TOWARDS ESTIMATING THE BRITTLENESS INDEX OF THE ECCA GROUP SANDSTONES BASED ON MINERALOGICAL CHARACTERISTICS

BY

SEKGONA TUMI LAURETTA

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MAIN SUPERVISOR: PROF FHATUWANI SENGANI

CO-SUPERVISOR: MS ELELWANI DENGE

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DECLARATION

I, **Sekgona Tumi Lauretta**, do hereby declare that this dissertation is the results of my investigation and research that has not been submitted in part or full for any other degree to any university or publication houses (journal, conference proceedings etc).

Sekgona TL

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ABSTRACT

Brittleness is a critical property in geological formations, influencing the extraction and production processes in the oil and gas industry, as well as in mining and civil engineering applications. Many methods were developed to determine the brittleness index; however, none is standardised. This study aims to estimate the brittleness index of the Ecca Group Sandstones, a significant geological formation in southern Africa, based on its mineralogical characteristics. Petrographic analysis, including thin section microscopy and image analysis, was conducted to quantify petrographic characteristics such as mineral composition, grain size distribution, and fabric properties (packing density, packing proximity, consolidation factor). Through detailed petrographic analysis, key mineralogical parameters, including the types and proportions of clay minerals, quartz, feldspars, and other constituents, were systematically evaluated. Statistical analyses, including correlation studies and multivariate analyses, were conducted to establish relationships between these petrographic attributes and the Brittleness Index. Results indicate that specific petrographic characteristics such as contact type and contact nature strongly influence the brittleness of the sandstones in the Ecca Group. Four brittleness indices, namely B3, B4, B5, and B6, were formulated. Brittleness indices B1 and B2 proved ineffective, hence equations were not formulated for them. Models with a high number of independent variables are deemed impractical and less reliable. Hence, among the developed models for predicting brittleness indices, the most optimal choice is the model that has fewer independent variables, along with higher values of R^2_a and F. These brittleness indices (B_3, B_4, B_5 and B_6) highlight the complex interplay of various contact types and packing arrangements in determining the brittleness of a material.

Keywords: Brittleness index, petrographic characteristic, sandstone, Ecca Group, hydraulic fracturing, Southern Africa.

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SYMBOLS AND ABBREVIATIONS

ANN	Artificial Neural Networks
BI	Brittleness Index
BTS	Brazilian Tensile Strength
E	Young Modulus
GSI	Geological Strength Index
LIBS	Laser-induced Breakdown Spectral
OCR	Over Consideration Ratio
RMR	Rock Mass Rating
RQD	Rock Quality Designation
SIPS	Surface Instability Peak Strength
TOC	Total Organic Carbon
TS	Tensile Strength
UCS	Uniaxial Compressive Strength
V	Poisson’s ratio
XRD	X-ray diffraction

CHAPTER ONE

INTRODUCTION

1.1. BACKGROUND

In recent years, the exploration and production of unconventional hydrocarbon resources, such as shale gas and tight oil, have gained significant attention in the energy industry. Understanding the mechanical properties of reservoir rocks, particularly their brittleness index, is crucial for optimizing hydraulic fracturing operations and maximizing hydrocarbon recovery (Rickman et al., 2008). Among these reservoir rocks, the Ecca Group sandstones present an intriguing yet challenging prospect due to their heterogeneous mineralogical composition and complex pore structure (Grosvenor et al., 2018).

The brittleness index, a key parameter in hydraulic fracturing design, quantifies the propensity of a rock to fracture under stress. Traditionally, this index has been determined empirically through laboratory experiments or inferred from well logs (Jarvie et al., 2007). However, these methods often lack accuracy and efficiency, especially when dealing with complex geological formations like the Ecca Group sandstones. For example, laboratory-based methods such as uniaxial compression tests can provide valuable data but are time-consuming and may not fully capture the heterogeneity of reservoir rocks (Shi et al., 2011). Similarly, well log-derived estimations often rely on empirical correlations that may not account for the intricate mineralogical variations present in formations like the Ecca Group (Wang et al., 2012).

This project aims to bridge this gap by proposing a novel approach to estimate the brittleness index of the Ecca Group sandstones based on their mineralogical characteristics. This study seeks to establish correlations between mineralogical composition and brittleness properties.

The significance of this research lies in its potential to enhance reservoir characterization and improve hydraulic fracturing design strategies in unconventional reservoirs. By providing a more accurate and efficient method for estimating brittleness index, operators can optimize well placement, stage spacing, and fracturing fluid selection, ultimately leading to increased hydrocarbon recovery and economic efficiency.

Through this project, we aim to contribute to the ongoing advancement of reservoir engineering practices and foster sustainable development in the energy sector.

1.2. PROBLEM STATEMENT

Brittleness Index (BI) is a term that is commonly used in rock mechanics which indicates whether a rock formation is brittle, tends to fracture easily or ductile, and tends to resist fracture growth (Mews et al., 2019). It is an important parameter that determines the rock mass failure characteristics which could be based on the lithology, mineralogy and the effective stresses acting upon the rock formation (Mews et al., 2019; Meng et al., 2020; Ryka, 2020; Zhang et al., 2022).

A rock's brittleness influences the mechanical responses and failure processes of the rocks to a certain extent such as cutability and drillability associated with the rock excavation; fractability during hydraulic fracturing mainly associated with oil and gas exploration; core disking during borehole drilling of rocks; and rock burst susceptibility in deep hard rock tunnels (Kang et al., 2020; Meng et al 2020; Zhang et al., 2021). Those problems have led to rock brittleness attracting a lot of interest in the literature with many researchers attempting to define what is brittleness. For instance, Rickman et al. (2008) introduced a method for calculating the brittleness index based on mineralogy and mechanical properties. Similarly, Li et al. (2018) developed an empirical approach that integrates well log data to estimate brittleness in shale reservoirs. More recently, Yang et al. (2021) proposed a combined petrophysical and mechanical model for brittleness estimation, addressing the variability in complex geological settings such as heterogeneity in mineral composition, variations in pore structure, and the presence of fractures in unconventional reservoirs like shale formations.

Despite the fact that the brittleness index is broad in literature to date, there is no standardized or universally accepted definition and analytical methods of the brittleness index. Different definitions and analysis methods apply to different fields (Meng et al., 2020). Furthermore, there is no study in South Africa about the estimation of the brittleness index of sandstones of the Ecca Group based on mineralogical

characteristics. Therefore, the proposed dissertation is anticipated to develop brittle indexes of the Ecca Group sandstones based on their mineralogical characteristics.

1.3. AIM

The aim of the study is to investigate and quantify the relationship between the mineralogical composition of the Ecca Group Sandstones and their brittleness index, with the goal of developing a reliable predictive model that can estimate the brittleness based on mineralogical characteristics such as mineral content, texture, and grain size.

1.4. OBJECTIVES

i. To determine the mineralogical composition of the sandstones of the Vryheid Formation using petrographic analysis.
ii. To evaluate the geomechanical properties of the Vryheid Formation sandstones (e.g., UCS, tensile strength and elasticity) required to calculate brittleness indices.
iii. To assess the relationship between mineralogical characteristics and the brittleness index of the Vryheid Formation sandstones.
iv. To develop a predictive model for estimating the brittleness index of the Vryheid Formation sandstones using mineralogical data.

1.5. RESEARCH QUESTIONS

I. How do mineralogical characteristics impact brittleness of the Vryheid Formation sandstones?
II. What mineralogical features most significantly affect brittleness of the Vryheid Formation sandstone?
III. How do variations in mineral composition impact the geomechanical properties of these sandstones?
IV. Can a reliable relationship be established between the mineralogical composition and the calculated brittleness index?

1.6. HYPOTHESIS

It is hypothesized that the brittleness index of the Vryheid Formation sandstones is significantly influenced by the mineralogical composition of the rock. Specifically,

higher concentration of quartz and feldspar are expected to correlate with an increased brittleness index, while the presence of more ductile minerals such as mica or calcite is likely to result in a lower brittleness index. The mineralogical components are expected to directly affect the rock's mechanical properties, thus influencing its ability to fracture under stress.

Another hypothesis is that the brittleness index of the Vryheid Formation sandstones varies across different stratigraphic columns, with certain layers showing higher or lower brittleness due to varying mineralogical content and diagenetic history. These differences are expected to reflect the varying depositional environments and subsequent geological processes that have influenced the mineralogical makeup of the rock over time.

A third hypothesis is that it is possible to develop a predictive model for the brittleness index of the Vryheid Formation sandstones using mineralogical data as input parameters. By analysing the relative proportions of key minerals, a robust model can be established that allows for accurate predictions of brittleness, aiding in the assessment of reservoir quality for hydraulic fracturing or other geotechnical applications.

1.7. JUSTIFICATION

The estimation of the brittleness index of the Vryheid Formation sandstones is of economic importance since the supergroup is associated with the shale gases and the coal seams hosted by the Early Permian Vryheid Formation and the Triassic Molteno Formation (Cadle et al., 1993). The Karoo strata could have economic ramifications for coal resource exploration since the coal is mostly used to generate electricity. However, it tends to behave brittle when subjected to high stresses with failure characterized by the complexity of fractures. The latter have a strong impact on the behaviour of the coal during mining operations (Singh, 1986). The brittleness index measures how well coal resists fragmentation under pressure (Zhang et al., 2021).

Furthermore, the possibility of a contentious fracking program to tap into its potential shale gas deposits has put South Africa's Karoo region in the news recently. For the fracking program also known as the hydraulic fracturing, which is the technique used to extract gases by injecting high-pressure liquids to fracture the reservoir to be a

success, the brittleness index of the rocks has to be determined. It is postulated that the use of hydraulic crackers is simpler in rock formations with high brittleness values (Mews et al., 2019). Thus, it is crucial to locate brittle zones in gas reservoirs to create effective fractures for hydraulic fracturing.

- Contribution to Geological Understanding

The Ecca Group, which is part of the Karoo Supergroup, is a significant geological formation known for its extensive sedimentary deposits. Estimating the brittleness index based on mineralogical characteristics contributes to a deeper understanding of its mechanical properties. This research can refine the geological models of the Ecca Group by integrating mineralogical data, offering insights into the formation's history, sedimentary processes, and tectonic influences.

- Advancement in Resource Exploration and Extraction

Understanding the brittleness of geological formations is crucial for industries such as oil and gas, coal mining, and geothermal energy. The Ecca Group is a notable target for these resources. By accurately estimating the brittleness index, this research can aid in identifying optimal drilling and fracturing strategies, enhancing the efficiency and safety of resource extraction. It can also reduce operational risks and costs associated with drilling in brittle formations.

- Environmental and Safety Implications

Accurate brittleness assessment can lead to better prediction and management of natural hazards, such as landslides and earthquakes. This research can contribute to the development of safer mining and drilling practices by identifying zones within the Ecca Group that are prone to failure. Furthermore, understanding brittleness can help in assessing the potential environmental impacts of extraction activities, leading to more sustainable resource management.

- Methodological Innovations

The approach of linking mineralogical characteristics to the brittleness index represents a methodological advancement. It can set a precedent for similar studies in other geological formations, promoting the use of mineralogical data in

geomechanical modelling. This research can also lead to the development of new analytical techniques and tools that can be applied broadly in geosciences.

- Academic and Educational Value

This research adds to the academic body of knowledge and serves as a valuable reference for future studies in geology, geomechanics, and related fields. It can provide a solid foundation for further research on the Ecca Group and similar sedimentary formations. Additionally, the findings and methodologies from this study can be incorporated into educational curricula, enhancing the training and skills of future geoscientists.

- Societal and Economic Benefits

By improving the efficiency and safety of resource extraction, this research can have significant economic benefits. It can contribute to the sustainable development of energy resources, which is crucial for energy security and economic growth. Moreover, by addressing environmental and safety concerns, this study supports the responsible stewardship of natural resources, which is essential for societal well-being.

In summary, estimating the brittleness index of the Ecca Group based on mineralogical characteristics holds significant implications for geological research, resource extraction industries, environmental safety, methodological advancements, academic knowledge, and societal and economic development.

1.8. Description of the study area

This section briefly describes the location, topography, and the climate conditions of the study area.

1.7.1. **Location of the study area**

The study area is located within the Ermelo Coal Mine, also known as the Spitskop Colliery, which is located near Breyten in the Msukaligwa Local Municipality, within the Gert Sibande District Municipality in Mpumalanga, South Africa (Mining Outlook) (Mapcarta, n.d.). The mine's geographical coordinates are approximately 26.13958°S latitude and 30.04497°E longitude (Mapcarta, n.d.).

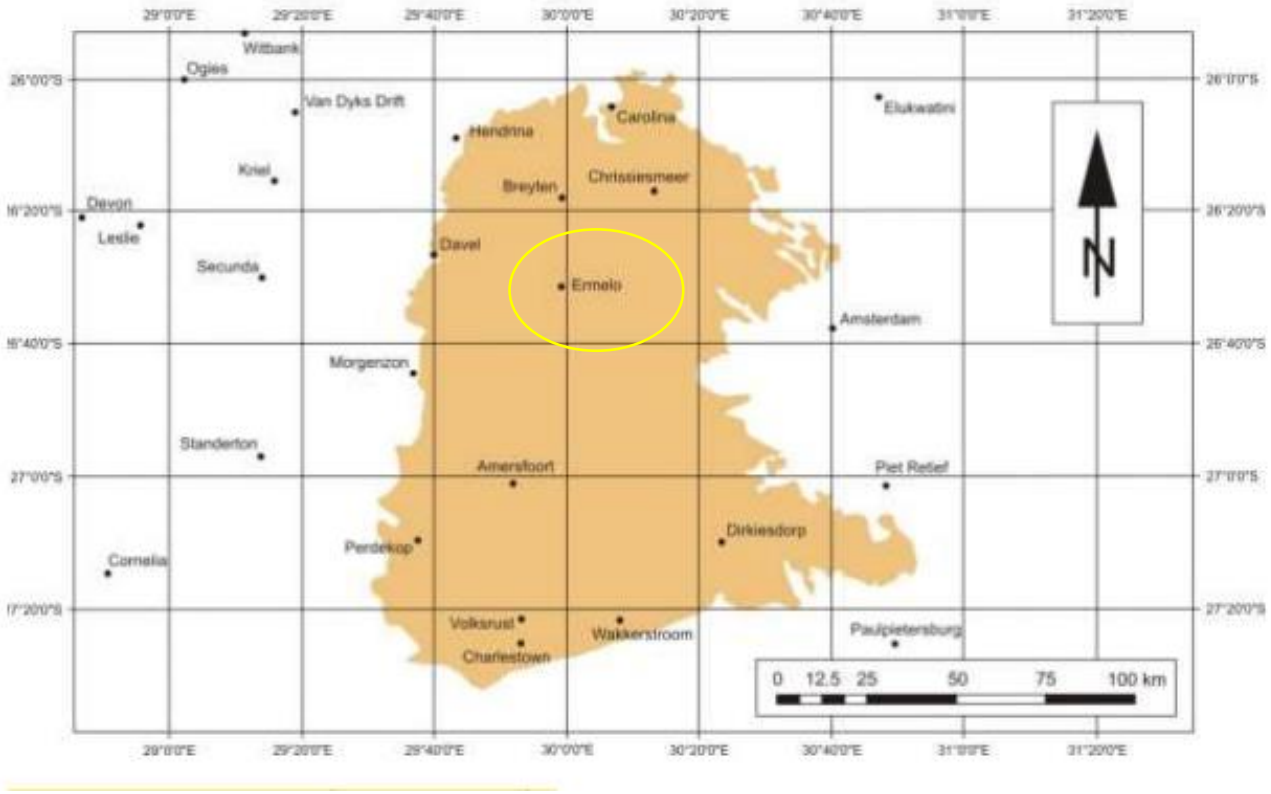

Figure 1.1 Geographic extent of Ermelo Coalfields (after Hancox and Gotz, 2014).

1.7.2. Brief Geology of the study area

The Ermelo coalfield is situated within the Ecca Group of the Karoo Supergroup in South Africa, specifically within the Vryheid Formation (Figure 1.2). The Ecca Group, which is part of the Karoo Supergroup in South Africa, is a significant geological formation primarily composed of sedimentary rocks deposited during the Early Permian period. It represents a sequence of shales, sandstones, and coal beds, indicating a variety of depositional environments ranging from deep marine to deltaic and fluvial settings. The Ecca Group is subdivided into several formations, including the Prince Albert Formation, Whitehill, Collingham Formation, Ripon Formation, Vryheid Formation and Volksrust Formation, each characterized by distinct lithological features. The Vryheid Formation, which hosts the Ermelo Coalfied, is a crucial unit

within the Ecca Group, predominantly composed of sandstones, shales, and coal beds (Johnson et al., 1996). The Whitehill Formation, in particular, is notable for its organic-rich shales, which are potential hydrocarbon source rocks (Cairncross., 2013). The deposition of the Ecca Group occurred in an extensive inland sea, known as the Karoo Basin, which was influenced by glacial and post-glacial processes (Johnson et al., 1996).

This geological formation is known for its significant coal reserves, primarily occurring in the Permian-aged strata of the Ecca Group (Wells et al., 1996). The coalfield is geologically characterized by alternating layers of shale, sandstone, and coal seams, which are typically inclined gently towards the south-southeast with angles ranging from 5° to 15° depending on local tectonic conditions (Lategan et al., 2009).

The geology of the Ermelo coalfield has been studied due to its economic importance as a coal-producing area in South Africa (Götz et al., 2015). These studies have focused on the coal-bearing stratigraphy, the structure of the coalfield, and its coal resources, which play a significant role in South Africa's energy sector. Understanding he structural and sedimentological aspects of the coalfield is crucial for effective exploration and exploitation of its coal resources (Eriksson et al., 2014). It consists of five coal seams namely A, B, C, D and E as depicted on Table 1. The coal deposits in the Ermelo coalfield are primarily bituminous and are associated with sedimentary environments indicative of ancient riverine and lacustrine settings during the Permian period. These environments facilitated the accumulation of organic-rich material that eventually formed the coal seams found in the region (Cairncross, 2013).

Table 1. 1 Ermelo Coal Seam (Hancox and Gotz, 2014)

Coal Seam	**Depth (m)**	**Description**
A Seam	0 to 1.5	The A seam is mostly eroded in the Ermelo coalfield, primarily found in the central and northern parts, with a thickness ranging from 0 to 1.5 m.
B Seam	1 to 2.7	The B seam is the thickest and most continuous in the eastern and northeastern regions of the Ermelo coalfield, with a thickness ranging from 1 to 2.7 m. It

		is subdivided into three parts: BL, BU, and BX. Although its quality is lower in rank than the C seam, it is still of good quality.
C Seam	0.5 to 4	The C seam is the most economically significant seam, divided into two parts: C-upper (CU) and C-lower (CL). The CU seam is well-developed throughout most of the coalfield, with a thickness ranging from 0.7 to 4 m, while the CL seam's thickness ranges from 0.5 to 2 m where it is developed.
D Seam	0.1 to 0.4	This seam contains good quality coal but is usually not thick (0.1 to 0.4 m) and not economically viable to mine. It has no partings, a large amount of vitrains, and some durain bands.
E Seam	3	The E seam is absent in most of the coalfield. Where it is developed in the north, its thickness can reach up to 3 m. The coal is bright, usually banded, often with sandstone layers above and below, and can sometimes be split by partings.

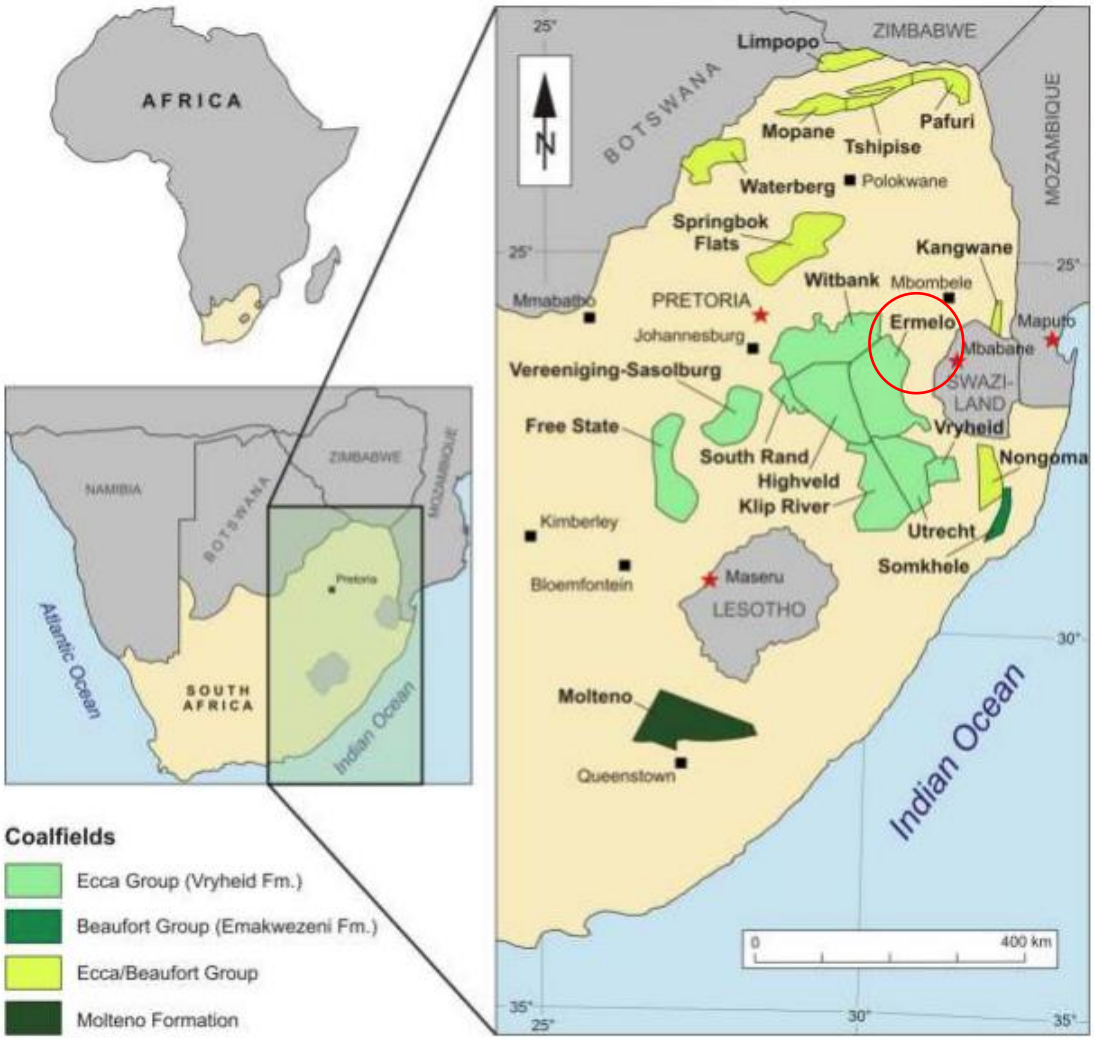

Figure 1.2 Geological Map of the Karoo Supergroup with the Ermelo Coalfield demarcated with red circle (modified after Hancox and Gotz, 2014)

1.7.3. Topography of the study area

The Msobo Coal Mine, located near Ermelo in Mpumalanga, operates within a region characterized by gently rolling terrain typical of the Highveld, with an elevation of approximately 1,700 m above sea level. The topography is predominantly flat to slightly undulating, which facilitates both surface and underground mining activities. The area is part of the broader Ermelo Coalfield, which is known for its bituminous coal reserves.

The region surrounding the mine includes agricultural land, with a landscape that supports both farming and mining operations. The climate in the area is generally temperate, contributing to the viability of coal extraction and other economic activities.

1.7.4. Climatic conditions of the study area

The Msobo Coal Mine in Ermelo, Mpumalanga, experiences a temperate climate with distinct seasonal variations. Ermelo's climate is characterized by warm, rainy summers and cool, dry winters.

During the summer months (November to February), temperatures are relatively high, averaging around 26°C during the day and dropping to about 14°C at night. This period is also the wettest, with December being particularly rainy, receiving up to 111 mm of precipitation, and high humidity levels around 80% (Weather and Climate, n.d.).

Winter months (June to August) are cooler and much drier. Daytime temperatures average about 21°C while nighttime temperatures can drop to around 7°C. This season sees minimal rainfall, with June typically receiving only about 24 mm of rain and lower humidity levels, averaging around 59% (Weather and Climate, n.d.).

Overall, the climate in Ermelo supports mining operations with favourable conditions for most of the year, though summer rains can occasionally impact activities. The dry and cool winters provide stable conditions for continuous mining operations with minimal weather interruptions.

1.8 Layout of the dissertation

The dissertation is divided into five chapters with distinct goals after the introductory chapter. The literature review portion comes after the introduction chapter. The geology of the study area is briefly summarized in the literature review, which is followed by a thorough analysis of the literature on brittleness index techniques. Although there are several brittleness index approaches, the literature chapter showed that each method has a number of drawbacks. Since none of the earlier techniques could reliably predict the brittleness of the sandstones, there is therefore potential to investigate the utilization of mineralogical features to build the brittleness index of the sandstones.

In chapter three, specific methods are reported after the knowledge gap has been outlined in chapter two. A reconnaissance survey, data collecting, laboratory analysis, numerical analysis, and statistical analysis are all included in these techniques. The findings chapter is on using empirical methods and mineralogical methods to generate new brittleness methods. In fact, a number of novel techniques were created to precisely calculate the brittleness of sandstones using both techniques. Finally, Chapter five reports the findings and recommendations.

CHAPTER TWO

LITERATURE REVIEW

2.1. GEOLOGY

2.1.1. Geological setting of the Main Karoo Basin

The Karoo Basin is a vast and intricate sedimentary basin located in South Africa that spans almost two-thirds of the country and covers an area of approximately 700,000 Km^2 (Johnson et al., 1996; Catuneanu et al., 2005). The basin was formed during the late Carboniferous and early Permian periods, approximately 300-250 million years ago, when much of the area was covered by a shallow sea (Catuneanu et al., 2005; Smith et al, 1993). The formation of the Main Karoo Basin in South Africa has been the subject of scientific study and debate, resulting in various proposed models. While the understanding of the basin's formation is still evolving, a few of the proposed models include the rift basin model, foreland basin model, flexural basin model, and combination model (Baiyegunhi and Liu, 2021; and Catuneanu et al., 2005).

The rift basin model suggests that the Main Karoo Basin formed as a result of extensional tectonic forces, leading to the opening of a rift. It proposes that the basin was formed during the breakup of the supercontinent Gondwana, when the landmass started to split apart (Catuneanu et al., 2005; Rubidge and McCarthy, 2005). The extensional forces caused the Earth's crust to stretch and thin, creating a basin where sedimentation took place. The foreland basin model suggests that the Main Karoo Basin was formed as a foreland basin in response to the collision of Gondwana with another landmass (Cadle et al., 1993; Catuneanu et al., 2005). As the collision occurred, compressional forces led to the uplift and deformation of the surrounding areas, causing the formation of a large basin in front of the rising mountain ranges. According to this model, sedimentation in the basin was primarily influenced by the erosion and deposition of materials from the adjacent uplifted regions.

The Flexural Basin Model proposes that the Main Karoo Basin developed as a flexural basin in response to the loading and subsequent bending of the Earth's crust due to the weight of the overlying sediments (Catuneanu et al., 2005). It suggests that during the accumulation of thick sedimentary sequences in the Karoo Basin, the underlying crust flexed downward, creating a depression where sediments were deposited. Some

researchers (Rubidge and McCarthy, 2005) suggest that the formation of the Main Karoo Basin involved a combination of tectonic processes. Similarly, Catuneanu et al. (1998) and Johnson et al. (2006) highlight the role of tectonic loading associated with the Gondwana orogeny and the subsequent influence of glacial and post-glacial dynamics on basin evolution. This combination model considers the interplay of rift tectonics, foreland basin tectonics, and flexural subsidence. It suggests that the basin's formation was influenced by a complex interplay of extensional, compressional, and flexural forces, resulting in the development of the basin and subsequent sedimentation. These models are not mutually exclusive, and the actual formation of the Main Karoo Basin may have involved a combination of these processes.

The Main Karoo Basin is packed with sedimentary strata that date back to the Carboniferous period and extend to the Early Jurassic period, between approximately 360 and 180 million years ago (Rubidge and McCarthy, 2005). These strata, which make up the Karoo Supergroup, have a thickness of over 5 km. The Karoo Supergroup is located in southern Africa, covering large parts of South Africa, Lesotho, Swaziland, Namibia, Zimbabwe, and Mozambique. Catuneanu et al. (2005) postulated that the sedimentation of the Karoo Supergroup was influenced by shifts in climate and tectonic activity, marking significant geological changes in the region.

Deposition within the Main Karoo Basin commenced with the formation of glacial deposits from the Late Carboniferous to Early Permian period, known as the Dwyka Group. This was followed by the deposition of the Permian Ecca Group, which contains significant coal deposits and is associated with deltaic, continental slope, and lacustrine sediments. Above the Ecca Group, we find the Beaufort Group, which consists of fluvial-lacustrine rocks from the Middle Permian to the Middle Triassic period. The Beaufort Group overlaps the Molteno Group, which dates back to the Late Triassic. The Molteno Formation was deposited in a broad, perennial braided system characterized by seasonally warm to humid climatic conditions, primarily in limited braid plain areas. In contrast, the Elliot Formation, which lies above the Molteno Formation, consists of argillaceous deposits. These were formed by high and low sinuosity fluvial systems under semi-arid climatic conditions. As the climate progressively became drier, the aeolian dune complexes of the Clarens Formation were deposited. These dunes were formed by wind activity. The sedimentation

process of the Karoo Basin came to an end with the deposition of the continental flood basalts of the Drakensberg Group, which are approximately 180 million years old (Smith and Keyser, 2003). This event coincided with the initiation of the breakup of the supercontinent Gondwana.

In general, the sedimentary layers within the Main Karoo Basin of South Africa were formed in various environments, including glacial, deep marine (including turbidite), shallow marine, deltaic, fluvial, lacustrine, and aeolian settings" and stratigraphically divided into five groups namely: Dwyka; Ecca; Beaufort; Stormberg, and Drakensberg from the oldest to the youngest.

2.1.1.1. The Dwyka Group

The Dwyka Group deposits accumulated about 300 to 260 Ma (duration of about 40 million years) during a time when the southern hemisphere experienced a major ice age known as the Gondwana glaciation (Rubidge and McCarthy, 2005). This glaciation occurred when the supercontinent Gondwana, which included present-day Africa, South America, India, Australia, and Antarctica, was located near the South Pole (Rubidge and McCarthy, 2005). Hence, the Dwyka Group consists of glacial and post-glacial sediments including glacial till, dropstones, diamictites, sandstones, and mudstones. These sediments were deposited as a result of glacial activity and subsequent meltwater runoff during the retreat of the ice sheets. The glacial deposits often contain boulders and cobbles that were transported and deposited by ice, while the post-glacial deposits consist of finer-grained sediments.

2.1.1.2. The Ecca Group

Another constituent of the Karoo Supergroup is the Ecca Group, which was formed during the Early Permian period about 260 to 250 Ma (Johnson et al., 1996; Catuneanu et al., 2005; Rubidge and McCarthy 2015). It is the Karoo Supergroup's second largest component spanning over 50% of South Africa's land area (Baiyegunhi et al., 2019). The Ecca Group dominates the Permian timeframe of the Karoo Supergroup and encompasses a significant portion of its geological record. The Ecca Group primarily comprises mudstones, siltstones, sandstones, and a few conglomerates, which were deposited in wetlands and shallow water deltas on the northern coast of the Karoo Sea. These sediments were formed in warm climatic conditions, which created a marshy or "paludal" environment. As a result, coal

deposits are abundant within the Ecca Group, mainly in the northern region of the basin, and they do not occur in the southern part of the basin. In South Africa, almost all coal resources are found in the Ecca Group in the provinces of Mpumalanga, Gauteng and KwaZulu-Natal.

In contrast, Raseroka and McLachlan (2009) postulated that the southwestern region was characterised by steep slopes that led up to the shoreline and created a deep environment that created favourable conditions for underwater sediment avalanches, also known as turbidites, to transport coarse and fine-grained material out into the distant, deep waters. These turbidites are formed by a fining-upward sequence that ranges from coarse sandstone at the bottom to siltstone and shale at the top. During transportation, the organic material in the turbidite remained in suspension, settling out under anoxic bottom conditions in the deepest part of the basin. Eventually, this organic material was buried and converted into oil and gas (Raseroka and McLachlan, 2009).

2.1.1.3. **The Beaufort Group**

The sediments within the Beaufort Group were deposited about 250 to 240 million years ago (Rubidge and McCarthy, 2005). The group represents a diverse range of depositional environments, including marine, deltaic, and fluvial (river) systems. The sedimentary rocks of this group include sandstones, siltstones, shales, and conglomerates. These rocks were formed from the accumulation of sediment eroded from pre-existing landmasses, transported by rivers, and deposited in various environments such as shallow seas, river deltas, and nearshore areas (Catuneanu et al., 2005; Johnson et al., 1996).

2.1.1.4. The Stormberg Group

The Stormberg Group is considered the uppermost unit of the Karoo Supergroup and is characterized by its distinct lithological and stratigraphic features. It was deposited about 240 to 193 Ma (Smith et al., 1993). The sedimentary rocks of the Stormberg Group consist of various formations, including sandstones, shales, siltstones, and mudstones. These rocks were deposited in a variety of environments, such as floodplains, river channels and deltaic systems. The lower part of the group, which is known as the “Molteno Formation”, consists of fluvial and lacustrine deposits, indicating the presence of rivers and lakes. These sediments were likely formed in a

lowland or floodplain setting, where erosion and deposition by rivers played a significant role. Above the Molteno Formation are the deposits of the Elliot Formation consisting of more deltaic and coastal plain deposits and semi-arid to arid alluvial plains (Johnson et al., 1996; Catuneanu et al., 2005). The uppermost unit of the Stormberg Group is the Clarens Formation consisting of wind-blown and dune-covered desert deposits. It is believed that during the Early Jurassic, the region experienced alternating marine transgressions and regressions, resulting in the deposition of these coastal sediments (Johnson et al., 1996; Catuneanu et al., 2005).

2.1.1.5. The Drakensberg Group

This group consist of about 1.6 km layer of lava which was formed by the continental scale rifting during the breakup of Gondwana about 183 Ma (Rubidge and McCarthy, 2005). This group marks the end of the Karoo Supergroup (Figure 2.1).

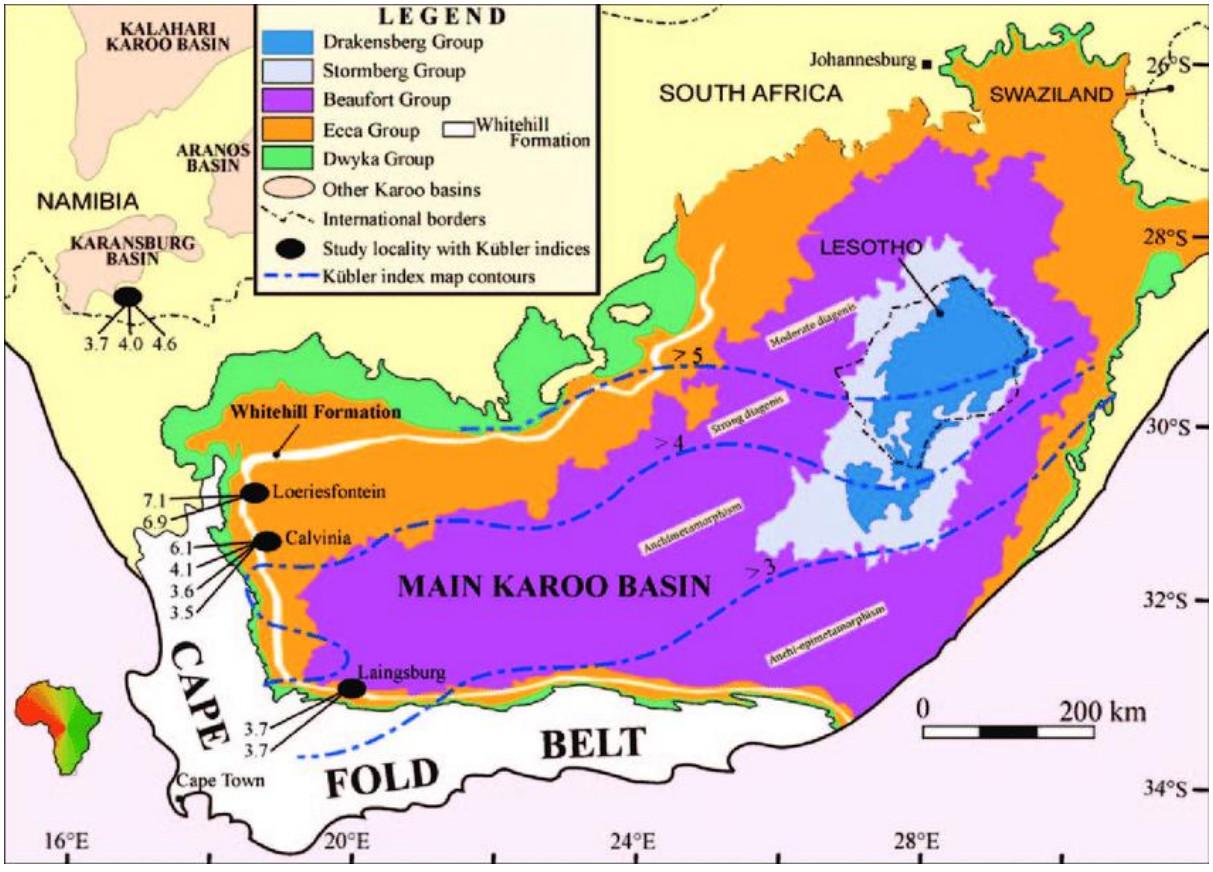

Figure 2.1. Geological map of the Karoo Supergroup in South Africa (after Smithard et al., 2015)

2.1.2. Lithostratigraphy of the Ecca Group

The stratigraphy of the Ecca Group exhibits intricacies that can be attributed to its diverse origins, including deep marine water, shallow marine turbidites, and submarine fan deposits sourced from specific areas (Baiyegunhi et al., 2019; Catuneanu et al., 2005). Despite these intricacies, the proposed stratigraphy which is about 3 km thick, is accepted by SACS (1980) and is divided into five formations namely, the Prince Albert Formation; the Whitehill Formation; the Collingham Formation; the Ripon Formation; and Fort Brown Formation (Baiyengunhi et al., 2019; Baiyengunhi and Liu, 2021; Catuneanu et al. 2005).

The Prince Albert Formation: The Prince Albert Formation overlies the Dwyka Group and represents a transition from glacial to non-glacial conditions constituting the lowest component of the Permian Ecca Group (Baiyegunhi and Liu, 2021; Smith and Keyser, 2003). The formation is predominantly a thinly to thickly bedded, dark grey to greyish-black and dark greenish-grey massive mudstone with subordinate siltstone and very fine-grained sandstone with a thickness ranging from 50 to 200 m (Smith and Keyser, 2003). The carbonaceous shales, located approximately 5 m from the uppermost part of the formation, exhibit a dark green to black coloration (Watkeys and Watkeys, 2004). In the Prince Albert Municipal area, Almond (2015) recorded a succession of dark greyish green to metallic grey, shaley mudrocks with a well-developed pencil cleavage and containing ferruginous and manganiferous diagenetic nodules.

The Whitehill Formation: The Whitehill Formation, often referred to as the "white band," gets its name from its distinctive white appearance on the surface (Baiyenguni et al. 2019). This whitening effect is a result of the weathering process, where the originally black carbon-bearing shales transform into soft, white sediments (Baiyegunhi et al., 2021; Catuneanu et al., 2008). The formation is up to 60 m thick, and is composed of interbedded sandstones, shales and coal seams with lenses of dolomite and organic-rich layers (Coffing and De Wit, 1994; Smith et al 1993). Baiyegunhi and Liu (2021) mentioned that the finely layered carbonaceous shale in the Whitehill Formation is believed to have been formed in an oxygen-depleted environment, indicating high anoxia. They further mentioned that it is likely that the sediments of the Whitehill Formation were deposited on an unstable slope, potentially leading to slumping.

The Collingham Formation: The Collingham Formation consists of six lithofacies namely, shale, siltstone, mudstone, sandstone, chert, tuff, phosphorite and terrigenous rocks (Baiyegunhi et al., 2019). It represents a deltaic environment with periodic marine incursions. Thin layers of siltstone and sandstone within the formation are the result of low-density turbidity currents occurring at a distance from the source (Baiyengunhi et al., 2019). This formation is rich in organic material.

The Ripon Formation: The Ripon Formation overlie the Collingham Formation and consists of sandstones, mudstone, and carbonaceous shale. It is poorly sorted, with fine- to very fine-grained litho-feldspathic sandstone alternating with dark grey clastic rhythmite and mudstone (Buick and Robert, 2007).

The Fort Brown Formation: The Fort Brown Formation is composed of sandstones, siltstones, and shales. It represents a transitional environment between fluvial and marine conditions. The Fort Brown Formation lies conformably above the Ripon Formation and has a thickness ranging from 0.5 to 1.5 km (Baiyegunhi et al., 2019). The lower portion of the Fort Brown Formation is predominantly argillaceous and is believed to have been deposited in a pro-delta environment (Baiyengunhi et al., 2019; Baiyegunhi and Liu, 2021). Both the Ripon and Fort Brown Formations together form a turbidite fan complex, with sediment movement occurring from the southeast in a deep to medium aqueous environment (Smith and Erikksson,1997) (Figure 2.2).

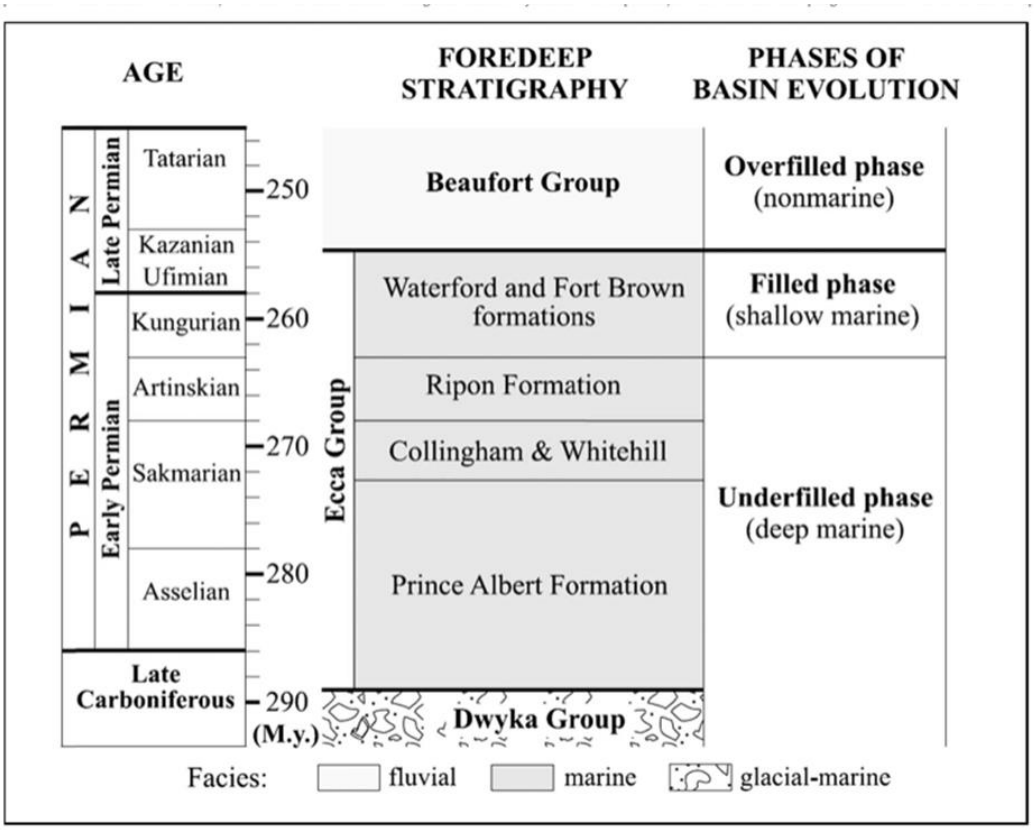

Figure 2.1 Lithostratigraphy of the Ecca Group (after Baiyengunhi et al., 2020)

2.1.3. Mineralogical characteristics of the Ecca Group

The Ecca Group in the South African geological context is characterized by a diverse mineralogical composition, offering valuable insights into its geological makeup. This stratigraphic unit encompasses both clay-rich (argillaceous) and sand-rich (arenaceous) rocks, presenting a spectrum of minerals that contribute to its distinctive features. The primary framework minerals identified within the Ecca Group include quartz, feldspar, micas, and lithics, as elucidated by Baiyegunhi et al. (2017).

Quartz, a prevalent mineral in various rock formations, significantly influences the texture and appearance of the Ecca Group rocks. Concurrently, feldspar, with its distinctive crystalline structure, contributes to the mineral diversity observed in this geological formation. The presence of micas and lithics further enriches the complexity of the rock composition, highlighting the intricate nature of the Ecca Group (Grosvenor et al., 2018; Wells et al., 1996; Botha et al., 2005).

An examination of the argillaceous rocks within the Ecca Group by Baiyegunhi et al. (2017) reveals a substantial clay content, indicating the prevalence of clay-rich formations. The detrital framework grains found in the sandstones of the Ecca Group comprise quartz, feldspars, rock fragments, matrix, and accessory minerals. Quartz, as the predominant mineral, occur as monocrystalline and/or polycrystalline quartz. Feldspar minerals, including alkali feldspar (orthoclase and microcline) and plagioclase feldspar (albite), are present within the Ecca Group sandstones. Alkali feldspar and albite are identified as the most prevalent feldspar components (Baiyegunhi et al, 2017)

The detrital framework grains are bound together by a combination of cement and matrix, where the matrix predominantly consists of clay minerals such as illite, kaolinite and chlorite (Cairncross et al., 2002; Gross et al., 2014). These clay minerals may exhibit either detrital or diagenetic forms, further influencing the overall diagenesis and reservoir properties of the Permian Ecca Group sandstones and mudrocks in the Eastern Cape Province, South Africa (Baiyegunhi et al., 2017).

2.1.4. Mechanical Properties of rocks of the Ecca Group

The sandstones of the Vryheid Formation exhibit significantly higher mechanical strength compared to other Ecca Group lithologies (Johnson et al., 1996). These sandstones are primarily composed of mechanically strong minerals such as quartz and feldspar, contributing to their overall durability and brittleness (Catuneanu et al., 2005). Their compressive strength typically ranges from 40 MPa to 150 MPa, depending on factors such as grain size and cementation (Smith, 1990). This strength allows them to withstand higher loads compared to shales and siltstones (Smith, 1990). Despite their high compressive strength, the tensile strength of Vryheid Formation sandstones remains relatively low, generally below 10 MPa (Malan, 2011). This makes them more susceptible to fracturing under tensile stress, a key factor in their geomechanical behaviour (Malan, 2011).

Additionally, they exhibit a higher Young's modulus, typically between 10 and 30 GPa (Hobbs, 1964), indicating greater stiffness and resistance to deformation under stress. Their Poisson's ratio, which reflects how they deform laterally when subjected to axial stress, ranges from 0.2 to 0.35 (Goodman, 1989).

The density of these sandstones varies between 2.4 and 2.7 g/cm³ (Carmichael, 1989), influenced by mineral composition and porosity. In terms of porosity and permeability, Vryheid Formation sandstones generally exhibit higher values than other Ecca Group rocks, with porosity ranging from 10% to 20% (Nelson, 1994). This characteristic enhances their ability to store and transmit fluids, making them important in subsurface reservoir studies (Nelson, 1994).

Although sandstones are more resistant to mechanical failure compared to weaker, fine-grained rocks such as shales, they still display relatively low fracture toughness, making them prone to fracturing under stress (Atkinson, 1984). This property is crucial for understanding their behaviour in geomechanical applications, particularly regarding stress regimes and fracture propagation.

These mechanical properties make the Vryheid Formation sandstones a crucial focus for geomechanical studies, particularly in applications related to rock stability, fluid flow, and energy resource exploration.

2.2. BRITTLENESS INDEX

2.2.1. Definition

The term "brittleness index" has different definitions depending on the context and the field of study. Hucka and Das (1974) and Hetenyi (1990) describe a brittle rock as a rock that easily ruptures without undergoing any permanent deformation under stress while Nonvendor et al. (2005) describe a brittle rock as a rock that fractures at approximately less than 2 to 5% deformation. In a study published in the Journal of Structural Geology by Hennings et al. (1993), the brittleness index is defined as the ratio of uniaxial compressive strength to the sum of uniaxial compressive strength and tensile strength.

Allaby and Allaby (1990) define brittleness as a condition in competent rocks when the elastic limit is surpassed under stress resulting in rocks losing their internal cohesiveness along specific surfaces. According to these authors, for a rock to be classified as brittle, it has to be competent. Zoback et al. (2012) refer to a brittleness index as a measure of a rock's mechanical response to stress. The Brittleness Index (BI) in shale reservoirs is a quantitative measure of the rock's propensity to undergo brittle failure. It is often calculated based on a combination of rock mechanical

properties such as Young's modulus, Poisson's ratio, and tensile strength. The higher the brittleness index, the more brittle the rock is expected to be (Ross et al., 2013). Furthermore, Yu et al. (2016) define brittleness as the degree at which a material (rock) fractures when subjected to stress.

These definitions demonstrate that the brittleness index is a multidisciplinary concept that varies slightly depending on the field of study. However, in general, it refers to a quantitative measure of a material's propensity to fracture or undergo brittle failure under stress.

2.2.2. Factors affecting brittleness indices of rock mass

The brittleness index of rocks, a crucial parameter in various fields such as geology, petroleum engineering and civil engineering, is influenced by a multitude of factors such as mineral composition, porosity, grain size and texture (Jaeger et al., 2009; Rickman et al., 2008; Zhang, 2016). Understanding these factors is essential for the accurate characterization of rock properties and for making informed decisions in applications such as hydraulic fracturing, rock excavation and underground construction.

Mineral Composition

Mineral composition plays a significant role in determining the brittleness of rocks. Minerals with strong intermolecular bonds, such as quartz and feldspar, tend to contribute to higher brittleness due to their resistance to deformation and fracture. In contrast, rocks containing minerals with weaker bonds, such as clays and micas, are generally less brittle (Tang and Rai, 2013). Studies have shown that the mineralogy of a rock sample can directly influence its mechanical properties and fracture behaviour, thus affecting its brittleness index (Yildirim and Sonmez, 2008).

Grain size and texture

Grain size and texture are additional factors that impact the brittleness index of rocks. Fine-grained rocks, such as shale and mudstone, often exhibit higher brittleness compared to coarse-grained rocks like sandstone and conglomerate. This is because smaller grain sizes result in a more homogeneous and isotropic material with fewer inherent weak planes, making it more prone to brittle failure under stress (Ghasemi et al., 2012). Additionally, the orientation and alignment of grains within a rock matrix can

affect its mechanical anisotropy and brittleness behaviour, with the preferential alignment of grains influencing fracture propagation paths.

Moreover, rock fabric and texture also impact brittleness. Rocks with a fine-grained and homogeneous texture often exhibit higher brittleness due to the uniform distribution of stress throughout the matrix (Warpinski et al., 2012). Conversely, coarse-grained, or heterogeneous rocks may display lower brittleness as stress concentrations can be mitigated by variations in mineralogy and grain size (Jaeger et al., 2009; Zhang, 2016).

Porosity

The porosity, which is the amount of void space within a rock, also affects a rock's brittleness. Rocks with higher porosity typically exhibit lower brittleness because the presence of pores reduces the material's overall strength and cohesion. Conversely, denser rocks with lower porosity tend to be more brittle due to their higher resistance to deformation and fracture propagation (Zhang et al., 2017). The distribution and connectivity of pores within a rock formation further influence its brittleness characteristics, with interconnected pores facilitating crack propagation and reducing overall rock strength. Furthermore, high porosity, especially in porous or vesicular rocks, can reduce brittleness by providing space for deformation and energy dissipation during stress application (Fjaer et al., 2008). Conversely, low-porosity rocks with a tight pore network tend to be more brittle as there is limited accommodation for strain, leading to rapid fracture propagation (Jaeger et al., 2009; Rickman et al., 2008; Zhang, 2016).

Elastic Parameters (Young Modulus and Poisson's Ratio)

Young's modulus, which measures the stiffness of rocks, influences their brittleness. Rocks with higher Young's moduli are typically more brittle (Jaeger et al., 2007). Poisson's ratio, representing the ratio of lateral strain to axial strain, affects rock brittleness. Rocks with lower Poisson's ratios are often more brittle (Goodman, 1989).

Effective stress and strain rate

The effective stress acting on rocks, which is the difference between the total stress and the pore pressure, plays a crucial role in determining their brittleness. Higher effective stress generally leads to increased brittleness by promoting the propagation

of fractures (Jaeger et al., 2009; Zhang, 2016; Wang et al., 2017). Strain rate, which refers to the rate at which stress is applied to rocks, plays a significant role in determining their brittleness, with higher strain rates typically leading to more brittle behaviour, while slower loading rates may allow for more ductile behaviour (Zhu et al., 2015).

Uniaxial Compressive (UCS) and Tensile Strength

The uniaxial compressive strength (UCS) of a rock refers to its ability to withstand axial loads pushing it together, representing the maximum stress it can endure before failure (Jaeger et al., 2009). Higher compressive strength often correlates with lower brittleness **(**Jaeger et al., 2009; Zhang, 2016). Tensile strength is the maximum stress a material can withstand while being stretched or pulled before breaking. Rocks with higher tensile strength tend to be less brittle (Jaeger and Cook, 1979).

Geological structures

The presence of natural fractures, faults, and discontinuities within a rock mass significantly influences its brittleness characteristics. Rocks containing extensive networks of fractures or faults typically exhibit higher brittleness due to increased stress concentration and reduced load-bearing capacity along these discontinuities (Wang et al., 2016). Conversely, highly competent rocks with minimal natural fractures may display lower brittleness and greater ductility, as they can better distribute and dissipate applied stresses (Jaeger et al., 2009; Rickman et al., 2008; Zhang, 2016).

Temperature and Pressure

In addition to intrinsic factors, external conditions such as confining pressure and temperature can influence the brittleness of rocks. High confining pressures tend to increase brittleness by promoting crack closure and enhancing the likelihood of brittle failure (Zoback and Kohli, 2019). Similarly, low temperatures can increase brittleness by reducing ductility and promoting brittle fracture mechanisms (Jaeger et al., 2009).

2.2.3. Brittleness Indices Analysis Methods

The term “brittleness index” has been a hot topic in the literature. Since 1967, numerous brittleness indices (e.g., B_1, B_2, B_3, B_4, B_5 and B_6 among others) have been suggested to meet different requirements, particularly in the field of rock mechanics (Meng et al., 2020). The most common proposed brittleness indices are based on the

rock parameters such as strength, stress-strain curves, mineral composition, elastic parameters (Young Modulus and Poisson's Ratio) and angle of internal frictions. Other parameters used to determine the brittleness indices of rocks include force penetration graphs, over-consolidation characteristics, conventional well logging and indentation tests (Meng et al., 2020).

2.2.3.1. Brittleness indices based on mineral composition:

A method for analysing the brittleness index of a rock based on its mineral composition involves assessing the relative proportions of different minerals present in the rock. By quantifying the mineral composition, researchers can establish correlations between specific minerals and the brittleness of the rock since the mechanical properties of rock materials can be significantly influenced by their mineral composition (Ai et al., 2016). This analysis typically considers the content of brittle minerals such as quartz and the content of ductile or less brittle minerals such as clay. In a study conducted by Rickman et al. (2008), the mineral composition of the Barnett Shale was analysed using X-ray diffraction and laser-induced breakdown spectral (LIBS) techniques. The findings of the study demonstrated that the brittleness of the rock exhibited a positive correlation with the amount of quartz and an inverse correlation with the amount of clay. According to Jarvie et al. (2007), the amount of quartz present in a rock material can impact its brittleness. Consequently, they introduced the variable B_1 (Equation 2.1) to represent the quartz content and calculate the brittleness of the rock. The relation incorporating Total Organic Carbon (TOC) and dolomite was then formulated by Wang and Gale (2009) denoted as "B_2" (Equation 2.2).

$$B_1 = \frac{Q}{\mathrm{Q} + C_c + \mathrm{Cl}} \qquad (2.1)$$

$$B_2 = \frac{Q + Dol + C_c}{Q + Dol + C_c + Cl + TOC} \qquad (2.2)$$

Where, Q is quartz content; Dol is dolomite content; C_c is carbonate content; Cl is the clay content and TOC is the total organic carbon content.

Ai et al. (2016) and Yang et al. (2020) argued that the above brittleness indices (B_1 and B_2) based on mineral composition lack a physical basis and may well yield contradictory results despite their simplicity and convenience. Ai et al. (2016) argument was based on the fact that evaluation indices based on the brittle mineral composition do not consider rock diagenesis, which has a great influence on

brittleness. They further explained that the brittleness of rock materials with similar mineral compositions that have experienced different diagenetic processes, may differ substantially. Moreover, there is still no universal standard for the weight of each brittle mineral. Yang et al's. (2020) argument stemmed from the fact that the brittleness indices (B_1 and B_2) does not consider the mechanical properties of the brittle minerals such as bulk and shear modulus. They further commented that it might be essential to distinguish the individual contributions of brittle minerals to the overall brittleness of the rock. This is because different brittle minerals possess their unique mechanical properties, leading to variations in mineral brittleness. Therefore, they introduced a novel brittleness index denoted as "B_3" (as given in Equation 2.3) that takes into account the weighting of brittle minerals.

$$B_3 = \frac{C_Q \times W_Q + C_F \times W_F + C_D \times W_D}{W_T} \quad (2.3)$$

Where C_Q, C_F, C_C and C_D represent the weighting coefficients associated with shale brittleness for quartz, feldspar, calcite and dolomite, respectively; W_Q, W_F, W_C and W_D denote the respective weights assigned to quartz, feldspar, calcite, and dolomite; W_T represents the total weight of minerals involved.

2.2.3.2. Brittleness Indices based on strength parameters:

Hucka and Das (1974) proposed the rock brittleness index based on the relationship between the uniaxial compressive strength and the tensile strength. They introduced a ratio called "B_4" as given in Equation 2.4, to describe rock brittleness and concluded that when the compressive strength-to-tensile strength ratio is higher, there is the likelihood of internal microcracks forming and the occurrence of brittle fractures within the rock also increase. An additional index, denoted as "B_5," is also formulated by considering the ratio of the sum of the UCS and tensile strength to the difference between the uniaxial compressive strength and the tensile strength as given in Equation 2.5 (Meng et al., 2020). Altindag and Guney (2010) also conducted research on the correlation between rock brittleness and rock strength. In their study, they developed functions "B_6" and "B_7", expressed in Equations 2.6 and 2.7 respectively, to characterize rock brittleness.

$$B_4 = \frac{\sigma_c}{\sigma_t} \quad (2.4)$$

$$B_5 = \frac{(\sigma_c - \sigma_t)}{(\sigma_c + \sigma_t} \quad (2.5)$$

$$B_6 = \frac{\sigma_c \times \sigma_t}{2} \quad (2.6)$$

$$B_7 = \sqrt{\sigma_c} \times \sigma_t / 2 \quad (2.7)$$

Where σ_c is the uniaxial compressive strength and σ_t is the Brazilian tensile strength.

Ozfirat et al. (2016) conducted a study to explore the connections between the drilling rate index and various brittleness values. Subsequently, they introduced a novel brittleness index denoted as "B_8" (as given in Equation 2.8), which is determined as the mean value of the uniaxial compressive strength (UCS) and tensile strength (TS). This new proposed formula exhibits strong agreement with the B_6 brittleness formula proposed by Altindag (2002), as well as Altindag and Guney (2010).

$$B_8 = (\sigma_c + \sigma_t) / 2 \quad (2.8)$$

Where σ_c is the uniaxial compressive strength and σ_t is the Brazilian tensile strength.

Khandelwal et al. (2016) came up with a new formula of estimating the brittleness of the rocks using two modelling techniques, NLMR (Nonlinear Multiple Regression) and GP (Genetic Programming). These analyses considered the inputs of UCS (unconfined compressive strength), BTS (Brazilian tensile strength), and unit weight, while setting brittleness as a dependent variable based on these factors. The brittleness index proposed based on NLMR method is denoted as B_9 and the one based on GP technique B_{10} are represented in Equation 2.9 and 2.10 respectively as follows:

$$B_9 = 0.59 \times \sigma_c^{0.78} - 5.09 \times \sigma_t^{0.53} + 0.009 \times y^{2.33} \quad (2.9)$$

$$B_{10} = \left[Ln\left(\tan(10.90 + \gamma) + \frac{53.27}{\sigma c} + \tan(\sigma c) + \sigma c + 6.66\right)\right] 2 + \left[\frac{6{,}65 - \sigma t + \tan(\sigma c) + \sigma c}{4.17 * TS * \sqrt[3]{\frac{\gamma 2}{\sigma c - 4.17}}}\right] 2 \quad (2.10)$$

Where σ_c is the uniaxial compressive strength; σ_t is the Brazilian tensile strength and γ is the unit weight of the rocks.

Tarokh et al. (2016) evaluated the brittleness indices of the rocks from the spalling and bending tests. Brittleness under compression was characterized as the ratio of the

uniaxial compressive strength by the peak strength of surface instability. Equation 2.11 was proposed B_{11} as follows:

$$B_{11} = UCS / SIPS \quad (2.11)$$

Where UCS is the Uniaxial Compressive Strength and SIPS is the surface instability peak strength. Tarokh et al. (2016) concluded that in the case of extremely brittle rock, the value of the brittleness index tends to be near one, while for ductile rock, brittleness index tends to approach zero.

2.2.3.3. Brittleness Index based on elastic parameters:

The brittleness indices acquired using elastic parameters are mainly used in fracturing design for the extraction of shale gas in unconventional reservoirs (Tarokh et al., 2016). Rickman et al. (2008) proposed that the brittleness index can be determined using dynamic Young's modulus and Poisson's ratio obtained from sonic wave velocity data. This empirical equation established on the principle that a rock is considered brittle when it exhibits high Young's modulus and low Poisson's ratio, is illustrated in Equation 2.12 below.

$$B_{12} = \frac{1}{2}\left(\frac{E - E_{min}}{E_{max} - E_{min}} + \frac{V_{max} - V}{V_{max} - V_{min}}\right) \quad (2.12)$$

Where, E is the Young Modulus and V is the Poisson's ratio of the rock. Li et al. (2013) showed a correlation between Young's modulus and Poisson's ratio of Barnett shale in Figure 2.3. The figure indicates an inverse relationship between Young's modulus and Poisson's ratio, aligning with the observation that brittle rock tends to exhibit high Young's modulus and low Poisson's ratio (Guo et al., 2012a; Li et al., 2013)

Kuanda and Asbury (2016) assessed the brittleness of hard rocks using the Artificial neural networks (ANN) model where P and S wave velocities, and Young's Modulus properties of the rock are used as inputs. Then, the following equation denoted as "B_{13}" was proposed:

$$B_{13} = 2.02\,\gamma + 0.008P + 0.007S - 61.12V - 2.62\,R - 21.76 \quad (2.13)$$

Where, γ is the unit weight; P is the P-wave velocity, S is the S-wave Velocity; V is the Poisson's ratio; and R is the rock type. They concluded that the model's applicability is constrained to predicting the performance of intact rock and cannot be directly employed to assess the conditions of rock masses during ongoing mechanical excavation.

Song et al. (2016) estimated the brittleness as well as the fragility of the rocks surrounding the borehole by using the profile of velocity variations. They proposed this analysis owing to the fact that during drilling the cracks form on rocks around the borehole. As a consequence of the created cracks, the elastic wave velocity of the rock surrounding the borehole diminishes. By evaluating the velocity alteration resulting from the cracks induced by drilling, it becomes possible to estimate the combined impact of rock brittleness and fragility (Song et al., 2016). They define the brittleness and fragility index using Equations 2.14 and 2.15 respectively, B_{14} for P-wave, and B_{15} for S-wave:

$$B_{14} = \frac{1}{R}\int_{r=R}^{\infty} \Delta V_p\ (r)/\ V_p dr \qquad (2.14)$$

$$B_{15} = \frac{1}{R}\int_{r=R}^{\infty} \Delta V_s\ (r)/\ V_s dr \qquad (2.15)$$

Where, R is borehole radius; r is the radius; V is the velocity.

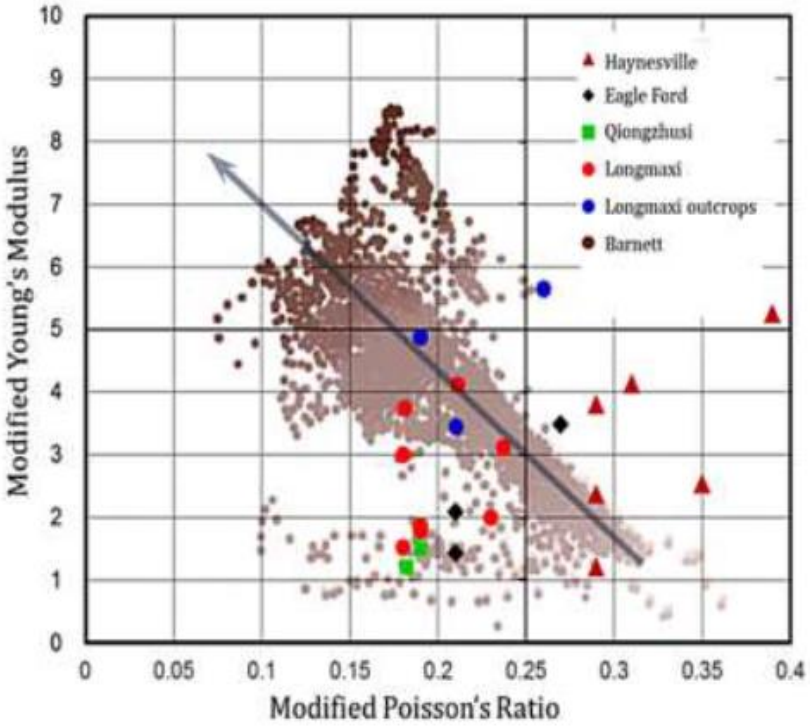

Figure 2. 2 Inverse relationship between Young's modulus and Poisson's ratio (after Li et al., 2013)

2.2.3.4. Brittleness Indices based on stress-strain Curves:

The stress-strain curves acquired through compressing rocks provide a direct representation of their mechanical behaviour, offering insights into the internal mechanisms of rock rupture (Ai et., 2016; Meng et al., 2020). By encompassing the entire testing process (uniaxial and triaxial compressive tests), from initial loading to specimen failure, the stress-strain curves can quantitatively describe the brittleness characteristics of various rocks under identical stress conditions, as well as the brittleness of the same rock under different stress conditions. Multiple brittleness indices have been developed based on the complete stress-strain curves of rock materials (Ai et al., 2016; Meng et al., 2020).

Huckas and Das (1974) proposed brittleness indices B_{16} and B_{17} (as presented in Equations 2.16 and 2.17 respectively) of the rocks that solely consider the stress-strain behaviour before the peak, specifically the strain ratio and energy ratios. B_{16} evaluates the brittleness index as the reversible strain divided by the total strain (DE/OE) as

illustrated on the stress-strain curve on Figure 2.4. Another brittleness Equation 2.8, denoted as B_{18} was proposed by Kidybinksi (1981) based on the stress strain curves.

$$B_{16} = \frac{\varepsilon_A - e}{\varepsilon_A - ir + \varepsilon_A - e} \quad (2.16)$$

$$B_{17} = \frac{U_e}{U_{peak}} \quad (2.17)$$

$$B_{18} = \frac{U_{pre}}{U_e} \quad (2.18)$$

Where ε_{A-e} is the elastic axial strain and ε_{A-ir} is the irreversible axial strain; U_{peak} is the strain energy until peak stress ($U_{peak} = U_{pre} + U_e$); Upre is the pre-peak strain energy and Ue is the elastic strain energy.

Munoz et al. (2016a) argued that the above-mentioned brittleness indices (B_{16}, B_{17} and B_{18}) rely solely on the pre-peak behaviour of rocks may not be adequate in developing a rock brittleness index that accurately describes the behaviour of rock failure with the necessary precision. Consequently, they proposed a new brittleness index "B_{19}" that considers the post-peak instability observed in uniaxial compression test from the complete stress-strain curve of rocks. Munoz et al. (2016b) further introduced the other two brittleness indices (B_{20} and B_{21}) that are formulated by considering energy quantities derived from both pre-peak and post-peak stress-strain characteristics, considering energy balance and post-peak instability. In general, Munoz et al (2016a, b) introduced three brittleness indices which are as follow:

$$B_{19} = \frac{U_e}{U_{peak}} \quad (2.19)$$

$$B_{20} = \frac{U_e}{U_{post}} \quad (2.20)$$

$$B_{21} = \frac{U_{peak}}{U_{total}} \quad (2.21)$$

Where, U_e is the elastic energy; U_{total} is the total fracture energy; U_{peak} is the strain energy until peak stress; U_{post} is the post strain energy (the strain energy after the peak stress has been reached). According to Munoz et al. (2016a) an increase in post-peak energy signifies improved stability (i.e., reduced brittleness), while a significant

decrease in post-peak energy indicates lower stability of the failure process (i.e., increased brittleness).

Wang et al. (2016) proposed the brittleness index "B_{22}" as represented in Equation 2.22, calculated based on the peak strength, axial peak strain, and the modulus of elasticity (E), which are determined from the stress-strain curves' pre-peak section in UCS tests. They demonstrated that 'm' which is the homogeneity index, incorporates the post-peak stress drop, indicating that this homogeneity parameter effectively assesses the brittleness of the rock. According to these authors a higher value of 'm' signifies increased brittleness in the rock and a greater likelihood of complete fracture. It is hypothesized that m is correlated with the level of strength concentration among the elements, meaning that as 'm' increases, more elements tend to fail near the average strength.

$$B_{22} = \frac{1}{In\,(E) - In\,(E_s)} \tag{2.22}$$

Where, E is the elastic modulus and E_s is the secant modulus through the stress-strain curve.

Ai et al. (2016) established two novel brittleness indices, namely B_{23} and B_{24}, which are derived from analyzing the energy changes within the stress-strain curves. The proposed indices B_{23} and B_{24}, are represented in Equations 2.23 and 2.24 respectively:

$$B_{23} = \frac{dw_f + dw_d}{dw_{ue} + dw_d} \tag{2.23}$$

$$B_{24} = \frac{dw_x}{dw_{ue} + dw_d} \tag{2.24}$$

Where, the fracture energy, dWf, refers to the energy associated with the process of fracturing. The total elastic energy, dWe, is the cumulative energy stored in the rock specimen up to its peak strength. The unloading elastic energy, dWue, represents the energy released during unloading. The extra energy required, or excess energy released is represented by dW_x, which is indicated by the yellow dotted lines for type

I behaviour or the yellow area for type II behaviour (Figure 2.3). Finally, the dissipation energy of the pre-peak stage is represented by dW_d.

These indices proposed by Ai et al. (2016) consider the complete range of transformation from absolute ductility to absolute brittleness as illustrated on Figure 2.4. According to Ai et al. (2016) the stress-strain curve of an entirely brittle rock should exhibit the following features: Lack of a significant yield platform in the stress-strain curve prior to reaching the peak stress; the post-peak modulus (M) must be equivalent to the elastic modulus (E); the elastic energy accumulated within the rock specimen must undergo minimal transformation into fracture energy during the post-peak stage. The characteristics of absolute ductility should be as follows: The dissipation energy during the pre-peak stage and the fracture energy during the post-peak stage should approach infinity; the yield platform and the post-peak curve must merge into a single straight horizontal line; the curve should lack distinct failure point, implying that the material continues to deform with increasing loading.

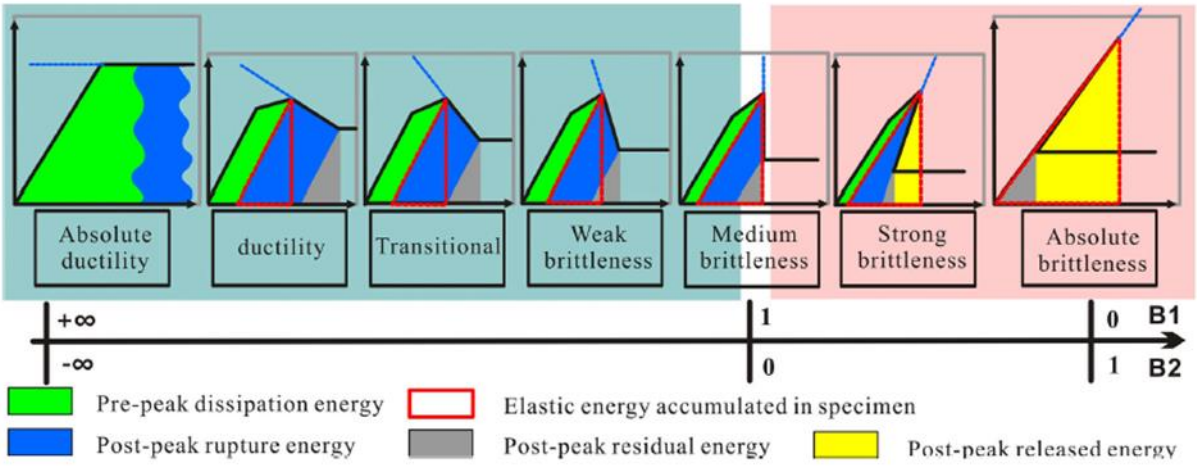

Figure 2. 3 Transition of rock from absolute ductility to absolute brittleness from stress-strain curve (after Ai et al., 2016).

2.2.3.5. Brittleness Indices based on internal angle of friction:

Oyeneyin (2015) defines the angle of internal friction denoted as “φ” as the angle between the axis of normal stress and the tangent line drawn on the Mohr Coulomb failure envelope at a specific point that represents a particular failure-stress condition

for a solid material. Huckas and Das (1974) proposed the following brittleness index (B_{25}) in relation to the angle of internal friction (Figure 2.5).

$$B_{25} = sin\varphi \qquad (2.25)$$

Where the value of φ is established by the gradient of Mohr's envelope at a stress condition where the normal stress (σ_n) equals zero illustrated on Figure 2.5.

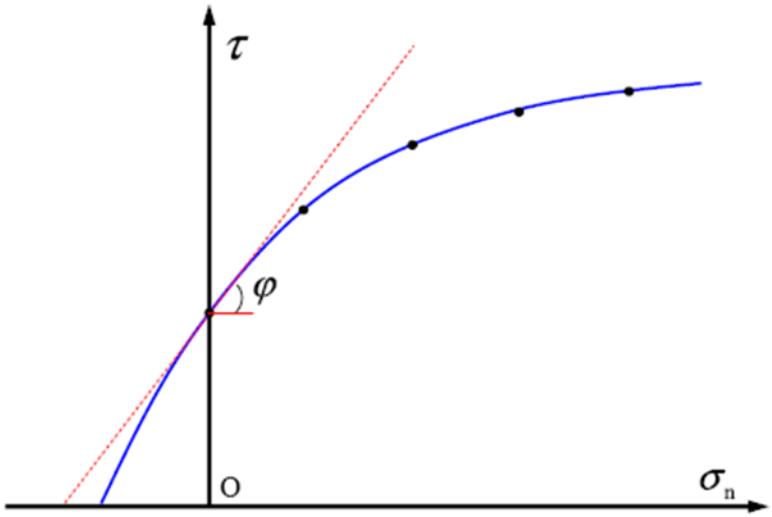

Figure 2. 4 Brittleness index based on the angle of internal friction (after Meng et., 2020).

2.2.3.6. Brittleness Index based on Indentation Test/ Punch Penetration test.

The punch penetration test, utilized for determining the brittleness index, is an assessment method developed to measure the relative brittleness of materials. It was originally developed in the late 1960s to directly evaluate certain physical properties of rocks, including penetration, toughness, and hardness (Munoz et al., 2016a, b). It involves applying a controlled force or load to a punch-like indenter, which then penetrates into the material's surface (Figure 2.8).

Copur et al. (2003) developed a new brittleness index (B_{26}) based on the penetration test (Figure 2.7) and is given by Equation 2.26 as follows.

$$B_{26} = \frac{P_{dec}}{P_{inc}} = \frac{\frac{1}{s}\sum_{t=1}^{s} P_{dect}}{\frac{1}{n}\sum_{p=1}^{n} P_{incp}} \qquad (2.26)$$

Where, P_{dec} is the decrement period (force decrease period) and P_{inc} is the increment period (force increase period) on the force- penetration graph (Figure 2.6).

Lately, Yagiz (2009), proposed a new brittleness index represented as B_{27} as given in Equation 2.27.

$$B_{27} = \frac{F_{max}}{P} \qquad (2.27)$$

Where, F_{max} is maximum applied force on a rock sample in kN, P is the corresponding penetration at maximum force in mm (Figure 2.7).

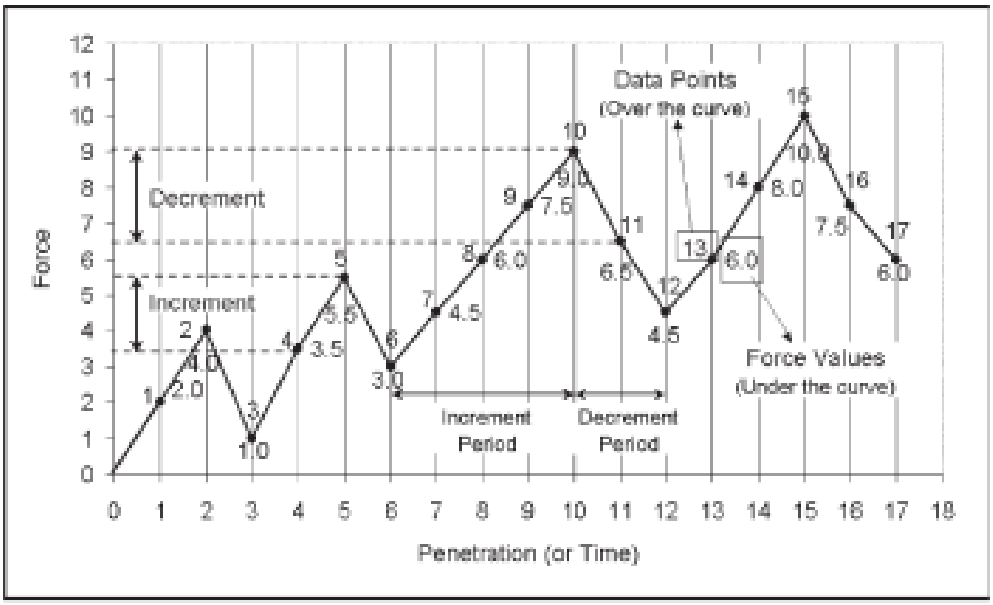

Figure 2. 5 Force penetration graph for force increment and decrement rates and periods (after Copur et al., 2003).

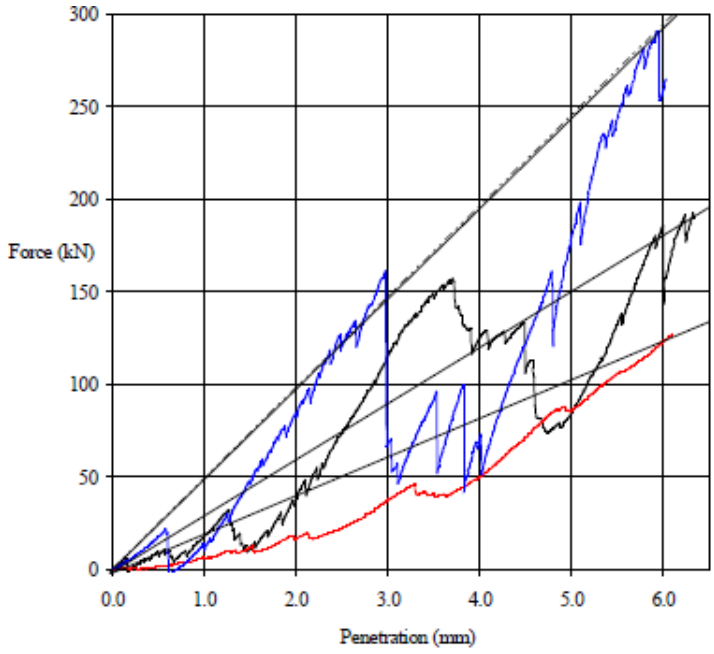

Figure 2. 6 Force penetration graph used to determine brittleness indices (after Yagiz, 2009).

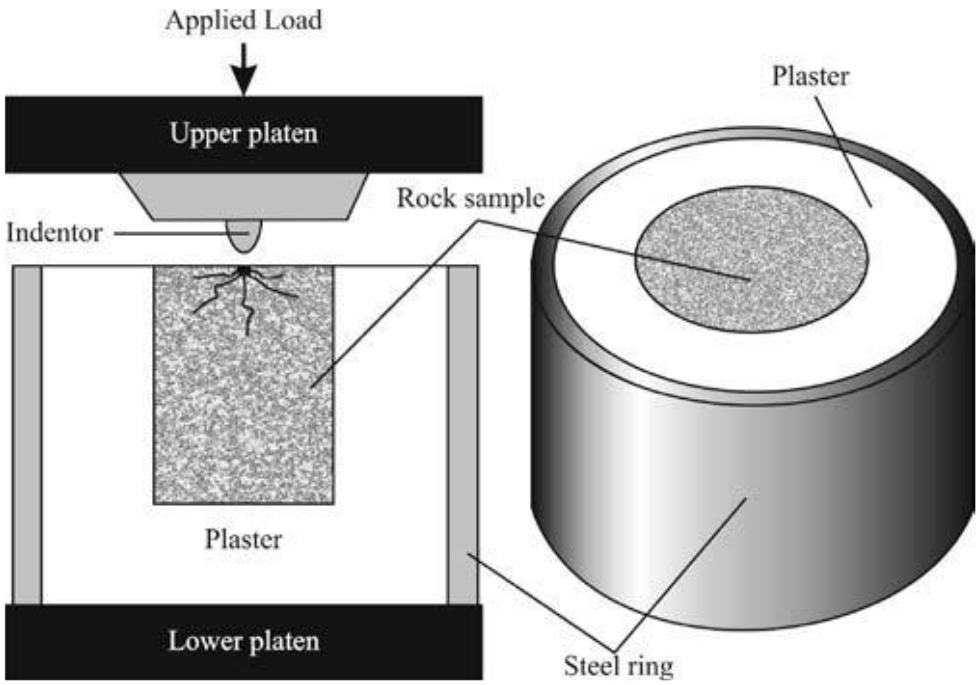

Figure 2. 7 Indentation test apparatus (after Yagiz, 2009).

2.2.3.7. Brittleness index based on finer impacts.

Protodyakonov (1962) introduced one of the initial concepts of rock brittleness B_{28}, utilizing the Protodyakonov impact test and uniaxial compressive strength (UCS) testing as given by Equation 2.28.

$$B_{28} = q\sigma_c \qquad (2.28)$$

Where, q represents the proportion of fines (particles smaller than -28 mesh equivalent to 0.595 mm) and σ_c denotes the UCS.

His approach centred on the observation that brittle rocks tend to produce a higher ratio of fine-to-coarse fragments when broken under impact. This characteristic, indicative of a finer formation, was employed to gauge rock brittleness since fines are typically generated through the loss of cohesive strength, a common occurrence during brittle failure.

2.2.3.8. Brittleness index based on over consolidation ratio.

Nygard et al. (2016) developed a brittleness index B_{29} based on the over consolidation ratio (OCR). The latter is a measure of the stress history of a rock, defined as the ratio of the maximum past vertical effective stress to the present vertical effective stress. According to Nygard et al. (2016), OCR definition is only valid for the horizontally layered formations where $\sigma_v=\sigma_1$ and $\sigma_h=\sigma_3$ (vertical stress is equivalent to sigma 1 stress and horizontal stress is equivalent to sigma 3 stress, respectively). Sigma 1 and sigma 3 represents the maximum and minimum principal stress acting on a rock, respectively (Nygard et al., 2016).

$$B_{29} = OCR^b = \left(\frac{\sigma_{vmax}}{\sigma_v}\right)^b \qquad (2.29)$$

Where, b is the empirical constant, σ_{vmax} is the maximum vertical effective stress it has experienced, σ_v is the current effective vertical stress.

2.2.3.9. Brittleness index based on geophysical methods

Guo et al. (2012) proposed a brittleness index B_{30} utilizing the crossplot proposed by Goodway et al. (2012) as illustrated in Figure 2.9.

$$B_{30} = \frac{\lambda+2\mu}{\lambda} \qquad (2.30)$$

Where, λ is the Lame's lambda and μ is the shear modulus or rigidity modulus.

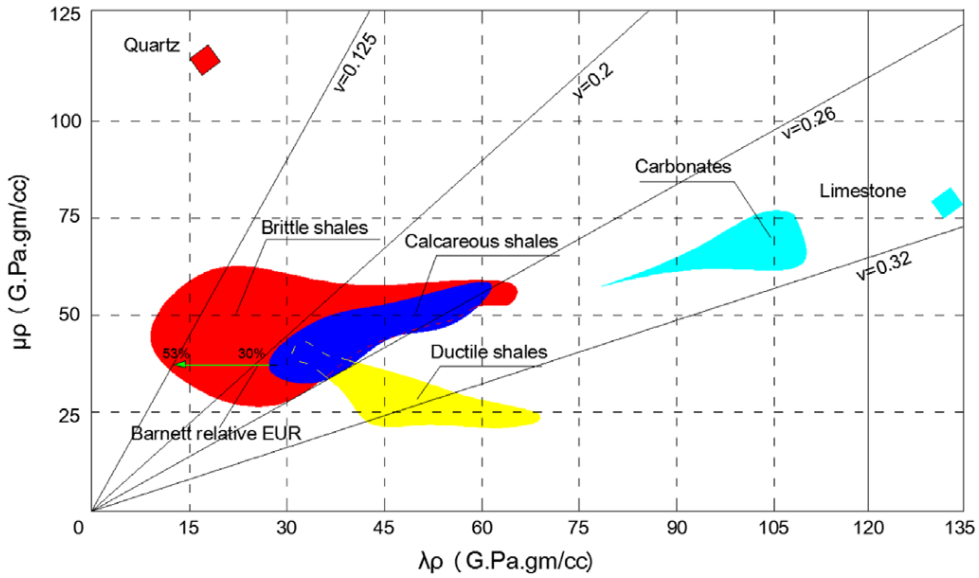

Figure 2. 8 Crossplot used to determine brittleness index (after Goodway et al., 2010).

2.2.3. APPLICATION OF BRITTLENESS INDEX

2.2.3.1. Hydraulic Fracturing

The brittleness index of rocks is instrumental in optimizing hydraulic fracturing processes. It assists in delineating the mechanical behaviour of reservoir rocks, thereby facilitating the design of effective fracturing treatments to enhance oil and gas recovery. By evaluating the brittleness index, engineers can identify zones within the reservoir that are more prone to fracturing. This information is crucial for determining the placement of hydraulic fractures and optimizing their orientation and propagation. Rocks with higher brittleness indices tend to fracture more readily under pressure, allowing for the creation of interconnected networks of fractures that improve fluid flow within the reservoir (Zhang et al., 2020; Zhao et al., 2019).

Furthermore, the brittleness index helps in selecting suitable proppants and fluids for hydraulic fracturing operations. Proppants are materials injected into the fractures to keep them open, and the choice of proppant depends on the brittleness of the rock formation. Additionally, understanding the brittleness index aids in mitigating operational risks such as screen out and fracture containment by optimizing the pumping parameters and treatment design (Smith & Montgomery, 2021; Wang et al., 2018).

In summary, the brittleness index serves as a fundamental parameter in hydraulic fracturing, guiding decision-making processes to maximize oil and gas production while ensuring the economic and environmental sustainability of the operation (Li & Zhang, 2022; Baker Hughes, 2017).

2.2.3.2. Wellbore Stability Analysis

In wellbore stability analysis, the brittleness index of rocks plays a crucial role in assessing the propensity of formations to failure during drilling operations. Understanding the brittleness of the rocks surrounding a wellbore helps engineers predict potential challenges such as borehole instability, formation collapse, and wellbore breakout. By incorporating brittleness indices into wellbore stability models, engineers can identify zones of heightened risk and implement appropriate mitigation measures to ensure safe and efficient drilling operations (Zoback, 2010; Chen et al., 2014).

One key aspect where the brittleness index is utilized in wellbore stability analysis is in determining the likelihood of borehole breakout. Brittleness indices help quantify the tendency of rocks to fail and deform under stress, providing insights into the mechanical behaviour of formations surrounding the wellbore. Rocks with higher brittleness are more susceptible to fracturing and spalling, increasing the risk of borehole instability. By evaluating the brittleness of formations, engineers can anticipate potential breakout zones and adjust drilling parameters such as mud weight, drilling fluid properties, and wellbore trajectory to minimize the risk of instability (Zhang et al., 2015; Plumb, 1994).

2.2.3.3. Rock Burst Susceptibility Indicator

The brittleness index of rocks is a key factor in assessing rockburst susceptibility in mining and underground excavation operations. Rockbursts are sudden, violent failures of rock mass that occur in deep underground mines, tunnels, or other excavations, often resulting from high stresses and rock mass instability. Understanding the brittleness of the rock mass helps engineers and geologists predict the likelihood and severity of rock burst events and implement appropriate measures to mitigate their impact. Rocks with higher brittleness are more prone to rapid, brittle failure, increasing the risk of rock burst occurrences, especially in regions with high stress concentrations (Cai et al., 2004; Morissette et al., 2010).

By integrating brittleness assessments into rock burst risk assessments, engineers can identify areas of heightened susceptibility within underground excavations and implement appropriate engineering controls and support systems to minimize the risk of rock bursts. These measures may include reinforcement with rock bolts and mesh, ground support with shotcrete or steel ribs, and careful excavation sequencing to manage stress concentrations (Kaiser & Kim, 2008; Brady & Brown, 2004).

2.2.3.4. Rock Cutting and drilling

In rock cutting and drilling operations, the brittleness index serves as a fundamental parameter for assessing the ease of fracturing and disintegration of rocks (Smith et al., 2021). Rocks with higher brittleness indices are more prone to fracturing and fragmentation, making them easier to cut and drill (Doe & Roe, 2019). Mining engineers leverage this knowledge to identify suitable rock formations for excavation, as well as to optimize drilling strategies in exploration and production activities (Johnson & White, 2020).

By incorporating the brittleness index into drilling equipment design and selection processes, engineers can tailor drilling operations to the specific characteristics of the rock formations encountered (Brown et al., 2018). For instance, in brittle rock formations, rotary drilling methods may be preferred over percussive drilling techniques to minimize tool wear and maximize drilling efficiency (Lee & Kim, 2022). Similarly, the choice of drill bits and cutting tools can be optimized based on the brittleness index to enhance penetration rates and reduce downtime (Green & Black, 2017).

Furthermore, the brittleness index guides the development of predictive models and simulation tools for rock cutting and drilling operations (Taylor & Adams, 2023). By correlating the brittleness index with other rock properties such as hardness, abrasiveness, and fracture toughness, engineers can forecast drilling performance and anticipate challenges associated with different rock types (Parker et al., 2019). This proactive approach enables better decision-making and risk management during drilling operations, leading to improved productivity and safety outcomes (Walker et al., 2021).

2.4 Conclusion of the chapter

Meng et al. (2020) reviewed various brittleness indices based on different parameters and concluded that the term "brittleness" is occasionally misused, and certain brittleness indices fail to accurately depict the brittleness of rocks. Moreover, brittleness indices developed for one specific field often prove ineffective when directly applied to other fields. Indices derived from a particular rock type, or a limited database may not be applicable to more general conditions, encompassing diverse rock types and stress states.

It may be inferred that even though a number of researchers have worked to address the issue of developing an appropriate brittleness index the developed techniques seem to face a number of difficulties. For instance, the majority of mechanical techniques fail to take into account how the mineralogical composition affects the brittleness of the rock units. Thus, among other things, brittleness methods that combine mechanical techniques with rock unit mineralogical composition must be developed.

CHAPTER THREE

MATERIALS AND METHODS

3.1. Preliminary Work

3.1.1. Desktop Study

During the desktop phase of the project, extensive research was conducted to gather existing literature, studies and data related to the Ecca Group sandstones and brittleness index estimation methodologies. This involved accessing academic journals, conference proceedings, geological databases, and relevant online resources to review previous research on the mineralogical characteristics of the Ecca Group sandstones and their relationship to the brittleness index.

3.1.2. Reconnaissance Visit

A reconnaissance visit was conducted at Msobo Coal Mine in Ermelo, South Africa, as part of the research. The primary objective of this visit was to gain a preliminary understanding of the study area, assess the accessibility of outcrops, and identify suitable locations for sample collection. Additionally, the visit aimed to establish communication with mine personnel and ensure that all necessary logistical and safety requirements were met before the detailed fieldwork commenced.

During the visit, key geological formations of the Ecca Group were examined to determine their spatial distribution and variability in lithology. Particular attention was given to sandstone units, as these formed the core of the study. The reconnaissance involved visual inspections of exposed rock faces, mine benches, and available drill cores to assess the degree of weathering, fracturing, and accessibility for sampling. Initial observations indicated significant variations in grain size, cementation, and bedding characteristics, which are crucial factors influencing the brittleness index.

Engagement with mine geologists and operational staff provided valuable insights into the geological setting and mining conditions. Discussions focused on the stratigraphy of the Ecca Group in the area, previous geological studies, and any existing geotechnical data that could support the research. The visit also facilitated the planning of sampling strategies, including the selection of representative sandstone units and determination of optimal sample extraction points.

In addition to geological assessments, logistical considerations such as site access, safety protocols, and sample transportation were evaluated. Compliance with the mine's safety regulations was emphasized, and necessary permissions for future fieldwork were secured. GPS coordinates of key locations were recorded to streamline subsequent visits and ensure efficient sample collection.

Overall, the reconnaissance visit was instrumental in refining the research approach and confirming the feasibility of the study. The preliminary findings provided a solid foundation for detailed field investigations, laboratory analyses, and the eventual estimation of the brittleness index of the Ecca Group sandstones based on their mineralogical characteristics.

3.2. Field Work

The fieldwork involved surface drilling to collect sandstone core samples from the Ecca Group (Vryheid Formation). The primary objective was to obtain high quality core samples of the Ecca Group sandstones for laboratory analysis to estimate the brittleness index based on mineralogical characteristics. This was achieved by drilling three boreholes (BH1, BH2 and BH3) at the selected sites in Ermelo, Msobo Coalfield Spitskop farm, where Ecca Group formations are well exposed and accessible (Figure 3.1). The drilling method employed was surface diamond drilling, a technique known for its precision and ability to recover continuous, high quality rock cores. This method was chosen due to its efficiency in penetrating various lithologies while maintaining the structural integrity of the extracted samples.

The boreholes were planned based on prior geological mapping and subsurface data to intersect key lithological units. Each borehole reached a predetermined depth (Table 3.1) sufficient to capture variations in grain size, mineralogy and cementation, which are critical factors in brittleness index estimation (Table 3.1).

Table 3.1 Location of the three drilled boreholes.

Borehole ID.	**Depth (m)**	**Location**
JGT202301	40	26°11'35.4"S 29°33'19.2"E
WE018	69.17	26°11'26.30"S 29°33'20.45"E
DF1669	62.54	26°11'36.54"S 29°33'34.63"E

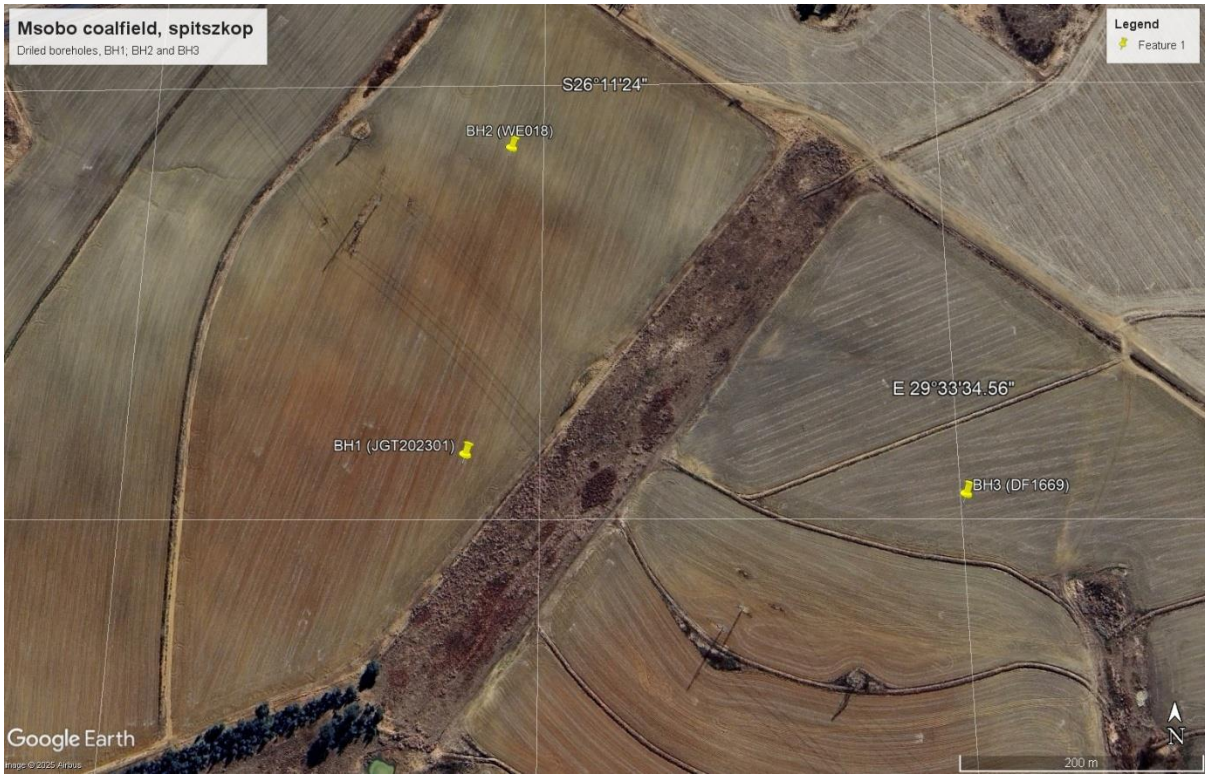

Figure 3.1 Location of drilled boreholes from Msobo coalfield.

3.2.1. Core logging

Core logging was conducted using a systematic approach to ensure the accurate documentation of geological features, following the guidelines outlined in the SAMREC Code (South African Code for the Reporting of Exploration Results, Mineral Resources, and Mineral Reserves) (SAMREC, 2016). Initially, the core was extracted from the drilling site and transported to the logging facility. The cores with diameter of 54 mm from boreholes JGT202301, WE018 and DF1669 were then carefully cleaned to remove any drilling fluids or debris that could obscure its features. Measurements of core length, diameter and core recovery were recorded to assess the quality and completeness of the samples.

Geological features such as rock type, mineralogy and structural characteristics were identified and described in detail. Core logging also included the identification of faults, fractures, and other discontinuities, as well as the assessment of their orientation and distribution. Rock properties, such as colour, texture, and grain size, were noted, and any observable variations or anomalies were documented.

Photographs (Figure 3.2) of the core samples were often taken to provide visual records that complement the written descriptions.

3.2.2. Sample Collection

After core logging, the core samples (Figure 3.2) from boreholes JGT202301, WE018 and DF1669 were collected from Ermelo Coalfields and transported to the University of Limpopo storage for further analysis**.**

Figure 3. 2 Core extracted from Ermelo Coalfield.

3.3. Laboratory Work

Geotechnical tests, such as point load tests, uniaxial compressive strength tests and Brazilian Tensile strength tests were performed on selected samples to provide additional data on the mechanical properties of the rock.

3.3.1. Point Load Test

The point load test was conducted following a standardized methodology to evaluate the rock's strength, in accordance with the ISRM Suggested Method for Determining the Point Load Strength Index (1985). Initially, rock samples were prepared by cutting them into cylindrical or cubic shapes, depending on the specific requirements of the test. The dimensions of each sample were measured accurately to ensure they met the test specifications.

The prepared samples were then placed in the point load testing apparatus. The apparatus consisted of two conical or spherical platens that applied a concentrated load to the sample (Figure 3.3). The sample was positioned between the platens with the loading points aligned according to the ISRM Suggested Method for Determining the Point Load Strength Index (1985).

A load was gradually applied to the sample until failure occurred. The load was increased at a controlled rate to ensure consistent application and accurate measurement. The failure load was recorded at the point where the sample exhibited significant deformation or fracture.

The test was repeated on five samples to obtain a reliable set of data. The point load strength was calculated using the recorded failure loads and the sample dimensions using Equation 3.1, as per the ISRM Suggested Method for Determining the Point Load Strength Index (1985). The results were then compared to standard reference values to assess the rock's mechanical properties (Table 3.1).

$$I_S = \frac{P}{D^2} \qquad (3.1)$$

Where, I_S is the Point load strength index (MPa), P is the Failure load (N) and D is equivalent core diameter (mm)

Table 3. 1 Point load Strength of rocks (ISRM,1985).

Rock Type	Point Load Strength (MPa)
Very Weak	≤ 0.25
Weak	0.25 - 1.0
Medium	1.0 5.0
Strong	5.0 – 15.0
Very Strong	>15.0

Figure 3.3 Point load test to determine point load strength.

3.3.2. Uniaxial compressive strength test

The uniaxial compressive test was performed using a standardized procedure to assess the compressive strength of rock samples, following the ISRM Suggested

Method for Determining the Uniaxial Compressive Strength of Rock Materials (1979). Initially, rock specimens were prepared by cutting them into cylindrical shapes with precise dimensions, typically measuring 54 mm in diameter and 100 mm in height. The specimens were then carefully examined to ensure they were free of visible defects and irregularities.

The samples were placed in a uniaxial compressive testing machine, which was equipped with two rigid plates that applied a unidirectional compressive load. The specimen was aligned between the plates to ensure even distribution of the load. The load was applied gradually at a constant rate until the sample failed. Throughout the test, the load and corresponding deformation of the specimen were continuously recorded by the testing machine.

Upon failure, the maximum load sustained by the specimen was noted. The compressive strength of the rock was calculated by dividing the maximum load by the cross-sectional area of the specimen. The test was repeated on a sample to obtain a consistent set of data. The results were analysed to determine the rock's UCS and were documented along with any observations regarding the failure mode, such as whether the failure was brittle or ductile. The findings were used to assess and describe the mechanical properties (Strength) with reference to standardized values (Table 3.2).

Table 3. 2 Standardized UCS description (ISRM, 1985).

Description	Uniaxial Compressive Strength (MPa)
Very weak	Less than equal 10
Weak	10 - 50
Medium	50 - 100
Strong	100 - 250
Very Strong	>250

3.3.3. Brazilian tensile strength test

The Brazilian tensile strength test was employed to evaluate the rock's tensile properties, which are essential for understanding its brittleness. This test measures

the indirect tensile strength of a rock specimen by applying a diametral compressive load.

The Brazilian test involved preparing cylindrical rock samples, typically with a diameter of 50–70 mm and a thickness approximately half the diameter. The sample was then placed between two flat loading platens, with the load applied along the diameter of the specimen. The loading configuration induced a tensile stress perpendicular to the applied compressive load, leading to failure along a vertical plane (Figure 3.4)

The test followed the standards such as ASTM D3967-08, which outlines the procedure for determining the Brazilian tensile strength of rock. The applied load was continuously increased until the specimen fails, and the maximum load was recorded. The Brazilian tensile strength was then calculated using the formula:

$$\sigma_t = \frac{2F}{\pi DL} \qquad (3.2)$$

Where σ_t is the tensile strength (MPa), F is the applied load (N), D is the diameter of the specimen (mm), and L is the thickness of the specimen (mm).

Figure 4.3 Brazilian tensile strength test.

3.3.4. **Elasticity tests (Poisson's ratio and Young Modulus)**

The elasticity of the Ecca Group sandstones was analysed through the measurement of Poisson's ratio and Young's modulus, both of which are essential for determining the mechanical properties and the brittleness index of the rock. These properties are influenced by mineralogical composition and understanding them is critical for evaluating the rock's behaviour under stress.

Poisson's ratio was determined experimentally by conducting uniaxial compression tests. The ratio was calculated by measuring the lateral strain to the axial strain during the application of stress. The standard used for Poisson's ratio determination was the ASTM D3148-15, which outlines procedures for measuring the elastic properties of rocks. Poisson's ratio provides insights into the material's response to deformation, where a higher value generally indicates more ductility.

Young's modulus (E) was also derived from the uniaxial compression tests, where stress-strain curves were obtained. The modulus was calculated by measuring the slope of the linear elastic region of the stress-strain curve, as defined in the ISO 14688-2:2017 standard. This modulus reflects the rock's stiffness and is a key parameter in the brittleness index formula.

3.3.5. Petrographic Analysis

Upon collection, rock samples were transported to the laboratory for further analysis. Care was taken to avoid contamination during transportation and handling. Each sample was cleaned to remove surface debris and weathering products that could affect subsequent analyses. Subsequently, samples were dried to remove any moisture content. Fourteen thin sections were prepared from fourteen rock samples (Figure 3.4) by mounting them with epoxy on glass slides and grinding them to a thickness of 35 microns. These sections were then examined under both reflected and transmitted light microscopes to analyse their mineral compositions, rock textures, grain shapes and sizes, and types of cement present (Figure 3.5).

The point-counting method introduced by Gezi-Dickenson (1974) was utilized for modal analysis, following the procedure outlined by Ingersoll et al. (1984).

Approximately 700 points from various random fields of view were counted for each thin section, with around 400-600 grains counted per section while the remaining points represented cement, matrix, and void.

Figure 3. 4 Example of sandstone sample used for petrographic analysis.

Figure 3. 5 Microscope used for petrographic analysis.

3.3.6. Textural Studies

In examining the texture of the thin sections, various aspects were considered, including mean grain size, grain shape (roundness and sphericity), sorting, textural interlocking, grain contact nature, packing properties (density and proximity), and consolidation factor.

- Mean grain size was determined by studying 100 randomly chosen grains from multiple fields of view per thin section, following the method prescribed by Hutchinson (1974), which measured the average length of two perpendicular axes crossing each grain's centre.
- Grain roundness (ρ) and sphericity (Ψp) were assessed using Powers' (1953) comparative chart for roundness and Rittenhouse's (1943) chart for maximum projection sphericity, based on visual analysis of 50 randomly selected particles in each thin section (Figure 3.6).
- The degree of sorting was determined using Beard and Weyl's classification chart (1973), with five random fields of view per thin section being evaluated (Figure 3.7).
- Packing density and packing proximity are two important parameters used to describe the arrangement and distribution of grains within a sedimentary rock, particularly in sandstones. They indicate how closely packed the grains are and can influence properties such as porosity and permeability. Packing density refers to the volume fraction of solid grains within a rock sample. Packing proximity refers to the spatial arrangement of grains within a rock sample, particularly the distance between neighbouring grains. Kahn's (1956) method was used to determine the packing density and packing proximity (Figures 3.8 3.9 and 3.10). The packing density was calculated by dividing the total length of grains intercepted by an imaginary line by the length of the entire imaginary line (Gezi and Dickinson, 1974) (Equation 3.3).

$$P_d = \frac{Total\ length\ of\ grains\ along\ the\ traverse\ imaginary\ line}{length\ of\ traverse\ imaginary\ line} \times 100\ (\%) \qquad (3.3)$$

Packing proximity was determined by comparing the quantity of grain-to-grain contacts along an imaginary line to the overall number of contacts between particles along the imaginary line (Gezi and Dickinson, 1974) (Equation 3.4).

$$P_p = \frac{Total\ number\ of\ grain\ to\ grain\ contact\ along\ the\ imaginary\ traverse\ line}{Total\ number\ of\ contacts\ along\ the\ imaginary\ traverse\ line} \times 100(\%) \qquad (3.4)$$

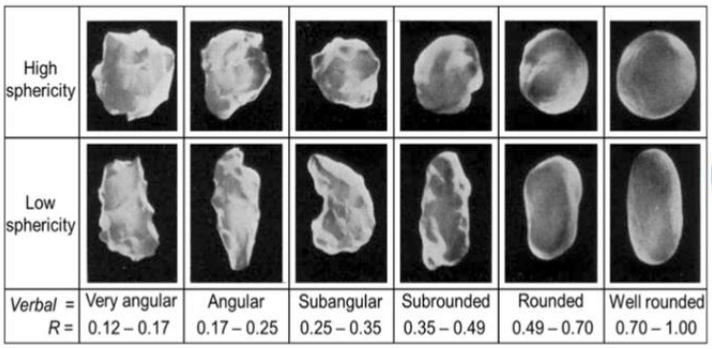

Figure 3.6 Roundness and sphericity comparative chart (after Powers, 1953).

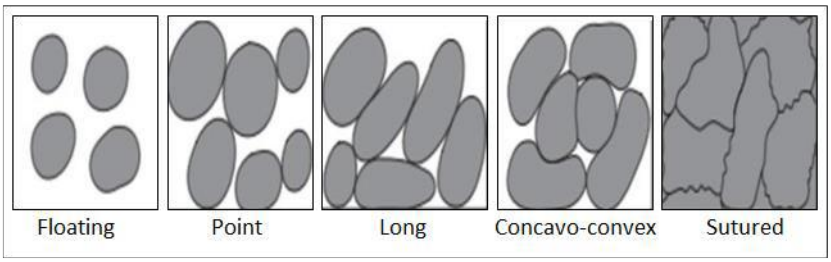

Figure 3.7 Grain to grain contact types (after Chima et al., 2014).

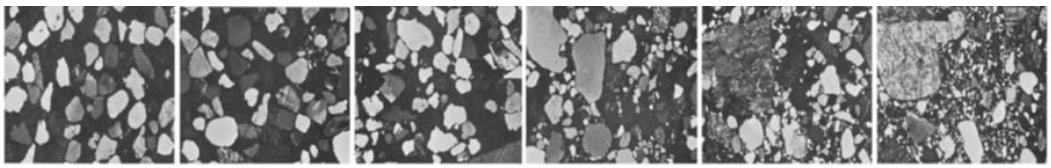

Figure 3.8 Sorting groups comparative thin section (after Beard and Weyl, 1973).

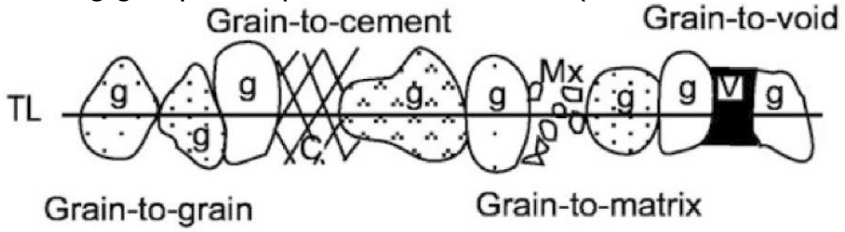

Figure 3.9 Contact nature (after Tamrakar et al., 2007).

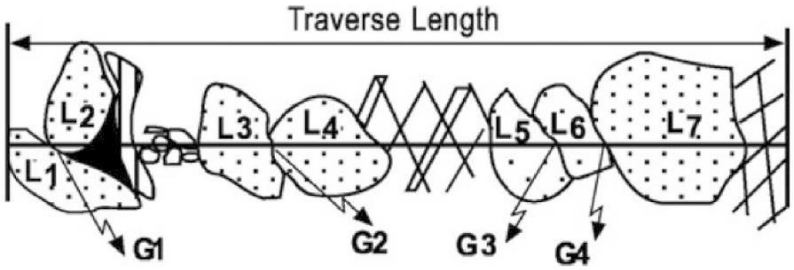

Figure 3.10 Methods for determining packing density and packing proximity (after Tamrakar et al 2007).

3.4. Statistical analysis

The Multivariate linear regression (MLR) was employed as a key statistical tool for analyzing the relationship between various mineralogical characteristics and the brittleness index. MLR allows for the modeling of multiple independent variables (predictors) and their combined effect on a dependent variable (in this case, the brittleness index). The MLR methodology involved several key steps:

Selection of Predictors: Various mineralogical characteristics such as mean grain size (Mz), sorting coefficient (So), mineral composition, packing density, packing proximity, and consolidation factor were selected as independent variables. These variables were hypothesized to influence the brittleness of the Ecca Group sandstones.

Model Development: The regression model was developed by considering multiple mineralogical predictors simultaneously. This enabled an understanding of how each predictor, in combination with others, contributed to the estimation of the brittleness index.

Backward Stepwise Selection: To refine the model, a backward multivariate linear regression approach was utilized. In this method, all potential predictors were initially included in the model, and then the least significant variables (those with the highest p-values) were removed step by step. This process continued until only the most statistically significant predictors remained, optimizing the model for simplicity and accuracy.

Statistical Significance: The significance of each predictor was assessed based on p-values obtained from the regression analysis. Predictors with p-values greater than 0.05 were considered statistically insignificant and were excluded from the final model.

Model Validation: The model's performance was validated by checking the residuals for normality, homoscedasticity, and independence. Goodness-of-fit measures such as R^2, adjusted R^2 , and the root mean square error (RMSE) were used to assess the accuracy and reliability of the final regression model.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1. Introduction

In this chapter, the findings from the mineralogical analysis of the Ecca Group sandstones are presented and discussed about their implications for estimating the brittleness index. The Ecca Group is a significant geological formation known for its complex lithological and mineralogical composition. It plays a crucial role in various industrial applications, particularly in resource extraction and geotechnical engineering. Understanding its brittleness is essential for optimizing extraction techniques, minimizing rock fall, slope instability, and ensuring structural stability in engineering projects.

The results presented herein were derived from comprehensive mineralogical assessments, employing a combination of petrographic microscopy to characterize the mineral constituents of the sandstones of the Ecca Group. Key petrographic parameters such as mineral composition, grain size, and textural features, have been meticulously quantified and analysed.

The mineralogical analysis of the samples was then followed by an in-depth discussion linking these findings to the brittleness index. The correlation between mineralogical characteristics and brittleness was explored, providing insights into how variations in mineral composition and texture influence the mechanical behaviour of the Ecca Group sandstone. Comparative analyses with existing models and literature were also included to validate the findings and highlight the contributions to the broader geological and engineering communities.

4.2. Stratigraphy description

The Ermelo coal formation is part of the larger Ecca Group within the Karoo Supergroup and a significant coal-bearing stratigraphic unit located in Mpumalanga, South Africa. This formation is renowned for its economic importance due to the substantial coal resources it hosts, which are pivotal for both local and national energy production (Hancox and Gotz, 2014).

4.2.1 Lithostratigraphy

From the cores that were logged (Figures 4.1, 4.2, 4.3 and 4.4), the Ermelo Coal Formation starts with basal units of sandstone and siltstone, marking the onset of coal deposition in the Ermelo coalfield. It features multiple coal seams interbedded with shale, siltstone, and minor sandstone. The Main Seam is the thickest and most valuable, with other seams including the Upper and Lower Seams. Between the coal seams are fine-grained sedimentary rocks, mainly shales and siltstones, often containing fossilized plant material, indicating peat-forming swamps and low-energy fluvial systems. The formation's upper parts transition into finer-grained sedimentary rocks like shales and mudstones, which lead into the overlying formations of the Ecca Group. Table 4.1 provides a detailed stratigraphic description of a geological core sample JGT202301, organized by depth intervals and corresponding rock or sediment layers encountered.

Table 4. 1 Lithostratigraphy of Ermelo Coalfield.

Depth (m)	**Layer**	**Thickness (m)**	**Description**
0-3	Topsoil (Regolith)	3	Yellow sandy colour, predominantly sand sized particles, mixed with silt and clay.
3-11	Sandstone	8	Light grey to grey, upward coarsening, visible grains, feldspathic minerals, quartz dominate, minor fractures.
11-12	Laminated shale	1	Black and grey colour, laminated, fractured.
12-13	Sandstone	1	Light whitish colour, intact, coarse quartz grains.
13-18.2	Laminated shale	5.2	Black and grey colour, joints with calcite and pyrite, black colour becoming dominate with depth.
18.2-19.05	Shale	0.85	Black, dull colour.
19.05-19.95	Coal seam D	0.9	Dull black colour, brighter than coal seam E, carbonaceous organic materials, pyrite present.

19.95-31.9	Sandstone	11.95	Fine grained, dark grey colour with organic layers, gradually transitioning to laminated shale.
31.9-33.9	Laminated Shale	2	Similar to the above-mentioned shale.
33.9 -34.4	Shale	0.5	Very dark dull black colour resembles coal from a distance.
34.4-35.1	Sandstone	0.7	Contains organic material layers.
35.1-37	Coal Seam E	1.9	Dull colour, sulphide veins present.
37-37.5	Sandstone	0.5	Thin, dark organic material layers.
37.5-38.5	Laminated shale	1.5	Laminated shale
38.5-40	Sandstone	1.5	Extends to the end of the core

Figure 4. 1 Core from borehole JGT202301.

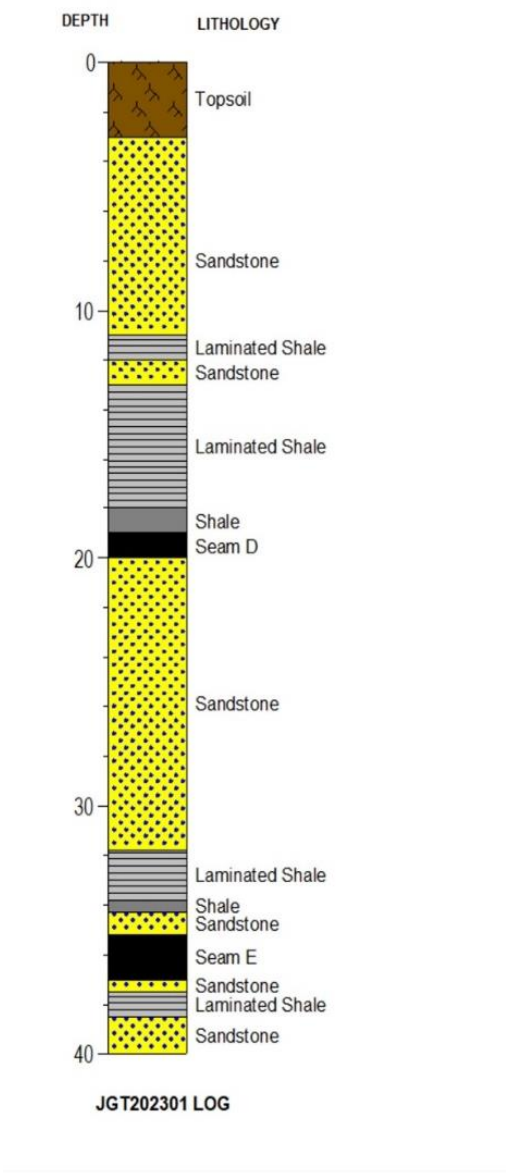

Figure 4. 2 Stratigraphic column from borehole JGT202301.

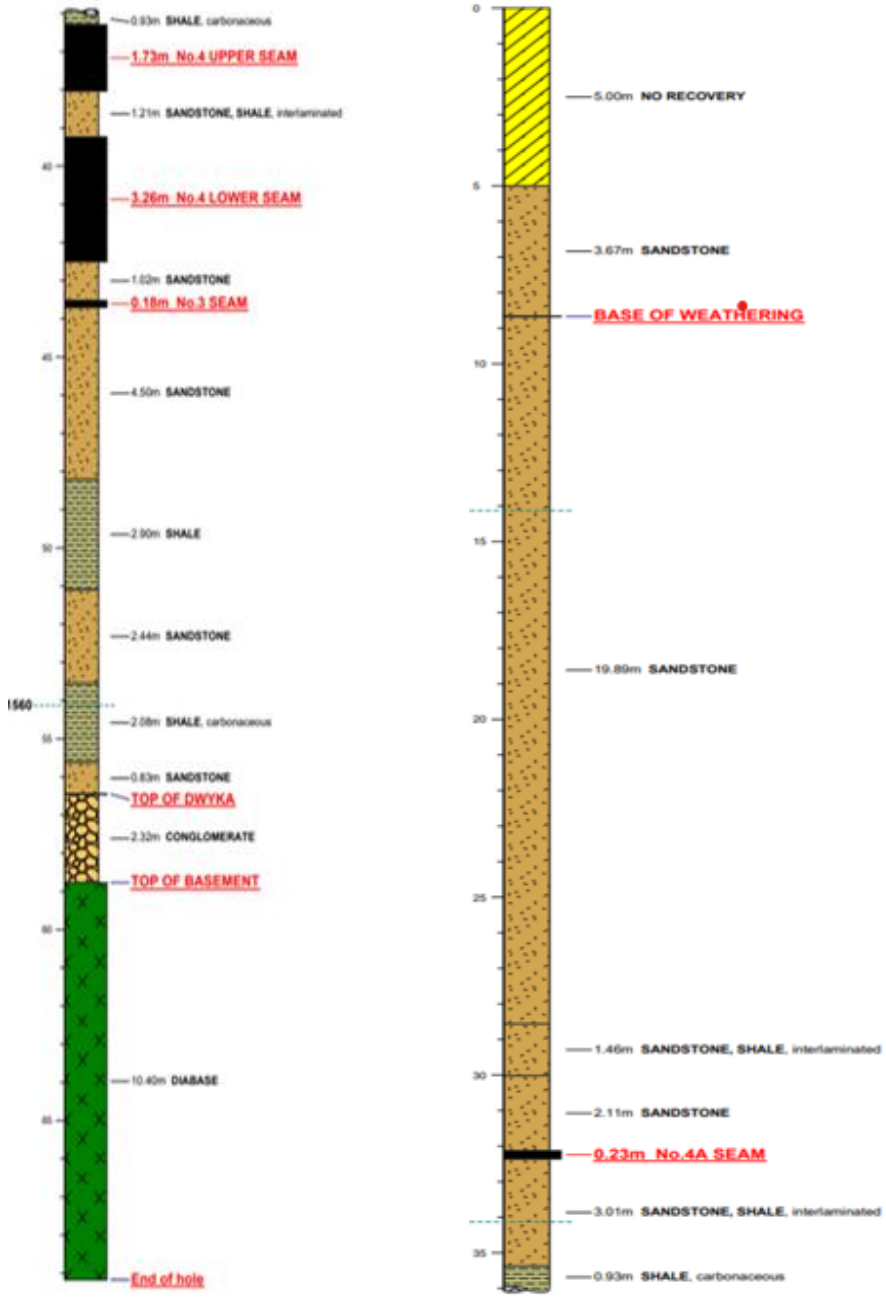

Figure 4.3 stratigraphic column from borehole WE018.

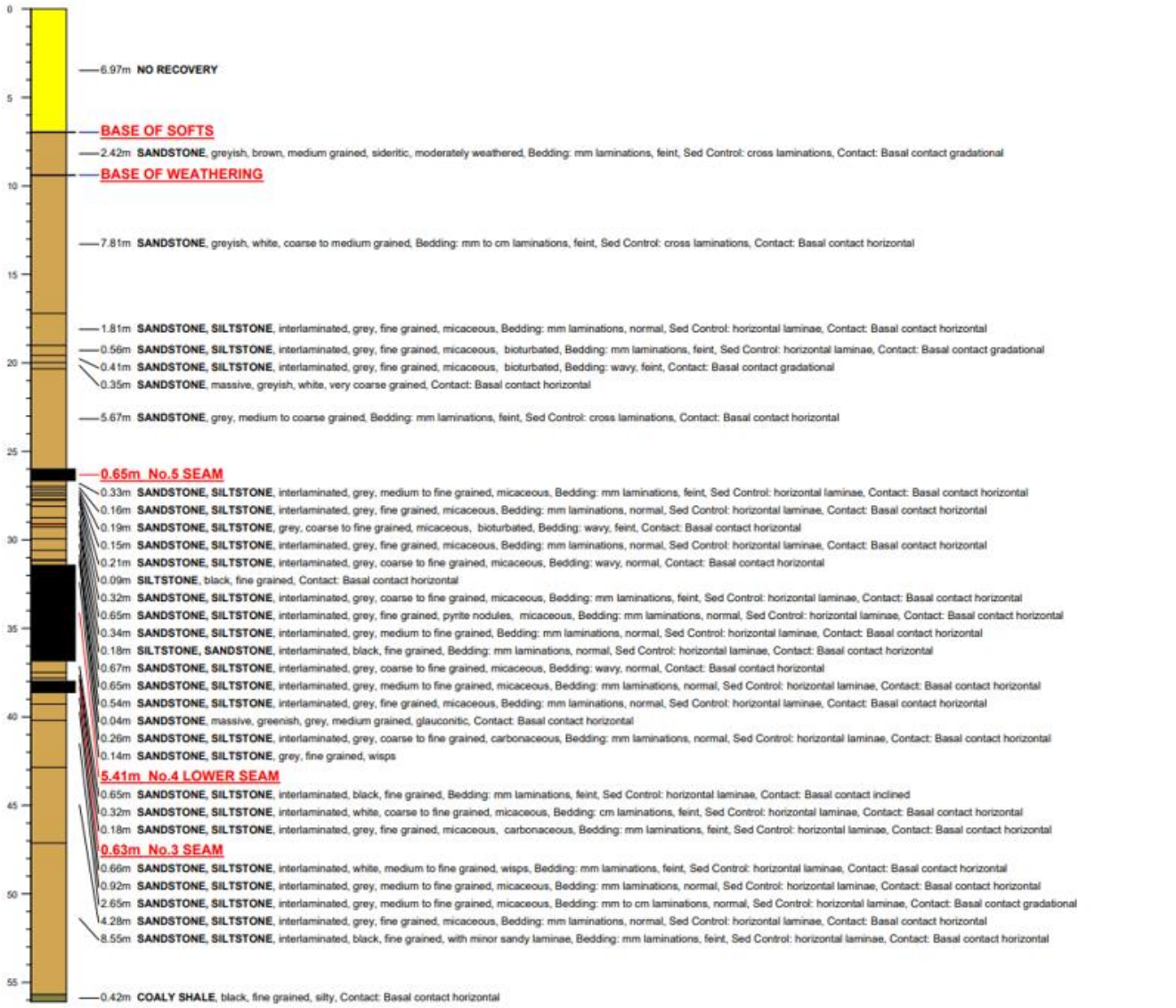

Figure 4.4 Stratigraphical column from borehole DF1669.

4.3. Petrographic Characteristics of the Studied Sandstones

Detrital framework grains commonly found in the studied Ecca Group sandstones include quartz, feldspar, lithic fragments, and accessory minerals. Quartz occurs as a dominate framework constituting more than 50% of the total framework grains due to its resistance to weathering and erosion. The remaining fraction includes a small but notable amount of lithic fragments, generally ranging between 5% and 10%, and minimal feldspar content. These lithic fragments could have been derived from a variety of rock types, including volcanic, metamorphic, and sedimentary rocks, adding to the rock's heterogeneity. The matrix, or finer-grained material that fills the spaces between the larger grains, is typically minimal, often less than 25% of the total rock volume. The matrix is made of mica group minerals such as muscovite and quartz fragments. The calcite cement fills the pores spaces within the rock. Based on the results obtained from point counting (Table 4.2) and in accordance with Folk Classification of sandstones (Figure 4.3) the studied sandstones could be classified as sublithic arenite.

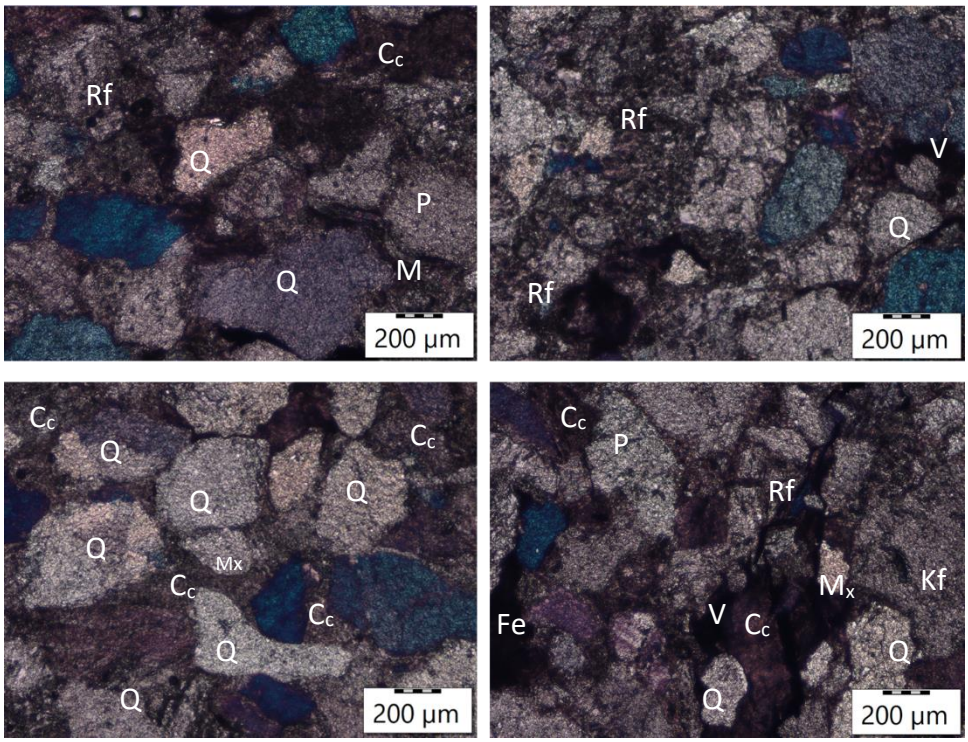

Figure 4.3 Photomicrographs of the studied sandstones under cross polarized light (XPL). (Q= quartz; P=plagioclase; C_c=Calcitic cement; Fe= iron cement; M_x=matrix; M=Muscovite; Kf=K-feldspar; Rf=rock fragments; V=void space).

Sample Number	*Sandstone Type*	*Framework (%)*					*Matrix (%)*	*Cement (%)*			*Void (%)*	*QFL*		
		Q	Kfs	Plg	L	OP		C_c	C_f	C_t		Q	F	L
S1	Sublithic Arenite	77	0	0	0	2	13	5	0	5	3	100	0	0
S2	Sublithic Arenite	69	0	0	10	3	10	3	0	3	5	91	0	9
S3	Sublithic Arenite	72	0	0	7	3	10	3	0	3	5	92	0	8
S4	Sublithic Arenite	64	0	3	7	3	13	4	0	4	6	90	2	8
S5	Sublithic Arenite	68	0	0	3	2	14	5	8	13	6	95	0	5
S6	Sublithic Arenite	66	0	0	0	3	11	12	0	12	8	100	0	0
S7	Sublithic Arenite	60	0	4	4	2	13	9	4	13	4	90	5	5
S8	Sublithic Arenite	69	0	0	0	2	13	11	0	11	5	100	0	0
S9	Sublithic Arenite	77	0	0	3	2	9	3	2	5	8	96	0	4
S10	Sublithic Arenite	73	0	0	2	2	10	4	3	7	6	97	0	3
S11	Sublithic Arenite	76	0	0	0	2	9	5	3	8	5	100	0	0
S12	Sublithic Arenite	75	0	0	0	1	11	10	0	10	3	100	0	0
S13	Sublithic Arenite	69	0	0	6	0	12	11	0	11	2	92	0	8
S14	Sublithic Arenite	74	0	0	0	2	13	8	0	8	2	100	0	0

Table 4.2 Modal composition of the studied Ecca Group sandstones. Q=quartz; Kfs=K-felspar; L-lithic fragments; OP=opaque minerals; C_c=calcite cement; C_f=iron cement; C_t=total cement; F=feldspar.

4.4. Textural Studies

The studied sandstones are very fine to fine-grained, moderately well-sorted to poorly sorted with angular to subangular. The type of grain-to-grain contacts encountered includes concavo-convex, tangential, long, sutured, and floating (Figure 4.4) with the consolidation factor ranging from 74 to 82%. The most common grain-to-grain contact is floating followed by sutured. The packing density P_d of the studied sandstones ranges from 69 to 88% (Table 4.3), high values indicate that the grains are interlocking, and low values of the packing density indicates that the grains are floating. Packing proximity values range from 59 to 70% and the consolidation factor ranges from 60 to 74% (Table 4.3).

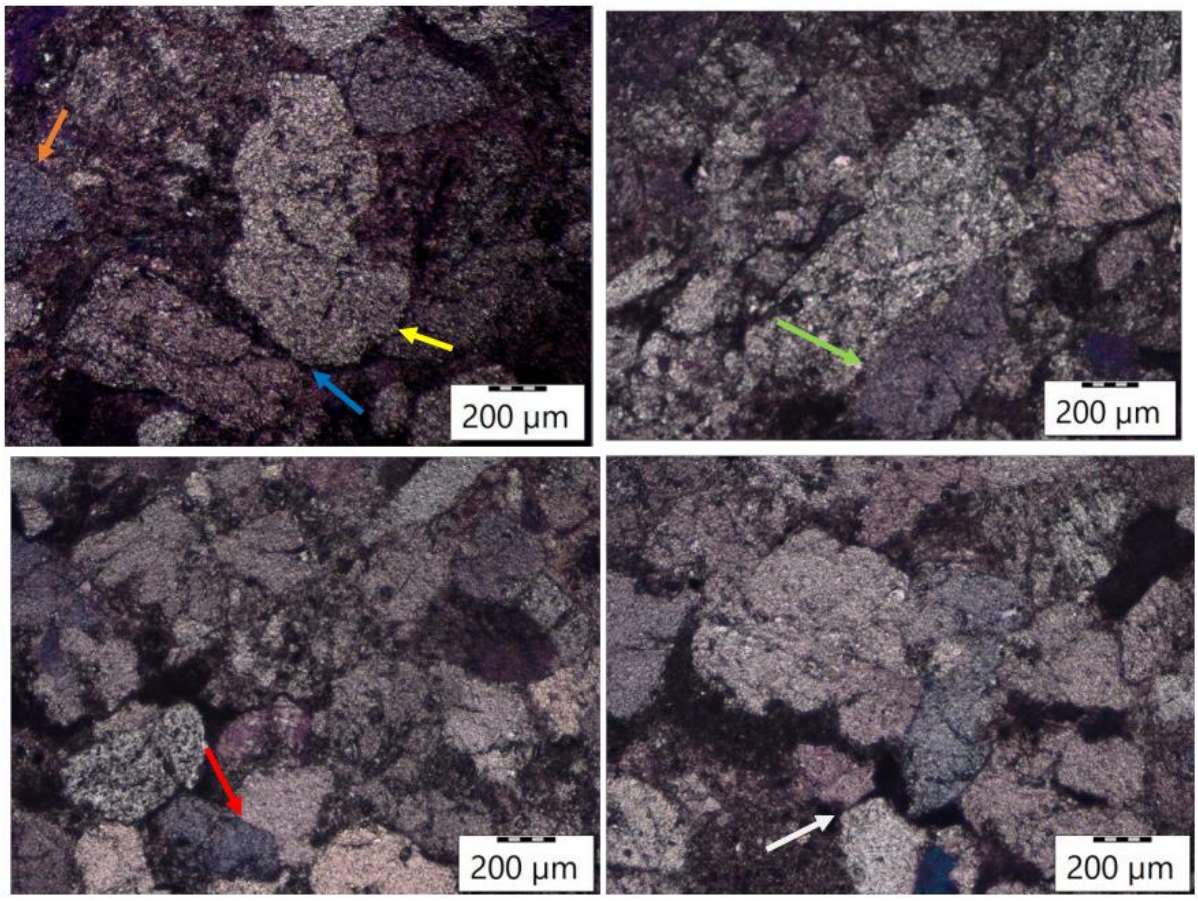

Figure 4. 3 Photomicrographs of the studied Ecca Group sandstones showing different contacts under cross-polarized light (XPL). Red arrow: concavo-convex contact; white arrow: tangential contact; green arrow: long contact; yellow arrow: sutured contact; orange arrow: floating contact.

Table 4.2 Textural characteristics of the studied Ecca Group Sandstones. (MC: matrix

Sample Number	***MC***	***Textural Parameters***		***Microfabric***											
				% Contact Type					**% Contact Nature**				**Pd%**	**Pp%**	**Pcc%**
	MC	**M_z**	**S_o**	**Su**	**Cc**	**Lo**	**Ta**	**Fl**	**GG**	**GC**	**GM**	**GV**	Pd%	Pp%	Pcc%
S1	18	0.21	1.11	2	2	15	25	56	80	4	16	0	78	63	62
S2	20	0.19	1.32	4	3	13	26	54	72	9	19	0	69	60	64
S3	20	0.19	1.32	5	3	12	30	50	72	9	19	0	69	60	64
S4	22	0.20	1.24	3	5	12	31	49	74	9	17	0	77	56	63
S5	27	0.19	1.18	3	4	11	30	51	73	9	18	0	71	53	72
S6	23	0.22	1.14	4	4	10	22	60	79	7	14	0	88	60	74
S7	26	0.20	1.22	5	3	9	29	54	78	7	15	0	76	59	70
S8	24	0.19	1.12	5	3	10	26	56	79	4	17	0	81	66	69
S9	19	0.21	1.10	6	5	11	29	49	82	7	11	0	89	69	74
S10	21	0.24	1.11	3	3	10	30	54	77	9	14	0	76	63	66
S11	20	0.19	1.10	4	3	11	32	50	76	8	16	0	74	70	68
S12	22	0.24	1.22	2	3	13	34	48	80	7	13	0	77	69	69
S13	21	0.20	1.32	2	3	12	34	49	79	6	15	0	76	66	60
S14	21	0.24	1.32	1	2	11	34	52	80	8	12	0	80	67	62

and cement content; Mz: mean grain size; So: sorting coefficient; Su; sutured contact; Cc: concavo- convex contact; Lo: longitudinal contact; Ta; tangential contact; Fl; floating contact; GG: grain to grain contact; GC; grain to cement contact; GM: grain to matrix contact; GV: grain to void contact; Pd: packing density; Pp: packing proximity; Pcc: consolidation factor).

4.5. Mineralogy and Modal Composition

The modal mineral composition of the Ecca Group Sandstones (Vryheid Formation) was obtained from point counting of the prepared thin sections (Figure 4.5), and the results are presented in Table 4.2. The most abundant mineral is quartz (monocrystalline and polycrystalline) constituting about 50% of the rock's framework with monocrystalline quartz being dominant. The total feldspar (K feldspar and plagioclase) ranges from 2 to 5%. Calcite cement and iron-hydroxide were observed. However, calcite cement is the most dominant cement (98%) than iron hydroxide cement (2%). The dominance of the calcite cement suggests that the rock might have underground chemical diagenesis resulting in the replacement of some framework grains by calcite cement. The void spaces comprise about 3 to 8% of the sandstone (Figure 4.4).

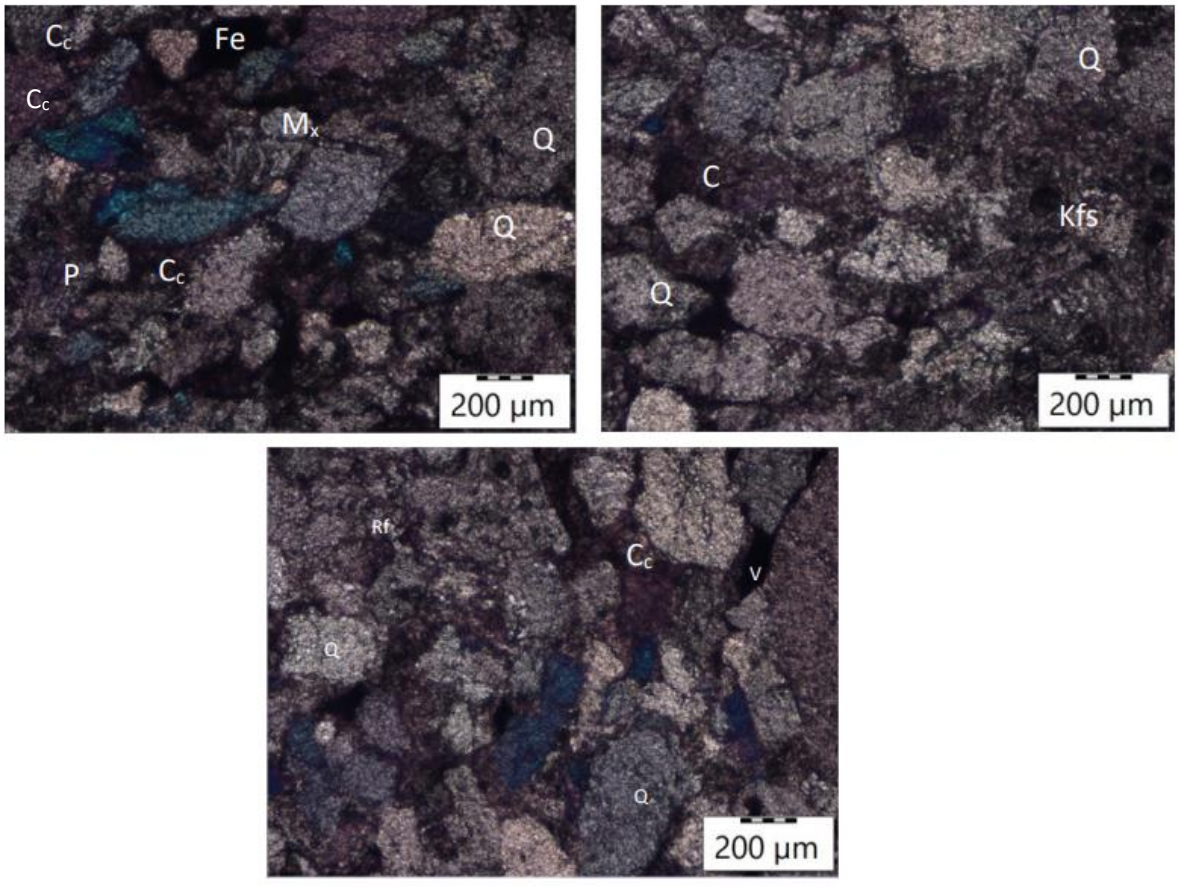

Figure 4.4 Photomicrographs of the studied sandstone under cross-polarized light (XPL). Q=quartz; P=plagioclase; Kfs=K-feldspar; Rf=rock fragments; C_c=Calcitic cement; Fe=iron cement; M_x=matrix.

4.6. Geotechnical properties of the sandstone

4.6.1. Rock strength testing

4.6.1.1. Point Load Test Result

The point load test is used to determine the strength and toughness of rock materials. Specifically, it measures the point load strength index of a rock, which is an indicator of its uniaxial compressive strength (UCS). Table 4.4 presents the results summary of the point load tests conducted on five sandstone samples taken from varying depths. The data includes measurements of depth, sample length and diameter, failure load, and calculations of the point load strength index (Is), the point load strength index corrected to a 50 mm diameter sample (Is(50)), and the unconfined compressive strength (UCS). Equation 4.1 was used for conversion of point load test to UCS of a rock.

$$UCS = a \times (I_s)^b \qquad (4.1)$$

where:

- UCS is the unconfined compressive strength (MPa),
- I_s is the point load strength index (MPa),
- a and b are empirically determined constants that vary depending on the rock type. For sandstones a=24 and b=0.5, as proposed by Bieniawski (1975) in his studies on rock strength.

The results revealed that the Ecca Group sandstones are characterized by high UCS ranging from 36.24 to 115.68 MPa.

Table 4. 3 Point Load Test Results

Sample no:	Rock type	Depth (m)	Length (mm)	Diameter (mm)	PLT Failure load (kN)	I_s (MPa)	$I_{s(50)}$ (MPa)	UCS (MPa)
1	Sandstone	4.00	185.00	60.00	5.00	1.39	1.51	36.24
2	Sandstone	10.86	250.00	60.00	16.00	4.44	4.82	115.68
3	Sandstone	15.12	130.00	60.00	6.00	1.67	1.81	43.44
4	Sandstone	26.74	145.00	60.00	9.00	2.50	2.71	65.04
5	Sandstone	31.17	120.00	60.00	11.00	3.06	3.32	79.68

4.6.1.2. Uniaxial Compressive Strength Test Results

Table 4.5 presents data related to the determination of Unconfined Compressive Strength (UCS) for a specific rock sample, identified as Sample No. 1. This sample belongs to the rock type sandstone and was extracted from a depth of 35.00 m below the surface, providing context about its geological origin and potential overburden pressure. The density of the sandstone sample is recorded as 2380 kg/m^3, indicating its mass per unit volume. Density is a critical parameter as it affects the overall strength and stability of the rock.

The failure load, measured at 151.5 kN, represents the maximum load the sandstone sample can withstand before it fails under compression. This load is essential in understanding the mechanical behaviour and stability of the rock when subjected to stress. The UCS of the sandstone is reported as 52.8 MPa. It is a fundamental measure of the sandstone's strength and is crucial for assessing its suitability for various engineering and construction applications where compression resistance is critical.

Table 4. 4 UCS test results

Sample No.	Rock Type	Depth (m)	Density (Kg/m^3)	Failure Load (kN)	UCS (MPa)
1	Sandstone	35.00	2380	151.5	52.8

4.6.1.3. Tensile Strength Test Results

Table 4.6 provides detailed information about a specific rock sample, identified as Sample No. 2. This sample was extracted from a depth of 35.15 m below the surface, indicating its geological context and potential overburden pressure. The rock type for this sample is sandstone, a prevalent sedimentary rock formed from compacted sand grains. The density of this sandstone sample is measured at 2360 kg/m³.

The failure load for this sample is reported to be 10.2 kN. The tensile strength of the sandstone is calculated as 3.4 MPa. Tensile strength measures the resistance of the rock to breaking under tension. A tensile strength of 3.4 MPa indicates that the sandstone can endure a significant amount of tensile stress before fracturing, which is essential for understanding its performance in various engineering applications.

Table 4. 5 Tensile Strength Test Results

Sample No.	Sample Depth (m)	Rock Type	Density (Kg/m^3)	Failure Load (KN)	Tensile Strength (MPa)
2	35.15	Sandstone	2360	10.2	3.4

4.6.2. Elasticity (Young Modulus and Poisson's Ratio)

Table 4.7 presents the results for Poisson's ration and Young's modulus for sample No.2. Poisson's Ratio for this sandstone sample is reported to be 0.47. This ratio is a measure of the material's tendency to expand perpendicularly when compressed longitudinally. A value of 0.47 is relatively high, indicating that the sandstone deforms significantly in the lateral direction when subjected to axial stress.

The Young's Modulus of the sample is given as 8.9 GPa. This modulus quantifies the stiffness of the rock, showing how much it will deform under a given stress. A Young's Modulus of 8.9 GPa suggests that the sandstone has moderate stiffness, implying that it is relatively more flexible compared to other rocks with higher modulus values.

The failure description for this sample indicates that it underwent sliding shear failure. This type of failure occurs when the rock fractures and slides along a plane of weakness due to shear stresses exceeding the rock's shear strength. In summary, Sample No. 2 is a sandstone characterized by a high Poisson's Ratio of 0.47 and a

moderate Young's Modulus of 8.9 GPa, which failed through sliding shear failure, reflecting its mechanical response to applied stress.

Table 4. 6 Elasticity test results

Sample No.	Rock Type	Poisson's Ratio	Youngs Modulus (GPa)	Failure Description
2	Sandstone	0.47	8.9	Sliding shear failure

4.7. Developing brittleness index

Statistical analysis was used to develop the brittleness indices. Statistical analysis serves as an effective modelling technique for developing predictive models of the mechanical properties of rocks. These analyses are instrumental in elucidating the relationship between petrographic characteristics and the mechanical behaviour of rocks. Models created through statistical analysis help identify the most significant independent variables that influence the dependent variables (Ali et al. 2014). In this study, SPSS version 25 was utilized to conduct regression analysis on the available data set. The original data comprised 14 models (petrographic variables) as independent variables, with the brittleness indices considered as dependent variables.

4.7.1. Preparation for Model Development

Table 4.8 presents a detailed analysis of various models, indicating the relationships between dependent and independent variables across different types of model structures. Each model is denoted by an index (B_1, B_2, B_3, B_4 and B_5) and is characterized by a set of independent variables and statistical parameters (Table 4.7). The table outlines multiple models examining the relationships between various independent variables and their respective dependent outcomes, with different statistical measures indicating the fit, significance, and predictive power of each model.

B1 Models

B_1 was predicted by 11 independent variables in the first step (Model 1) with R^2=0,669. The number of variables was reduced to three in the last model (Model 1) with R^2 =0,300. All the two models used (Table 4.8.) the petrographic characteristics do not significantly correlate with B_1. They give p-values which is greater than 0,05. Similarly, for B_2, all the two models, the petrographic characteristics do not significantly correlate with B_2.

Main Model: This model includes the independent variables M_z (mean grain size), S_o (sorting coefficient), SU (sutured contact), CC (concavo-convex contact), LO (long contact) , Ta (tangential contact), Fi (floating contact), GG (grain-grain contact), GC (grain to cement contact), GM (grain to matrix contact), and GV (grain to void contact). It has an R-value of 0.818, indicating a strong correlation. However, the adjusted R-squared (R_a^2) is negative (-0.765), which suggests that the model may not be well-suited for prediction. The standard error (SE) is 0.075, and the significance of F (Sig.F) is 0.763, indicating that the model is not statistically significant.

Microfabric Model: This model focuses on variables Pp (packing proximity), Pd (packing density), and Pc (consolidation factor). It shows an R-value of 0.547 and an R-squared (R^2) of 0.300, suggesting a moderate fit. The SE is 0.060, and the Sig.F is 0.291, which is not statistically significant.

B2 Models

Main Model: Similar to B1, this model includes the same independent variables with an R-value of 0.789 and an R^2 of 0.622. The negative R_a^2 (-0.968) again points to potential issues in the model's predictive ability. The SE is 0.000, and the Sig.F is 0.821, indicating no statistical significance. Microfabric Model: With variables Pp (packing proximity), Pd (packing density), and Pc (consolidation factor), this model shows an R-value of 0.569 and an R^2 of 0.323. The SE is 0.000, and the Sig.F is 0.251, indicating no statistical significance.

B3 Models

To estimate B_3, in the first step (Model 1), three variables were used with the R^2= 0,156. The variables were reduced to three in the last step (Model 3), with R^2 =0,470. Out of four models developed to estimate B_3, model 3 is the only one that significantly

correlates with brittleness index B_3 at 95% confidence (P-value range from 0,020 to 0,025). As a result, model 3 is the most efficient to estimate B_3.

Microfabric Model: This model has an R-value of 0.395 and an R^2 of 0.156. The negative R_a^2 (-0.096) and SE of 0.089 suggest a weak model fit. The Sig.F is 0.619. Main Model: With the same variables as B1 and B2, this model shows a high R-value of 0.989 and an R^2 of 0.979, indicating an excellent fit. The SE is 0.025, and the Sig.F is 0.066. Contact Type Models: Model 1: Includes variables SU (sutured contact), CC (concavo-convex contact), LO (long contact), Ta (tangential contact), and Fi (floating contact), with an R-value of 0.880 and an R^2 of 0.774. The R_a^2 is 0.633, SE is 0.051, and Sig.F is 0.017, indicating statistical significance. Model 2: With variables SU and LO, it shows an R-value of 0.685 and an R2 of 0.470. The R_a^2 is 0.374, SE is 0.067, and Sig.F is 0.030, indicating statistical significance.

B4 Model

To estimate B_4, seven models were developed. In the first step (Model 1) with eleven variables, with R^2 =0,925 were employed to predict B_4. The number of variables was then reduced to three in the last step (Model 3) with R^2 =0,821. Model 3 petrographic characteristics significantly correlate with (B_4 at 95% confidence level). Out of all seven models developed, Model 3 is the most efficient to predict B4.

Main Model: This model shows an R-value of 0.962 and an R2 of 0.925. The R_a^2 is 0.508, SE is 0.001, and Sig.F is 0.092. Microfabric Model: With Pp (packing proximity), Pd (packing density), and Pc (consolidation factor), this model has an R-value of 0.590 and an R^2 of 0.349. The SE is 0.002, and Sig.F is 0.212. Contact Type Models: Model 1: Variables SU (sutured contact), CC (concavo-convex contact), LO (long contact), Ta (tangential contact), Fi (floating contact), show an R-value of 0.861 and an R^2 of 0.742. The SE is 0.001, and Sig.F is 0.028. Model 2: Ta (tangential contact) and Fi (floating contact) variables yield an R-value of 0.790 and an R^2 of 0.624. The SE is 0.001, and Sig.F is 0.004. Contact Nature Models: Model 1: Variables GG (grain-grain contact), GC (grain to cement contact), GM (grain to matrix contact), and GV (grain to void contact), with an R-value of 0.689 and an R^2 of 0.475. The SE is 0.001, and Sig.F is 0.077. Model 2: Variable GG (grain-grain contact) alone has an R-value of 0.578 and an R2 of 0.334. The SE is 0.001, and Sig.F is 0.030. Contact Model 3: Variables

GG (grain-grain contact), Ta (tangential contact), Fi (floating contact), show an R-value of 0.906 and an R^2 of 0.821. The SE is 0.001, and Sig.F is 0.000.

B5 Models

The index B_5 was predicted by three variables in the first step (Model 1) with R^2 = 0,135. The number of independent variables was reduced to three in the last step (Model 3) with R^2 = 0,652. Out of all the four models developed, Model 3 is the only that significantly correlate with B3 at 95% confidence level. It is the most efficient model to predict B5.

Microfabric Model: With Pp, Pd, and Pc, the R-value is 0.368 and R2 is 0.135. The SE is 0.030, and Sig.F is 0.675. Main Model: This model shows an R-value of 0.953 and an R2 of 0.908. The SE is 0.018, and Sig.F is 0.277. Contact Type Model 2: Variables SU (sutured contact), CC (concavo-convex contact), LO (long contact), Ta (tangential contact), Fi (floating contact), show an R-value of 0.888 and an R2 of 0.789. The SE is 0.017, and Sig.F is 0.013. Contact Model 3: Variables CC (concavo-convex contact), Ta (tangential contact), Fi (floating contact), show an R value of 0.807 and an R^2 of 0.652. The SE is 0.019, and Sig.F is 0.011.

B6 Models

To estimate B_6, in the first step (Model 1), eleven variables were used with the R^2= 0,460. The variables were reduced to one in the last step (Model 2), with R^2 =0,459. Out of three models developed to estimate B_6, model 2 is the only that significantly correlates with brittleness index B_6 at 99% confidence (P-value = 0,007). As a result, model 2 is the most efficient to estimate B_6. Main Model: This model shows an R-value of 0.987 and an R^2 of 0.975. The SE is 0.006, and Sig.F is 0.079. Microfabric Models: Model 1: With Pp, Pd, and Pc, this model has an R-value of 0.678 and an R^2 of 0.460. The SE is 0.016, and Sig.F is 0.091. Model 2: Variable Pp (packing proximity) alone shows an R-value of 0.677 and an R^2 of 0.459. The SE is 0.015, and Sig.F is 0.007.

Table 4. 7 BMLR analysis to estimate brittleness indices.

Index		*Model no.*	*Independent variables*	*R*	R^2	R^2_a	*SE*	*Sig.F*
B1	Main	1	$M_z,S_O,S_U,C_C,L_O,T_a,F_i,G_G,G_C,G_M,G_V$	0,818	0.669	-0,765	0,075	0,763
	Microfabric	1	P_p,P_d,P_c	0,547	0,300	0,090	0,060	0,291
B2	Main	1	$M_z,S_O,S_U,C_C,L_O,T_a,F_i,G_G,G_C,G_M,G_V$	0789	0,622	-0,968	0,000	0,821
	Microfabric	1	P_p,P_d,P_c	0,569	0,323	0,121	0,000	0,251
B3	Microfabric	1	P_p,P_d,P_c	0,395	0,156	-0,096	0,089	0,619
	Main	1	$M_z,S_O,S_U,C_C,L_O,Ta,F_i,G_G,G_C,G_M,G_V$	0,989	0,979	0,578	0,025	0,066
	Contact Type	2	$S_U,C_C,L_O,T_a,F_i,$	0,880	0,774	0,633	0,051	0,017
	Contact Type	3	S_U,L_O	0,685	0,470	0,374	0,067	0,030
B4)	Main	1	$M_z,S_O,S_U,C_C,L_O,T_a,F_i,G_G,G_C,G_M,G_V$	0,962	0,925	0,508	0,001	0,092
	Microfabric	1	P_p,P_d,P_c	0,590	0,349	0,153	0,002	0,212
	Contact Type	1	$S_U,C_C,L_O,T_a,F_i,$	0,861	0,742	0,581	0,001	0,028
	Contact Type	2	$T_a,F_i,$	0,790	0,624	0,556	0,001	0,004
	Contact Nature	1	G_G,G_C,G_M,G_V	0,689	0,475	0,218	0,001	0,077
	Contact Nature	2	G_G	0,578	0,334	0,279	0,001	0,030
	Contact	3	$G_G,$ Ta,Fi,	0,906	0,821	0,768	0,001	0,000
B5	Microfabric	1	P_p,P_d,P_c	0,368	0,135	0,123	0,030	0,675
	Main	1	$M_z,S_O,S_U,C_C,L_O,T_a,F_i,G_G,G_C,G_M,G_V$	0,953	0,908	0,270	0,018	0,277
	Contact Type	2	$S_U,C_C,L_O,T_a,F_i,$	0,888	0,789	0,657	0,017	0,013
	Contact	3	$C_C,$ $T_a,F_i,$	0,807	0,652	0,547	0,019	0,011
B6	Main	1	$M_z,S_O,S_U,C_C,L_O,T_a,F_i,G_G,G_C,G_M,G_V$	0,987	0,975	0,560	0,006	0,079
	Microfabric	1	P_p,P_d,P_c	0,678	0,460	0,299	0,016	0,091
	Microfabric	2	$P_p,$	0,677	0,459	0,413	0,015	0,007

From Table 4.8, the following terms are defined: Mz represents the mean grain size, and S_o refers to the sorting coefficient. SU denotes sutured contact, while CC represents concavo-convex contact. LO stands for long contact, Ta is tangential contact, and Fi refers to floating contact. GG indicates grain-to-grain contact, GC represents grain-to-cement contact, and GM stands for grain-to-matrix contact. GV refers to grain-to-void contact. Additionally, Pp is the packing proximity, Pd is the packing density, and Pc represents the consolidation factor.

4.7.2. Developed brittle indices

The given brittleness indices B_3, B_4, B_5, and B_6 are equations formulated to quantify brittleness in a geological or materials context. B1 and B2 equations were not developed, and all the models do not significantly correlate, as evidenced by p-values greater than 0.05. The p-value is a statistical measure that indicates the probability of obtaining results at least as extreme as the ones observed, given that the null hypothesis is true. A p-value greater than 0.05 suggests that the results are not statistically significant. These brittleness indices presented are mathematical models that aim to quantify brittleness using various predictors.

The results include the equations, correlation coefficients (R), coefficients of determination (R^2), adjusted R^2 (R^2a), and significance levels (Sig.F) (Table 4.8). Correlation Coefficient (R): Indicates the strength and direction of the linear relationship between variables. Coefficient of Determination (R^2): Represents the proportion of the variance for the dependent variable that is explained by the independent variables. Adjusted R^2 (R^2a): Adjusted for the number of predictors in the model; useful for comparing models with different numbers of predictors. Significance of F (Sig.F): Indicates the overall significance of the regression model.

Table 4.8 Developed brittleness indices.

Equations	R	R^2	R^2_a	Sig.F
$B_{3=}0{,}796 - 0{,}037Su - 0{,}036Lo$	0,685	0,470	0,374	0,030
$B_4 = -0{,}066 + 0{,}000Ta + 0{,}000FI + 0{,}000GG$	0,906	0,821	0,768	0,000
$B_5 = -0{,}907 + 0{,}025C_C + 0{,}012Ta + 0{,}009FI$	0,807	0,652	0,547	0,011

$B_6 = 0{,}323 - 0{,}002P_p$	0,677	0,459	0,413	0,007

$B_{3=}0{,}796 - 0{,}037Su - 0{,}036Lo$ (4.1)

Equation (4.1) incorporates parameters Su (sutured contact) and Lo (long contact). The correlation coefficient (R) is 0.685, indicating a moderate relationship between the brittleness index and these variables. The R^2 value of 0.470 means that about 47% of the variability in brittleness is explained by Su and Lo. An adjusted R^2 (R^2a) of 0.374 suggests that when adjusting for the number of predictors in the model, about 37.4% of the variance is explained. Sig.F = 0.030 indicates that the model is statistically significant at a 3% significance level. Su (sutured contact**)** term has a negative coefficient, indicating that an increase in sutured contact decreases the brittleness index B_3. Sutured contacts are strong interlocks between grains, typically reducing brittleness by providing structural integrity. Similarly, (Lo) long contacts also have a negative effect on the brittleness index. Long contacts involve elongated grain boundaries that likely contribute to stability and less brittle behaviour. Intercept (0.796) is the baseline value of the brittleness index when Su and Lo are zero.

$B_4 = -0{,}066 + 0{,}000Ta + 0{,}000FI + 0{,}000GG$ (4.2)

Equation (4.2) includes Ta (tangential contact), Fi (floating contact), and GG (grain-grain contact) with an R of 0.906, which indicates a very strong relationship. The R^2 value of 0.821 suggests that 82.1% of the variability is explained by these predictors. An adjusted R^2 of 0.768 still represents a strong explanatory power. A very low Sig.F value (0.000) shows high statistical significance. Ta (tangential contact) coefficient is very small, suggesting that tangential contact has a minimal positive effect on brittleness. Fi (floating contact**)**, where grains are not in direct contact but suspended in the matrix, have a similarly negligible positive effect on brittleness. Direct GG (grain-to-grain contact) slightly increases brittleness, although the impact is very small. Intercept (-0.066) indicates an initial non-brittle baseline, but the contact types minimally increase the brittleness index.

$B_5 = -0{,}907 + 0{,}025C_C + 0{,}012Ta + 0{,}009FI$ (4.3)

Equation (4.3) uses C_c (concavo-convex contact) , Ta (tangential contact), and Fi (floating contact). An R of 0.807 suggests a strong relationship. The R^2 value is 0.652, indicating that 65.2% of the variability in brittleness is explained. An adjusted R^2 of 0.547 indicates that 54.7% of the variance is explained after adjusting for the number of predictors. Sig.F = 0.011 suggests the model is statistically significant at a 1.1% level. C_c (concavo-convex contact) has a relatively larger positive coefficient, indicating that concavo-convex contacts significantly increase the brittleness index. These contacts are strong and interlocking but can create points of weakness where stress is concentrated. Ta (tangential contact) has a positive coefficient, though smaller than C_c indicating a contribution to brittleness. Fi (floating contact) also increases brittleness but to a lesser extent. Intercept (-0.907) indicates a strongly non-brittle baseline, with contributions from C_c (concavo-convex contact), Ta (tangential contact), and Fi (floating contact) making the material more brittle.

$$B_6 = 0{,}323 - 0{,}002P_p \qquad (4.4)$$

Equation (4.4) with Pp has an R of 0.677, indicating a moderate relationship. An R^2 value of 0.459 means 45.9% of the variability is explained by Pp (packing proximity). An adjusted R^2 of 0.413 indicates that 41.3% of the variance is explained. Sig.F = 0.007 indicates statistical significance at a 0.7% level. P_p (packing proximity) has a negative coefficient, suggesting that increased packing proximity reduces brittleness. Tight packing generally enhances structural stability and reduces the tendency to fracture. Intercept (0.323) is the baseline brittleness index, which decreases as P_p increases.

4.8. Discussions of the results

In this research, multivariate regression analysis was employed to investigate how the mineralogical characteristics influence the brittleness indices of the Ecca Group sandstones. The textural properties under examination were categorized into three distinct subgroups: grain packing properties, contact nature, and contact type. This approach allowed for a comprehensive examination of how these specific aspects of sandstone composition and structure contribute to its overall brittleness. By statistically analyzing these relationships, the study aimed to deepen understanding of how different petrographic features impact the mechanical behaviour and fracture

propensity of sandstone, crucial for applications in geotechnical engineering, construction, and resource exploration.

Backward multivariate linear regression (BLMR) was employed to determine the relationship between the petrographic characteristics as independent variables and brittleness indices (B_1, B_2, B_3, B_4, B_5 and B_6) as dependent variables. Backward multivariate linear regression is a method of model selection in multiple linear regression where multiple independent variables are considered, and the least significant variables are removed step-by-step (Akaike, 1973). This method helps to identify the most significant predictors for the dependent variable. The process involves starting with all candidate variables and iteratively removing the least significant variable until only statistically significant variables remain.

To analyse the given brittleness index equations and compare them with established indices by other authors, we should focus on their statistical metrics (R, R^2, R^2a, and Sig.F) and understand their components in the context of brittleness. Rickman et al. (2008) proposed a brittleness index using mineralogical content, emphasizing quartz and carbonate content as primary indicators. They found high R^2 values (up to 0.85) when correlating brittleness with mineralogical content, suggesting a strong explanatory power. Grieser and Bray (2007) developed a brittleness index based on dynamic elastic properties (Young's modulus and Poisson's ratio). This method showed high correlation coefficients ($R > 0.8$), indicating strong relationships between brittleness and elastic properties. Hucka and Das (1974) defined brittleness based on mechanical properties such as compressive strength and tensile strength, yielding moderate R^2 values (0.5-0.7).

B_4 has the highest R^2 (0.821) among the given equations, which is comparable to the models by Rickman et al. (2008) and Grieser and Bray (2007). This indicates a strong predictive capability when using parameters Ta, FI, and GG. B_3, B_5, and B_6 show moderate R^2 values (0.459 to 0.652), which are similar to the brittleness indices by Hucka and Das (1974). These models may not be as robust but still offer significant predictive power.

The given brittleness indices generally show moderate to strong predictive power. The B_4 equation stands out with its high R^2 and statistical significance, making it

comparable to the robust models developed by Rickman et al. (2008) and Grieser and Bray (2007). The other indices, while useful, have lower explanatory power but still provide valuable insights into the brittleness characteristics of the materials studied.

4.8.1 The effect of microfabric on brittleness index

The effect of microfabric on the brittleness index (BI) of rocks is significant and multifaceted. Microfabric characteristics such as pore proportion (Pp), packing density (Pd), and consolidation factor (Pc) proposed by Kahn (1956) play crucial roles in determining BI Packing Proximity and Density. These properties primarily describe how closely the particles in a rock are packed together (Smith et al., 2018). They are influenced by depositional processes and subsequent compaction. High packing density indicates closely packed grains, but it does not necessarily imply that the rock will behave in a brittle manner (Jones & Brown, 2020).

Consolidation Factor: This refers to the degree to which a sediment has been compacted and lithified into a solid rock (Williams, 2019). It includes the reduction of pore space and the cementation of grains. While more consolidated rocks might be less porous and potentially stronger, this does not directly indicate brittleness (Lee, 2021). While packing proximity, packing density, and consolidation factor provide important information about the physical arrangement and compaction state of a rock, they do not encompass all the factors that contribute to brittleness. Brittleness is a complex property influenced by a combination of mineralogical, structural, and environmental factors (Smith et al., 2018), making it possible for rocks with similar packing and consolidation characteristics to exhibit different levels of brittleness.

This is evident from Table 4.7, Microfabric models (using Pp, Pd, Pc) have a moderate correlation with the brittleness indices but generally fail to provide a statistically significant explanation for the variability in the indices. This is evident from their lower R and R^2 values, higher standard errors, and higher significance F values compared to main models using a broader set of variables. Model B1: The microfabric model shows an R-squared (R^2) value of 0.300, indicating that 30% of the variability in the brittleness index is explained by variables Pp, Pd, and Pc. However, the Sig.F value is 0.291, which is not statistically significant ($p > 0.05$). This suggests that the relationship between these microfabric variables and the brittleness index could likely be due to random chance rather than a true predictive relationship. Other Microfabric

Models (B2, B3, B4, B5, B6): Similar patterns are observed across different models where microfabric variables (Pp, Pd, Pc) show moderate to weak correlations (R^2 ranging from 0.135 to 0.460). Importantly, many of these models also have non-significant Sig.F values ($p > 0.05$), indicating that the relationships observed may not be reliable. These models have lower R and R^2 values, indicating a weaker fit. Most microfabric models are not statistically significant, with some exceptions (e.g., Microfabric Model 2 in B_6). Therefore, while microfabric variables may contribute to understanding the brittleness index, they are insufficient on their own and must be supplemented with other significant variables to create a robust predictive model.

4.8.2 The combined effect of contact nature, contact type and mean grain size on brittleness index.

The combined effects of contact nature, contact type, and mean grain size on the brittleness index (BI) of rocks are complex and interdependent. Contact nature, which includes grain-to-grain (GG), grain-to-matrix (GM), and grain-to-cement (GC) contacts, significantly influences the mechanical behaviour of rocks. GG contacts often lead to higher brittleness due to the direct stress transmission and minimal energy absorption during deformation (Smith et al., 2019). Conversely, GC contacts can reduce brittleness by increasing energy dissipation through the cementing material, which acts as a buffer during stress application (Jones & Brown, 2020).

Contact type further modulates this relationship. For instance, sutured (SU) contacts typically result in stronger and less brittle rocks because of the interlocking nature of the grains (Williams, 2019). On the other hand, planar contacts may lead to higher brittleness due to the ease with which fractures can propagate along these planes (Lee, 2021).

Mean grain size (Mz) also plays a critical role. Generally, finer-grained rocks tend to be more brittle as the smaller grain size facilitates fracture initiation and propagation through numerous grain boundaries (Smith et al., 2018). Larger grain sizes might lead to a reduction in brittleness due to the increased path length for fractures, which can hinder crack propagation (Jones & Brown, 2020).

When these factors interact, their combined effects on BI become apparent. Rocks with GG contacts and finer grain sizes are typically the most brittle, as both factors enhance fracture propagation (Smith et al., 2019). Rocks with SU contact types and

larger grain sizes tend to exhibit lower brittleness due to the combined effects of strong grain interlocking and increased crack path length (Williams, 2019).

Overall, the interplay between contact nature, contact type, and mean grain size is crucial in determining the brittleness of rocks, with each factor either amplifying or mitigating the effects of the others. Understanding these combined effects is essential for predicting rock behaviour in various geological and engineering applications.

The effect of the main model on the brittleness index (BI) is evident from Table 4.7 across several models. In Model B1, the main model, which includes variables such as Mz, SO, SU, CC, LO, Ta, Fi, GG, GC, GM, and GV, shows a significant effect on BI with a correlation coefficient (R) of 0.818 and an R-squared (R^2) value of 0.669, indicating that these variables explain about 66.9% of the variance in BI. However, the adjusted R-squared (R_a^2) value is -0.765, suggesting potential overfitting or multicollinearity issues. Similarly, in Model B_2, the main model has a correlation coefficient of 0.789 and an R^2 value of 0.622, explaining 62.2% of the variance in BI. The adjusted R_a^2 value is -0.968, again indicating possible issues with the model's fit. In contrast, Model B_3 exhibits a very strong correlation with BI (R = 0.989) and an R^2 value of 0.979, explaining 97.9% of the variance in BI. The adjusted R_a^2 value of 0.578 suggests a better fit compared to Models B_1 and B_2. Model B_4 shows a high correlation (R = 0.962) and an R^2 value of 0.925, explaining 92.5% of the variance in BI. The adjusted R_a^2 value is 0.508, indicating a reasonable fit. In Model B5, the main model has a correlation coefficient of 0.953 and an R^2 value of 0.908, explaining 90.8% of the variance in BI. The adjusted R_a^2 value is 0.270, suggesting some degree of overfitting. Finally, Model B6 displays a very high correlation with BI (R = 0.987) and an R^2 value of 0.975, explaining 97.5% of the variance in BI. The adjusted R_a^2 value is 0.560, indicating a good model fit. Overall, the main model across these various scenarios consistently shows a strong correlation with the brittleness index, with high R and R^2 values indicating that the included variables significantly influence BI. However, the negative or low adjusted R_a^2 values in some models suggest potential issues with overfitting, multicollinearity, or other model specification problems.

4.8.3 The effect of contact type (SU, CC, LO, Ta, Fi) on brittleness index

The types of grain-to-grain contacts in rocks, including tangential, long, sutured, concavo-convex, and floating contacts, significantly influence their brittleness

characteristics. Tangential contacts occur when grains touch along their edges, promoting interlocking and potentially enhancing the rock's strength and resistance to fracturing (Blatt et al., 2006). Long contacts involve grains touching over a larger area, which can provide stability but may also create planes of weakness, influencing the rock's fracture behaviour (Fossen, 2016). Sutured contacts feature grains with interlocking edges that are tightly bound together, contributing to higher rock strength but potentially increasing brittleness due to localized stress concentrations (Fossen, 2016). Concavo-convex contacts occur when one grain fits into a concave recess of another, which can enhance load distribution but may also contribute to brittleness under certain stress conditions (Blatt et al., 2006). Floating contacts, where grains are not in direct contact but are surrounded by matrix material, can weaken the rock structure and increase brittleness by reducing overall cohesion (Fossen, 2016). Understanding these types of contacts is crucial in assessing the mechanical behaviour and fracture propensity of rocks, aiding in geological studies, engineering designs, and resource exploration.

The results (Table 4.7) demonstrate that contact types are significant predictors of the brittleness index. For instance, in Model B3, Contact Type 2 (SU, CC, LO, Ta, Fi) exhibits a high R-value of 0.880 and an R^2 value of 0.774, indicating that approximately 77.4% of the variability in the brittleness index can be explained by this contact type. Similarly, Model B4 shows that Contact Type 1 (SU, CC, LO, Ta, Fi) has an R-value of 0.861 and an R^2 value of 0.742, further emphasizing the strong correlation between these contact types and the brittleness index.

The significance values (Sig.F) for these contact types are generally low, indicating their statistical significance in predicting the brittleness index. For example, in Model B4, Contact Type 2 (Ta, Fi) has a Sig.F of 0.004, suggesting a robust significance in predicting the brittleness index. This trend is consistent across various models, reinforcing the importance of contact types in estimating the brittleness index. Additionally, the adjusted R^2 values, though lower than the R^2 values, still demonstrate significant predictive power even after adjusting for the number of predictors. In Model B5, for instance, Contact Type 2 (SU, CC, LO, Ta, Fi) shows an adjusted R^2 of 0.657, meaning the model still explains a substantial portion of the variability in the brittleness index after accounting for the number of variables used.

Overall, the analysis underscores that contact types, such as SU, CC, LO, Ta, and Fi, play a crucial role in predicting the brittleness index. This is evident by high R and R^2 values and low Sig.F values across multiple models, highlighting the importance of these contact types in geological studies and related fields. Research has shown that different contact types impact rock strength and brittleness due to varying degrees of grain interlocking and stress distribution, significantly influencing the mechanical properties of rocks (Chang & Haimson, 2012). These findings align with the statistical models presented, which quantify the impact of petrographic characteristics, including grain contacts, on the brittleness index.

4.8.4 The effect of contact nature (GG, GV, GM, GC) on brittleness index.

The nature of contact (GG, GV, GM, GC) significantly influences the brittleness index (BI) of rocks. GG contacts, where grains are in direct contact without any interstitial material, typically result in higher BI due to increased mechanical interlocking and reduced energy absorption during deformation (Smith et al., 2019). In GV contacts, where grains are in contact with a thin film of interstitial material, BI may vary depending on the thickness and nature of this film; thicker films can dampen deformation, potentially lowering BI compared to GG contacts (Jones & Brown, 2020). GM contacts, characterized by a matrix material surrounding grains, tend to exhibit intermediate BI, influenced by the strength and compliance of the matrix itself (Williams, 2018). GC contacts, involving cemented grains, often result in lower BI as cementation can reduce the grain-to-grain interactions and increase energy absorption during deformation (Lee, 2021). Thus, contact nature plays a crucial role in determining the brittleness index of rocks, reflecting variations in mechanical behaviour and energy dissipation mechanisms (Smith et al., 2019).

From Table 4.7, the effects of contact nature GG, GC, GM, and GV on the brittleness index (BI) are discernible across several models, namely B_1, B_2, and B_4. In Model B1, these contact types are included among the independent variables influencing BI (R = 0.689, R^2 = 0.475), with GG alone showing a significant effect (R = 0.578, R^2 = 0.334). Similarly, in Model B_4, GG, GC, GM, and GV are categorized under Contact Nature 1, contributing to BI variation (R = 0.689, R^2 = 0.475), with GG again demonstrating a notable impact (R = 0.578, R^2 = 0.334). These findings underscore the differential influence of contact nature on BI: GG contacts tend to correlate with higher BI, likely

due to direct grain-to-grain contact that enhances mechanical interlocking and reduces energy absorption during deformation. In contrast, GC contacts often associated with lower BI, possibly attributable to the presence of cementation between grains, which increases energy dissipation and lowers the overall brittleness of the rock.

4.9. Conclusion of the chapter

In this chapter, the results of the mineralogical analysis of the Ecca Group sandstones were presented and discussed with the aim of estimating the brittleness index of the formations. Through comprehensive petrographic analysis and statistical modelling, significant correlations were identified between specific petrographic parameters and the brittleness index. Models with a high number of independent variables are deemed impractical and costly. Hence, among the developed models for predicting brittleness indices, the most optimal choice is the model that has fewer independent variables, along with higher values of R^2_a and F.

Four brittleness indices, namely B3, B4, B5, and B6, were formulated. Brittleness indices B1 and B2 proved ineffective, hence equations were not formulated for them. These brittleness indices (B_3, B_4, B_5 and B_6) highlight the complex interplay of various contact types and packing arrangements in determining the brittleness of a material. Key takeaways include:

- Sutured and long contacts tend to reduce brittleness by providing structural integrity and stability.
- Concavo-convex contacts significantly increase brittleness, likely due to stress concentration at interlocking points.
- Tangential and floating contacts have minor contributions to increased brittleness.
- Packing proximity reduces brittleness by enhancing the stability of the granular structure.
- These indices are valuable for understanding and predicting the mechanical behaviour of geological materials based on their microstructural characteristics.

These findings underscore the importance of mineral composition, texture, and fabric in determining the mechanical properties and fracturing behaviour of the Ecca Group.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Summary of Research

The aim of this study was to estimate the brittleness index of the Ecca Group sandstones based on mineralogical characteristics. This was driven by the need to better understand the rock mechanics of the sandstones of the Ecca Group (Vryheid Formation) in order to enhance reservoir characterization and exploration strategies.

The methodology employed in this research involved a comprehensive analysis of petrographic data from samples of the sandstones of the Ecca Group (Vryheid Formation). This included microscopic examination, image analysis, and statistical modelling to establish correlations between petrographic features and brittleness.

The results of the study revealed significant correlations between specific petrographic characteristics such as mineral composition, grain size distribution, and porosity, and the brittleness index of the sandstones of the Ecca Group (Vryheid Formation). These findings provide valuable insights into the mechanical properties of the rock formation.

In conclusion, the study confirms the feasibility of estimating the brittleness index of the sandstones of the Ecca Group (Vryheid Formation) based on mineralogical characteristics. This approach enhances the predictive accuracy of brittleness assessments in similar geological settings, benefiting geological and engineering applications.

Recommendations from this study include further validation of the developed models across different reservoirs within the Ecca Group and exploration of additional petrographic parameters that may influence brittleness. This will contribute to refining predictive models and improving reservoir characterization techniques.

5.2 Conclusions of the Study

This study aimed to estimate the brittleness index of the Ecca Group sandstones based on their mineralogical characteristics. The research was conducted through petrographic analysis, geomechanical testing, and statistical modeling to establish relationships between mineralogical composition and brittleness. The findings contribute to a better understanding of the mechanical behaviour of these sandstones,

which is crucial for applications in rock engineering, hydrocarbon exploration, and geotechnical assessments.

The first objective focused on determining the mineralogical composition of the Ecca Group sandstones using petrographic analysis. The results revealed that these sandstones primarily consist of quartz, feldspar, and clay minerals such as muscovite and cementing materials such as calcite and silica. Quartz content was found to be the dominant factor influencing rock strength. Quartz, being a hard and chemically stable mineral, enhanced the rock's overall strength and brittleness, making them more resistant to deformation but more susceptible to fracturing under stress. This can be observed in the point load test results of sandstone samples, where higher quartz content within the sublithic arenite correlates with increased unconfined compressive strength (UCS).

The second objective involved evaluating the geomechanical properties of the Ecca Group sandstones, including uniaxial compressive strength (UCS), Brazilian tensile strength (BTS), and elasticity parameters. Laboratory tests showed high values of UCS and BTS values, ranging from 36.24 to 115.68 MPa due to high quartz content of the Ecca Group sandstones. The elasticity modulus was directly linked to the degree of compaction and cementation. These results provided the necessary input parameters for calculating various brittleness indices, allowing for a comparative assessment of the mechanical behaviour of different sandstone samples.

The third objective assessed the relationship between mineralogical characteristics and brittleness indices. The variation in grain size and distribution affected the rock's strength and stability. Well-sorted samples, with more uniform grain sizes, generally exhibited better mechanical stability and higher compressive strength due to more efficient packing and interlocking of grains. Poorly sorted samples, with a wider range of grain sizes, had lower mechanical stability because the varying grain sizes created more void spaces and less effective grain contact, leading to weaker structural integrity.

Microstructural characteristics, particularly the type of grain-to-grain contacts, were critical in determining the mechanical behaviour. The predominance of floating contacts suggested weaker mechanical interactions between grains, making the rock more susceptible to deformation under stress. Sutured contacts, which indicate more

interlocked and tightly bound grains, contributed to higher mechanical strength and resistance to deformation.

The consolidation factor, indicating the degree of compaction and grain contact, ranged from 74% to 82%, which impacted the rock's strength and stability. Higher consolidation factors generally indicated better compacted and more cohesive rocks, leading to higher strength. Conversely, lower consolidation factors suggested less compacted rocks with more void spaces, resulting in weaker mechanical properties.

In conclusion, the mechanical behaviour of the Ecca Group Sandstones is a complex interplay of grain size, distribution, and microstructural characteristics. Well-sorted, angular grains with high packing density and sutured contacts led to higher mechanical strength and stability, while poorly sorted, subangular grains with low packing density and floating contacts resulted in weaker, more deformable rock structures. Grain shape significantly influenced fracture propagation and rock strength in sublithic arenites, where the grains were angular to subangular. The angularity of the grains created more points of contact and interlocking mechanisms, which generally enhanced the rock's overall strength. The sharp edges of angular grains contributed to higher frictional resistance, which increased the rock's ability to withstand applied stresses without significant deformation. This resistance to deformation often resulted in a higher brittleness index, as the rock was more prone to fracturing rather than deforming plastically under stress.

Moreover, the angular to subangular grain shapes facilitated the development of micro-cracks along the grain boundaries, which acted as pathways for fracture propagation. These micro-cracks tended to follow the planes of weakness around the angular grains, promoting more rapid and pronounced fracturing under applied loads. The combination of increased interlocking and the presence of micro-cracks due to angular grain shapes ultimately contributed to a higher fracture propagation rate and greater overall rock strength.

The fourth objective aimed on developing the predictive models of rock brittleness index of the Ecca Group sandstones. From statistical analysis the main models consistently showed higher R-squared values and lower SE values compared to the microfabric models, indicating better overall performance in explaining brittleness indices. Microfabric models, despite their moderate fit (lower R^2), consistently failed to

achieve statistical significance (higher Sig.F values), suggesting that the variables related to microfabric (Pp, Pd, Pc) did not reliably predict brittleness. Models incorporating variables related to contact type or nature showed varying degrees of explanatory power but tended to have statistically significant results, indicating that these variables were more predictive of brittleness indices compared to microfabric variables. Therefore, when assessing brittleness indices, models that included a broader set of variables (such as those related to contact type/nature) tended to provide more reliable predictions compared to those focused solely on microfabric characteristics.

The study successfully created models B_3, B_4, B_5, and B_6 that incorporated various petrographic characteristics, such as mineral composition, grain size, grain shape, grain distribution, and microstructural features. Brittleness indices B_1 and B_2 proved ineffective, hence equations were not formulated for them. These models provided a robust framework for predicting the brittleness index based on observable and measurable properties of the rock samples.

By integrating data from point load tests, unconfined compressive strength (UCS) measurements, and detailed petrographic analysis, the models demonstrated a strong correlation between these characteristics and the brittleness index. For instance, the presence and proportion of quartz, the degree of grain interlocking, and the nature of grain-to-grain contacts were all critical factors influencing the brittleness of the Ecca Group sandstones. The predictive models allowed for accurate estimation of brittleness index values, aiding in the assessment of mechanical behaviour and potential fracture propagation within the Karoo Supergroup.

5.3 Overall Conclusions

Overall, this study achieves its aim of estimating the brittleness index of the Ecca group sandstones based on mineralogical characteristics. The study aimed to enhance the understanding of the rock mechanics of the Ecca Group sandstones to improve reservoir characterization and exploration strategies. Through comprehensive petrographic analysis, including microscopic examination, image analysis, and statistical modeling, the research established significant correlations between mineralogical features and the brittleness index.

The results demonstrated that mineral composition, particularly the presence of quartz, feldspar, clay minerals, and carbonates, significantly influenced the brittleness of the Ecca Group. Higher quartz content, known for its hardness and strength, correlated with increased unconfined compressive strength (UCS) and brittleness, making the rock more resistant to deformation but more susceptible to fracturing under stress.

The study also highlighted the impact of grain size, distribution, and microstructure on the rock's mechanical behaviour. Well-sorted samples with more uniform grain sizes exhibited better mechanical stability and higher compressive strength due to efficient packing and interlocking of grains. Conversely, poorly sorted samples with a wider range of grain sizes had lower mechanical stability. Microstructural characteristics, such as the type of grain-to-grain contacts and the consolidation factor, were critical in determining mechanical behaviour. Sutured contacts and higher consolidation factors indicated stronger, more cohesive rocks, whereas floating contacts and lower consolidation factors suggested weaker mechanical properties.

Grain shape also played a crucial role in fracture propagation and rock strength. Angular to subangular grains created more points of contact and interlocking mechanisms, enhancing the rock's overall strength and brittleness. These grains facilitated the development of micro-cracks along grain boundaries, promoting rapid and pronounced fracturing under applied loads.

The fourth objective involved developing predictive models for the rock brittleness indices of the Ecca Group sandstones. The main models consistently showed better performance in explaining brittleness indices compared to microfabric models, indicating that variables related to mineral composition, grain size, shape, and grain-to-grain contacts were more predictive of brittleness. The study successfully created robust models that allowed for accurate estimation of brittleness index values, aiding in the assessment of mechanical behaviour and potential fracture propagation.

In conclusion, the research confirmed the feasibility of estimating the brittleness index of the Ecca Group sandstones based on mineralogical characteristics. This approach enhances the predictive accuracy of brittleness assessments in similar geological settings, benefiting geological and engineering applications. Recommendations for further research include validating the developed models across different reservoirs

within the Ecca Group and exploring additional petrographic parameters to refine predictive models and improve reservoir characterization techniques.

5.4 Recommendations

Based on the findings of this study, the following recommendations are proposed to further advance the understanding and application of brittleness index estimation in the Ecca Group sandstones:

- One of the key recommendations for enhancing the reliability and applicability of this study's findings is to increase the number of samples used in both mineralogical and geomechanical analyses. The current research was limited by a relatively small sample size, which may not fully capture the variability present within the Ecca Group sandstones. Expanding the sample set would allow for a more comprehensive understanding of how different mineralogical compositions influence brittleness and improve the robustness of the predictive model developed in this study.
- Utilize more advanced mineralogical techniques, such as X-ray diffraction (XRD) and scanning electron microscopy (SEM), to further refine the understanding of mineral composition and its impact on brittleness. These techniques could provide more precise quantification of clay minerals and cementing phases.
- Validate the developed models across different sections and wells within the Ecca Group to improve predictive accuracy.
- Explore additional petrographic parameters such as cementation, fracturing, and organic content to further refine brittleness assessments.
- Implement these refined models and parameters in reservoir characterization and exploration activities to optimize hydrocarbon recovery and minimize operational risks.
- Researchers and practitioners should consider these findings when selecting variables for predicting brittleness indices. Microfabric variables, while potentially informative, may not independently or significantly explain brittleness compared to broader sets of geological or geomechanical factors included in main models.

- Further exploration or refinement of microfabric models might be necessary to strengthen their predictive power or to integrate them more effectively with other influential variables.

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APPENDIX 1

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.818209
R Square	0.669465
Adjusted R Square	-0.76565
Standard Error	0.075656
Observations	14

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	11	0.034779	0.003162	0.60762	0.763012
Residual	3	0.017171	0.005724		
Total	14	0.05195			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	41.634	10.0091	4.159613	0.02527	9.780561	73.48743	9.780561	73.48743
Mz	0.390003	2.770755	0.140757	0.896982	-8.42778	9.207783	-8.42778	9.207783
So	0.049333	0.319922	0.154204	0.887239	-0.9688	1.067467	-0.9688	1.067467
Su	-0.10413	0.092794	-1.12217	0.343504	-0.39944	0.191181	-0.39944	0.191181
Cc	-0.05486	0.114672	-0.47845	0.665038	-0.4198	0.310072	-0.4198	0.310072
Lo	-0.09505	0.100446	-0.94631	0.413803	-0.41472	0.22461	-0.41472	0.22461
Ta	-0.09255	0.103708	-0.89242	0.437917	-0.4226	0.237493	-0.4226	0.237493
Fl	-0.08926	0.103713	-0.86065	0.452722	-0.41932	0.240801	-0.41932	0.240801
GG	-0.00079	0.020584	-0.03848	0.971721	-0.0663	0.064716	-0.0663	0.064716
GC	0.008639	0.036394	0.237381	0.827649	-0.10718	0.124461	-0.10718	0.124461
GM	0	0	65535	#NUM!	0	0	0	0
GV	0	0	65535	#NUM!	0	0	0	0

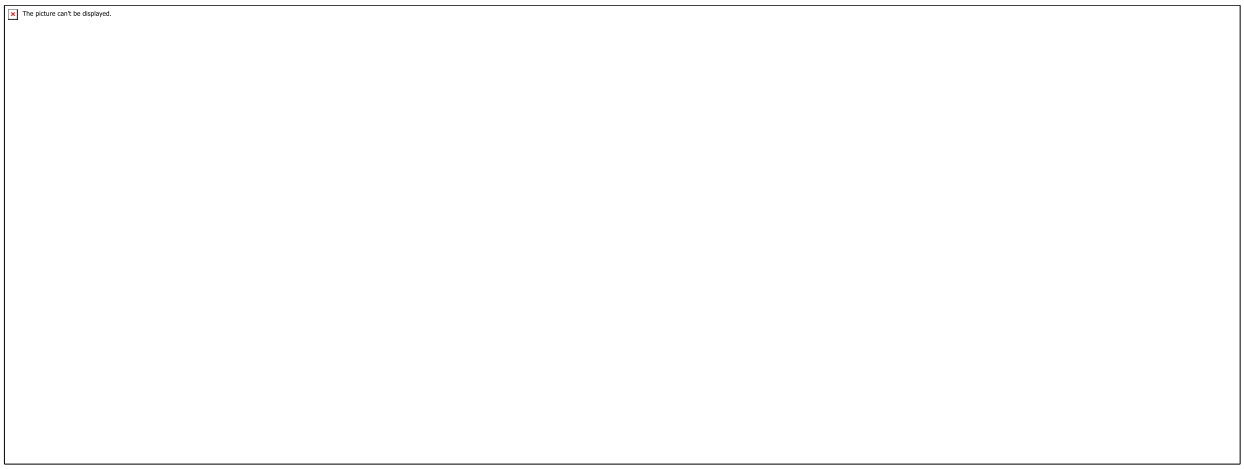
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.547914
R Square	0.30021
Adjusted R Square	0.090273
Standard Error	0.060294
Observations	14

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.015596	0.005199	1.430002	0.291432
Residual	10	0.036354	0.003635		
Total	13	0.05195			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	32.78927	0.324255	101.1218	2.19E-16	32.06678	33.51176	32.06678	33.51176
Pd%	0.000947	0.00354	0.267468	0.794545	-0.00694	0.008834	-0.00694	0.008834
Pp%	-0.0057	0.003619	-1.57422	0.146513	-0.01376	0.002367	-0.01376	0.002367
Pcc%	0.003753	0.004275	0.877845	0.400627	-0.00577	0.013278	-0.00577	0.013278



APPENDIX 2

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.789046
R Square	0.622594
Adjusted R Square	-0.96876
Standard Error	0.000131
Observations	14

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	11	8.54E-08	7.76E-09	0.494899	0.821104
Residual	3	5.18E-08	1.73E-08		
Total	14	1.37E-07			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.953134	0.017377	54.84901	1.33E-05	0.897831	1.008437	0.897831	1.008437
Mz	0.000861	0.00481	0.178992	0.86935	-0.01445	0.01617	-0.01445	0.01617
So	9.75E-05	0.000555	0.175583	0.871804	-0.00167	0.001865	-0.00167	0.001865
Su	-0.00015	0.000161	-0.91372	0.428237	-0.00066	0.000366	-0.00066	0.000366
Cc	-6.2E-05	0.000199	-0.30897	0.777558	-0.0007	0.000572	-0.0007	0.000572
Lo	-0.00013	0.000174	-0.76739	0.498718	-0.00069	0.000421	-0.00069	0.000421
Ta	-0.00013	0.00018	-0.71421	0.526653	-0.0007	0.000444	-0.0007	0.000444
Fl	-0.00012	0.00018	-0.68047	0.545021	-0.0007	0.000451	-0.0007	0.000451
GG	-4.4E-06	3.57E-05	-0.12293	0.909938	-0.00012	0.000109	-0.00012	0.000109
GC	8.28E-06	6.32E-05	0.130988	0.904076	-0.00019	0.000209	-0.00019	0.000209
GM	0	0	65535	#NUM!	0	0	0	0
GV	0	0	65535	#NUM!	0	0	0	0

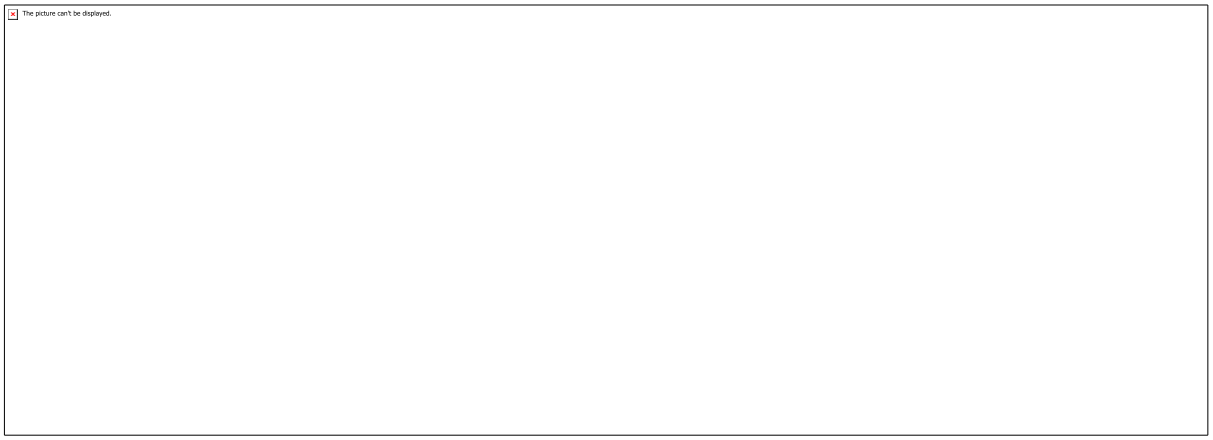
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.569181
R Square	0.323967
Adjusted R Square	0.121157
Standard Error	9.63E-05
Observations	14

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	4.44E-08	1.48E-08	1.597394	0.2512
Residual	10	9.27E-08	9.27E-09		
Total	13	1.37E-07			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.940797	0.000518	1816.832	6.28E-29	0.939643	0.94195	0.939643	0.94195
Pd%	2.53E-06	5.65E-06	0.447151	0.664295	-1E-05	1.51E-05	-1E-05	1.51E-05
Pp%	-1E-05	5.78E-06	-1.74976	0.110724	-2.3E-05	2.76E-06	-2.3E-05	2.76E-06
Pcc%	5.36E-06	6.83E-06	0.785666	0.450277	-9.8E-06	2.06E-05	-9.8E-06	2.06E-05



APPENDIX 3

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.685882916
R Square	0.470435375
Adjusted R Square	0.374150898
Standard Error	0.067662347
Observations	14

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	0.044737	0.022369	4.88589	0.030307793
Residual	11	0.05036	0.004578		
Total	13	0.095097			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.796163391	0.18265	4.358959	0.001138	0.39415368	1.198173	0.394154	1.198173
Su	-0.037255219	0.014386	-2.58976	0.025149	-0.068917692	-0.00559	-0.06892	-0.00559
Lo	-0.036539886	0.013456	-2.71541	0.020096	-0.066157418	-0.00692	-0.06616	-0.00692

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.880175
R Square	0.774708
Adjusted R Square	0.633901
Standard Error	0.05175
Observations	14

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	0.073673	0.014735	5.501898	0.017267
Residual	8	0.021425	0.002678		
Total	13	0.095097			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	11.29576	5.707991	1.978939	0.083185	-1.86689	24.45841	-1.86689	24.45841
Su	-0.12749	0.055282	-2.30612	0.049991	-0.25497	-6.2E-06	-0.25497	-6.2E-06
Cc	-0.11299	0.065487	-1.72543	0.12273	-0.26401	0.03802	-0.26401	0.03802
Lo	-0.13833	0.056818	-2.43458	0.040909	-0.26935	-0.00731	-0.26935	-0.00731
Ta	-0.09812	0.057162	-1.71655	0.124394	-0.22994	0.033694	-0.22994	0.033694
Fl	-0.1102	0.05726	-1.92449	0.090486	-0.24224	0.021845	-0.24224	0.021845

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.989736
R Square	0.979578
Adjusted R Square	0.578172
Standard Error	0.025443
Observations	14

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	11	0.093155	0.008469	14.39013	0.066726268
Residual	3	0.001942	0.000647		
Total	14	0.095097			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	15.0238	3.366089	4.46328	0.020946	4.311401021	25.7362	4.311401	25.7362
Mz	-0.58379	0.931813	-0.62651	0.575412	-3.54923691	2.381651	-3.54924	2.381651
So	-0.0532	0.107591	-0.49443	0.654943	-0.39559725	0.289205	-0.3956	0.289205
Su	-0.1702	0.031207	-5.45382	0.01211	-0.26950973	-0.07088	-0.26951	-0.07088
Cc	-0.16796	0.038564	-4.35532	0.022365	-0.29068922	-0.04523	-0.29069	-0.04523
Lo	-0.17982	0.03378	-5.32335	0.012952	-0.28732742	-0.07232	-0.28733	-0.07232
Ta	-0.14592	0.034877	-4.1837	0.024887	-0.25691019	-0.03492	-0.25691	-0.03492
Fl	-0.15809	0.034879	-4.53263	0.020096	-0.26909443	-0.04709	-0.26909	-0.04709
GG	0.014332	0.006922	2.070418	0.13019	-0.00769799	0.036363	-0.0077	0.036363
GC	0.00894	0.012239	0.73046	0.517984	-0.03001077	0.047892	-0.03001	0.047892
GM	0	0	65535	#NUM!	0	0	0	0
GV	0	0	65535	#NUM!	0	0	0	0

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.395228
R Square	0.156205
Adjusted R Square	-0.09693
Standard Error	0.089578
Observations	14

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.014855	0.004952	0.617073	0.619555249
Residual	10	0.080243	0.008024		
Total	13	0.095097			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*
Intercept	-0.31068	0.48174	-0.64492	0.533494	-1.3840678	0.7627	-1.38407
Pd%	-0.00286	0.005259	-0.54348	0.598702	-0.014576	0.00886	-0.01458
Pp%	0.006333	0.005377	1.177736	0.26617	-0.00564805	0.018313	-0.00565
Pcc%	0.005693	0.006351	0.896421	0.391097	-0.008457828	0.019844	-0.00846