by their names in the study and all information will be destroyed after being used.

Your participation is highly valued in advance.

Yours in education

B. K. MAPHUTHA (Researcher)

APPENDIX K: Acceptance letter from the participant (Lerato)

P O Box
Polokwane
0700
04 August 2010

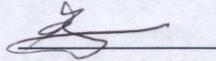
The researcher
P O Box
Lebowakgomo
0737

Dear researcher

Re: Acceptance for a request of being a participant

Your request for my participation in your study is accepted. I am happy to inform you that I am available to be participant in your study as I am prepared to learn more in the Mathematics field. I understand that I will not be mentioned by my name and that all the information will be destroyed immediately after being used.

Yours in education



(Maths teacher)

APPENDIX L: Acceptance letter from the participant (Thabiso)

P O Box
Lebowakgomo
0739
06 August 2010

The researcher
P O Box
Lebowakgomo
0737

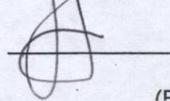
Dear colleague

Re: Acceptance for taking part in research

This serves to confirm that I hereby accept to be part of the participants in the study to be conducted. I understand that I will not be mentioned by my name and that all the information will be destroyed immediately after being used.

Hoping the above matter will be found in order.

Yours faithfully



(FET Mathematics teacher)