IMPROVING GRADE 9 LEARNERS' MATHEMATICAL PROCESSES OF SOLVING WORD PROBLEMS

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IMPROVING GRADE 9 LEARNERS' MATHEMATICAL PROCESSES OF SOLVING WORD PROBLEMS

by

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DECLARATION

I declare that the mini-dissertation hereby submitted to the University of Limpopo, for the degree of Master of Education in Mathematical Problem Solving has not previously been submitted by me for a degree at this or any other university; that it is my work in design and in execution, and that all material contained herein has been duly acknowledged.

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Maluleka B.K (Mr) **Date**

DEDICATIONS

TO MY PARENTS

RAMADUMATJA AND THE LATE LETSOALO

AND

MY CHILDREN

BOITSHEPO, THABO AND MMATHABO

TO GOD BE THE GLORY

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ABSTRACT

This study intended to improve Grade 9 learners' mathematical processes of solving word problems. It was an action research study in my own classroom consisting of 64 Grade 9 learners. Learners were given learning activities on word problems to carry out as part of their normal classroom mathematics' lessons. Data were collected in two stages: first, through passive observation, that is, without my intervention, and later through participant observation thus provoking their thinking as they attempt the given questions. The learners' responses were analyzed through checking the mathematical processes they used without my intervention. Learners also submitted their post-intervention responses for analysis of progress after interventions. The scripts were reviewed based on four problem-solving stages adopted from George Polya (1945). Those stages are, namely understanding the problem, devising the plan, carrying out the plan and looking back. It became evident from the findings that learners attempt solving word problems with no understanding. Communication, reasoning and recording processes appear to be key factors in assisting learners to make sense of word problems and, finally, proceeding towards an adequate solution.

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CHAPTER 1: INTRODUCTION AND BACKGROUND

1.1 Setting the Scene

Extensive work has been done in problem solving for a substantial period of time and problem solving as an area in mathematics showed significant bearing in addressing issues at different levels. I felt it was still important to do a search on problem solving because it still emerges to be a pillar in the South African school's mathematics curriculum which according to my experience is still a challenge to most of our learners and teachers.

From my personal experience as a Grade 9 mathematics teacher for the past 22 years, I realized that Grade 9 learners are not performing up to standard when solving mathematical word problems. Learners tend to make use of processes that are not relevant to mathematical problem-solving when solving problems and in certain cases they are unable to merge their mathematical knowledge into the contexts of the mathematical word problems assigned.

Shuard (1991) describes a process as something we do with mathematical ideas. Frobisher (1994) discusses the role of mathematical processes in investigations and suggests that the aim of problem-centred mathematics curriculum should be to develop in learners a 'knowledge of the relationships which exists between mathematical processes, as the one leads naturally into another'. Frobisher (1994) further outlined different processes that can be applied in mathematics, which amongst, others include guessing, pattern-searching, interpolating, predicting, conjecturing, hypothesizing, generalizing and proving. When these processes are properly selected and made use of in computation of mathematical word problems, constructive learning will be enhanced. The interrelationships of these processes are as reflected in figure 1 hereunder.

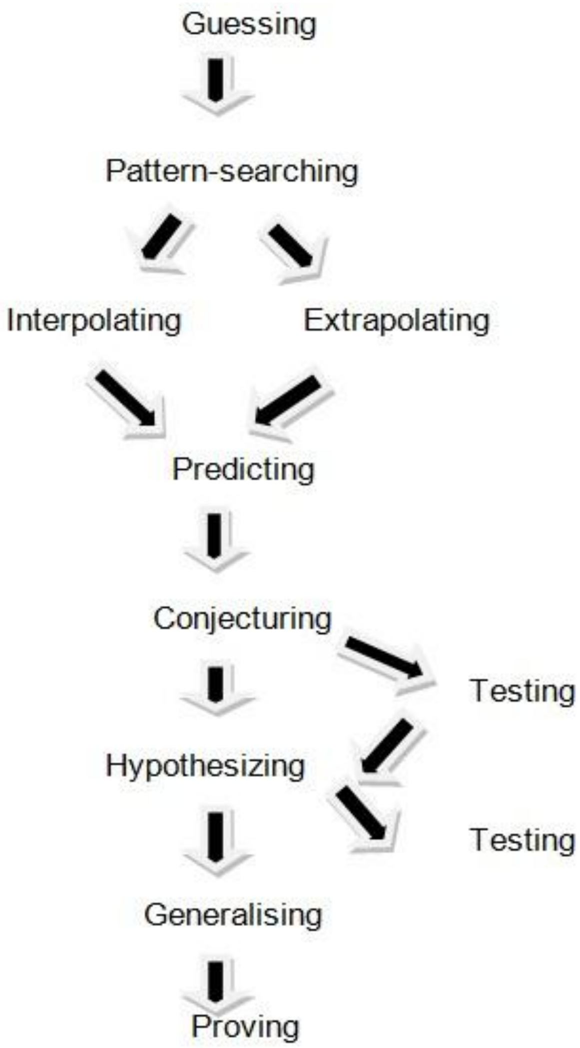


Figure 1: Processes involved in mathematical problem solving

My experience in facilitating learning in a Grade 9 class is that learners at this stage of development are not yet familiar with the skill of interrogating a word problem for understanding. This is reflected in how they respond to given word problems. A big challenge in working with word problems is that real-life problems are to be translated into mathematical word problems requiring understanding of the problem first, before attempting to respond to it. It is through communication, recording and reasoning that the mathematical solution could be translated back into a real-life solution.

The teaching and learning of mathematics aims to develop a critical awareness of how mathematical relationships are used in social, environmental, cultural and economic relations (Department of Education, 2002). Challenged by this aim, I intended in this study to improve learners' use of mathematical processes in solving mathematical word problems, which will further empower learners with skills of solving real-life problems accountably and thus develop them into responsible citizens of the country.

1.2 The South African Curriculum and Problem Solving

The curriculum change became an issue of main concern in 1994 when Curriculum 2005 (C2005) was introduced. The introduction posed a range of challenges to teachers with regard to the new curriculum's underlying assumptions and goals; subject demarcations; the content; the teaching approach; and the method of assessment. C2005 is a form of Outcomes-Based Education and from its philosophical background; it is, according to Chisholm (2003), a pedagogical route out of the apartheid education. C2005 was since revised, resulting in what is today called the National Curriculum Statement (NCS), which became policy in 2002.

The underpinning features of the revised New Curriculum Statements (NCS) are Learning Outcomes (LO) which are explicitly explained through Assessment Standards (AS). Some critical outcomes of Outcome-Based Education (OBE) envisage learners who are able to identify and solve problems and make decisions using critical and creative thinking skills and also who are able to demonstrate an understanding of the world as a set of related systems by recognizing that problem-solving contexts do not exist in isolation.

The study focused on the exit phase of the senior phase in the General Education and Training band (GET) where the problem was identified. This phase, according the South African schools' context, stretches from Grade 7 to Grade 9. It is in this phase where learners, according to LO 1- (Numbers, Operations and Relationships), are expected to be exposed to ample opportunities of solving a variety of problems, using increased range of numbers and the ability to perform multiple operations correctly and fluently (Department of Education, 2002). LO 4 on measurements expects learners to solve problems in a variety of measurement contexts, through the selection of appropriate formulae while in LO 5, the focus is on the application of techniques already learned in order to investigate and solve problems.

1.3 Approach

In order to study the improvement of learners' processes when solving mathematical word problems, one has to be immersed in the situation hence an action research in my own classroom.

Problem solving activities were administered to a class of 64 Grade 9 learners who were heterogeneously grouped according to their intellect, behaviour and gender. These activities were administered in two phases, first through passive observation and secondly through participative intervention. In passive observation, learners were exposed to learning activities to work independently on their own, whereas in participative intervention learners were assisted with some sort of eliciting questions and some leading inputs. These were procedures that resulted in the data that were later narratively analysed.

1.4 Research Questions

Research questions that guided my focus in this study are:

- What challenges do learners encounter with regard to the processes of solving word problems?
- How to improve Grade 9 learners' mathematical processes of solving word problems?
- What is the impact of exposing learners to problem solving strategies on their performance in solving word problems?

1.5 Structure of the Dissertation

A comprehensive discussion on the South African background of the need for problem-solving skills was outlined in this chapter. This chapter further orientates the reader on identified problems with regard to mathematical-problem solving and mathematical processes that can have a positive bearing on the computation of mathematical word problems. Questions that guided the focus in this study are also outlined in this section.

Chapter 2 covers literature review. Reference is made to previous studies with the main focus on secondary school level. Problem-solving models were also investigated in this chapter, and this was the stage at which the study aligned itself to one of the models identified.

Chapter 3 discusses the research methodology. The research design adopted is explicitly discussed and captured in this section. The participating group and their location are also dealt with in the chapter. Many of the opinions and findings expressed in the readings are reflected in the research designs, data gathering techniques and data analysis of this section.

Chapter 4 reports on findings and discussions from reviewed data segments that are presented in a form of episodes in a teaching experiment. A teaching experiment referred to in the paragraph is, according to Steffe and Thompson (2000), a living methodology designed initially for the exploration and explanation of students' mathematical activity. It involves a sequence of teaching episodes that include a teaching agent; one or more students; witness; and method of recording what transpires.

Teaching experiments in this study were informed by the cyclic nature of action research which works through various iterations of planning, acting, observing and reflecting. They were captured in two iterative phases, firstly through passive observations to identify problematic areas that needs attention and secondly through participative interventions as described in the methodology section below. These experiments are presented in this chapter through iterations of planning, devising the plan, implementing the plan and looking back. The iterations as informed by Polya's theory are directly linked with action research cycles identified.

Chapter 5 summarises all the concluding remarks and recommendations in this study. This section is also responding to the research questions posed in Chapter 1 of this report.

A list of references and appendices form part of the closing sections of this dissertation.

CHAPTER 2: LITERATURE REVIEW

2.1. Introduction

Mathematical problem-solving has been the subject of substantial and often controversial research for a number of decades (English & Sriraman, 2010). Problem solving as a term used in this study is used in a broad sense to cover a range of activities that challenge and extend one's thinking. Problem solving in an academic field involves being presented with a situation that requires a resolution (Snodgrass, 1988).

2.2. Definitions

The conceptual definition of problem solving in the mathematics classroom has become rather convoluted for several reasons. Perhaps the most significant reason is because no formal conceptual definition has ever been agreed upon by experts in the field of mathematics education. According to Chamberlin (2004), there seem to be some overlap in most definitions, but there is rarely an agreed upon definition of mathematical problem-solving.

Lester and Kehle (2003) attribute mathematical problem-solving to a thinking process in which a solver tries to make sense of a problem situation using mathematical knowledge he has and attempts to obtain new information about that situation till he can resolve the tension or ambiguity. They (Lester & Kehle, 2003) further suggest that reasoning and or higher order thinking must occur during mathematical problem solving. The existence of mathematical reasoning suggests that automaticity (Resnick & Ford, 1981) is absent. Hence, a pre-learnt algorithm cannot simply be implemented for successful solution. It is important to note that an algorithm may be used to solve some part of a mathematical problem-solving task.

However, in mathematics, problem solving is explained in terms of Polyas' steps as something that generally involves being presented with a written out problem in which the learner has to interpret the problem, devise a method to solve it, follow mathematical procedures to achieve the results and then analyse the results to see if it is an acceptable solution to the problem presented (Polya, 1945).

Literature is littered with a number of problem-solving models which, in some cases have contradictory views.

Mathematical problem solving as used in this study refers to a classroom situation wherein one is presented with a written out problem in which a learner has to interpret, devise the solution plan, apply the plan through mathematical procedures and finally analyse the results.

2.3. Problem Solving Models

The first model reviewed is that of Polya (1945) on which other models seem to be centred around. The model from his book, *“How to solve it”* is the one wherein heuristics and strategies of solving mathematical problems were emphasized. In the “*How to Solve it”* as captured in Polya (1945: xxxvi-xxxvii), he explicitly outlines the following captions and explanations of his model:

First. UNDERSTANDING THE PROBLEM.

- You have to understand the problem.
- What is the problem?
- What are the data?
- What is the condition*?*

Second. DEVISING THE PLAN

- Find the connection between the data and the unknown.
- You may be obliged to consider auxiliary problems if an immediate connection cannot be found.
- You should obtain eventually a plan of the solution.

Third. CARRYING OUT THE PLAN.

- Carrying out the plan of your solution, check each step.

Fourth. LOOKING BACK.

- Examine the solution you obtained.

The outlined problem-solving phases of Polya seem to be the foundation to all other models reviewed.

Polya (1957) also conceptualized a problem solving process as constituted by four key features, namely: understanding the problem, designing the plan, executing the plan and testing the solution. Meanwhile, Johnston (1994) viewed the process as comprising six critical steps, namely:

- Represent the unknown by a variable.
- Break the word problem into small parts.
- Represent the pieces by an algebraic expression.
- Arrange the algebraic expression in an equation.
- Solve the equation.
- Check the solution.

What appears to be core in both Polya's and Johnston's models is understanding of the given problem before solving it, for a learner cannot represent the unknown by a variable if he or she cannot make sense of the problem. After making sense of the problem, the learner needs to select or order the processes available to her or him. If the processes are not available to the learner, then it would be hard to move forward in solving given problems. Communication, operational, recording, and reasoning processes appear core or essential in the solving word problems.

The operational, communication, recording and reasoning as investigative processes and the mathematical processes, as highlighted, are used to put concepts, knowledge and skills to work in developing new ideas and exploring relationships (Orton & Frobisher, 1996). Communication processes include: explaining, talking, agreeing and questioning. Operational processes include: collecting, sorting, ordering and changing. Recording processes include: drawing, writing, listing and graphing. Reasoning processes include: collecting, clarifying, analyzing and understanding. All these processes contribute towards attaining mathematical processes as suggested by Frobisher. Little is known about how learners select a process that they feel is appropriate to the circumstances, or how they order processes to form a strategy. Communication and reasoning, as processes, actualize the understanding of any problem given in any context. It is actually important to understand the problem before one could plan for its solution.

Johnston's model seems not to differ much from what Polya has already outlined because it encompasses all the four problem-solving phases as outlined by Polya.

The other model that was identified was that of Descartes (1994) wherein extensive elaborations on Polya's methods were made. In the paper reviewed, Descartes' personally expressed his theory in this way:

> "*I believe that I should find the four (steps) which I shall state quite sufficient, provided that I adhered to a firm ,and constant resolve never on any single occasion to fail in their observance".* (p516)

The **first** of this was to accept nothing as true, which I did not clearly recognize to be so: that is to say, carefully to avoid precipitation and prejudice in judgments, and to accept in them nothing more than what was presented to my mind so clearly and distinctly that I could have no occasion to doubt it;

The **second** was to divide each part of the difficulties that I examined into as many parts as possible, and as seemed requisite in order that it might be resolved in the best manner possible;

The **third** was to carry on my reflections in due order, commencing with objects that were the most simple and easy to understand, in order to rise little by little, or by degree, to knowledge of the most complex, assuming an order, even if a fictitious one, among those that do not follow a natural sequence relatively to one another.

The **last** was, in all cases, to make enumerations so complete and reviews so general that I should be certain of having omitted nothing."

All the four cases as presented reflect back to what was initially proposed by Polya. When trying to check the theory presented, it becomes very clear that aspects of heuristics and strategies are emphasized. Descartes intended, in his third stage, to carry his reflections, commencing with objects that are most simple and easy to understand and thereafter proceeding to the knowledge that is more complex to understand. At this stage of problem solving, he actually emphasised the aspect of building knowledge on ones' own understanding as the difficulty of the problem rises.

However, D'Ambrosio (2007) further provided a new framework of problem solving which according to him is incorporated in the current research and is being referred to as "new thinking" in human activity. The conceptual problem-solving transition theory as presented by him is as follows:

- Given problems to Identifying the problems (problem posing).
- Individual work to Cooperative work (Teams).
- One solution problems to Open ended problems.
- Exact solutions to Approximate solutions.

This "new thinking" in mathematics education calls for what is sometimes called "story problems" (D'Ambrosio, 2007). A story problem, which is sometimes referred to as a word problem, appeals to the imagination of the solver. Word problems are normally open-ended or have multiple solutions and they usually call for cooperative work.

Harskamp and Suhre (2007) found that secondary education mathematics teachers teach students to solve mathematical problems by having them copy standard solution methods provided by the textbooks. They also noted that little time is devoted to the teaching of how to solve the problem. Problem-solving failures were found not to be the result of lack of mathematical knowledge, but rather the result of ineffective use of the knowledge. Schoenfeld (1992) stated that students need to learn to define goals and to self-regulate their problem solving behaviour in order to improve solving of non-standard mathematics problems.

Schoenfeld further identified the following as crucial episodes in problem solving*:*

- the problem
- making a plan
- carrying it out
- checking the answers against the question asked.

However, when looking at the episodes as named, they cannot be divorced from problem solving theory of Polya. I therefore feel it is important to explore further how this theory was used in various studies to enhance the computation of mathematical word problems. Even though the theory was negatively interrogated by Lesh and Zawojewski (2007), it still remained to be the underpinning theory in mathematical problem solving.

Jeremy Kilpatrick (1969) however presented a report on what he calls the twin issues of problem solving, which according to him are, how the problem is learned and also how it can be solved. He raised several issues around the teaching and learning of problem-solving which includes, problem-solving ability, problem-solving tasks, problem-solving processes, instructional programs and teachers influence. When he reviewed problem solving-processes, he viewed a mathematical problem as a prior index of processes used to arrive at the solution. He further recommended that problem-solving processes be studied by getting subjects to generate observable behaviour.

Kilpatrick further emerged with what he refers to as “the thinking aloud technique” which is according to him the primary source of informationprocessing approaches in the study of problem solving. Information rich heuristic rules of Polya were also recommended in his report.

2.4. Application of Models Identified

In an article by Nunokawa (2005), a problem was given to students and the problem was an introduction of multiplication of two-digit numbers. The paper had the primary purpose of re-examining the relationships between mathematical problem-solving and learning mathematics. Students have already learnt the algorithm of multiplication of three-digit number and one-digit number (e.g., 128 x 8) and multiplication of multiples of 10 (e.g., 4 x 30 or 40 x 30).

The approach to the problem was to construct small towers using plastic blocks. Students were given 23 blocks each and for them to determine how many blocks were needed if there were 12 children. In this problem, it was proposed that students could either add the number of blocks for each student 12 times or rather draws the situation consisting of blocks which are arranged in a certain pattern as a way of trying to unpack the problem. Nunokawa (2005) further suggests that if students partition 12 children into 10 children and the rest, the number of blocks for those 10 children can be calculated by 23 x 10. The remaining 2 children's blocks can be calculated easily, 23 x 2. Both of these multiplications are included in the mathematical knowledge the students already have. The total number of blocks can be figured out by adding these two products. The paper revealed that there are also some children using other ways of decomposing 12 (e.g., 6 + 6) or transforming the problem situation. It is further believed that through such reasoning, students obtain the new information about the situation that the total number is 276. This is in line with Polya's heuristic strategy of unpacking the problem for understanding whereby students were allowed to discover or learn on their own through discovery.

The paper revealed through discussions and conclusion, four problem-solving approaches in which much emphasis was based on the following: application of pre-knowledge; understanding the problem situation; new mathematical methods or ideas for making sense of the situation; and management of solving processes themselves. The latter aspect helps learners to become aware of the process nature of problem solving. This way they become equipped with skills that can be used in approaching other problem solving situations.

This paper finally exposes the fact that usual experiences of problem solving use to have complex features that correspond to more than one type. It also highlights the fact that even if we are aware of what we expect our students to obtain through problem solving, it is also important to choose appropriate problem situations, thus providing a suitable way of supporting students.

English and Sriraman (2010) revealed that a larger amount of research has focused primarily on word problems of the type emphasized in school textbooks or tests. This type of problem solving include “routine” word problems that require application of standard computational procedure, as well as “non-routine” problems, which involves getting from a given goal when the path is not evident or clear. It was found that in the non-routine problems, students actually struggled a lot in completing such. Upon realizing this problem, Polya’s book, *How to Solve it* (1945) was welcomed because it introduced the notion of heuristics and strategies. The book was highly valued by mathematics educators as a resource for improving abilities to solve unfamiliar problems, that is, to address the usual question of “What should I do when I’m stuck?”

It became explicitly evident that a large number of studies have gone into attempts to find out what strategies students use in attempting to solve mathematical problems, but no clear-cut directions for mathematics education are provided and that is part of what informed me to come up with a title on: “Improving Grade 9 learners’ mathematical processes of solving word problems”. Research revealed that problem-solving strategies are both problem- and student-specific often enough to suggest that hopes of finding one or few strategies which should be taught to most of the students are far too simplistic.

Schoenfeld’s (1992) review of problem-solving research also concluded that attempts to teach students to apply Polya-style generally had not proven to be successful, and he attributed the failure to the fact that heuristics and strategies are “descriptiveness rather than being prescriptive”. This was viewed as just names for large categories of processes rather than being well-defined processes. Even though some contradictory thoughts emerged with regard to the Polya-style, the overriding statement is that all are informed by Polya’s theory. I am of this opinion because Schoenfeld developed his model, which is extracted from the above stated theory, based on recommendations of problem solving research that “he” himself developed. He felt that problem-solving research should: help students develop a larger repertoire of more specific problem-solving strategies that link more clearly to specific classes of problems, foster meta-cognitive strategies so that students learn when to use their problem-solving strategies and content knowledge and to develop ways to improve students’ beliefs about the nature of mathematics, problem solving, and their own personal competencies.

English, Lesh and Fennewald (2008) were convinced that knowing when, where, why, and how to use heuristics, strategies and meta-cognitive actions lies at the heart of what it means to understand them. It was elaborated that, in the very early phase of complex problem-solving, students do not apply any specific heuristics, strategies, or metacognitive actions. They tend to simply brainstorm ideas in random fashion without understanding and following any predetermined way. When progressing towards a solution, however, effective reasoning processes and problem-solving tools are then needed. This study also emphasized the fact that students need to know what tools to apply, when to apply them, and how to apply them.

The recognition of the underlying structure of a problem was also seen to be fundamental in selecting the appropriate tools. For example, the strategic tool, draw a diagram, can be effective in solving some problems where structure lends itself to the use of this tool. In this case, the solver needs to know which type of diagram to use, how to use it, and how to reason systematically in executing their actions. However these claims cannot in one way or the other, be divorced from the theory of heuristics and strategies.

In concluding this argument, problem solving abilities are assumed to develop through the initial learning of basic concepts and procedures that are then practiced in solving word problems. The exposure to a range of problem-solving strategies and applications of these strategies to novel or non-routine problems usually follows.

Palm (2007) presented a study that sought to investigate the impact of authenticity on the students' disposition to make necessary real-world considerations in their word problem-solving. The aim is to also gather information about the extent to which different reasons for the students' behaviour are responsible for not providing solutions that are consistent with the 'real' situations described in the word problems. In this study, it was found that when elementary and secondary school students from different parts of the world are presented with word problems, they often provide solutions that are inconsistent with 'real' situations described in the task. The general conclusion is that students have the tendency not to make proper use of their real-world knowledge and to suspend the requirement that their solutions must make sense in relation to the real situations.

Several reasons for the 'unrealistic' answers have been suggested in the literature, and one of those is that students' solution strategies comprise mindless calculations and do not include considerations of real life aspects of the situations described in the task. They also showed that a major reason for the difference in the type of realistic considerations made is a different interpretation of what the task demand. Tasks used also seem to be a source of "unrealistic' solutions. Tasks themselves are often 'unrealistic' in the sense that important aspects of real situations described in the task are not well emulated in word problems. Thus, when students are faced with 'unrealistic' mathematical school task they in most cases turn to provide unrealistic responses.

In this study, data were collected through tests and interviews and before tests were administered, students were told of the intentions of the test as part of the ethical considerations. Students were also encouraged to reflect on their understanding of solutions made as part of their commentary remarks. Interviews were structured and were based on questions. The purpose of the interviews, in this case, was to learn from students about mathematical task solving.

Finally, this study provided evidence about the reasons that students provide solutions that are not consistent with realities of the 'real' situations described in the word problems used in investigations. It showed that not all 'unrealistic' answers provided by the students in the study stem from total 'suspension of sense-making'.

2.5. Theoretical Framework for the Study

This study is framed by George Polya's theory which views mathematical problem solving as being presented with written out problems which requests one to interpret, devise the solution method, follow mathematical procedures to achieve the results and then analyse the results. It is through the outlined steps that this study turned to adopt the theory because it addresses problems related to mathematical word problem. Primary sources of data in this study were mathematical word problem activities that, according to Polya (1945), require heuristic and strategies theory for better solution.

The heuristics and strategies theory of Polya seem to be the foundation theory of all theories reviewed. In this theory, he (Polya) initially noticed that students don't know how to solve problems and he further accentuates that the difficulty was not associated with learners not knowing mathematics but rather lacked the ability to their thought processes along fruitful channels. The claim is still a challenge to most learners studying problem solving. It however resulted in the overwhelmingly accepted heuristics strategies which he developed.

The theory requests learners to understand the problem, devise the plan, carry out the plan and interpret the results. The aspirations of this theory seem to be in line with what the study envisages. The study envisages a learner who is able to understand what the given problem is all about; the given data; and the conditions attached to each problem, before proceeding to the next stage, and this is also what the theory is advocating.

It is through the understanding of connections between the data and the unknown in a given problem that one can arrive at a relevant plan for the problem. The given mathematical word problems also request learners to come up with the plans for the solutions to the problems assigned.

The implementation stage of Polya requests learners to implement the proposed plan and check each step as procedures unfolds. Finally, learners are expected to interpret the results and check as to whether they make sense to the problem. Those are also expectations from everyday mathematics problem-solving lessons and that is why the theory was deemed fit for the study.

2.6. Chapter Summary

It is through the current chapter that I tried to draw the attention to the substantial amount of work done in the area of mathematical problem solving. Different perspectives from which mathematical problem-solving is defined were also catered for in this section of the report.

Mathematical problem-solving seems to be influenced by a number of problem solving models. Most of those problem-solving models were discussed in the chapter and a choice of the most proficient one for the study was made, resulting in a theoretical framework for the study. That is the choice that guided methodological issues and findings in chapters that follow in the report.

CHAPTER 3: RESEARCH METHODOLOGY

3.1. Introduction

Research is a systematic process of collecting and logically analysing data for some purpose. This inquiry of making informed decisions can either adopt one of the following methodological paradigms, the qualitative approach or quantitative approach. The word “paradigm” as used in this context is, according to Maree (2010), referring to a set of assumptions or beliefs about aspects of reality which gives rise to how we view reality.

This study is a qualitative study, which is described by Denzin and Lincoln (1994) as a multi-method in focus, involving an interpretive, naturalistic approach to its subject matter. The paradigm was chosen because the focus of the study is on exploration of processes that are used in solving mathematical word problems. It gives thick descriptions of the case rather than the degree to which generalisation claims can be made. The choice was informed by my experience of working with Grade 9 learners who are mostly not performing satisfactorily when solving mathematical word problems. The approach, as Creswell (2007) has alluded to, is justified as it addresses the silent voices or the complexities that often mar the understanding of issues being explored.

Denzin and Lincoln (1994) attribute qualitative research to deploy a wide range of interconnected methods such as case study, personal experience, life story, interviews, observations and interactions that describe routine and problematic moments and meanings in individuals’ lives. According to Creswell (2007), qualitative research begins with assumptions, a worldview, the possible use of a theoretical lens, and the study of research problems inquiring into the meaning individuals or groups ascribe to a social or human problem. It is because of the underlying assumptions, personal experiences and the need for the inquiry in research problems, that I align this study with qualitative approaches.

Through this chapter, I discuss the research methodology wherein issues of research design in qualitative studies are explicated and captured. The participants and their location are also dealt with in the chapter. Issues of data collection and data analysis are also addressed in this chapter.

3.2. Research Design

A research design is a plan or strategy that moves from underlying philosophical assumptions to specifying the selection of respondents, the data gathering techniques to be used and the data analysis to be done (Maree, 2010). In the research literature (Cohen, Manion & Morrison, 2000; Leedy & Ormrod, 2005 and Creswell, 2007, 2009 & 2012) six types of qualitative research designs are often discussed: Conceptual Studies, Historical Research, Action Research, Case Study Research, Ethnography and Grounded Theory.

Conceptual research is largely based on secondary sources and it critically engages with understanding of concepts and it aims to add to the existing body of knowledge, whereas historical research is a systematic process of describing, analysing and interpreting the past based on information selected from sources as they relate to the topic under review.

Case study design is, according to Bromley (1990), a systematic inquiry into an event or a set of related events which aims to describe and explain the phenomenon of interest and it does not differ much from Action Research which draws attention to collaborative or participative dimensions and to the focus on practical problem experienced by participants for which the practical solution is sought.

Ethnography, as a term, has traditionally been associated with anthropology and more specifically social and cultural anthropology. In the field of anthropology, ethnography means the description of a community or a group that focuses on social systems and cultural heritage.

Action Research draws attention to collaborative or participative dimensions and to the focus on practical problem experienced by participants for which the practical solution is sought. As a research design, it often utilize both quantitative and qualitative data, but they focus more on procedures useful in addressing practical problems in schools and in classrooms (Creswell, 2012). It is a systematic procedure done by teachers or other individuals in the education setting to gather information about and subsequently improve the ways their particular educational setting operates, their teaching and their student learning (Mills, 2011)

However, Maree (2010) describes Action Research as being typically cyclical in terms of data collection and analysis. It starts with identifying the problem, collecting data (through the use of a variety of data gathering techniques), analysing data, taking action to resolve the problem and assessing/evaluating the outcome of the intervention.

Informed by the above attributes the study took the shape of an action research through which practitioners study their own practice to solve their personal problems. It was an Action Research study in my own classroom with which I have contact almost everyday. It is through this Action Research that I aim to improve the practice of education by studying issues or problems faced and reflect about these problems.

Teacher Action Research, as described by Nixon (1987), is concerned with the everyday practical problems experienced by the teachers, rather than the “theoretical problems” defined by pure researchers within a discipline of knowledge. Research is designed, conducted, and implemented by the teachers themselves to improve teaching in their own classrooms. The prevailing focus of a teacher research is to expand the teacher’s role and to inquire about teaching and learning through systematic classroom research (Cooper, 1990).

A review of many major writers in educational research revealed that there are two basic Action Research designs, which are, namely, practical action research and participatory action research. Participatory Action research has, according to Creswell (2012), a long history in Social Inquiry which concentrates more on organisations outside education whereas in practical action research teachers seeks to research problems in their own classrooms so that they can improve their students' learning and professional performance. It is against the above sentiments that I consider this study follows a practical Action research design since its purpose and participants are located within the education perimeters at a school. However, data management and reporting use teaching experiments concepts and techniques. That is, individual designs were not used in their purest forms but, in a way that fitted with my experiences as a classroom teacher and a researcher.

3.3. Site and Participants

Sampling in qualitative research refers to a process used to select a portion or participants for study. Qualitative research is based on non-probability and purposive sampling rather than probability, which can, in simple terms, be described as Simple Random sampling. Purposive sampling decisions are not only restricted to the selection of participants but also involve the settings, incidents, events and activities to be included in data collection. Patton (1990) argues that the logic and power of purposeful sampling lies in selecting information-rich cases for study in depth. These cases are the ones from which one can learn a great deal about issues of central importance to the purpose of the research. The following three sampling procedures are commonly used in qualitative research, that is, Stratified Purposive sampling, Snowball sampling and Criterion sampling.

Participants in this study were Grade 9A learners at a school in the Mogodumo cluster of the Capricorn District in the Limpopo Province. The school had 263 Grade 9 learners who were evenly distributed in five classrooms of which one of those classrooms is Grade 9A with 64 learners of which 28 are males and 36 are females.

The site was purposefully chosen based on its extreme characters of heterogeneous combination in terms of gender, intellect, background and it was my class wherein daily activities were taking place. That was supported by Merriam (1998) when she accentuated that purposive sampling is mostly based on the assumptions that the investigator wants to discover, understand and gain insight and therefore must select a sample from which most can be learned. The background referred to above includes a number of factors that, amongst others, is the place where learners are coming from, their respective families in which they grew up and the type of education they were exposed to at primary levels. Those, I claim, can have a bearing on how learners view things. Learners in this group come from different villages around the school.

This study therefore adopted purposeful sampling procedures in selecting information rich cases for analysis. In purposeful sampling, information rich cases for the study in-depth are selected and these are selected when one needs to understand something about those cases without needing or desiring to generalize to all such cases (McMillan & Schumacher, 1997). Since 64 learners were engaged in learning activities in two stages, their responses were also purposefully sampled for analysis. Information-rich transcripts of participants to study more about emerging mathematical processes of solving word problems were looked into. The sampling by case type strategy of purposeful sampling was used in the study whereby extreme cases were chosen after knowing typical or average cases. In this context, “case” refers to an in-depth analysis of a phenomenon and not the number of people sampled.

3.4. Data Gathering Techniques

Collecting data always involves generating data, selecting data, and the techniques for data collection. All this has effect on what finally constitutes ‘data’ for the purposes of research. The data collection techniques used, as well as specific information considered to be data in a study, are according to Merriam (1998) determined by the researcher’s theoretical orientation, by the problem and purpose of the study, and by the sample selected. This therefore justifies the learning activities that are used as tools for data collection in this study, which were deemed suitable for the problem identified.

Data in qualitative research can be collected through four basic data gathering techniques, which are, namely, observations, interviews, documents and audio-visual material. Observations are those in which the researcher takes field notes on the behaviour and activities of the individuals at the research site whereas interviews intend to elicit views and opinions from the participants through face to face interactions or telephonic conversations.

Audio and visual material may take the form of photographs, arts objects, videotapes or any form of sound, whereas documents are regarded as written communication that may shed light on the phenomenon that one is investigating. According to Maree (2010), documents or written data sources may include published and unpublished documents; company reports; memoranda; agendas, administrative documents; letters; reports; email messages; faxes; newspaper articles; or any other document that is connected to the investigation.

Learning activities as primary sources of data were given to learners in two teaching experiments. A teaching experiment is, according to Czarnocha and Maj (2008), a classroom investigation of teaching and learning process that is conducted simultaneously with teaching aimed at the improvement of learning in the same classroom and beyond. A teaching experiment, as presented in this study is not holistically used as a tool for data collection but rather as a mechanism for data management.

The activities were chosen because they serve as written evidence, which saves the researcher time and expense for transcribing. Even though they require the researcher to search out the information in hard-to-find locations, they yield the data that are thoughtful and which enable the researcher to draw conclusions about the hidden views and opinions of the respondents.

In the first teaching experiment, learners were to respond to the given activities prior my intervention and thereafter submitted their written responses for assessment. This experiment had three learning activities which unfolded over the period of three days. My intentions at this phase were to identify processes learners might use prior to my intervention when solving mathematical word problems.

Learning activity 1 was completed in groups (four groups of eight members per group) and attempted to address the following three assessment standards:

- solve equations using trial and improvement;
- solve equations using algebraic processes; and
- use equations to solve everyday-life problems.

Learning activity 2 was completed individually and attempted to address the following two assessment standards:

- solve problems in context including contexts that may be used to build awareness of other learning areas; and
- calculate by selecting and using operations appropriate to solving problems and judging the reasonableness of results.

Learning activity 3 was completed in pairs and attempted to address the following assessment standards:

- solve the equation using algebraic processes;
- use equations to solve everyday problems; and
- use proper techniques to plan for the solution of the problem.

The second teaching experiment involved the completion of same learning activities given in the first phase with a bit of assistance. The experiment took place two days after learners made submissions of their first attempts. The procedure was to start by attaching meaning to the given problems by explaining. This was followed by asking learners questions based on their attempts and, in certain cases, gave leading inputs. My intention was to attempt guiding learners towards using mathematical processes involved in the activities. Finally, learners were requested to submit their written responses after the intervention. This resubmission of responses was to allow me to find out if learners had improved on their thinking and whether or not they had changed ways of responding to the questions.

3.5. Data Analysis

Data analysis involves making sense out of the text and image data. It involves, according to Creswell (2009), preparing data for analysis; conducting different analysis; moving deeper and deeper into understanding the data; representing the data; and making an interpretation of the larger meaning of data. In qualitative data analysis, the researcher collects qualitative data, analyse it for themes or perspectives, and reports on the emergent themes. The analysis of data in this case went through three entwined stages, which are, namely, the selection stage, the preparation stage and the evaluation stage.

3.5.1. The selection stage

Qualitative studies are not at all times subjected to analysis of all the data collected, but in some instances information-rich cases for the study in-depth are selected. That is done when one needs to understand something about those cases without needing or desiring to generalize to all such cases.

I preferred at this stage to purposively select information-rich cases from the data that respondents provided. Consistency was not always maintained in terms of the number of selected cases. An account for such is provided in the next chapter whereby I deal with specific teaching experiments.

3.5.2. The preparation stage

Qualitative data collected sometimes tend to be very lengthy and require extensive examination, understanding and reading. This section therefore presents different levels of organisation through which large volumes of collected data from 64 respondents went through and finally resulting in meaningful themes for presenting arguments and drawing conclusions. Thick descriptions of participants and site, which are regarded as valuable for introducing this section, were dealt with in the introductory sections of this chapter. The collected data were then organised in three levels as presented below:-

3.5.2.1. Level 1

Participants were subjected to three learning activities whereby activity one was completed in groups, activity two individually and activity three in pairs. From the chunk of data that were presented, information-rich cases from all activities were selected. The selection was informed by the extent to which learners responded to activities. The said criterion is considered because, in some cases, learners just resorted to writing answers only or copying the activity as presented and such responses could not present anything for discussion.

3.5.2.2. Level 2

This is the level through which the information-rich cases underwent review as per priori coding system developed from literature. Priori codes are codes developed from literature related to the topic under review. The codes are developed from Polyas' problem-solving strategies, which are sequentially progressing as per the illustration below:

Understanding the problem ➨ Devising the plan ➨ Carrying out the Plan ➨ Looking back

There are some criteria within each problem-solving strategy and each case was subjected to review under the said criteria. For example, a script from one group, pair or child went through review of the stage on understanding the problem guided by the following criteria:

- Do they understand the problem?
- What is the problem?
- What are the data?
- What is the condition?

The said system was maintained throughout with all other selected cases as they progressed on to other problem-solving strategies. This system emerged with narratives or stories for categorisation and identification of common trends.

3.5.2.3. Level 3

This stage involved the clustering of common themes together for discussions. Themes that emanated from review stage were then clustered accordingly for meaningful discussions.

3.5.3. The evaluation stage

The analysis of the selected data was done in two distinct phases. In phase 1, I focused on the evaluation with respect to problem-solving stages. In phase 2, the attention was on the emergent issues from phase 1, with the view of identifying categories of challenges learners had.

3.5.3.1. Phase 1- Evaluation of information rich cases

Information rich cases were subjected to review guided by criteria emanating from Polyas' problem-solving stages. The said criteria can be referred to in subsection 2.3 of Chapter 2 above on literature review. For example, the first stage of problem solving goes with understanding of the problem. Amongst the criteria within this stage we have: What is the problem? What is the data? and What is the condition?

All the selected responses were subjected to a review using the criteria, and the pattern of evaluation was maintained throughout in the phase with other problem solving-stages. The purpose of this phase was to identify challenges learners encountered across the four stages of problem solving. The challenges in each stage can be regarded as narratives of learners experience which are the subjects or the focus of the next phase.

3.5.3.2. Phase 2 – Categorisation of emergent data

In this phase, I now turn my focus to challenges or narratives emanating from each problem solving stages in phase 1. The emergent themes showing common trends were then grouped together into categories, which assisted me in, amongst others, to establish stories from each category, relate the shift between two stages of data analysis and select pieces of evidence to use in the main report. These narrative threads are, according to Creswell (2009), emergent themes covering finer issues raised.

These finer issues or categories were then analyzed qualitatively using narrative analysis and, subsequently, analysis of narratives, that is, the analysis of stories emerging from main ones. The word narrative in this context refers to a particular type of a discourse, which is a story, but not an ordinary one, because, Polkinghorne (1995) cautions that a story carries a connotation of falsehood or misrepresentations as in the expression: “that is only a story” (p.7). The two stages of data analysis of this segments alluded to refer pre-intervention analysis of threads and post- intervention analysis of threads.

3.6. Quality Criteria

Learning activities were validated and checked for reliability through piloting. The second class of Grade 9 learners was used as a pilot group to validate this instrument (learning activities) before the actual study could be carried with the sampled participants. At this stage, students’ respond once to learning activities with the primary aim of checking whether or not the activities will yield what is expected.

It is generally accepted that engaging multiple methods of data collection which is commonly known as triangulation, enhance trustworthiness in research. The study adopted document review as a primary source for data collection and informally used observations in stage two when respondents were responding to learning activities with interventions. This was done to capture exactly what transpired during interventions.

The study also adopted member checking as a strategy to enhance design validity. Member checking is usually used in participant observation studies to check informally with participants for accuracy during data collection. I therefore decided to use this strategy to informally interact with learners' responses during the second stage of data collection as they were engaged in activities and also responding to verbal questions for clarity. This eliminated misconceptions as I was interacting with them for justification of all inputs made in their responses.

Maintaining a record of data management techniques and decision rules that documents the chain of evidence account for audibility in research. Procedures for choosing information-rich cases, evaluation of cases and categorisation of emergent themes were maintained throughout the process of data management that served as an account for audibility in this study, which is a strategy to eliminate bias in research.

3.7. **Ethical Considerations**

Ethics are generally considered to deal with beliefs about what is right or wrong, proper or improper, good or bad. Openness and honesty were catered for in this study by informing participants about the plans and aspirations of the study.

Taking into account that the class that undergone the study is my own classroom; I informed the management and parents of the affected group through letters of my intentions to undertake the study. Through the permission granted, I then informed the participants of my intensions and benefits of the study to be carried in order to draw their attention to the study. [see Appendix A] The information obtained about the participants is at all times held confidential and the only people who will be exposed to it will be the researcher and the subjects. Names that are used in the narrated stories as the analysis unfolds are not real names of participants. That is done as a way of protecting anonymity of the participants.

3.7. Limitations of the study

Post intervention assessment was very limited in the sense that learners were not tested on how they would approach new problems. Instead, the same set of problems that were used in the pre-intervention stage and during the intervention stage were used. Prolonged exposure of the problems might have impacted on the ultimate performance.

3.9. Chapter Summary

In this chapter, I addressed qualitative methodological issues. Different research designs were discussed with the view of aligning the study to the most suitable one. Issues of selecting the location and participants for this study were also catered for in this section. Data gathering techniques; the how part of analysing data; ethical considerations; and quality criteria of the study were also fully captured.

CHAPTER 4: FINDINGS AND DISCUSSIONS

4.1. Introduction

In an attempt to answer the research questions as outlined in Chapter 1, learners were exposed to learning activities in two stages. In the first stage, I remained a passive observer, whereby learners worked on the learning activities on their own. In the second stage I became an active participant and guided the learners by asking them guiding questions. In Chapter 3, I gave an account of how the data emanating from those two encounters were captured. Through the current chapter, I present the findings and discussions in relation to the research questions informed by the theory already identified in my literature review.

Findings at this stage are presented through two problem-solving teaching experiments that are unpacked into episodes. The first teaching experiment captured responses to learning activities prior my formal interventions whereas the second one captured post intervention responses. Both teaching experiments comprised three episodes each.

Names of the groups presented in this chapter are not always the names of the group leaders in each individual group. The “Merlyn group” adopted its name from Thutse Merlyn who acted as the group leader for that group whereas the “Charlo-moro group” adopted its name from the first letters of the names of the group members. “Charlo” is from the name Charlotte and “Moro” is from the name Morongoa. Both of them are members of the “Charlo-moro group”

The two teaching experiments were analysed using Polya’s four stages. The results are also reported in a similar way. Before concluding the chapter, I present reflections from two teaching experiments and also the post intervention review that is answerable to questions raised.

4.2. The Problem Solving Teaching Experiments

Teaching experiment is used in this study as it presents us with a structure through which we organised data. The format allowed me to capture exactly how learners responded to questions and also how the intervention unfolded. For reflective purposes, however, Polya's problem solving stages were used. The repeated iterations of reflections as the study proceeded from one teaching experiment to another and one episode to another allowed me to integrate action research processes into the study. That is, the iterating cycles of planning, acting, observation and reflecting fitted well into the teaching experiments and their associated episodes. At the end, the study design became an integration of teaching experiments and action research.

The two teaching experiments used in this study are based on the same set of learning activities. They are differentiated in terms of my role during the classroom sessions. In the first teaching experiment, I let the groups work on their own without any form of interference. Learners discussed their work and agreed on their responses to each of the three assigned activities. They were required to apply their skill of interrogating a question for understanding and even develop mechanisms through which problems could be solved. Applications of the planned strategy are further demanded to check the credibility of the strategy.

In the second teaching experiment, the learners were now assisted by the teacher to solve the problems as presented in the three activities.

The three learning activities that were used were as follows:

Learning Activity 1

(a) Tina and Nomusa are knitting squares for a blanket. Tina knits four times as many squares as Nomusa. They need 100 squares to complete the blanket. How many squares do they each knit.

(b) The perimeter of an equilateral triangle is 210mm. calculate the lengths of the sides.

(c) A mother is seven times as old as her daughter. In five years' time she will be four times as old as her daughter. How old are they now? (Hint: let the daughter's age be x.)

Figure 2: Learning Activity 1

Activity 1(a) requested learners to calculate the number of squares for the blanket that the girls would knit given that one girl (Tina) was knitting four times the squares of the other one (Nomusa) and the total number of the squares needed amounted to 100.

Activity 1(b) was entirely based on determining the sides of a triangle given the perimeter, with the hope that learners will develop the plan and apply it properly. It was further hinted in the given statement that the triangle is an equilateral triangle with the perimeter of 210mm.

Learning activity 1(c) requested learners to calculate the ages of the child and the mother given the two domains. At first, it was highlighted that the mother was seven times as old as the child, and as the statement continued it was indicated that, in five years, the mother will be four times as old as the daughter. The two as presented in the problem needed to be read concurrently and be separated into various parts as per conditions.

The second episode of the two teaching experiments is based on the learners' attempts to undertake learning activity 2.

Learning activity 2

A boy cycles from home to school in the morning and back home in the afternoon. He cycles from home to school at 32km/h and back at 24km/h. It takes him 15 minutes longer in the afternoon than in the morning. Find the distance between his home and the school.

(a) Let the distance between his home and the school be x km.

(b) Copy the table in your workbook and complete it.

	Distance	Speed	Time
Morning	x km	32 km/h	$\frac{x}{32}$ h
Afternoon	...	...	...

Figure 3: Learning Activity 2

The activity requested learners to determine distance from two points given the speed and time that are different. That is, the time taken from home to school differs because of different speed at which the distance was travelled.

The third episode emanated from the learners' engagements with the third activity as outlined hereunder.

Learning activity 3

Mr Tsewete decides that at the end of each year he will take his Grade 9 class on an outing to Ratanga Junction. He decides to pay for the bus fare, entrance ticket and meal for his learners.

Mr Tsewete has 48 learners in his class and wants to calculate the total cost of the outing.

1. Work out the total cost of the outing and then compare answers and methods in class.
2. What is different and what is the same about the methods or calculations used in class.
3. Will these methods always work even if the number of learners or prices changes.
4. Which or whose method is best? Discuss this in class.

Figure 4: Learning activity 3

This episode was entirely based on making connections between the gained mathematical knowledge and real-life. The main objective was to identify mathematical concepts that could be useful in addressing real life situations.

4.2.1. ***Teaching experiment 1 – Prior intervention stage***

At this stage, I present the findings and discussions of narratives that emerged from data presented prior interventions. This is done to track, amongst others, the extent to which learners understand the subject;, their strengths and weaknesses; and processes employed when solving mathematical word problems. The discussion itself is framed by the four problem-solving stages of George Polya.

The teaching experiment is used in this context as a diagnostic measure for learners' problems and a determining agent for issues rose in this section. It is used to capture exactly how learners respond to questions in relation to mathematical word problems and also to identify challenges and approaches learners had. This experiment unfolded through three episodes that spread over a period of three days. The subsequent discussions in this teaching experiment are informed by George Polya's problem-solving strategies, which are explicitly appearing as:

- Understanding the problem;
- Devising the plan;
- Implementing the plan; and
- Looking back.

4.2.1.1. Understanding the problem

Understanding the problem entails working through the problem of which one desires the solution. Learners at this stage should be able to point out the principal parts of the problem, the unknown, the data, and the conditions attached to the problem.

As the first step of problem solving in Polyas' strategies, it can always be marked under the following conditions:

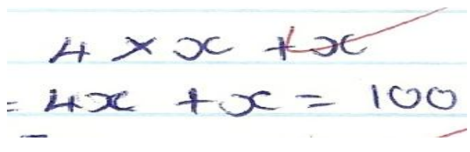
- What is the unknown?
- What are the data?
- What is the condition?

The analysis of learners' responses at this stage emerged with three issues. At first, learners were directly picking up numbers from the given problems letting alone words which goes with the problem. Secondly, learners were not unpacking the problem for better understanding. Emergence of little or no understanding of the problem was also identified as a problem. Communication and reasoning, as investigative processes in problem solving, appeared to be at a lower level at this stage.

The issue of direct translation of numbers from the problem into mathematical statements developed, relates to a situation wherein learners look for numbers as they appear in a statement. The words that provide contexts to the numbers are ignored or omitted. These are words that carry the meaning of the whole statement. The operational words, such as times, plus, etc., are also targeted. This is demonstrated in responses from groups of learners who repeatedly committed the same mistakes in two of the activities in various episodes.

<u>Activity 1a</u>

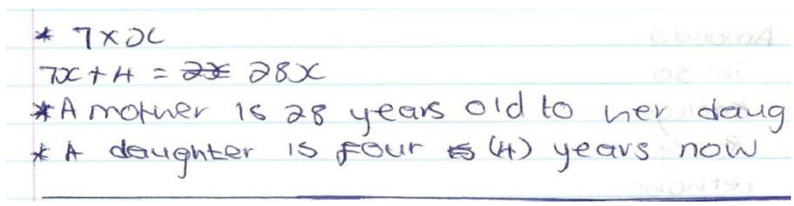
BALTYMORE GROUP


$4 \times x + x$

$= 4x + x = 100$

<u>Activity 1c</u>

BALTYMORE GROUP

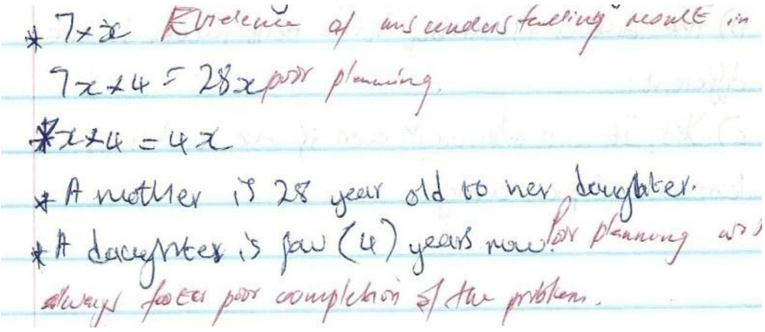

* 7 × x

7x + 4 = 28x

*A mother is 28 years old to her daug

* A daughter is four (4) years now

GO-KAT GROUP



* 7×x Evidence of misunderstanding results in
7x+4 = 28x poor planning.
*x+4 = 4x
* A mother is 28 year old to her daughter.
* A daughter is four (4) years now. Poor planning will always foster poor completion of the problem.

Learners at this stage tend to directly translate the question into a mathematical statement. They just resorted to picking up numbers only from the given problem and try to formulate the linear mathematical statements from them. That is justified by ways in which the Baltymores and the Go-kats groups responded to this activity.

When checking through what the Baltymore conferred in activity 1(a), the four as captured in the response is a direct translation from the statement that directed learners that one girl was knitting four times the squares of the other one. The times that followed was captured as the multiplication sign after the four. That in itself justifies direct translation.

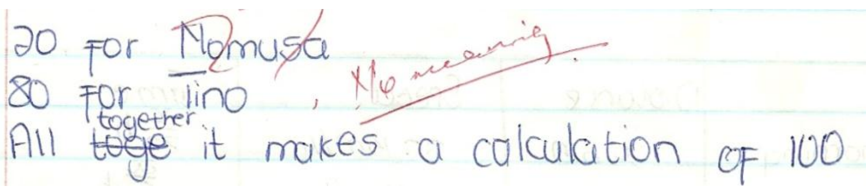
The *x*in this case, as presented by the Baltymores in activity 1(a), was taken from the normal mathematical trend of representing the unknown with a variable. X is the most common letter to be used in most expressions. I could hardly trace any understanding in what was presented. It was fortunate that they could realise that the 4x should be added to the x to make up 100, and that in itself revealed that 100 was just directly ordered from the problem into the equation developed. The practice in itself does not display any sense of understanding the problem but rather direct linear translation.

The Baltymores and Go-kats gave common responses to this activity as captured above, also revealed an element of linear translations. The 7 and 4 in the equations of the two groups were directly extracted as they appeared in the problem statement. The seven was ordered from the statement that reads:“the daughter is seven times as old as the mother", while the four was ordered from the fact that over a stated period of time the mother will be four times as old as the child. Realising that the five years in the statement could not be placed in the equations, they resorted to multiplying the seven by four in order to come up with 28.

Learners' responses as captured above did not reveal any element that justifies the understanding of the problem for better solution. Their intentions could not be traced in their plan. Learners did not even bother of what the problem was all about but they rather resorted to linearly translate the problem.

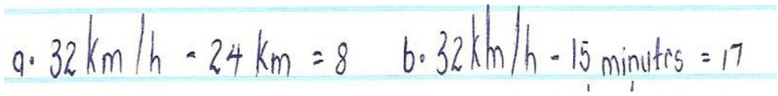
The act of unpacking the problem for better understanding still appeared to be missing in how the Mbongeni group responded to learning activity 1(a). This refers to breaking the problem into different meaningful parts for better understanding.

MBONGENI GROUP



The 20 and 80 in their presentations, as recorded above, could hardly tell a story on how learners unpacked the problem for understanding. The group can, however, be credited for the correctness of 20 and 80 which sum up to 100. Yes, indeed 80 is four times 20 and that accounts for the fact that one learner knits four times the squares of the other one. The challenge in this presentation is that the group only considered the number of squares knitted to be 100. An account for the 20 for Nomusa and 80 for Tina, as presented in their workings above, could not be traced. They did not even consider context within which the problem was presented.

The issue can also be traced in episode 2 when a learner called Thaetso presented confusing statements as follows:

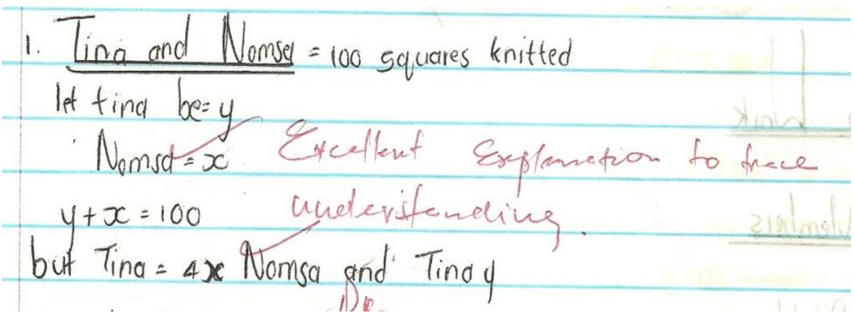


The direct translations were also spotted in these workings. The 32 and 15, as placed in the two equations, were directly picked up from the problem. The sums 8 and 17 did not have any meaning in the statements.

Thaetso appeared to have little understanding of what was expected of him. In presenting the equation numbered (a), it could not really be traced what prompted him to subtract 24km/h from 32km/h. The response was really confusing in itself.

However, one group from the rest (the Charlo-moro group) presented a positive response which positively addressed the understanding stage of problem solving. The initial part of their algorithm which responded to the aspect on understanding was explicitly captured in this way:-

CHARLO-MORO GROUP



Looking at their first statement, they tried to bring the understanding that both Nomusa and Tina are knitting squares that, when completed, will together amount to 100. They even decided to differentiate Tina and Nomusa's squares with different variables x and y. In the third statement, it was indicated that the sum of the two is 100. Finally, they declared their understanding by unpacking the problem thereby represented Tina with 4x and Nomusa by y. These really depicted the understanding borne in this group. The presentations seem to be the explanation of factors used in developing the plan for the solution.

It is a general trend from most groups that they work out the problem without attaching meaning and the practice always leads to literal translation of the problems. Learners display that they are still lacking the skill of interrogating the problem for better understanding. They fail to unpack the problem in the three domains of understanding the problem. Their inputs did not display their understanding of the data provided in the problem and also the context within which the data were provided. The claims justified above retarded learners' capability of devising the plan for the solution which is a problem solving stage to be discussed in the next problem solving level.

4.2.1.2. Devising a plan

In addressing this second stage of problem solving, one needs to find connections between the data and the unknown, thereby applying operational processes which includes amongst others, collecting, sorting, ordering and changing. In some cases auxiliary problems may be considered if an immediate connection cannot be established. These are problems related to the problem in question. All of the above attributes eventually lead to the establishment of the plan for the solution.

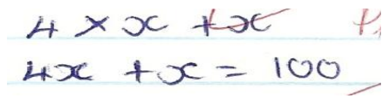
According to Polya (1945: 8), "We have a plan when we know, or know at least in outline, which calculations, computations, or constructions we have to perform in order to obtain the unknown".

At this stage of problem solving issues of plans which were not accounted for and issues of attaching meaning to the plan in relation to the problem were marked. That is checking whether or not the developed plan is relevant to the question. In most cases learners just embark with the calculations before even getting to know what the problem is all about. Sometimes plans provided revealed that learners did not devote time to check the authenticity of the plans to the problem. This stage has a direct link with the first one because ones' understanding can be traced in the plan.

I therefore decided to present narratives in a form of positive stories (the ones showing a bit of understanding in devising the plan) and negative ones (the ones showing no understanding of devising the plan). The negative and positive side as referred in the above sentence only refers the position in which the response is clustered. The said responses are therefore captured in this way:-

<u>Activity 1a</u>

BALTYMORE GROUP

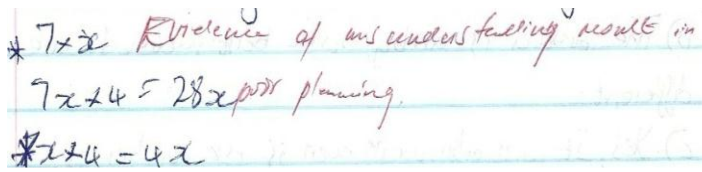


<u>Activity 1b</u>

BALTYMORE GROUP

<u>Activity 1c</u>

GO-KAT GROUP



The Baltymores' plan, as presented for activity 1a, cannot clearly reveal the group's intensions because their understanding of *4xc*annot be traced from the plan. What could be told is that their sum is 100.

The *4 xx + x* that serves as the initial plan for the question is just presented as a mathematical statement with no explanations of what the *4* and x in the equation stood for. Even though the groups' understanding of an equilateral triangles can be spotted in the plan presented above for activity 1b, one could hardly tell what the P, 3 and s stand for in the plan. It would be better if the group went to an extent of explaining that the P stands for perimeter;, 3 stands for the number of sides of a triangle; and s is for the common sides of an equilateral triangle. The statement in itself will have accounted for the proposed plan.

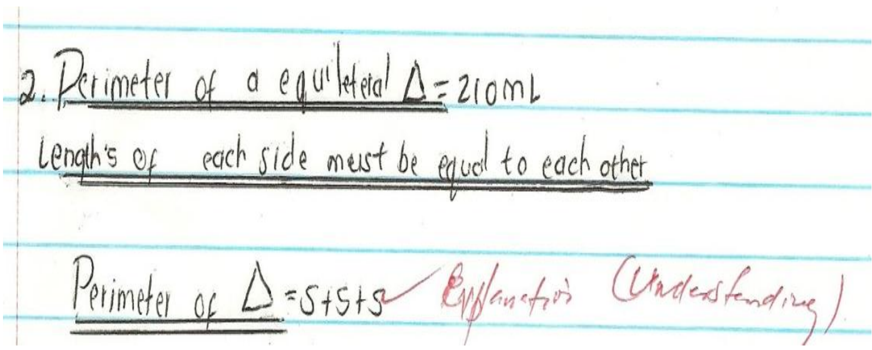
The *7* and the *x* as presented by the Go-kat group above are not accounted for and the group further expanded their plan to *7x x 4* which is equated to *28x*. The meaning behind the introduction of 4 and the sum of 28x can hardly be traced in their plans. These also revealed that the plan was not traced back into the given problem for authenticity.

An account for every term introduced in a mathematical statement as a plan for solving a particular problem needs to be clearly reflected. That will further assist the problem solver to have a clear picture of the problem and even realise connections between the data and the unknowns.

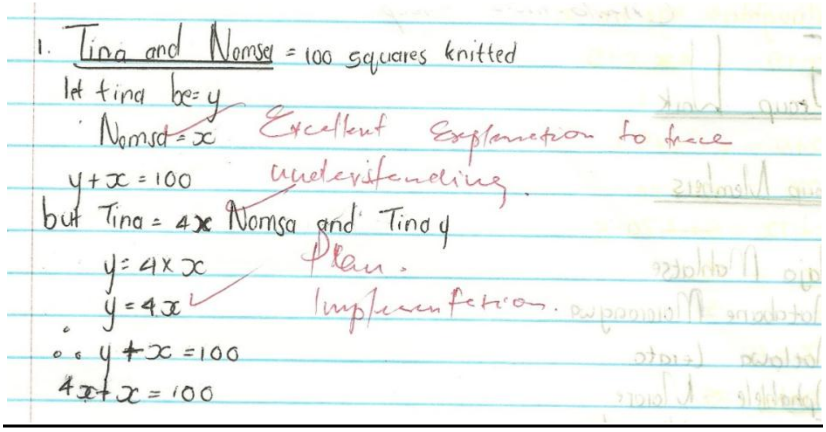
Contrary to negative aspect from a number of groups reflected above, here emerges the Charlo-moro group with detailed plans for learning activity 1c and 1b. Their detailed plans are as follows:

CHALO-MORO GROUP

Activity 1b

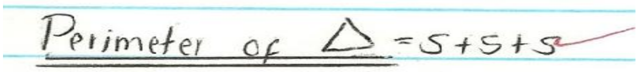

2. Perimeter of a equilateral Δ = 210mL
Length's of each side must be equal to each other
Perimeter of Δ = S+S+S
Explanation (Understanding)

Activity 1c



When going through the plans for activities 1b and 1c as presented by the Charlo-moros, one could be tempted to conclude that Grade 9 learners are matured to an extent of interrogating the problem for better understanding. That claim could however be disputed by negative discussions captured above. The group started by introducing the two key role players in the context of the problem and at a later stage decided to represent the two with variables. Their use of x and y is explained in their plan by the manner in which they equated a letter to a name.

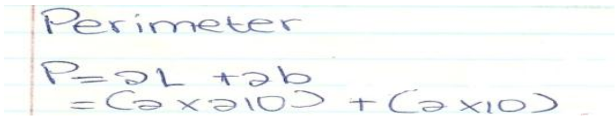
The accountability to the plan could again be noticed when they responded to learning activity 1b. Their endeavours of explaining that the sides of an equilateral triangle are equal had a significant impact on the understanding of their proposed plan.



The sum of the 3s' carried more weight in the plan because explanations were provided in preceding statement.

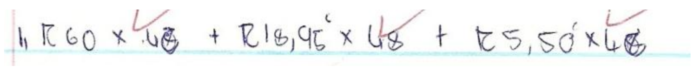
In an attempt to unpack the second issue relating to the development of the plan in relation to the context within which the problem is presented, the Mbongeni group provided detailed evidence in this way:

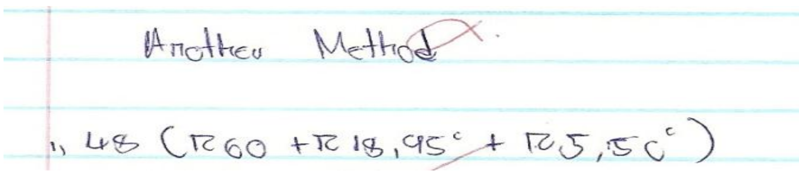
MBONGENI GROUP



There is no indication that the group understood what a triangle was all about. From their undertakings, it would mean that the triangle has four sides with each pair of sides equal in length. The group failed to relate their plan into the problem thereby not realising that a triangle has three sides.

It is again picked up in Tshepiso and Siswe's plans for episode 3, that the skills of planning were developed in some of the learners. That could be inferred in their workings when they presented their plan for activity three in this fashion.





The two learners were able to provide convincing plans in different ways for solving the problem in question. Tracing back in the plans as presented above, it could evidently be seen that the pair was able to take out the common factor from the first plan and that assisted in developing the second plan. The mathematical algorithm used in developing the second plan really makes sense to the problem in question.

It can therefore be convincingly concluded from the undertakings reviewed that there are still some learners who are not able to account for the plans developed. They failed to make connections between the data and the unknowns. In some cases, they could not relate their plans into the contexts of the problem. Even though there are those learners who explicitly presented their plans much still has to be done to correct the anomaly. That which needs to be corrected will emerge as the workings in the preceding problem solving stage on implementing the plan.

4.2.1.3. Carrying out the plan

This stage of problem solving is focused on implementing the plan that was developed in the initial plan. It involves checking each step when the calculation unfolds and proving that the steps are correct. Recording and operational processes appeared to be key in this stage of problem solving.

It was revealed from most undertakings reviewed that proposed plans were properly executed. Issues raised at this stage were those of the mathematical algorithms not coming out very clear in some of the calculations and misconceptions as working unfolds. In some cases, learners were tempted to complete some given tables without even showing workings of how they arrived at solutions fitted in tables.

Misconceptions could also be depicted from tables completed by some of the learners in response to episode 2. The table presented by Pertunia was as follows:

Table 1: Distance, Time and Speed

	Distance	speed	Time
Morning	40 km	32 km / h	$h = 1\ \frac{8}{32}h$
Afternoon	8 km	24 km / h	$\frac{8}{24} = \frac{1}{3}h$

This also proves that learners are just giving themselves assumptions without even making justifications to such assumptions. Justifications referred to in this paragraph are detailed workings resulting in solutions placed on the table. It is evident from the table that the child did not understand the relationship between the distance from home to school and that from school to home. The two distances cannot differ because they are of the same radius. The direction from which the child is coming does not have an impact on the distance.

Understanding of the problem as an issue raised in problem solving stage one seems to cut across almost all stages of problem solving and is still missing this presentation.

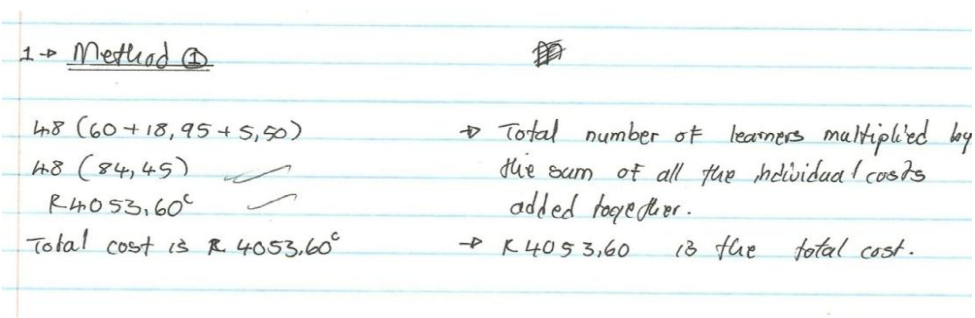
Contrary to what is presented above, there emerges this learner,Morema, who successfully completed the table with understanding, and that further proved that the recording process of problem solving was developed. The completed table was as follows:

Table 2: Distance, Time and Speed

	Distance	Speed	Time
Morning	x km	32km/h	$\frac{x}{32}h$
Afternoon	x km	24km/h	$\frac{x}{24}h$

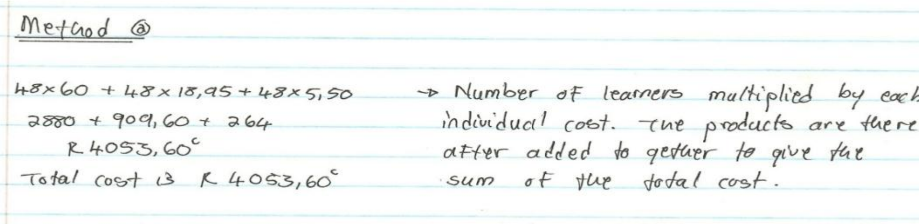
One could really depict that the learner was able to spot the relationship between distance, speed and time as given in the morning row on the table and that guided him to come up with what is presented in the afternoon row.

The understanding that this learner had in execution of devised plan can also be tracked from his undertakings in episode 3. Everything that is presented below is always supported with reasons. His workings of the said episode are captured as follows:


1 → Method ①

48 (60 + 18,95 + 5,50)
48 (84,45)
R4053,60c
Total cost is R4053,60c

→ Total number of learners multiplied by the sum of all the individual costs added together.
→ R4053,60 is the total cost.

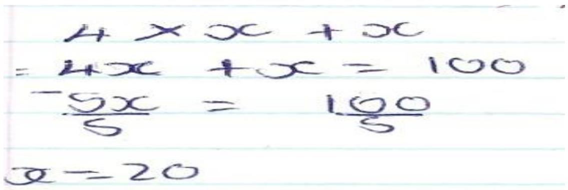


Method ②

48×60 + 48×18,95 + 48×5,50 → Number of learners multiplied by each individual cost. The products are there after added to getuer to give the sum of the total cost.
2880 + 909,60 + 264
R 4053,60c
Total cost is R 4053,60c

In the first method, he started by adding all addends in the brackets. That is a clear indication of the knowledge of the mastery of the bodmas rule in algebra. He further continued by multiplying the calculated sum with the factor 48 and in every workings supportive reasons were furnished. From the workings presented, one could really be informed that the plan as proposed could really be implemented.

The Baltymore group, who at all times in the previous stages above seemed not to be addressing every level, correctly now emerged with the correct implementation of the plan. Their full response to learning activity 1a of episode 1 was captured as follows:



4 × x + x
= 4x + x = 100
5x/5 = 100/5
x = 20

In tracking their implementations from their workings, one could really tell that the mathematical algorithms of solving linear equations were developed. It is also important at this stage to remember what the group presented as an initial plan for the problem. They really managed to equate the sum of 4x and x to 100. The 4x in their workings seemed to be representing the number of squares for the blanket knitted by a girl who knits four times the squares of the other one.

Realising that x was to be made the subject of the formula, the group eliminated the 5 by multiplying both sides with the reciprocal of five and the practice was absolutely relevant. The group correctly employed the concept of reciprocals because they realised that the 5 was the numerical coefficient of the term. Finally, they found the answer to be 20.

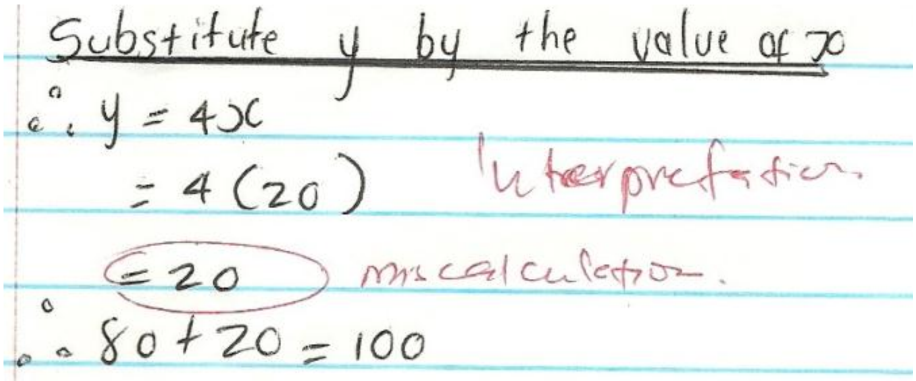
This really uncovered as the proper application of the plan devised, but still missing, appeared to be the skill of checking the authenticity of the answer and that will be addressed in the next stage.

4.2.1.4. Looking back

This stage involves the cross examination of the solution found in an attempt to check its authenticity. It is the stage through which the results of each problem are interpreted. George Polya attributes this stage to a level through which one checks the arguments presented in the previous problem-solving stage and expect one to visualise the solution from a glance. Clarification, analysis and understanding of solutions attained as skills in the reasoning process of problem solving are also taken into consideration.

The failure to trace back the calculated values into the problem seemed to be the resultant issue at this stage. In most cases, learners tend to correctly determine solutions to problems but fail to check whether or not answers are making sense to the problem. Referring back to the extract from the Baltymore group as reflected in the implementation stage above, it became evident that the group just ended their calculation with the answer 20, and that was the only level at which their calculations ended. The 20, as the calculated value, was not verified in the problem.

Tracking the presentations of the Charlo-moro group at this stage, one could really be convinced that the stage is far more credible than others. The conviction can be checked against this concluding presentation of the groups' workings.



This excerpt is the concluding part of what the group presented in the stage on carrying out the plan. The group tried to justify their answer by first determining the values of y with the calculated values of x. Even though the group mistakenly captured the product of 4 and 20 as 20, instead of 80, that did not impact much on the meaning.

Their interpretations of results came out very clear when they added 80 to 20 to come up with 100. That in itself revealed that one girl was knitting 80 squares and the other one was knitting 20 squares. Still missing appeared to be an account of clarity for the two domains. That is clarity on who was knitting 20 squares and who was knitting 80.

The teaching experiment was used as a diagnostic measure for learners' problems and a determining agent for issues emerged with a number of issues that needs immediate attention. Issues identified are amongst others the following:

- Direct linear translations;
- Failure to break the problem into component parts for better understanding;
- Little or no understanding of the problem;
- Unaccounted plans;
- Irrelevant plans to problems;
- No plans;
- Unclear mathematical algorithms;
- Misconceptions; and

- Failure to interpret the results.

It is therefore important, as proposed in this study, that constructive intervention be carried out in an attempt to address all issues identified. The intervention as informed by the theory already identified is presented in the subsequent teaching experiment and its developments are captured as the experiment unfolds.

4.2.2 Teaching experiment 2: Intervention stage

This teaching experiment served as a casement through which one could peruse on actual undertakings when teaching and learning unfolded in class. It is used in this context supported by Zarnocha and Maj (2008), as an experimental tool that seeks to address research questions and inquiring about the nature of learning mathematics. This teaching experiment is reported following the same system of problem-solving stages that was adopted in the preceding teaching experiment. The experiment further captured events as teaching and learning progressed in class. This stage proved to be worthy in the teaching and learning of mathematical word problems.

4.2.2.1. Understanding the problem

In an attempt to be answerable to issues raised in teaching experiment 1 at this stage of problem solving, learners were given time to share their understanding of problem with others. In some other instances, eliciting questions and leading inputs to foster constructive arguments were posed.

Most learners still seemed not to be clear of what was expected of them in the initial stage of interacting with the problem when they were interrogating episode 1. When they were requested to share their understanding of how they understood the question, a learner by the name of Gledwin made the following contribution:

Gledwin: "If Nomusa has 1 square then Tina has 4 squares more than Nomusa".

The input as presented only revealed the understanding of the fact that one girl is knitting four times the squares of the other one. Little is known about the number of squares for both girls. The response seemed to be the avoidance of direct translations because the learners were trying to attach meaning to the problem through arguments.

In trying to make a follow-up to the input made, Andiswa asked the following question as a way of trying to understand what was posed.

> *Andiswa: "Does it mean that 1 + 5 + 100?*

Other members of their group collectively disputed what their counterparts were saying by stating that: "*but it was given that the total number of squares is 100*". That triggered a moment of critical thinking, which also required reasoning as an essential process in mathematics. Morema, who at all times showed to be having a bit of understanding on what was required, responded thus:

> *Morema: If one learner knits 20 squares for the blanket the other one should knit 80 squares to make the total of 100.*

It became evident during this phase that learners started communicating with the given statement. The skill of communication appeared to be developing. It was after Moremas' contribution that I requested the learners to relate the shared understanding of 80 squares and 20 squares to Tina's squares, which were said to be greater than those of Nomusa by 4.

In an attempt to unpack the sense of Moremas' understanding, Nikkie belonging to the same group as Morema, responded thus:

> *Nikkie : "Four(4) times 20 squares is 80 squares plus 20 squares is equal to 100 squares".*

The input as presented by "Nikkie" revealed the understanding borne in this learner. The multiplication of 20 by 4 is an indication that the learner can communicate with the statement that one girl is knitting four times the squares of the other one. The addition of the product 80 to 20 clearly reveals that the learners understand that the sum of the squares is equal to 100.

In assisting the learners to unpack question 1(b), which was on the calculation of sides of an equilateral triangle where its perimeter was given to be 210mm, I requested them to share their understanding of what perimeter is. Sizwe's response was that a perimeter refers to the sides of a triangle.

> *The teacher asked: How are the sides of a triangle related to the perimeter of a triangle?*

Learners revealed not to be well conversant with the concept of perimeter. I then gave them the following leading input: *"A rectangle is a figure with four sides and its perimeter is the sum of all four sides".*

> *Morema: "Oh yes, then the perimeter of a triangle will be the sum of the three sides since the triangle has three sides".*

Communication and reasoning processes appeared to be integrated and being developed in this argument. Learners realized that a triangle has three sides and its perimeter is the sum of all the three sides. I further requested the learners to describe an equilateral triangle. Tshepiso described an equilateral triangle as a triangle with three sides that are equal. I requested the learners to plan for the solution using Tshepiso's input. Boitumelo, a slow learner in their group, had: $x + x + x = 210mm$

> *The teacher asked: Why do you use x only in your equation? Will I be true to write the equation as x + y + z = 100?*

> *Boitumelo : The three x's which are similar indicates that the 3 sides are equal since the triangle is an equilateral triangle.*

The argument presented above proved that the mathematical statement presented was understood and constructed according to the specifications of the question. Inputs that learners were giving are a clear indication that learners started understanding what the problem was all about.

For a change, episode 2 was to be carried out in groups. In the previous teaching experiment, the episode was completed individually. The purpose was to monitor learners who hid behind others and to identify problem areas within the learning content. I requested learners to read through the learning activity and to share their understanding with other members of the class. Lesedi form the Charlo-moro group presented as follows:

Lesedi: The time at the lower speed seem to be more than the time at a Higherspeedby15minutes.

Lesedi revealed that he analysed the statement with understanding. Reasoning process emerged to be improving to a desirable level of development.

This episode was carried out in the similar way as episode 2. Learners had to first interpret the given question, read through the statement, and share the understanding with others. Merlyn quickly identified the key elements that constituted the expenditure for an outing from the extract, that is:

entrance fee – R60
meals - R18,95
bus ticket - R 5,50

At this stage learners were able to unpack the statement through communication. Learners were able to break the given problem into principal parts for better understanding.

The actions as presented in the above discussions addressed direct translation and understanding issues raised in the preceding teaching experiments.

Dynamic shifts in terms of understanding were marked when learners were interrogating problems. Learners at this stage refrained from their usual norm of directly picking up numbers from the problem and formulating mathematical statements without understanding the context within which the numbers were given. I therefore recommend that when problem-solving activities are given to learners, they must be encouraged to interrogate the problem for better understanding. Learners must also be encouraged to identify the data provided and also look into the conditions given for the problem in question.

4.2.2.1. Devising the plan

Learners were, at this stage, requested to make use of the shared understanding in developing the plan for the solution to the problem. In an attempt to give them the leading input when responding to learning episode 1(a), I highlighted that they should represent the unknowns by any variable of their choice.

> *The teacher suggested: Let Nomusa's squares be x and Tina's squares be 4x where both of them add up to 100.*

My input assisted the learners to conclude that the mathematical statement for the given problem was $x + 4x = 100$. I requested the learners to explain the meaning of the statement.

> *Morema : If one child knits x squares then the other one will knit four times the squares of the first one and all will amount to 100.*

This explanation was convincing that the learners were now able to identify what was required. They could explicitly communicate with the problem and that led them to plan properly for its solution with valid reasons.

Tshepiso, who responded to episode 1(b), described an equilateral triangle as a triangle with three sides that are equal. The class was then requested to plan for the solution using Tshepiso's input. Boitumelo, a slow learner in their group, had:

$$x + x + x = 210mm$$

The teacher asked: Why do you use x only in your equation? Will I be true to write the equation as x   +   y   +   z   =   100?

Boitumelo : The three x's which are similar indicates that the 3 sides are equal since the triangle is an equilateral triangle.

The argument presented above proved that the mathematical statement presented was not as a result of picking up letters and numbers but was rather understood and constructed according to the specifications of the question.

Even though learners seemed to understand the statement through explanations given in problem solving stage 1 of this experiment, they still failed to come up with the mathematical statement for the solution. I decided to intervene when they were struggling with episode 2 by providing them with the following formulae:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

The provided formulae were just to show them the relationship that exists between speed, time and distance. It was after the input that the distance from home to school and from school to home are the same, when Morema came up with a proposed plan for the solution.

After a while, Morema seemed to have realized how to calculate distance with the given formulae and decided to substitute time with variable x. He had:

$$s \times x = s \times (x + 15)$$

The teacher asked: Why did you add 15 to x *because the initial statement was,* $s \times t = s \times t$?

Morema : The 15 in the equation represents the extra time travelled at a lower speed.

The above deliberations resulted in the group developing the initial plan of solving the problem. The plan that indicated that the time from home to school was not similar to that from school to home was captured as follows:

$$(H-S) \quad s\times x = s\times x\,(S-H)$$
$$32km/h\times x = 24km/h\times(x+15\min)$$

This was the plan to calculate the time that will further assist in determining the distance between the two points.

It is through discussions, leading inputs and eliciting questions that the planning stage of mathematical problem-solving can be properly achieved. Mathematics educators are, at this stage, encouraged to ask more eliciting questions that may guide learners to come up with plans for the solutions of mathematical word problems. Learners need to be encouraged to understand the plan in relation to the context within which the problem is given.

4.2.2.2. Carrying out the Plan

The carrying out stage, implementation stage or the execution stage of mathematical problem-solving involves implementations of the devised plan through correct mathematical algorithms. It also includes the checking of each step against the correct mathematical procedures applied.

I requested Laura, during interventions, to show how she would solve the problem on the board using the devised plan. Then she responded in this way:

$$Laura: \quad x+4x=100$$
$$5x=100$$
$$x=\frac{100}{5}$$
$$x=20$$
$$\text{Nomusa's squares} = x = 20$$
$$\text{Tina's squares} = x\times 4 = 20\times 40 = 80$$

The statement as presented by Laura is an indication that the child is able to apply the correct procedures when solving problems. The addition of x to 4x is a clear indication that the child mastered the aspect of adding like terms in mathematics. The elimination of 5 in an attempt to make x the subject of the formula through multiplicative inverse is an indication of the adaptive knowledge borne in this child. The mistake of multiplying 20 by 40 in the last statement does not have a significant impact on the procedural knowledge of the child but rather on the problem solving skills, that is, the skill of looking back.

The Charlo-moro group also responded through Morema to learning activity 1b as follows:

$$x + x + x = 210mm$$

$$3x = 210mm$$

$$\frac{3x}{3} = \frac{210mm}{3}$$

$$x = 70mm$$

$$\therefore \text{each side is equal to } 70mm \text{ and } x + x + x = 210mm$$

$$\text{i.e. } 70mm + 70mm + 70mm = 210mm$$

This response is still having the same bearing as the one above.

From this response, it could be learned that learners started applying skills of reasoning and interpretation. That was observed when they tried to justify that each side is equal to 70mm. They gradually developed skills of interrogating the problem, planning for the solution, implementing the plan and checking their solutions through substitution.

In an attempt to proceed with the devised plan for episode 2, the class responded in this away:

$$
\begin{aligned}
32km/h.x &= 24km/h.x + 360km/h \\
32km/h.x - 24km/h.x &= 360km/h \\
8km/h.x &= 360km/h \\
x &= \frac{360km/h}{8km/h} \\
x &= 45
\end{aligned}
$$

These workings have followed correct mathematical procedures. Even though we acknowledge that the procedures applied were correct, what seemed to be lost was the understanding of what they were calculating. Upon realising that, I then challenged the learners to determine the time taken from school to home and from home to school at different speed, and also the distances, using the calculated time and speed.

This episode was rather a challenge to most learners in the sense that they could not clearly have a way of going through the activity. Reasoning at this stage became gradually developed. It became evident the from learners' responses to questions posed during this stage of interventions that they could really make sense of the question.

Learners seemed to have acquired mathematical procedures of solving word problems. Procedural knowledge, conceptual understanding, adaptive knowledge and problem-solving skills seemed to be developed at this stage of mathematical problem-solving.

4.2.2.3. Looking back

Looking back is a very crucial stage of mathematical problem-solving which requires the problem solver to have the understanding of the solution found in relation to the devised plan and the problem with its given context. Calculated values need to initially be substituted in the devised plan and thereafter checked against the context of the problem.

After the implementation of the devised plan by Laura in learning activity 1(a) above, I requested learners to substitute their calculated values into the planned equations to check as to whether or not they made sense to the question. Their responses were captured as follows:

The teacher suggested: *Substitute the calculated value of x into the initial statement to check as to whether the solution is correct/ not.*

Toko: x = 5 and 20 x 5 = 100

The teacher asked: "Does it mean that x is equal to five squares and 5 times Twenty squares is *equal to 100"?*

The whole class responded in chorus form, disagreeing with Toko. This response is a clear indication that the learner did not really understand the answer in the context of the problem. Even though the product of 20 and 5 is 100, that does not account for the number of square each learner would knit. The multiplication of 20 by 5 does not reveal any sense of understanding that one learner knits four times the squares of the other one.

Morema immediately stood up and put forth his understanding as follows,

Morema:the first statement is $x + 4x = 100$*, then it will be*

$$20 + 4 \times 20 = 100$$
$$20 + 80 = 100$$
$$\therefore 100 = 100 \text{ since the calculated value of } x \text{ is } 20.$$

The whole class agreed with Morema's response to the question with reasons. It proved that learners started interpreting the solution of the problem. It was through Morema's input that most learners started realising that the 20 is substituted in place of x in the equation. Still missing in the response is the second level of looking back, which takes into account the substitution of the calculated value into the whole problem to check as to whether the answer makes sense to the problem in its context.

This response from the Charlo-moro group, when responding to learning activity 1b, carried a lot regarding the looking back stage of problem solving

$$x + x + x = 210mm$$
$$3x = 210mm$$
$$\frac{3x}{3} = \frac{210mm}{3}$$
$$x = 70mm$$
$\therefore$ each side is equal to $70mm$ and $x + x + x = 210mm$

i.e. $70mm + 70mm + 70mm = 210mm$

The last but one statement of the input is a clear indication that 70mm is a representation of only one side of an equilateral triangle. This was captured when the learner was trying to indicate that each side is equal to 70mm. It is further indicative of the fact that the sides of the triangle are equal. When the three sides are added the total is 210mm which is according to the given data, the perimeter of the triangle.

It is upon the above findings that I feel it is quite important to encourage learners to look back in an attempt of interpreting the results. This problem-solving stage is important because it makes learners aware of mistakes that might have been committed in the execution stage. It also develops learners' confidence when defending their calculated values because they will be understanding their answers in contexts given.

It is generally viewed that when learners are assisted in interpreting mathematical word problems, they turn to do that with understanding. That is supported by they way in which they responded to eliciting questions as posed above. Communication as a process in mathematics seems to be developed because learners started interrogating statements for understanding. Recording is also at this stage receiving attention; hence learners were able to capture their thinking in writing on the chalkboard and in their workbooks as reflected in Appendix C2.

4.2.3. *Reflecting on the two teaching experiments*

Teaching experiment 1 entirely requested learners to work through learning activities without the facilitators' interventions. That implies learners were to independently work on the activities without the assistance from the facilitator. It became explicitly evident form their presentations that they work through the activities with no understanding. It is further revealed from their workings that the skill of breaking the problem into smaller parts for better understanding, as recommended by Polya, still needed to be developed. Learners further work on activities without attaching meaning and even understanding the expectation of the activity itself. That could be traced in their presentations of all activities.

4.2.3.1. Understanding the problem

Understanding the problem as an initial stage of problem solving involves communicating with the problem for better clarity. That could be achieved by knowing what the problem is all about; the provided data; and also the conditions attached to the problem. Communication, as an investigative process in mathematical problem solving, still appeared to be missing. Learners' responses in teaching experiment 1 revealed that most were just embarking on problem calculations without even communicating to the problem for better understanding. That could be achieved by unpacking the problem into small meaningful parts that make sense to the problem solver.

It, however, became convincingly evident from their responses after interventions that communication plays an integral part in the understanding of a problem. Eliciting questions posed forced learners to communicate with the problem before the actual computation could be done. They also assisted learners in making sense of the problem and prompted them to break it into principal parts for better understanding.

Discussions in which learners were engaged reduced the risk of directly translating the problem as identified in teaching experiment 1. That could be traced through the way in which Gledwin responded when trying to attach meaning to episode 1 [Teaching experiment 2 – understanding the problem].

The above attributes positively impacted most learners in executing the planning stage of problem solving. The revelations of teaching experiment 2 justifies that it is better to know what the problem is all about before one could actually try to work on it. The claim is further supported by theories already identified in literature review. The challenge is still on how learners think about the problem itself.

4.2.3.2. Devising the plan

The planning stage of teaching experiment 1 revealed that learners were still in most cases trying to plan for the solution with minimal or no understanding. In most cases, plans that were provided were not accounted for. Their intentions could not be fully traced from their plans. That made it difficult to trace the understanding and intentions that learners had. The improper plan in problem solving is always attributed to lack of understanding which finally affects the solution of mathematical word problems.

From the leading inputs given during interventions in teaching experiment 2, learners were reminded about representing unknowns with variables, which led to the choice of x as a variable to be used in episode 1. The said practice assisted learners in developing the plan as, $x + 4x = 100$ which carried more meaning because they could realise from the plan that one learner knits four times the squares of the other one. In some cases, learners could tell that if one child knits x squares then the other one would knit 4x squares.

The planning stage problem solving cannot be divorced from the understanding stage. The understanding of an equilateral triangle in episode 1(b) gained after interventions assisted learners in developing the plan for calculating the perimeter of equilateral triangle as, $s + s + s$. This was marked as a dynamic shift in terms of understanding. Learners were now able to tell that an equilateral triangle had three sides and they could further relate that the sides were of the same magnitude.

The intervention stage positively impacted most learners in planning for the solution and even sharing their understanding of the plan. This stage further developed confidence in learners when discussing issues around the problem in question. Learners could now defend their inputs with reasons. Little is known about the choice of the domains to be included in plans.

4.2.3.3. Carrying out the plan

This stage of problem solving involves carrying out the plan in an attempt to answerable to the problem in question. It became evident form responses that most learners seemed to be familiar with the algorithms of solving linear equations from the previously learnt mathematical concepts. Such algorithms are very much important when implementing proposed plans. There were still a few who could not work out problems because of their failure to understand what was expected from them. Post intervention responses revealed that the anomaly can be corrected through communication. If learners are granted an opportunity to justify their workings with reasons, it makes it easier for them to proceed with understanding.

4.2.3.4. Looking back

The looking-back stage, which is sometimes referred to as the stage of interpreting the results, can be viewed from different perspectives. This is a very crucial stage of problem solving that actualises the understanding borne in the problem under review.

Firstly, the looking-back stage of problem solving involves the substitution of the calculated values in the devised plan to check whether the answers are relevant to the problem. This is done with the primary aim of determining whether the calculated values are correct.

Secondly, the stage involves the interpretation of the results in relation to the problem as a whole. This involves understanding the answers from the context within which the problem is given. For example, in learning activity 1(a) of episode 1 learners were expected to calculate the number of squares which each learner would knit for the blanket given that one learner (Tina) was knitting four times the squares of the other learner (Nomusa). It was also hinted that 100 squares were needed to complete the blanket.

In some of the devised plans, learners captured their plans as $4x + x = 100$. In cases were mathematical algorithms were properly applied, the value of x was found to be 20. When the calculated value is substituted in the plan, the left-hand side and the right-hand side of the equation should be equal to make the statement true, and that involves the first phase in the looking back stage.

When the 20 is checked from the perspective that one learner knits four times the squares of the other one, it brought us to the second phase of the looking back stage which involves understanding the answer from a bigger picture. This understanding will prove to us that one learner knits 80 squares whereas the other one knits 20 squares. This will also make us aware that 80 is a product of four times 20.

Two interrelated-problem solving models from models identified in the literature review section above also concluded by examining the solutions found or checking the answers against the questions. It emerged as a general trend from most responses reviewed in teaching experiment 1 that no time is devoted for checking the validity and reliability of the answers found, but however, when interventions are done, the skill seemed to be developing.

It was only after the intervention stage that learners became aware of the importance of this stage because they started realising that some calculated values can be wrong. I, therefore, present a brief discussion of post intervention responses as evidence to claims on improvement marked.

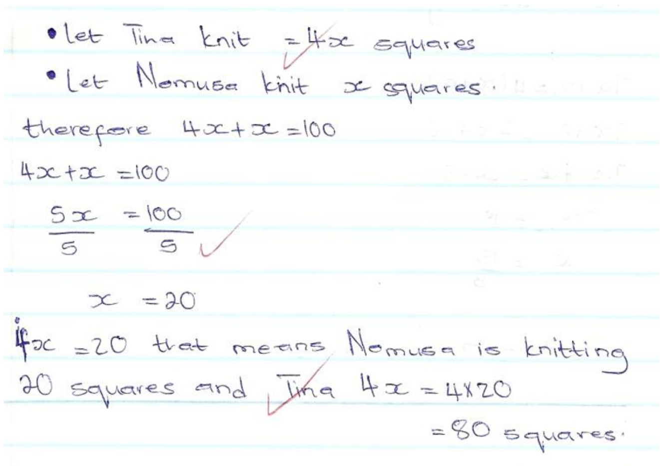
4.3. Review of Post Intervention Responses

This stage is used to capture the impact of interventions in teaching and learning of mathematical problem solving. Due to the limited nature of this study I am presenting only one group that revealed significant improvement in all problem solving stages. This is the group which presented much work for discussions in the first teaching experiment. This section therefore reviews the improvements under the planning stage and looking-back stage of problem solving. I, therefore, present my arguments on the two problem-solving stages as identified above.

4.3.1. Devising the plan

It was already outlined in the pre-intervention stage that planning should be informed by connections between the data and the unknown. That is subsequently followed by the application of operational processes. The above aspirations seemed to be developing when interventions were done and that is evident from post intervention responses to learning activity 1(a) as presented by the Baltymores below. This is the group that presented much work for review under the prior-intervention stage and the improvements are marked following the groups work.

BALTYMORE GROUP

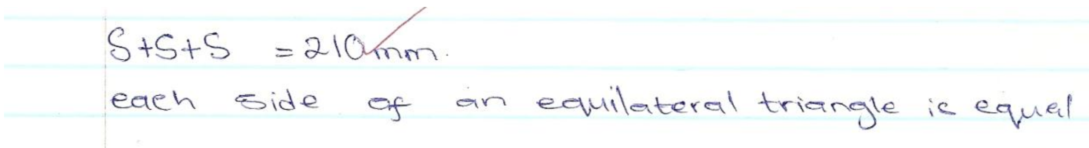

• let Tina knit = 4x squares
• Let Nomusa knit x squares.
therefore 4x+x =100
4x+x =100
5x/5 = 100/5
x = 20
4x = 20 that means Nomusa is knitting 20 squares and Tina 4x = 4x20
= 80 squares.

In teaching experiment 1, the 4 x *x* + *x,* which served as the initial plan for the question, were just presented as a mathematical statement with no explanations of what 4 and x stood for. Post- intervention response gave an account of what g the *x* and 4*x* stands for in the equation. The connection between 4*x* and *x* in the presentation can be seen when the group equates 4*x* to Tina's squares and *x* to Nomusa's. This is an indication that one child knits four times the squares of the other one. When the 4*x* and *x* are placed in the equation, they are understood when their sum is 100.

The explanation above can also be attributed to the understanding stage of problem solving that requests one to break the problem into small meaningful parts. It is also becoming evident that the group is communicating with the problem for better understanding, which finally resulted in a plan that is accounted for.

The presentation below is also an indication that learners are no longer directly picking up numbers from the provided information in an attempt to come up with the reasonable plan for the solution. This is justified when they are bringing in the idea that the sides of an equilateral triangle are equal hence s + s + s = 210mm

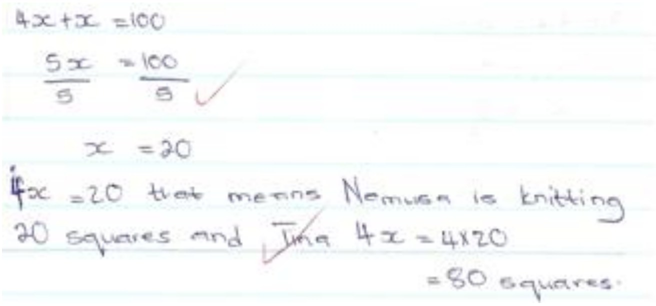
BALTYMORE GROUP


S+S+S = 210mm.
each side of an equilateral triangle is equal

4.3.2. Looking back

The looking back stage was viewed to be a very crucial stage in problem solving because it actualises the knowledge gained in all problem solving stages before. At this stage, we are expecting a learner to have sense of the calculated answer in terms of the plan and the problem as a whole.

The Baltymores showed a significant improvement at this stage of problem solving when they tried to attach meaning to the calculated value of learning activity 1(a).


4x+x = 100

5x/5 = 100/5

x = 20

4x = 20 that means Nomusa is knitting 20 squares and Tina 4x = 4x20

= 80 squares.

Their plan was properly executed and yielded the answer 20. In interpreting the answer 20, the group managed to bring the idea that Nomusa was knitting 20 squares and Tina was knitting 4 times 20, squares which is equal to 80. This revealed that the group understood the answer from a bigger perspective. Still missing is the aspect of understanding the answer 20 in terms of the context. They still failed to make us aware that if one learner knits 20 squares, the other one will then knit 80 squares, which will then imply that the squares for the blanket are 100. [see Appendix C1 for more developments]

4.4. Chapter Summary

This chapter covered the analysis of learner's responses which unfolded in two teaching experiments. The analysis in all stages was critically informed by the four problem solving stages as adopted from George Polya. A detailed comparison of output and solutions is captured in the reflection part of this chapter.

Reponses to two teaching experiments revealed that reasoning processes, which inter alia include collecting, clarifying, analysing and understanding, still desires much to be done.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

English and Sriraman (2010) viewed mathematical problem solving as a subject of substantial and controversial research for a number of decades. This study managed to present findings that will in a way alleviate the controversy alluded to. It is through multiple research methods of practical Action Research and teaching experiment employed in this study that the results responded to the research questions raised. The concluding remarks and recommendations in this section are therefore presented as responses to questions raised in Chapter 1.

5.1. What Challenges Do Learners Encounter with Regard to Processes of Solving Word Problems?

The study revealed that much still has to be done in class in an attempt to develop or to improve the use of mathematical processes when solving word problems. The learners' inputs revealed that they had a tendency of responding to word problems even before they made sense of the given problem. That led them to directly translate word problems into mathematical statements before they could even understand what was expected from them.

Attaching meaning to given statements also appeared missing as a skill. Learners were still failing to break the problem into small meaningful parts for better understanding. Learners could not make connections between real-life problems and the mathematical content learnt in class. This was made clear by the failure to plan for the solution to the problems, which is identified by Polya in his second phase of problem solving as very important. In some instances plans are not accounted for. This is a very crucial phase of devising the plan for attack, which is a much more complex part of the problem-solving process.

5.2. How to Improve Grade 9 Learners' Mathematical Processes of Solving Word Problems?

Interventions in mathematical problem solving teaching experiments are very much important because they turn to reveal best approaches to be applied which are answerable to the question on, "How to improve Grade 9 learners' mathematical processes of solving word problems

In most cases, when learners were expected to attach meaning to word problems, they struggled to justify their inputs with reasons and they were unable to account for their submissions. It appeared that there was no way a problem could be amicably solved if it was not properly understood. Learners need to be encouraged to read through the problem with understanding before attempting for the solution. I attempted that during my intervention by requesting learners to go through the statement in question and sometimes asked questions to elicit understanding of the problem. Communication, as a process in mathematics, plays an integral part when solving mathematical word problems. Learners should be encouraged to reason out every step taken when solving mathematical word problems.

When planning for the solution to a problem, learners should be challenged to furnish a reason for any step adopted. In that way, they would guard against plans that might mislead them in their solutions. It should be encouraged that connections be made between the problem and the mathematics learnt. In most cases, mathematical word problems are given in real-life contexts and connections needs to be made. Mathematical solutions should be translated back into the real-life solutions.

Finally, when mathematical word problems are solved, results need to be checked. Results could be checked by substituting the calculated values in the initial statement and also against the context within which the problem is given. I presume, that if the above-mentioned techniques are adopted, mathematical word problems could be computed with ease using relevant processes.

5.3. What is the Impact of Exposing Learners to Problem Solving Strategies on their Performance in Solving word Problems?

The impact of exposing learners to problem-solving strategies was revealed in the section on review of post-intervention responses in the chapter above. When learners are encouraged to communicate with the problem by breaking the problem into meaningful parts for better understanding, they seemed to be gaining a better insight into the problem as a whole. This intervention even assisted learners in planning for the solution as the second stage for problem solving in the adopted theory. It is, therefore, recommended that learners be encouraged to communicate with the problem for better understanding before embarking on the solution.

The plans that were generated after interventions were relevant to problems in question. In most cases, plans were accounted for. It is through interventions that learners also started justifying each domain used in mathematical equations developed. They also refrained from their norm of directly picking up numbers from mathematical word problems into equations. Learners could now realise the connections between the data and the unknown, which is subsequently followed by the application of operational processes.

The looking-back stage of problem solving seemed to be impacted a lot through exposure to problem solving strategies because learners were now able to substitute their calculated values in planned equations to check whether answers are relevant to questions. Still missing is the second stage of looking back, which, according to me, is not fully appearing to be developed, which is the stage whereby one needs to check the answer against the context within which the problem is given. Most responses are not capturing that stage. It is further recommended that solutions to problems must, at all times, be interpreted against the planned strategy for solution and the problem as a whole.

Problem solving ability does not develop in a vacuum but it rather needs a rich background of knowledge and intuition before it can be taught and learnt effectively.

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APPENDICES

APPENDIX A: LETTER OF ACCEPTANCE TO CONDUCT RESEARCH

DEREK KOBE SEN SEC SCHOOL

SEDES SAPIENTIAE

PO Box 1230
Lebowakgomo
0737

Tel: 015 633 7748 Fax: 015 633 7748 E-mail: kobe1@telkomsa.net

30.07.2010

Enq: Maloba L.M

To: Maluleka B.K

Subject:Permission to conduct research.

The above matter bears reference that

1. We acknowledge receipt of your request for conducting research at this institution.
2. The institution gladly informs you that your request was accepted.

We hope that you will enjoy conducting your study at this institution.

Yours Faithfully

Maloba L.M

(Deputy Principal)

DEPARTMENT OF EDUCATION
DEREK KOBE S.S. SCHOOL
P.O. BOX 1230 LEBOWAKGOMO 0737
2010 -07- 30
TELEFAX: 015 633 7748 CELL: 072 299 1863
LIMPOPO PROVINCE

APPENDIX B: EXPECTATIONS FROM LEARNING ACTIVITIES

In an attempt to solve all learning activities, I expect learners to adopt George Polya's problem solving strategies of,

- Understanding the problem;
- Planning for the solution;
- Implementing the plan; and
- Interpreting the results.

Learning activity 1

(a) Let the number of squares that Nomusa knits be x , then

Tina will knit 4x squares

$$x + 4x = 100$$

$$5x = 100$$

$$x = \frac{100}{5}$$

$$x = 20$$

Nomusa will knit 20 squares and Tina will knit (4 x 20) 80 squares.

(b) All the sides of an equilateral triangle are equal. Let the length of the sides be x.

$$x + x + x = 210mm$$

$$3x = 210mm$$

$$x = \frac{210mm}{3}$$

$$x = 70$$

The three sides are of an equilateral triangle are equal and each is 70mm if the perimeter is 210mm.

(c) Let the daughters' age be x.

The mothers' age will the be 7x.

In five years, the daughters age will be x + 5, then the mothers' age will be 7x + 5. To determine their ages, we will then develop the equation

$7x + 5 = 4(x + 5)$ *since the mother will be four times as old as the daughter in 5 years.*

$$7x + 5 = 4x + 20$$

$$7x - 4x = 20 - 5$$

$$3x = 15$$

$$x = \frac{15}{3}$$

$$x = 5$$

the daughters' age will be 5yrs and the mothers' age will then be

7 x 5 =35yrs.

Learning activity 2

(a) Let the time be represented by letter *t*

Time from : home – school = *t*

school – home = *t* + 15 minutes

distance = speed x time

distance (home – school) = distance (school – home)

$$s \times t = s \times t$$

$$32km/h \times t = 24km/h \times (t + 15)$$

$$32km/h.t = 24km/h.t + 360km/h$$

$$32km/h.t - 24km/h.t = 360km/h$$

$$8km/h.t = 360km/h$$

$$t = \frac{360km/h}{8km/h}$$

$$t = 45$$

time from home to school = 45minutes

time from school to home = (*t* + 15min.)

= 45min. + 15min.

= 60min.

Distance from home to school, where *t* = 45min.

Distance = speed x time

Distance = 32km/h x 45min.

Distance = 1440km/h

Distance from school to home, where *t* = 45min. + 15min.

Distance = speed x time

Distance = 24km/h x (45 + 15)

Distance = 24km/h x 60

Distance = 1440km

(b)

	Distance	Speed	Time
Morning	*x* km	32km/h	$\frac{x}{32h}$
Afternoon	*x* km	24km/h	$\frac{x}{24h}$

Learning activity 3

1.

METHOD A	**METHOD B**
= (48 x R60) + (48 x R18.95) + (48 x R5.50) = R2880 + R909.60 + R264 = R4053.60	= 48 x (R60 + R18.95 + R5.50) = 48 x R84.45 = R4053.60

2. Method A - Determine the total cost for the journey by multiplying each amount by the number of learners, which is constant, i.e,. 48 learners.

Method B - Add up all the amounts for the journey and multiply the total amount for each learner by the constant number of learners.

3. Yes, it will work out if the number of learners increased to 60.

METHOD A	**METHOD B**
= (60 x R60) + (60 x R18.95) + (60 x R5.50) = R3600 + R1137 + R330 = R5067.00	= 60 (R60 + R18.95 + R5.50) = 60 x R84.45 = R5067.00

4. No. B, because it takes out the common factor and adds up the totals or any other relevant input for the correct choice.

APPENDIX C1: PRIOR-INTERVENTION RESPONSES

Group work "Mbongeni Group"

20 for Nomusa

80 for Tino , No meaning.

All together it makes a calculation of 100

Perimeter ??

P=2L +2b No understanding.

=(2x210) + (2x10) No rectangle or triangle.

=420mm + 420mm

=840mm

In five years time she will be four times as old as her daughter

They will be 83 years

Members

Mnisi mbongeni

Mahlo Thabiso

Lekoona Toko

Motimela Regodile

Cathrine Makwarela

Thutse Merlyn

Matlakala Amos

Grade 9A Mathematics 15 September 2010

Groupmember "Baltymore Group"

Mphahlele Baltymore
Mphahlele Koketjo
Maesela Jonas
Mogowe Amanda
Molokwane Thuso
Lekgau Katlego
Thobejane Rocky
Maoto lethabo

Classroom activity

A Tina and Nomusa are knitting squares for a blanket. Tina knits four times as many as Nomusa. The need 100 Squares to complete the blanket

* Still owe explanation ??

$* \ 4 \times x + x$ Plan

$= 4x + x = 100$

$\frac{5x}{5} = \frac{100}{5}$ Implementation

$x = 20$

B The perimeter of an equilateral triangle is 210mm. calculating the length of the sides

$* \ P = 3 \times S$

$= 3S$

$\frac{3S}{3} = \frac{210}{3}$

$S = 70$mm each Sides

c A mother is seven times as old as the daughter. In five years time She will be four times as old as her as her daughter. How old are they now?

* $7 \times x$
$7x + 4 =$ ~~2x~~ $28x$
*A mother is 28 years old to her daughter
* A daughter is four ~~s~~ (4) years now

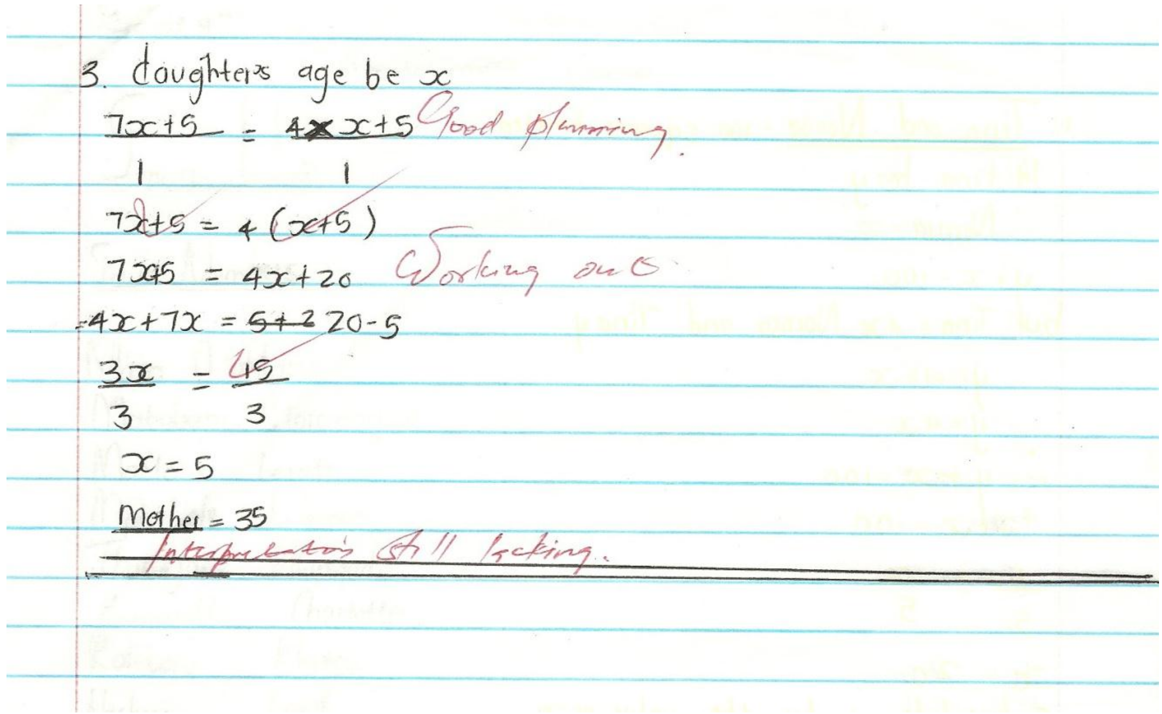
CHARLO-MORO GROUP: ACTIVITY 1 — 8/2

1. Tina and Nomsa = 100 squares knitted
let tina be = y
Nomsa = x — Excellent Explanation to trace understanding.
y + x = 100
but Tina = 4x Nomsa and Tina y
y = 4 × x — Plan.
y = 4x — Implementation.
∴ y + x = 100
4x + x = 100
$\frac{5x}{5} = \frac{100}{5}$
x = 20
Substitute y by the value of x
∴ y = 4x
= 4 (20) — Interpretation
= 20 — miscalculation.
∴ 80 + 20 = 100

2. Perimeter of a equilateral Δ = 210 mL
Length's of each side must be equal to each other

Perimeter of Δ = S + S + S — Explanation (Understanding)
210 mL = 3S
$\frac{3S}{3} = \frac{210\,mL}{3}$ — Implementation.
S = 70 mL

lacking interpretation



3. daughter's age be x

$$\frac{7x+5}{1} = \frac{4x+5}{1}$$ Good planning.

$7x+5 = 4(x+5)$

$7x+5 = 4x+20$ Working out

$-4x+7x = 20-5$

$$\frac{3x}{3} = \frac{15}{3}$$

$x = 5$

Mother = 35

Interpretation still lacking.

"Rocket group" A

Lekgau Keitlegó — Grade 9 — 16 September 2010

Mathematics — Classroom activities.

1. Groupwork

a. Tina and Nomusa are knitting squares for a blanket. Tina knits four times as many squares as Nomusa. The need 100 squares to they each knit.

* $4 \times x + x$ - Plan ✓

$= 4x + x = 100$ - Working ✓

$= \frac{5x}{5} = \frac{100}{5}$ The problem is solved but results could not be traced back into the problem.

$= x = 20$

b. The perimeter of an equilateral triangle is 210 mm. Calculate the length of the sides.

* $P = 3 \times S$ Plan ✓

$= 3S$ 3S

$\frac{3S}{3} = \frac{210}{3}$ Plan correctly worked out.

$S = 70$ mm each sides Checking not adequately done.

c. A mother is seven times as old as her daughter. In five year time she will be four times as old as her daughter. How old are they now?

* $7 \times x$ Evidence of misunderstanding result in

$7x \times 4 = 28x$ poor planning.

* $x \times 4 = 4x$

* A mother is 28 year old to her daughter.

* A daughter is four (4) years now. Poor planning will always foster poor completion of the problem.

Group Work "Merlyn Group"

(a) Tina nits 4 time as many as Nomusa *No understanding of the problem is displayed*

They need 45+2 squares to knit $(45^2+2 = 45+45+3^2$

$45+45 = 90+3^2$

$= 90+2+2 = 96$

$= 94$

b) Perimeter

P = 2L *This plan is not linked to the problem. Actually*

=(210mm x 210mm) + (210mm x 210mm) *it is evident*

= 420mm + 420mm *that the group does not*

= 840mm *understand the problem*

c) 7(x+5) = 4(x−5)

7x+5 = 4x+20 - *Mathematical idea cannot be*

7x − 4x = 20−5 *traced. The formula developed in the*

3x = 15 *second step shows a bit of understanding*

$x = \frac{15}{3}$ *even if it could not be manipulated fully*

Group Members

Thutoe Merlyn

Mnisi Mbongeni

Cathrine Makwarela

Lekoona Poko

Motimela Reqodile

Mahlo Thabiso

Activity 2 B/I Moloisi Morema

Mathematical Development Individual work Grade 9A 16 September 2010

	Distance	Speed	Time
Morning	x km	32 km/h	$\frac{x}{32}$ h
Afternoon	x km	24 km/h	$\frac{x}{24}$ h

Your understanding cannot be traced even though your substitutions seem to be correct.

Mathematical Development Work in Pairs 16 September 2010

1 → Method ①

48 (60 + 18,95 + 5,50)
48 (84,45)
R4053,60c
Total cost is R 4053,60c

→ Total number of learners multiplied by the sum of all the individual costs added together.
→ R 4053,60 is the total cost.

Method ②

48×60 + 48×18,95 + 48×5,50
2880 + 909,60 + 264
R 4053,60c
Total cost is R 4053,60c

→ Number of learners multiplied by each individual cost. The products are thereafter added together to give the sum of the total cost.

Activity 2 B/7

Name: Selemi Thaetšo

Grade: 9A

Date: 16 September 2010

Mathematics

Individual work

a. The distance between his home and the school is

	Distance	Speed	Time
Morning	x km	32 km/h	$\frac{x}{32}$ h
Afternoon	(a) 3 km ?	(b) 1,8 km/h ?	(c) ?

a. 32 km/h − 24 km = 8

$= \frac{8}{8} = \frac{24\text{km}}{8}$

= 24 ÷ 8 = 3

= 3 km/h

b. 32 km/h − 15 minutes = 17

$= \frac{17}{17} = \frac{32\text{ km/h}}{17}$

= 32 km/h ÷ 17 = 1,8 km/h

= 1,8 km/h

c. $\frac{x}{32}$h $\frac{x}{15}$

= 480 h

Your statement are not interrelated to each other. You cannot really convince one of what you really understand.

CA

Activity 3 B/I

pair work 22september 2010

maths

1. Ralebofu Lebogang
2. mojo Kgaugelo

1. The total cost of the outing and then compare answer and methods in class.

-the methods in class is 6253.5

2. The different and what is the same about the method of calculation used in class

-The different is the same of the learners they have calculated their methods by dividing and we calculated by multiplying the methods.

3. The method always work even if the numbe of learners or price change.

-the learners chang in the year and the price change of the year.

4. The methods that is the best ~~bus ticket~~ is the method that I have used

Montjane Thakgiso Activity 3 B/I
Hlengo Sizwe
Grade 9A
15 September 2010

CA

Pair Work

1. R60 × 48 + R18,95c × 48 + R5,50c × 48
= R2 880 + R9 09,60c + R 264
= R4053,60c

Another Method

1. 48 (R60 + R18,95c + R5,50c)
= 48 (R84,45)
= R4053,60c

Learning Activity 3 B/7

Mathematics

pair work.

Names

* Thipe Siyabonga.
* Mokobela Yuventine.

Mr Tshewete has 48 leaners in his class and want to calculate the total cost of the outing.

1. R60 + R18,95 + R5,50
= R84,45 × 48 Learners
= R4053,6

2. They never taught us in class.

3. yes.

4. The teacher's method is the best because he/she has a teacher's guide.

Pair work Activity 3 B/I
Maja manlatse
Thobejane Hlapogadi

Mr Tshewete has 48 learners in his class and want to calculate the total cost of the outing

1 Ratanga entrace fee R60
48 × 60
= 2880
meal R18,95
48× 18,95
= 909,6
Busticket R5,50
48× 5,50
= 264
2880 + 909,6 + 264
= 4053,6
2
3 Yes
4 My method is best

Grade 9A
mathematics
PAIR WORK : Activity 3 B|I
Names
1. Lekgothoane Mapeu.
2. Mood Boitumelo.

1. R60
R18.95
R5.50
= R45,35

2. Different could be there if the prices can be change,, but is not going to be the same.

3. yes,, methods works on every numbers of the prices over if it can change.. methods will still work.

APPENDIX C2: POST INTERVENTION RESPONSES

A/1 Baltymore Group

Group members

Lekgau katlego
Mogowe Amanda
Mphahlele Baltymore
Mphahlele Koketjo.
Thobejane Rockey

21 September 2010

1. Tina and Nomusa are knitting squares for a blanket. Tina knits four times as many squares as Nomusa. They need 100 squares to complete the blanket. How many squares do they each knit.

- let Tina knit $= 4x$ squares
- Let Nomusa knit x squares.

therefore $4x + x = 100$

$4x + x = 100$

$\frac{5x}{5} = \frac{100}{5}$

$x = 20$

if $x = 20$ that means Nomusa is knitting 20 squares and Tina $4x = 4 \times 20$
$= 80$ squares.

2. The perimetor of an equilateral is 210mm. Calculate the length of the side

$S + S + S = 210mm.$

each side of an equilateral triangle is equal

$3s = 210\ mm$

$\therefore S = \frac{210mm}{3}$

$S = 70mm$

∴ Each side of a triangle is 70mm.

3. A mother is seven times as old as her daughter. In five years time she will be four times as old as her daughter. How old are they now? (Hint, let the daughter's age be x)

x - Child

7x - mother's age.

$7x+5 = 4(x+5)$

$7x+5 = 4x+20$

$7x-4x = 20-5$

$3x = 15$

$x = \frac{15}{3}$

$x = 5.$

A/I.- Group Work Yuventine Group.

Mathematics Classroom Activities

Grade : 9A

Date: 21 September 2010

Group Members

1. Mabosheg Tlhologelo
2. Makobela Juventine
3. Modiba Kgothatso
4. Thipe Siyabonga
5. Selemi Thaetso

a. Tina and Nomusa are knitting squares for a blanket. Tina knits four time as many squares as Nomusa. They need 100 squares to complete the blanket. How many squares do they each knit

$$(x \times 4) + x = 100$$

$$\frac{5x}{5} = \frac{100}{5}$$

$$x = 20$$

b. The perimeter of an equilateral triangle is 210 mm. Calculate the lengths of the side

$$S \times S \times S = 210$$

$$\frac{3S}{3} = \frac{210}{3}$$

$$S = 70$$

c. A mother is seven times as old as her daughter. In five years time she will be four times as old as her daughter. How old are now? (Hint let the daughter's age be x)

$7x$ = mother age

$$7x + 5 = 4(x + 5)$$

$$7x + 5 = 4x + 20$$

$$7x - 4x = 20 - 5$$

$$3x = 15$$

$$x = \frac{15}{3}$$

$$x = 5$$

The child's age is 5

The mother's age is 35

Activity 2 – A|I   Molisi Morema

Mathematics Development   Individual   Grade 9A   22 September 2010

Distance from home - school = Distance from school - home

$D = S \times t$

$S \times t = S \times t$

~~32km/h × t = 24km/h 15min~~

$32km/h \times t = 24km/h (t + 15min)$

$32km/h \cdot t = 24km/h \cdot t + 360\ km/h \cdot m$

$\frac{8km/h \cdot t}{8km/h} = \frac{360\ km/h \cdot m}{8km/h}$ (45)

$t = 45min$

$\therefore D = S \times t$

$= 32km/h \times 45\ min$

$= 32km/h \times 0,75h$

$= 24km/h$

The distance between his home and the school is 24km/h

	Distance	Speed	Time
Morning	~~24~~ x km	32km/h	$\frac{x}{32km/h}$
Afternoon	24 km	24km/h	$\frac{24km}{24km/h} = 1h$

$D = S \times t$

$\therefore t = \frac{D}{S}$

$t = \frac{24km}{24km/h}$

$t = 1h$ it takes him one hour to cycle back home in the after-noon

Activity 2 - AfI

Name: Selemi Thaetšo
Grade: 9^A
Date: 23 September 2010
Mathemetics

- A boy cycles from home to school in the morning and back home in the afternoon. He cycles from home to school at 32 km/h and back at 24 km/h. It takes him 15 minutes longer in the afternoon than in the morning. Than find the distance between his home and the school

1. The distance between his home and the school is 42,67 km/h

Calculations $d = \frac{S}{t}$

$d = \frac{32 \text{ km/h}}{0{,}75 \text{ h}}$

$= \frac{3200}{75}$

$d = 42{,}67 \text{ km/h}$

	Distance	Speed	Time
Morning	x km	32 km/h	$\frac{x}{32}$ h
Afternon	42,67 km/h	60 min/h	45 minutes

(a) $d = \frac{S}{t}$

$d = \frac{32 \text{ km/h}}{0{,}75 \text{ h}}$

$= \frac{3200}{75}$

$d = 42{,}67 \text{ km/h}$

(b) $S = \frac{d}{t} \times \frac{t}{1}$

$= 45 + 15$

$S = 60 \text{ min/h}$

(a) $d = S \times t$

$32 \text{ km/h} \cdot x t = 24 \times (t \times 15)$

$32 \text{ km/h} \cdot t = 24 \text{ km/h} t + 360$

$32 \text{ km/h} - 24 \text{ km/h} = 360$

$= 8 = 360$

$t = \frac{360 \text{ /h min}}{8}$

$t = 45 \text{ minutes}$

ACTIVITY 3 - A/I

Molobsi Morema

Mathematics Development Task in pairs Grade 9A 22 September 2010

(a) $(48 \times R60) + (48 \times R18,95) + (48 \times R5,50)$

$= R2880 + R\,909,60^c + R\,264$

$= R4053,60^c$

or $48(R60 + R18,95^c + R5,50^c)$

$= 48(R84,45^c)$

$= R4053,60^c$

(b) The ~~Ba~~ First method is longer than the second.

(c) Yes they will work.

(d) The ~~f~~ seond method is better than the First.

PAIR WORK

1. Makobela Yuventine.

2. Thipe Siyabonga.

Mathematics Work

Grade 9A

22 September 2010

1. R60 (Ratanga Entrance Fee)

+R18,95 (Meal)

+R5,50 (Bus ticket)

= R84,45 (Total cost. ~~of the outing~~)

84,45 (Total cost. ~~of the outing~~)

× 48,00 (Total number of learners.)

4053,60 (Total cost of the outing with all the learners.)

2. The different is that other pairs used the method of calculating all the prices and then multiplyed by the total number of learners. The same is that the answer is the same.

3. Yes.

4. All the methods are best because they all found the same answer at the end.

ACTIVITY 2 – A/L

Ralebofu Khutšo
Grade 22 September 2010
Maths

Mr Tshewete decides that at the end of each year he will take his Grade 9 class on an outing to Ratanga Junction. He decides to pay for the bus fare, entrance ticket and meal for his learners

Mr Tshewete has 48 learners in his class and wants to calculate the total cost of the outing

1. 48 x R60 = R2,880 48 x R18,95 = R909,6 48 x R5,50 R264

The total cost of the outing is R4053,6

2. The different is that when you are calculating you will not get the same answers and the method or calculation is that you just have to muitiply the number of the learners by the prices given

3. Yes, the method will always work even if the numbers of learners or prices

4. The best method is just by muitiplying the number of the learners by the prices given and that is what I choose on question 1

ACTIVITY 3 – A/I

Pair work Mathematics 22september2010

1. Lekgothoane mapeu.
2. Mooa Boitumelo.

1. 48 (R60 + R18,95 + R5,50)
48 × R84,45
= R4053,6

2. (48 × R60) + (48 × R18,95) + (48 × R5,50)
= R 880 = 909,6 = 264

3. Yes.

ACTIVITY 3 A/I.

Pair Work

Members: Hlonga Siewe
Monjeine Thakgiso

Grade 9th 22 September 2010

1. 48 (R18,95 + R60 + R5,50)
48 (R84,45)
= R4053,60c

2. (48 × R60) + (48 × R18,95) + (48 × R5,80)
= R2880 + R909,60c + 264
= R4053,60c

3. Yes

4. The first one.