EXPLORING MATHEMATICAL CONCEPTS EMBEDDED IN THE MECHANICS AND OPERATIONS OF THE CENTRE PIVOT IRRIGATION SYSTEM

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EXPLORING MATHEMATICAL CONCEPTS EMBEDDED IN THE MECHANICS AND OPERATIONS OF THE CENTRE PIVOT IRRIGATION SYSTEM

BY

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ABSTRACT

The advent of a new mathematics curriculum in South Africa requires a sound Pedagogical Content Knowledge (PCK) for both novice and experienced educators. Central to this is the challenge of identifying and exploring “rich and appropriate” contexts that may serve as “scaffolds” in the understanding and internalization of school level mathematics concepts. This exploratory, inductive study focused on a real-life irrigation technology in the farming sector with a view to “exploring” the general school level mathematics concepts that might be “grounded” in the machine’s mobility and water spread mechanisms. Data was generated through two stages of theoretical and practical approaches. This was in accordance with Alasuutari’s (1993) phases of simplification of observations and “solving the enigma” during an exploratory research project. In the theoretical approach, the operations of a linear move irrigation machine and a circular move center pivot irrigation system were mimicked through sketches which were explored for the general school level mathematics concepts embedded therein. The practical approach centrally focused on hands-on activities that aimed at verifying the theoretical mathematics models that were perceived to explain how the CPIS moves and spread water across the entire irrigation field. An intense observation of the actual Centre Pivot Irrigation System (CPIS) at the research site formed the spine of the latter data collection stage. Finally a document analysis, which focused on mathematics documents such as the National Curriculum Statement and Curriculum and Assessment Policy Statement documents for grades R-12, was done to ascertain the school level at which the grounded general mathematics concepts are applicable. The findings of this study indicated that

certain mathematics concepts might be “constructed” and consolidated in the CPIS context or setting.

DECLARATION

I declare that the mini-dissertation: *Exploring Mathematical Concepts Embedded in the Mechanics and Operations of the Centre Pivot Irrigation System*, hereby submitted by me to the University of Limpopo for the degree Master of Education in Mathematics Education, has not previously been submitted by me for a degree at this or any other university; that it is my work in design and execution, and all the material contained herein has been duly acknowledged.

08 September 2016

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Morongwana Elias Tau Date

Children [learners] will understand mathematical concepts and procedures more thoroughly if they are allowed to use their own thinking processes to explore mathematics as if it were a dark room, and eventually find the light switch and come to an answer (Kamii, Lewis, & Jones, 1993 cited in Geist, 2001). Mathematical exploration, therefore, allows learners to make connections to what they already know and to real life experiences. However, this requires a special combination of the educator's subject matter (content) knowledge and knowledge of multiple teaching strategies (pedagogy) to ensure learners' cultural backgrounds, prior knowledge and experiences are effectively tapped into and maximally utilized (Geddis, 1993).

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CHAPTER 1

THE THEORETICAL OVERVIEW OF THE STUDY

1.1 Introduction

The use of authentic, real-world contexts, as advocated by the National Curriculum Statement (NCS) documents (Department of Education [DoE], 2007), helps to extend learners' conceptual grasp throughout the concrete-to-representational-to-abstract sequence of mathematical understanding. According to Fleener, Westbrook and Rogers (1995), conceptual understanding is enhanced when learners "construct" understanding of a concept from their interaction with real-life settings, not from a telling method or repetitive drill.

Furthermore, Frykholm and Glasson (2005) recommend a contextual learning theory according to which learning occurs only when learners process new knowledge in ways that make sense to their inner worlds, memory, experience and response. This clearly means that the newly acquired knowledge must resonate and "fit-in" with what learners already learnt or experienced in their actual life-worlds. Buxton (1978) concurs with this view when he indicates that understanding a topic of study is a matter of being able to perform in a variety of thought-demanding ways with the topic, for instance: explain, find examples, generalize, apply concepts, analogize, represent in new ways, and muster evidence. The prerequisite for all these assertions on effective teaching and learning mathematics concepts, however, is the use of "scaffolds" or contexts from learners' immediate environments that encourage processes of mathematics practice such as persistence, curiosity, flexibility, thoroughness, creativity, communication, reasoning and problem solving (Montgomery, 2001).

One of the aims of the NCS, as reiterated in the Curriculum and Assessment Policy Statement (CAPS) documents is to ensure that learners acquire and apply knowledge and skills in ways that are meaningful to their lives by grounding knowledge in local contexts while being sensitive to global imperatives (DoE, 2011). This simply means that contexts that learners are exposed to while learning and being taught mathematics have to be drawn from genuine and realistic situations, be relevant, and relate to daily life, the work place and the wider social, political and global environments.

1.2 **Purpose of the study**

The purpose of this exploratory, inductive study, therefore, was to explore and explicate the general school level mathematics concepts that are "grounded" in the mobility and water spread mechanisms of the Centre Pivot Irrigation System (CPIS) throughout the irrigation acreage or field.

1.3 **Research Questions**

Creswell (2009) directs that a research question is a clear, focused, concise, complex and arguable question around which a research study centers. It, therefore, helps in focusing a study by providing a path through the research as well as the writing process involved. The central, guiding research questions for this exploratory, investigative process were:

1.2.1 **Research Question #1**

What are the general mathematics concepts that are "grounded" in the CPIS' mobility and water spread patterns per sections of the irrigation field?

The way the CPIS moves as well as the way it spreads water to avoid swelling or water-logging of the centre were the bases of this investigative study.

1.2.2 **Research Question #2**

What are the "potential" authentic tasks within school level mathematics that may be designed and packaged in the context of the CPIS mobility and water spread mechanisms?

The focus was limited to interpretation of the "potential" tasks using mathematics policy documents and did not involve administering and/or piloting of the tasks in the actual school context.

1.3 **Significance of the study**

The study was firstly significant to me as a classroom-based mathematics educator who ventured into the "unusual" research territory outside the classroom setting, away from the common classroom interactions with learners during lesson presentations. I had the real experience of "doing" mathematics through the generation of endless sketches that, theoretically, were inspired by the irrigation machine's mobility and water spread patterns. This was a meaningful, real experience that I believe should be transferred to mathematics learners in a quest to make the subject more practical and interesting to them. In addition, the mathematics education research community may appreciate and value, through the outcomes of this study, the wealth of teaching and learning opportunities prevalent in real settings outside the normal classroom interactions.

Furthermore, the package of “potential” authentic teaching and learning tasks, comprising of interlinked activities, might assist in addressing the much emphasized use of real-world contexts and integration across learning areas and subjects in the NCS documents. These tasks might minimize the “widespread teacher frustration and dissatisfaction” on the use of projects and investigations in learner assessment as reported in The Report on the Implementation of the NCS (DoE, 2009, p. 31).

Consequently, mathematics policy makers may have, through the results of this study, more grounds to demand or encourage the vision of “using real contexts” in mathematics learning and teaching in modern classrooms.

1.4 **The Structure of the Report**

The main aim of this study was to explore and unearth the general school-level mathematics concepts that might be grounded in the mobility and water spread mechanisms of the Centre Pivot Irrigation System (CPIS), thereby rendering the machine a possible rich context for mathematical exploration. The research questions that steered this study ensured that this aim was realized.

This research report is arranged in six interwoven chapters as follows:

Chapter 1 is the introductory background of the study that alludes to the emphasis on the importance and use of real-world context in mathematics learning and teaching as advocated by National Curriculum Statement (NCS) and Curriculum and Assessment Policy Statement (CAPS) documents as well as by related literature. Furthermore, the chapter recaps the purpose, the research questions as well as the significance of the study.

Chapter 2 reviews the literature regarding irrigation systems, particularly the lateral move and circular move machines that might provide real contexts for mathematical exploration. In addition, the chapter zooms into exploratory, creative mathematics and problem solving, in a quest to establish the link and relevance to the irrigation systems context.

Chapter 3 outlines the methodology adopted in this investigative study and also encompasses the description of the research design, sampling or selection, data collection and analysis techniques and instruments used, ethical considerations, and the limitations of the study.

Chapter 4 reports the findings and analysis of data gathered from the operations of a linear move irrigation system that provides a springboard for the entire mathematical exploration in this study. Two basic types of structures of a linear move irrigation machine, namely, ground-bound pipes with sprinklers spraying water upward and suspended or hanging sprinklers on a mobile tower spraying water onto crops, are thoroughly explained. The sketches that were inspired by these systems' manual operations are used to provide a "visual" explanation and clarity. Furthermore, the type of sprinkler nozzles assumed at this stage, the mobility of a single tower linear move machine as well as a reflection on the gathered data are captured in this chapter.

Chapter 5 focuses on a circular move irrigation system's sprinkler nozzles, mobility and water spread patterns in a quest to unpack its operations with regard to the research questions posed earlier. The chapter gradually unfolds from single tower to the complex mimicked operations of a multi-tower machine and concludes with a reflection on the data gathered. A practical approach, in which data gathered from intense observation of the machine at the research site, also forms part of this chapter. The intention is to address the research questions of the study by verifying, comparing and linking the theoretical approach data about the CPIS mobility and water spread patterns with the data obtained from hands-on sessions at the research site.

Chapter 6 provides the overall summary and conclusion of the study by outlining the package of “possible” teaching and learning activities emerging from exploring the linear move and the circular move irrigation machines. It further proposes recommendations for future follow-up studies on mathematical exploration within the Centre Pivot Irrigation System context.

CHAPTER 2

LITERATURE REVIEW

2.1 **Introduction**

According to Lamb (1998), the review of literature in a research project is a discursive prose or a form of expository writing that seeks to situate the study within a body of literature or write-ups by offering critical insights on the topic or problem under scrutiny. The key issue in this section of academic writing is to critically peruse and dissect documents relevant to the field or topic under research and also to offer an argumentative thesis for the current study.

The following expository piece of writing under subheadings “Irrigation systems” and “Exploratory, creative mathematics and problem solving”, therefore, provide a critical appraisal of the literature related to the topic of this study. The literature relevant to various irrigation techniques or methods and the embedded opportunities that cater for creative, exploratory mathematics and problem solving approach to mathematics teaching and learning are henceforth outlined. This literature, however, is of a limited nature because the area of this study is relatively new, with a paucity of related academic write-ups within school level mathematics teaching and learning.

2.2 **Irrigation systems**

Snyder and Melo-Abreu (2005) define irrigation as the controlled application of water to land or soil for purposes of assisting the growing of crops, maintenance of landscapes and re-vegetation of disturbed soils in dry areas and during periods of inadequate rainfall.

The general purpose of irrigation, therefore, is to supply the entire field uniformly with water in a predetermined pattern so that each plant has the amount of water it needs, neither too little nor too much. This “controlled application” of water, is achieved through artificial or man-made irrigation systems whose design, installation and operations require and involve a mammoth of creativity, innovation and problem solving skills that are most basic or fundamental in mathematics teaching and learning in schools.

Snyder and Melo-Abreu (2005) indicate that irrigation techniques evolved from surface systems where water moves over and across land by simple gravity flow in order to wet the land and infiltrate soil; through localized irrigation where water is distributed under low pressure through piped network in a predetermined pattern and applied as a small discharge to each plant or adjacent to it; then through drip irrigation where water fall drop by drop or trickles just at the position of roots; and presently to sprinkler or overhead irrigation where water is piped to one or more central locations within the field and distributed by overhead high-pressure sprinklers or guns. This evolution of irrigation systems had both merits and demerits with regard the cost of labour, fuel and time required to spread water uniformly across the entire acreage.

The common denominator for these irrigation systems, for this study, is that their operations generally offer opportunities that invite “construction”, “exploration” and consolidation of school level mathematics concepts. The operations of pipes that require manual labour to move them from one spot to the next, for example, involve considering the circular wetted area achieved earlier by rotating sprinklers at the initial spot so that there would not be over-irrigation of a spot when moving pipes to the next spot. This means making sure that the distance between linear arrangements of sprinklers from one spot to the next is more feasible to cater for uniformity of water spread on areas between the sprinkler arrangements. The diagram in Figure 2.1 clearly shows how the water pipes are arranged in an irrigation field in an attempt to achieve water spread uniformity.

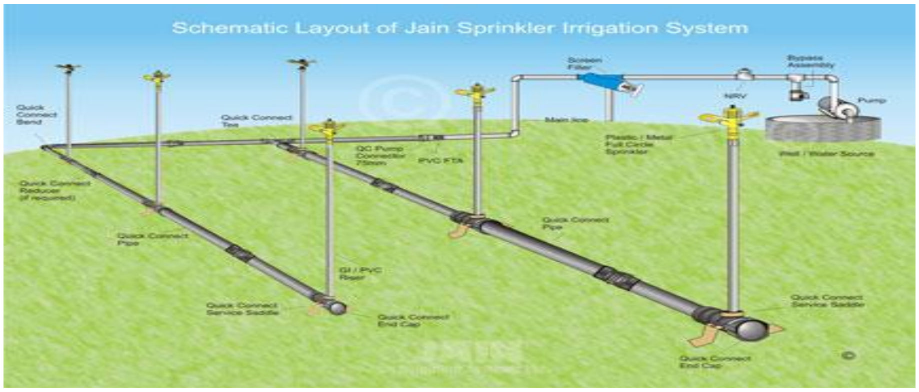


Figure 2.1: A picture of schematic layout of irrigation pipes in a field (*www.shutterstock.com*.)

Furthermore, the use of words and phrases such as “controlled application”, “artificial or manmade”, “predetermined pattern” and “uniformly” in the preceding paragraphs suggests that the planning, design, lay-out, installation and the operation of the irrigation systems is not a haphazard or clumsy activity, but rather requires a carefully thought-out, calculated, innovative and creative process that is accommodative of vital process skills envisaged in mathematics teaching and learning in schools. The knowledge and skills that may be constructed and nurtured in the settings or contexts of the different irrigation systems can be sourced out from cross curricula fields of, inter alia, Mathematics, Agricultural sciences and Physical sciences. For example, setting up furrows for irrigation purposes required thorough consideration of such factors as direction of crop rows, gravity flow and soil topography (slope). In addition, the concepts of pressure required in the pipes to ensure sufficient wetting around the crops, the amount of wetted area achievable under a certain pressure for a sprinkler nozzle as well as the speed of wheel sets carrying overhead irrigation systems, clearly lend themselves for critical and intensive enquiry within these irrigation systems contexts.

The vital aspects of irrigation systems' operations for this study, particularly the operations of the Centre Pivot Irrigation System (CPIS), are its mobility and water spread patterns between identified points on the irrigation field. According to Snyder and Melo-Abreu (2005), a centre pivot irrigation systems is a form of overhead sprinkler irrigation consisting of several segments of pipe (usually galvanized steel or aluminium) joined together and supported by trusses, mounted on wheeled towers with sprinklers positioned along its length. This definition clearly indicates that, with the CPIS, labour costs are minimized since the cumbersome duty of having to carry pipes from one point to the other across the irrigation field is discarded. Furthermore, Frenken (2006) alleges that a centre pivot irrigation system moves in circular pattern and that its overall speed is governed by the tower on the outside, which has to travel farther and faster than the towers towards the pivot centre. This simply means that when one tower gets behind, its motor turns on or runs faster to catch up, and this kind of movement is achieved through control sensors which were initially simple mechanical linkages but presently being electric sensors under computer control.

However, the complex mobility and water spread mechanisms of the CPIS provide a rich and authentic setting for the construction of mathematics concepts and nurturing of mathematics process skills in a creative, innovative and exploratory manner. In addition, the National Curriculum and Assessment Policy Statement (CAPS) document for Mathematics grades R-12 (DoE, 2007, p. 8) indicates that mathematics, as a human activity, involves observing, representing and investigating, and thus assist in the development of mental processes that enhance logical and creative thinking, accuracy and problem solving necessary for decision-making.

2.3 **Exploratory, creative mathematics and problem solving**

Bunt and Conati (2003) assert that exploratory environments or contexts seek to encourage active learning of mathematics through discovery and exploration. It is in these authentic, real-world settings that learners learn mathematics from their own experiences obtained in environments in which the mathematics they learn is rooted in real-life. This augurs well for this study as it stresses the “power” of irrigation systems, particularly the linear move machine and the centre pivot irrigation system (CPIS), in serving as “scaffolds” in the construction, internalization and consolidation of mathematics concepts. Furthermore, these irrigation systems have recently become a popular constant feature in many areas of the South African society, particularly learners in remote, rural schools.

According to Lester, Masingila, Mau, Lambdin, dos Santon and Raymond (1994), a problem solving approach to mathematics teaching focuses on teaching mathematics topics or concepts through enquiry-oriented environments which are characterized by teachers helping learners to construct a deep understanding of mathematical ideas and processes by engaging them in “doing mathematics”: creating, conjecturing, exploring, testing and verifying. Again, this concurs with the current study’s advocacy that irrigation systems offer teaching and learning opportunities in a relaxed context “familiar” to learners who have become accustomed to their visibility and daily operations in their (learners’) periphery.

Another supportive assertion to a problem solving approach in authentic contexts is by Fleener et al (1995), who indicate that conceptual understanding is enhanced when learners “construct” understanding of a mathematics concept from “their interaction with real-world settings”, not from a lecture or repetitive drill (rote learning). The clear mandate for teachers here is to let learners to, personally and independently, grapple with real-world contexts in order to find out solutions to problems, make conjectures and rules that they may test so as to check if they are feasible in a given context. Teachers should, therefore, know when it is appropriate to intervene and when to step back and let learners make their own way. Piggot and Pumfrey (2005) summarize the joy provided by problem solving to learners as:

> “The joy of confronting a novel situation and trying to make sense of it – the joy of banging your head against a mathematical wall, and then discovering there may be ways of either going around or over the wall” (p. 27).

However, Cobb, Wood and Yackel (1991) identify specific characteristics of a problem solving approach to mathematics teaching that include: teachers providing just enough information or “scaffolds” to establish background or interest of the problem, and learners clarifying, interpreting and attempting to construct one or more solution processes; and teachers encouraging learners to make generalizations about rules and concepts. This means that a problem solving approach to mathematics teaching and learning using authentic, real contexts like the linear move and circular move irrigation machines does not only develop learners’ confidence in their own ability to think mathematically, but also serve as a vehicle for learners to construct, evaluate and refine their own theories about mathematics and the theories of others so that they can develop their own “rules” that rarely involve standard algorithms. It is, therefore, a contextualized problem solving approach to mathematics teaching and learning that has the potential to enhance conceptual understanding than non-contextual problem solving that only targets regurgitation of mathematical “facts and correct procedures or answers” at all times.

A valuable contribution on a creative problem solving approach to mathematics teaching and learning is from the Osborn-Parnes process model, hinted in Hurson (2007). This model is defined as a simple process that involves breaking down a problem to understand it, generating ideas to solve the problem and evaluating those ideas to find the most effective solution to the problem.

2.4 **The Osborn-Parnes process model**

The Osborn – Parnes process model to creative problem solving was founded by Alex Osborn and Sidney Parnes in the 1950s. The model initially had six steps, which Watson and Mason (1998) condensed to three stages of: exploring the challenge; generate ideas and prepare for action. They further used the diagram in Figure 2.2 to recap the original six steps of the Creative Problem Solving process:

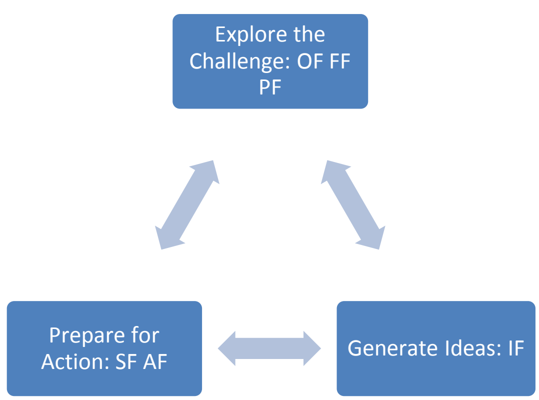


Figure 2.2: The condensed version of the Osborn–Parnes process model for Creative Problem Solving (Watson and Mason, 1998)

The three flexible, cyclical stages or phases of the Creative Problem Solving model in Figure 2.2 can clearly be outlined, with their respective original steps, as follows:

2.4.1 **Explore the challenge**

This phase involves three steps of: objective finding (OF) wherein one identifies a wish goal or challenge; fact finding (FF) wherein one gathers data and considers it in order to review the objective and begin to innovate; and problem finding (PF) wherein one makes sure that they are focusing on the right problem by exploring facts and data to find all the problems and challenges inherent in the situation together with all the opportunities they present.

2.4.2 **Generate ideas**

This phase involves a single step of idea finding (IF) wherein one becomes vigilant about differing judgment and comes up with wild, outrageous, out-of-the-box ideas that are possible solutions in a quest to finding a potentially innovative, novel solution to the problem at hand.

2.4.3 **Prepare for action**

This final phase of the Creative Problem Solving (CPS) process model involves two steps of: solution finding (SF) wherein one selects and strengthens a solution, bearing in mind that “a creative idea is not really useful if it won’t be implemented”; and acceptance finding (AF) wherein one adopts and embarks on a plan of action looking at who is responsible, what has to be done by when, and what resources are available to realize a fully-fledged, activated solution.

In a nutshell, the unique feature of the Osborn – Parnes process model is that each step first involves a divergent phase in which one generates lots of ideas (facts, problem definitions, evaluation criterion, implementation strategies) and then a convergent phase in which only the most promising ideas are selected for further exploration. Forbes (1995) refers to this act of generating lots of ideas to a problem as “multiple-idea facilitation” technique wherein the intention is that a larger number of ideas increase the chances that one of them has value. This, therefore, connects well with the purpose of the current exploratory study as it commenced with gathering lots of ideas on the mobility and water spread patterns of both linear move machine and centre pivot irrigation system through mimicked sketches and figures, and then a selection of the most promising data (theoretical mathematical models) thought to be useful in explaining the machines’ mobility and water spread patterns, and lastly a verification phase of checking the feasibility of the possible theoretical models at the research site using a hands-on or practical approach.

Furthermore, the National Council of Teachers of Mathematics (NCTM, 1980, p. 2-3) recommend a mathematics curriculum organized around and focusing on: developing skills and ability to apply these skills to unfamiliar situations; gathering, organizing, analyzing and conceptualizing problems; defining problems and goals; discovering patterns and similarities; seeking out appropriate data; experimenting, transferring skills and developing curiosity, confidence and open-mindedness. This resonates well with what the new mathematics curriculum in South Africa advocates through emphasis on both the use of authentic or real-world contexts and an integrated learning of theory, practice and reflection (Department of Education [DoE], 2007).

The research works quoted above make it clear that an integrated approach to mathematics teaching and learning in real-world, familiar settings like irrigation systems, particularly the linear move and CPIS machines in this study, is not optional for a constructivist educator who aspires to nurture creativity and critical thinking in learners. The most basic responsibility for constructivist educators, therefore, is to learn the mathematical knowledge of their learners and to learn how to harmonize their teaching methods with the nature of that mathematical knowledge (van Oers, 1996), and this requires planning whereby educators "plant" powerful mathematical ideas in personally meaningful contexts for learners to investigate. It must be borne in mind that creativity in mathematics classrooms is not just about what learners do, but also what we do as teachers. If we think creatively about the mathematical experiences we offer our learners, we can then open up opportunities for them (the learners) to be creative. This reaffirms the need for educators to have a set of special attributes that would help them to transfer their content knowledge to learners in a manner that is personally meaningful to the learners (Geddis, 1993). Shulman (1987) summarizes these attributes as follows:

> "The key to distinguish the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content knowledge he/she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (p. 9).

2.5 Conclusion

The value and emphasis of the above-quoted research works on the need for a contextualized teaching and learning of mathematics concepts, the paucity of literature on the exploration of the general school level mathematics concepts embedded in the irrigation systems context signaled a “grey area” that needed intensive inquiry. The CPIS setting provided a scaffold for the researcher to employ his creativity in garnering school level “possible” teaching and learning activities in chapter 6 of this report. These activities differ from the “traditional application-recall textbook exercises” wherein learners’ imagination and curiosity are stifled by demanding regurgitation of routine procedures and “mathematical facts”.

Furthermore, the condensed version of the Osborn-Parnes process model in Figure 2.2 was also applicable during the entire study. The challenge of “unearthing” and “ungrounding” the possible school level activities and concepts in the CPIS setting was explored through direct observation of the machine’s mobility and water spread patterns as well as through sketches inspired by such mobility and water spread patterns. The verification act in chapter 5 yielded the data in Table 5.4 that intended to ascertain the assertion that sprinkler nozzle radii seem to differ from closer to the pivot point going to the extreme end of the machine tower.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 **Introduction**

According to Kruger, Michelle, and Welman (2005, p. 2), a research methodology considers and explains the logic behind any research project. In addressing the afore-stated central research questions in the introductory part of this exploratory investigative study, the following overall procedures (outlined under sub-headings: research design, sampling, data collection, data analysis, ethical considerations, significance and limitations of the study) were adopted. Furthermore, the study adopted a qualitative methodology, which according to Franklin (2012) focuses on natural settings and artificially constructed data as well as data derived from narratives, images and observation.

3.2 **Research Design**

The study used an emergent, exploratory, inductive approach to "unearth" the general school level mathematics concepts that are grounded in the mobility and water spread mechanisms of the Centre Pivot Irrigation System. It was an exploratory study which, according to Yin (1994), tries to look for patterns in collected data and come up with a model within which to view the data. This clearly meant or implied that the researcher in this study just had to begin with a rather vague impression of what should be studied, that is, with a "preliminary theoretical notion" of how linear move and circular move CPIS operate in a quest to spread water evenly across the entire irrigation field. Denzin and Lincoln (1994) echo supporting sentiments when they highlight that an exploratory study is an approach of developing concepts, establishing connections or links between concepts, thereby creating an understanding of the entire setting to explicate the central phenomenon by relating diverse concepts in order to account for complex relationships, and to define multiple realities in a research setting.

Furthermore, Lambin (2000) asserts that an exploratory project is conducted to help researchers to have a better understanding of a situation or problem. However, the researcher in this type of study ought to be willing to change direction as a result of revelation of new data and new insights (Saunders, Lewis & Thornhill, 2007).

Perhaps a better summary of an exploratory research study is provided by Pride and Ferrell (2007) when they indicate that it should: have a general purpose of intending to generate insights about a situation or problem; involve a flexible, open-ended and rough data collection form with no set of defined procedures: and use a relatively small, subjectively selected sample in order to maximize generalization of insights. This study's findings and results, however, are not generalized for all CPIS technologies in the farming fraternity. Furthermore, Cuthill directs that exploratory design is often conducted about a research problem when there are fewer or no earlier studies to refer to or rely upon to achieve an outcome.

3.3 **Sampling**

Govindarajulu (1999) defines sampling as the process of selecting units from a wider population of interest so that by studying the subset one may fairly generalize results back to the population from which the units were chosen. This implies that sampling is necessarily vital when outcomes of a research project are to be "blanketed" for the entire population of interest. However, the selection of the CPIS in this study, may, in real life, be likened to a “raw selection” of a car from a group of cars of the same model and design. Cuthill (2002) supports this by arguing that an exploratory research is often conducted with a view to gaining insights and familiarity rather than for prediction purposes.

The selection criterion and technique for the intensely observed CPIS was not essentially necessary since generalizations were not going to be made to all unobserved and unstudied machines in the farming sector. However, the researcher adopted a typical case sampling, which is a purposive sampling technique that is used when the researcher is interested in the typicality of a setting or context (Patton, 1990). The CPIS mobility and water spread mechanisms were a typical context, which needed to be zoomed into in a quest to explicate the general school level mathematics concepts that might be embedded therein.

3.4 **Data Collection**

Yin (1994) listed six sources of evidence for data collection in an exploratory study protocol as: documentation; archival records; interviews; direct observation; participant observation and physical artifacts. In this study an intense direct observation of the CPIS mobility and water spread mechanisms per sections of the field was used as a follow-up or back-up to a theoretical observation approach that purported to explore the setting through sketches that mimicked both the linear move and the circular move irrigation machines. In addition, video recording of the observed CPIS in operation was done as it had an advantage of capturing data more faithfully than hurriedly written notes at the research site (Lincoln & Guba, 1985).

However, the data collection process in the study begun when the researcher observed the CPIS at a distant. This was followed by visits to the research site for an up-close, direct observation and video recording of the machine's mobility and water spread patterns. The process of data collection then resulted in the generation of many sketches inspired by the CPIS mobility and water spread patterns (see Figures 4.3, 4.5, 4.6, 4.7, 4.8, 4.9, 5.3, 5.4, 5.5 and 5.6). In summary, data collection involved gathering primary data through exploratory observation of CPIS mobility and water spread patterns and then gathering secondary data through sketches inspired by CPIS mobility and water spread patterns.

3.5 **Data Analysis**

According to Alasuutari (1993, p. 22), in an analysis of empirical findings of an exploratory study, the researcher should distinguish the following two overlapping phases: simplification of observations and interpretation of results or "solving the enigma". In the simplification phase, the material is inspected from a theoretical point of view of the study project and only the points relevant from this angle are noted while differing or random details are omitted and set aside so that the general lines of the data can be discerned more easily. The aim here is to find a general rule or model that is valid in all or most of the observations. This means that analysis starts from separate cases and aspires to create one or a few general models.

"Solving the enigma", however, does not always mean answering exactly those questions that were posed at the outset of the study project. Sometimes most interesting questions are found at a stage when the researcher has become an "expert" of the subject of study, where it is often said "data teach the researcher".

The study adopted both phases of data analysis advocated above since drawings or sketches inspired by the mobility and water spread mechanisms of both a single tower linear move machine and a single tower circular move machine (CPIS) were initially used (in a theoretical approach) to generate mathematical models applicable in both irrigation technologies' mobility and water spread patterns. The analysis deepened when the complex mobility and water spread patterns of the actual multi-tower CPIS had to be practically or literally observed at the research site with a view to ascertaining the theoretical mathematical models that were perceived to explain how the CPIS moves and spreads water uniformly across the entire irrigation field. In addition, the numerical data generated during the data collection process was captured vividly using Tables 4.1-4.4 and 5.1-5.4, followed by elaborative, clarification notes that deepened insight into the CPIS mobility and water spread patterns. Clifton (2010) refers to "Data mining", which is an analysis technique that focuses mainly on extraction of patterns and knowledge from large amounts of data.

Lastly, a document analysis of mathematics policy documents such the National and Curriculum and Assessment Policy Statement documents for grades R-12 was employed in identifying the general school level mathematics concepts "grounded" in the CPIS context as well as packaging, per level of schooling, "potential" authentic teaching and learning activities from the CPIS setting or context. Weber (1990) concurs with this use of document analysis by indicating that it is a systematic examination of instructional documents such as syllabi, assignments, lecture notes, and evaluation of course results in order to identify instructional needs and challenges. However, the document analysis in this study was only limited to the school level mathematics policy documents mentioned earlier. The grade levels, within the school level, for each "possible" activity in chapter 6 are clearly suggested at the start of each activity (sections 6.3.1 to 6.3.9).

3.6 **Ethical Considerations**

During the unfolding of this study, all ethical codes such as asking for permission from the gate-keeper, honesty and use of pseudonyms when reporting (Kruger, Michelle & Welman, 2005), were carefully observed. In particular, I requested for permission to conduct my research work at the research site (see Appendix A). Furthermore, the main focus was on the machine's structure and operations resulting in more work on generating more sketches based on and inspired by the structure and operations (mobility and water spread patterns).

3.7 **Limitations of the Study**

The lack of powerful cameras or videotapes that could vividly capture the subtle and complex operations and mechanics of the multi-tower Centre Pivot Irrigation System had a negative impact on the study. A very closer look at the machine at the research site revealed that the successive machine towers actually moved together for a while before the preceding one came to a halt, and the available videotape's ability could not clearly capture such observation.

In addition, the sprinkler nozzle type assumed for this study, as having a rigid 360 degrees circular pattern, had inherent limitations with regard to water spread. It must be understood at a micro level of exploration that within a single circular sector area watered by a sprinkler nozzle, there exist sectors between the circular pores which do not receive water directly from the nozzle pores.

Furthermore, videotaping or recording of the observed CPIS could not always coincide with days of irrigation at the site and this led to some reluctance from the farm manager to make unplanned operations of the machine. The data in Table 5.4 was generated at the time when soya beans at the research site only needed enough moisture for them to be easily uprooted by the farm labourers. This meant that the amount of water had to be restricted in accordance with soil structure and aeration of the research site and this affected the overall mobility of the CPIS.

CHAPTER 4

LINEAR MOVE IRRIGATION SYSTEM

4.1 **Introduction**

This chapter seeks to offer a theoretical basis and springboard for the entire study by first looking at a linear move machine's mobility and its water spread patterns. The intention is to conform to the general mathematics teaching principles of moving or commencing from: the simple to the complex, the easy to the difficult, and the known to the unknown. After all, effective mathematics teaching and learning should enable learners to make connections between their prior knowledge and new ideas or concepts generated in rich contexts. Furthermore, learners must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (Hiebert & Carpenter, 1992).

According to Evans and Sneed (1996), a linear or lateral move machine is designed to be used on rectangular-shaped fields and moves at right angles to the field row direction. The machine moves continuously in a straight line from one end of the field to the opposite end while spreading or releasing water.

4.2 **Getting to know the linear move machine**

There are two common basic types of linear move irrigation structures that quickly come to mind. The first type of structure comprises of ground pipes with vertically arranged sprinklers that spray or 'throws" water upwards as shown in Figure 4.1 below. It is important to note that this type of structure requires a lot of manual labour when pipes are physically moved from one spot to the other within the irrigation field.

Figure 4.1: A linear arrangement with ground pipes and vertical sprinklers spraying water upward (*www.shutterstock.com*.)

The second type of a linear move irrigation machine comprises of suspended or hanging sprinkler nozzles that throw water downward onto plants and crops as shown in Figure 4.2. It is also important to indicate that this type of an irrigation machine actually comprises of a tower of pipes wheeled along the field length. The wheeled tower spans the irrigation field while the hanging sprinklers release water while the machine is operating along the field length.

Figure 4.2: A wheeled linear move tower with hanging sprinkler nozzles spraying water (www.shutterstock.com.)

The latter type of linear move irrigation machine, which perfectly fits Evans and Sneed's (1996) description of a linear move machine, has been assumed for this investigative inquiry. The diagram in Figure 4.3 is a schematic representation of a linear move machine in a 10m x 10m irrigation field.

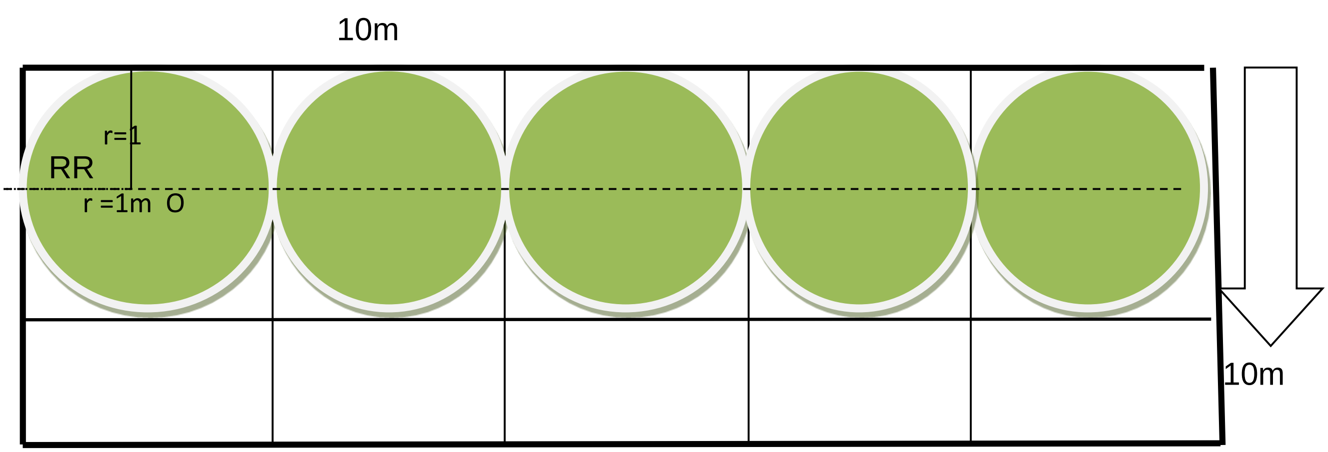
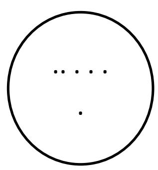


Figure 4.3: Circumference-to-circumference arrangement of sprinkler nozzles for a stationary single tower linear move machine

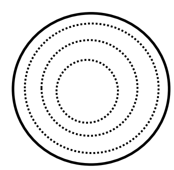
4.3 **Sprinkler Nozzles**

The purpose of any sprinkler is to take water from the mainline and distribute it uniformly over an area in droplets form. However, the ability of a sprinkler nozzle to cover a large area depends on the pressure, which for sprinkler nozzles on a mainline that moves in a linear way, is the same (Evans & Sneed, 1996). This means that as the pressure increases, the area wetted by a sprinkler nozzle also increases and vice versa.

The sprinkler nozzles used in a linear move irrigation machine may, from a theoretical perspective, be imagined as being one of the two types shown in Figure 4.4. They may either be a "shower-like" type with many random pores/openings or the one with "circular uniformly-arranged pores or openings". The latter type of sprinkler nozzle was imagined or assumed for this exploratory research project because of its perceived ability to spread or "throw" water uniformly outward from its centre. Furthermore, this type of sprinkler nozzle delivers water in a fixed 360-degrees spray pattern.



Shower-like sprinkler nozzle



Sprinkler nozzle with circular uniform pores

Figure 4.4: Types of perceivable sprinkler nozzles

4.4 **Stationary Single Tower Linear Move Machine**

Suppose each sprinkler nozzle has a radius of one metre (1m) and five (5) sprinkler nozzles are fitted on a single horizontal tower as shown in Figure 4.3. When the stationary single tower machine starts to release water in the first horizontal field strip, the area of the field strip that is wetted by each sprinkler nozzle at this stage may be determined as follows: Area wetted by each nozzle = $\pi r^2 = \pi(1m)^2 = \pi\ m^2$, where r=1 m.

This means that the wetted area within a single rectangular horizontal field strip, as more sprinkler nozzles are fitted along the tower length, may be calculated as follows: A $=\pi r^2$

times the number of sprinkler nozzles. And if the number of sprinkler nozzles fitted along the length of the tower is represented by letter x, then this wetted area within a single rectangular field strip will be A= πx m^2.

However, the arrangement of sprinkler nozzles in Figure 4.3, for a stationary single tower machine, clearly shows areas within the rectangular horizontal field strip that do not receive water when the machine begins to irrigate the field. The area that does not receive water directly from a single sprinkler nozzle in a 2m x 2m area/cell may be determined as follows: Area not receiving water = (2m x 2m) – π $(1m)^2$ = (4 -π) m^2

Table 4.1 captures and summarizes both sets of areas (receiving or not receiving water) as the number of sprinkler nozzles along the length of the irrigation tower increases.

Table 4.1: The data for a single stationary lateral move tower that begins to irrigate a single horizontal rectangular field.

Number of sprinkler nozzles	1	2	3	4	5	x
Wetted area (m^2)	π	2π	3π	4π	5π	πx
Area not wetted (m^2)	(4 -π)	2(4 -π)	3(4 -π)	4(4 -π)	5(4 -π)	(4 -π)x

Furthermore, when a stationary single tower linear move machine starts to irrigate the field in the first horizontal field strip with a side by side or circumference-to-circumference horizontal arrangement of sprinkler nozzles along the field width, it is clear that the “nodal points”, where two sprinkler nozzles or circumferences meet, receive more water from the adjacent sprinkler nozzles than any other section within the field strip. These “nodal points” within the first horizontal field strip increase with an increase in the number of sprinkler nozzles along the field width. Table 4.2 then captures the numbers of sprinkler nozzles and nodal points receiving more water as a result of the co-incising “water throw” from adjacent sprinkler nozzles.

Table 4.2: The number of nodal points receiving more water from adjacent horizontally arranged sprinkler nozzles in the first horizontal field strip

Number of sprinkler nozzles	1	2	3	4	5	x
Number of nodal points	0	1	2	3	4	x – 1

The scenario of a single stationary tower irrigation machine in a rectangular horizontal field strip (see Figure 4.3) triggers the question of how the sprinkler nozzles should be arranged in order for the entire field strip to receive water at the start of irrigation, that is, to avoid having areas within the horizontal field strip which do not receive water at all. This may clearly be addressed by starting irrigation outside the field boundary or gradually moving the machine towards the field edge as shown in Figure 4.5. But the question here still remains: *How far outside or towards the field edge should the sprinkler nozzles or the tower be moved?*

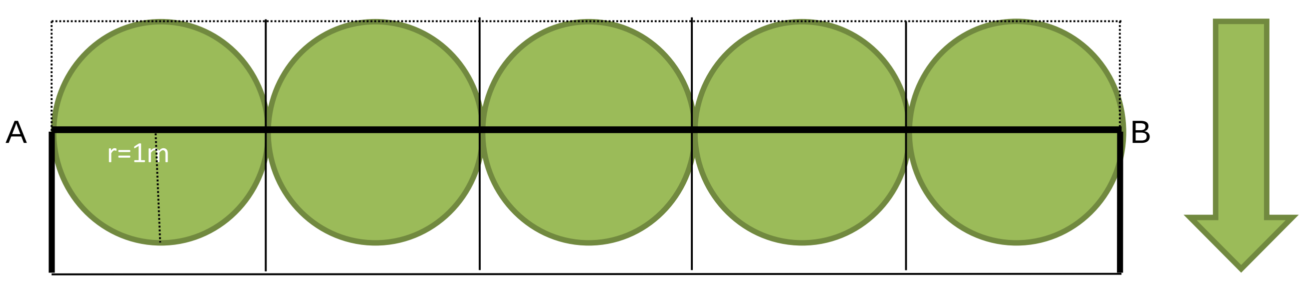


Figure 4.5: Shifting the sprinkler nozzles a metre outside the field boundary/edge AB

For simplicity sake, Figure 4.5 assumes moving the sprinkler nozzles by one metre (1m) towards the outside of the field edge or boundary indicated by the bold line segment AB.

Moving or shifting the sprinkler nozzles, as in Figure 4.5, results in each nozzle irrigating half of its initial wetted area inside the first field strip and half wetted area outside the field. Table 4.3 captures and shows how the initial wetted area inside the first field strip will be affected when the sprinkler nozzles are shifted a metre outside the field boundary AB, as opposed to the normal way of starting irrigation inside the field boundary.

Table 4.3: The data for a single stationary tower with sprinkler nozzles moved or shifted a metre outside the field edge AB

Number of sprinkler nozzles	1	2	3	4	5	x
Inside wetted area (m^2)	$\pi/2$	π	$3\pi/2$	2π	$5\pi/2$	$\pi x/2$

The data in Table 4.3, however, is just a simplified version of the area changes that occur inside and outside the first horizontal field strip when the machine is moved towards the outside of field edge AB (see Figure 4.5). The emerging question then becomes: *How will the inside wetted area compare to the outside wetted area if the machine is moved by fractions of a metre towards field edge, AB?*

The logical answer would then indicate that the sum of both inside wetted and outside wetted areas should always give π square metres. For example, if the inside wetted area from the first sprinkler nozzle is $\frac{\pi}{2}$ square metres, and then the same amount of area will be wetted outside the field edge.

However, this equality or congruence of both areas holds only for a shift and translation towards the field edge by half the diameter of the circular area wetted by a sprinkler nozzle. The data in Table 4.4 clearly shows both sets of areas when a machine nozzle is moved by fractions of a metre (the radius) towards the outside of field edge AB.

Table 4.4: The data from moving or shifting sprinkler nozzles by fractions of a metre towards field edge AB

Machine nozzles shifted by	0 m	$\frac{1}{4}$ m	$\frac{1}{2}$ m	$\frac{3}{4}$ m	1 m	$\frac{5}{4}$ m	$\frac{3}{2}$ m	$\frac{7}{4}$ m	2 m
Inside wetted area (m^2)	π	$\frac{7\pi}{8}$	$\frac{3\pi}{4}$	$\frac{5\pi}{8}$	$\frac{\pi}{2}$	$\frac{3\pi}{8}$	$\frac{\pi}{4}$	$\frac{\pi}{8}$	0
Outside wetted area (m^2)	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$	$\frac{5\pi}{8}$	$\frac{3\pi}{4}$	$\frac{7\pi}{8}$	π

It is, therefore, evident that gradually moving the sprinkler nozzle circumference towards the outside of the field edge (that is, by fractions of the radius) will still achieve the objective of ensuring that the entire horizontal field strip receives water at the start of irrigation.

However, while the translations of sprinkler nozzles hinted above will ensure that each section of the horizontal field strip receives water when the machine starts irrigating the field, a considerable portion of uncultivated land will be watered, thus resulting in an unnecessary wastage of water. As one of Africa's water-stressed countries, South Africa already has irrigation as a dominant water use activity (Department of Water Affairs and Forestry (DWAF), 2004). Furthermore, Evans and Sneed (1996) concede that about 70% of the earth's water is used for irrigation worldwide. The global water conservation campaigns, therefore, discourage irresponsible or inconsiderate use of water such as irrigating areas not meant for economic production purpose.

4.5 **Manually moving the linear machines**

As a theoretical basis to the mobility and water spread patterns of the linear machines in Figure 4.1, the sketches in Figure 4.6 were produced to mimic how the water spread patterns would look like when the machines are manually lifted and moved between points within the irrigation field. The sprinkler nozzle arrangements in these sketches were assumed to be circumference-to-circumference (see left side of the arrow) and circumference-to-centre (see right side of arrow) as shown in Figure 4.6:

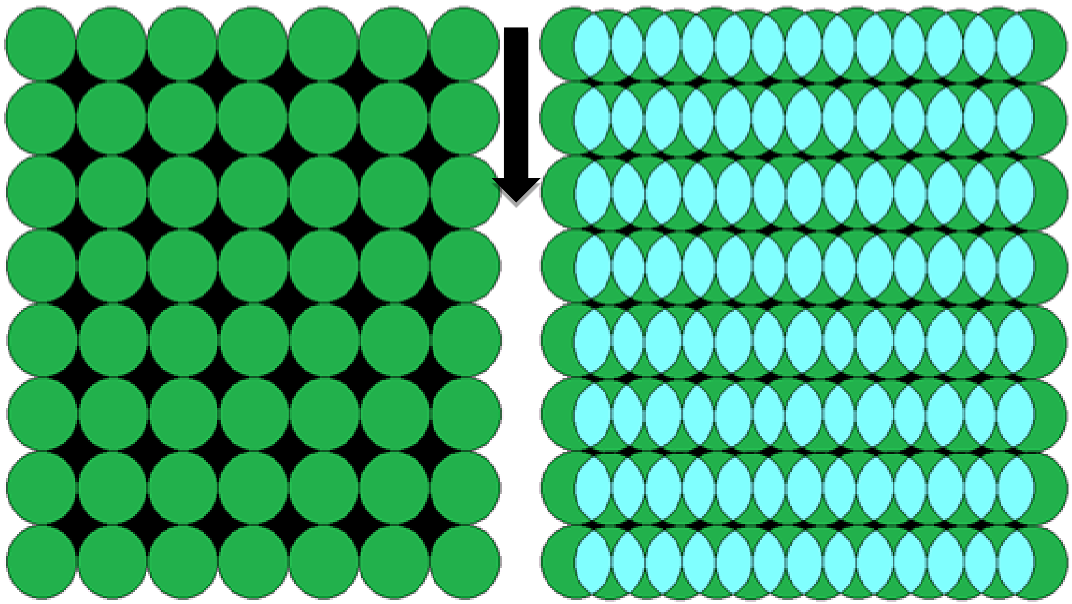


Figure 4.6: Field coverage in a circumference-to-circumference and horizontal circumference-to-centre arrangements of sprinkler nozzles

4.6 **Circumference-to-circumference manual machine movement**

The rows of the green horizontal circles on the left side of the arrow in Figure 4.6 represent sprinkler nozzles arranged in such a way that the circumference of one nozzle touches the circumference of the adjacent nozzle. It is therefore, apparent that the green areas within the circles receive water directly from individual sprinkler nozzles while the black shaded and blank areas within the field boundary do not receive water directly from any of the sprinkler nozzles. This clearly means that more water is received by the green areas than the black and blank portions of the field.

However, these black shaded and blank areas within the field only receive water as a result of water seepage or possibly blowing wind that might cause some droplets to fall there. Furthermore, the common pattern of the green circles and the blank and black portions repeat itself throughout the field from one end of the field to the opposite end. From a mathematics exploratory perspective, which was the objective of this study, it is crystal clear that area calculations within the irrigation field might be performed. It would be interesting for learners to determine the amount of field area receiving water directly from the sprinkler nozzles as well as the area that do not receive water directly from the nozzles. These areas could be calculated as reflected in table 1 above for data from a single stationary linear machine considering that a sprinkler nozzle still has a metre radius.

4.7 **Circumference-to-centre manual machine movement**

The right side of the arrow in Figure 4.6shows a circumference-to-centre arrangement of the sprinklers wherein the circumference of one nozzle waters or touches the centre of the adjacent nozzle. When these sprinkler nozzles are manually lifted and moved as per the direction of the arrow in Figure 4.6, such that circumferences touch each other going downward, it is clear that three different areas (green, black and sky blue) receive various volumes of water during irrigation. As in circumference-to-circumference arrangement of sprinkler nozzles on the left side of the arrow in Figure 4.6, the green shaded portions of the field receive water once directly from one sprinkler nozzle and the black shaded and blank portions receive no water directly from a nozzle. However, the sky blue portions of the field each receive water directly from two sprinkler nozzles. The area calculations in this case begun to elevate to a higher order intensity than in the circumference-to-circumference vertical manual shifting of sprinkler nozzles on the left side of the arrow in

Figure 4.6. This, once again, confirmed the need for exploring area calculations and water spread volumes in this exploratory research study.

The diagram in Figure 4.7 is a schematic of two identical one metre radius sprinkler nozzles with a circumference-to-centre in a 3m x2m field cell adapted from the right side of the arrow in Figure 4.6.

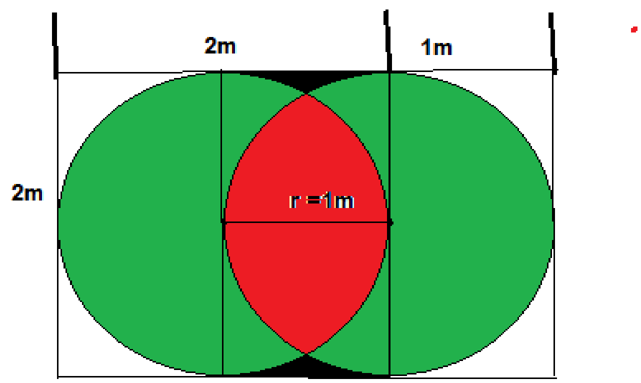


Figure 4.7: A schematic of two identical 1m radius sprinkler nozzles arranged circumference-to-centre in a 3m x 2m field area.

When one considers the scenario in Figure 4.7, it is crystal clear that the blank(white) and the black areas of field receive no water directly from the two sprinkler nozzles, the green areas receive water directly from one of the two sprinkler nozzles, and the red “lens-shaped” area receive water directly from the two sprinkler nozzles simultaneously.

The area calculations in this scenario can, therefore, be performed as outlined below:

The value of the area of each blank corner portion of the field may be deduced from the calculations whose results were captured in Table 4.1for data from a stationary single tower CPIS as: $(\frac{4-\pi}{4}) = (1 - \frac{\pi}{4})$ m^2. This area then translates to a total of $(4 - \pi)$ m^2 for all the blank areas within the 3m x 2m field area in figure 9 above.

The red shaded area, the area of common overlap of the two sprinkler nozzles, can be calculated using the formula: $A = 2R^2\text{Cos}^{-1}(\frac{d}{2r}) - \frac{1}{2} d\sqrt{R^2 - d^2}$, where d is the overlapping distance between the centres of the sprinkler nozzles and r and R the radii of radii of the wetted areas by the individual sprinkler nozzles (Weisstein, 1999). Furthermore, the central angle between the radii forming area sectors in the red area, given by $\theta = \text{Cos}^{-1}(\frac{d}{2r})$, must be converted from degrees to radians in the interval $0 \leq \theta \leq 2\pi$ during calculations involving the formula above.

It is important, therefore, to realize that the variables d, r and R in Figure 4.7 have the same value of one metre (1m) each. This then means that the red shaded area in Figure 4.7 can be calculated as follows:

- $A_{\text{red part}} = 2(1)^2\cos^{-1}(\frac{1}{2(1)}) - \frac{1}{2}(1)\sqrt{4(1)^2 - (1)^2}$

$$= 2\text{Cos}^{-1}(\tfrac{1}{2}) - \tfrac{1}{2}(\sqrt{3})$$

$$= 2 \times 60^0 - \tfrac{1}{2}(\sqrt{3})$$

$$= 2(\tfrac{\pi}{3}) - \tfrac{1}{2}(\sqrt{3}) \qquad [60^0 = \tfrac{\pi}{3} \text{ in radians}]$$

$$= \frac{4\pi - 3\sqrt{3}}{6} \text{ m}^2$$

This area of $\frac{4\pi-3\sqrt{3}}{6}$ metres within the 3m x 2m field in Figure 4.7 receives water directly from two sprinklers simultaneously. However, the green shaded areas of the field in Figure 4.7 that receive water from only one of the two sprinkler nozzles are linked to the red lens-shaped area calculated above in that it is the area of common overlap of the two nozzles. This red area falls in both wetted areas of the two sprinkler nozzles. The value of one of these green areas can be calculated as: $A_{\text{one sprinkler nozzle}} - A_{\text{red part}} = \pi - \frac{4\pi-3\sqrt{3}}{6} = \frac{2\pi+3\sqrt{3}}{6}$ m^2. This, therefore, translates to a total of $\frac{2\pi+3\sqrt{3}}{3}$ square metres within the 3m x 2m field area that receives water once directly from both sprinkler nozzles in Figure 4.7.

But the two black shaded areas in the 3m x 2m field area in Figure 4.7 are of equal magnitude. If the variable x is used to represent each of these areas (resulting in 2x for both areas), then the total area that does not receive water directly from any of the sprinklers nozzles within the 3m x 2m field area in Figure 4.7 will be $(4 - \pi + 2x)$ m^2. This makes it easier to calculate x and hence 2x within the entire $6m^2$ field area as follows:

- $A_{\text{field}} = A_{\text{black \& blank areas}} + A_{\text{green total}} + A_{\text{red part}}$

  $6 = (4 - \pi + 2x) + \frac{2\pi+3\sqrt{3}}{3} + \frac{4\pi-3\sqrt{3}}{6}$

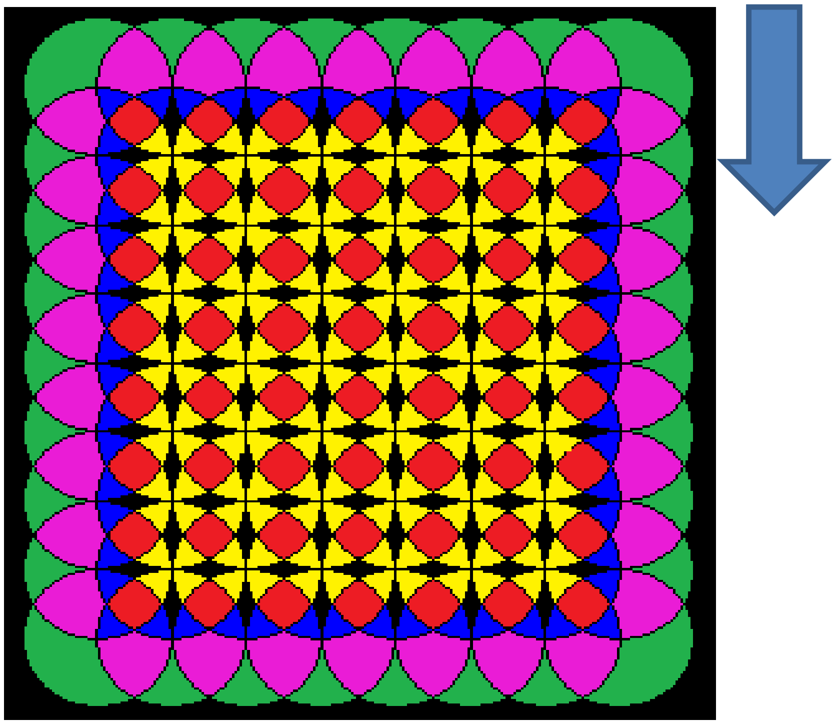
This ultimately results in $x = (1 - \frac{\pi}{6} - \frac{\sqrt{3}}{4})$ m^2 for each black shaded area in Figure 4.7 and $(2 - \frac{\pi}{3} - \frac{\sqrt{3}}{2})$ m^2 for both black shaded areas in the six square metre field. This clearly means that the total area not receiving water directly from any of the two sprinkler nozzles in Figure 4.7 will be: $(4 - \pi + 2 - \frac{\pi}{3} - \frac{\sqrt{3}}{2}) = (\frac{36-8\pi-3\sqrt{3}}{6})$ m^2.

These area calculations were further expounded on under “packaging the activities” section toward the end of this report as the types of “possible” authentic teaching and learning tasks. The intention at this stage was to show the “richness” of the linear irrigation context with regard to mathematical area exploration.

4.8 Circumference-to-centre: both horizontal and vertical manual machine movement

The complexities regarding the areas receiving water directly from individual sprinkler nozzles further took a more advanced level of exploration when a horizontal circumference-to-centre sprinkler arrangement, with a vertical manual lifting and placement in the direction of the arrow in Figure 4.8, was considered. Unlike in the previous scenario on the right side of the arrow in Figure 4.6, the lifting and the vertical manual movement of the sprinkler nozzles in this case was such that the circumferences of the first row of sprinklers were translated downward to the centres of the sprinkler nozzles until the far end of the field was reached.

The areas receiving water directly from the sprinkler nozzles were deduced from Figure 4.8 as follows: the green shaded portions of the irrigation field receive water directly from one sprinkler nozzle; the pink and black portions receive water directly from two nozzles; the yellow and blue areas receive water directly from three nozzles while the red field portions receive water directly from four sprinkler nozzles. The complexities of the possible area calculations for this study, therefore, became more and more apparent at this stage of theoretical exploration through sketches that the object of the study could be realized. The crucial thing to realize with regard this variation is that the area not receiving water directly from the sprinklers was drastically reduced.



Green: Serviced by 1 Nozzle	Black & Pink: Serviced by 2 Nozzles
Yellow & Blue: Serviced by 3 Nozzles	Red: Serviced by 4 Nozzles

Figure 4.8: Field coverage in a vertical and horizontal circumference-to-centre arrangements of sprinkler nozzles

4.9 **Reflection and Conclusion on Manual Linear Movements of Irrigation Machines**

The theoretical exploration of the manual lifting and movement of the linear machines in Figure 4.1, through sketches generated in Figures 4.6, 4.7 and 4.8, laid a solid foundation for the study in that it explicated area calculation possibilities and opportunities within the irrigation context. This ensured that the exploration activity had a starting point and direction in line with the objective of “unearthing” the general mathematics concepts grounded in these irrigation machines setting.

Figure 4.6 clearly indicates that the areas that do not receive water directly from the individual sprinkler nozzles, particularly the black shaded portions, are minimized or reduced in size when the sprinkler nozzle arrangement was assumed to be circumference-to-centre as opposed to circumference-to-circumference. Furthermore, these areas seemed to be eliminated with a theoretical perspective depicted in Figure 4.8 where all field areas receive water directly from at least one sprinkler nozzle.

The area calculation opportunities embedded in these manual movements intensify in order of difficulty in line with mathematics teaching principles of moving from the simple-to-complex as well as from the easy to the difficult. The calculations in these scenarios occur within a rectangular irrigation field and tap into fundamental knowledge of the section of mensuration in mathematics, especially formulae involving rectangles and circles. For example, the calculation of the area not receiving water directly from a sprinkler nozzle on the left side of the arrow in figure could be determined as the sum of four quarter circles forming that shape/cell (see Table 4.1).

The downside, for water spread uniformity, of these manual movements seemed to be with the availability of areas "not receiving" water directly from the individual sprinklers, especially in Figure 4.6. This might have rendered these linear move machines less popular in the irrigation fraternity. The fact that these assumptions meant intense manual labour as well as the need for a stricter precision with water pipe arrangements also might have discouraged the use of these tedious systems. However, the question that came up at this stage was: *Would it make any difference if continuous linear movement irrigation was used to curb the alluded demerits of the manual movement arrangements?*

4.10 **Continuous Mobile Single Tower Linear Move Machine**

When the single tower linear move machine starts to move continuously while releasing water, the question of how the released water is spread with a linear movement of the machine comes to mind. However, factors such as the speed of the machine and the amount of water that needs to be used for daily irrigation are determined by the user of the machine.

According to Sanders (2001), a continuous linear move system requires a guidance mechanism to guide it in a straight line down the field and this usually means following an above-ground cable via a radio signal, an under-ground cable or a Global Positioning System (GPS). Furthermore, the speed of the machine is controlled through the use of a flow control in an attempt to run it at a “given speed”. This clearly means that a lateral or linear move irrigation machine moves continuously at a constant or same speed across the length of the irrigation field.

In an attempt to determine the rate of water spread for a linear move single tower irrigation machine, the speed of the tower as well as the amount of water to be used for the entire field were assumed to be constant. Figure 4.9, which is adapted from Figure 4.3, shows the same information, but indicates the complexities for a mobile single tower system in a portion of the rectangular 10m x 10m field.

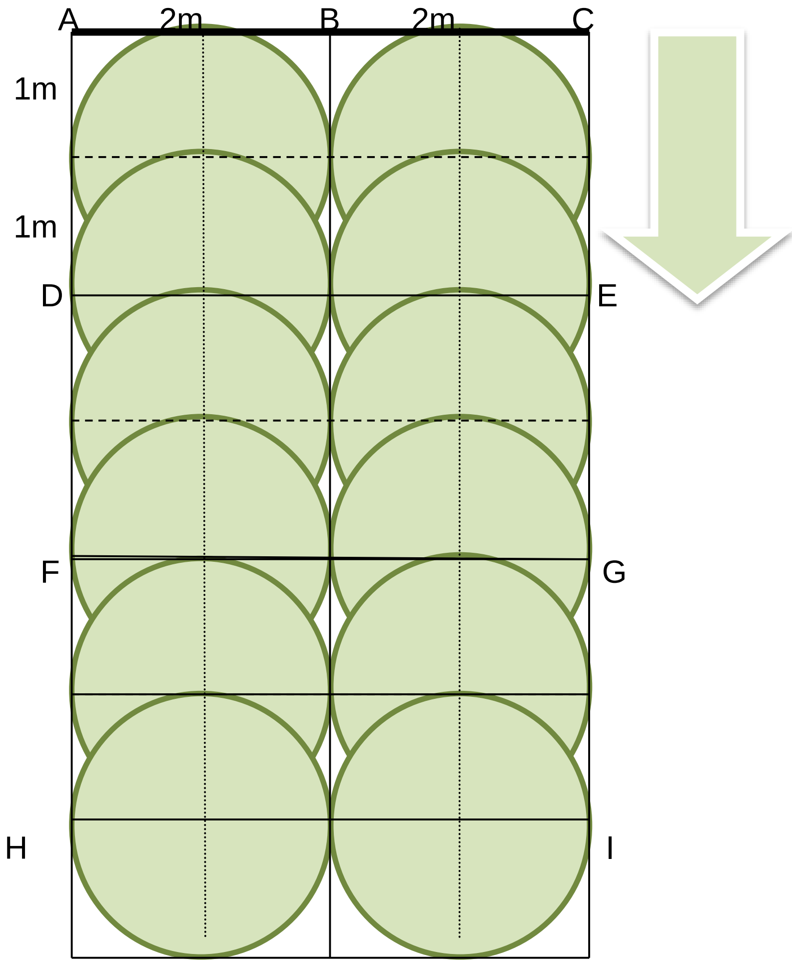


Figure 4.9: An adapted schematic of a linear move machine moving in units of 1metre

When the single tower linear move machine starts to operate, from inside the first horizontal field strip as it normally does, at a constant speed down the field while releasing water, some areas within the first horizontal field strip receive no water while other areas, even in the next horizontal field strips, receive water either once or twice in the same pattern until the machine reaches the opposite end of the field (see Figure 4.6). If irrigation commences inside the field boundary, as in Figure 4.9, while the machine has moved a metre from its starting position, an area of $(\frac{4-\pi}{2})$ square metres does not receive water from individual sprinkler nozzles and this area replicates to $(\frac{4-\pi}{2})$x square metres as the number of sprinkler nozzles increases to x units along the field width. The same amount of area will, thus, also not receive water at the opposite end of the field.

What Figure 4.9 shows is that when the machine's sprinkler nozzle is a metre (1m) away from the field edge (line segment AB) within a 2m x 2m cell, an area that either receives water once or twice is affected as follows:

- Area within first 2m x 2m cell receiving water once = $(\frac{\pi}{2} + \frac{4-\pi}{2}) = 2\ m^2$
- Area within first 2m x 2m cell receiving water twice = $\frac{\pi}{2}\ m^2$

This means that out of a four square metre ($4\ m^2$) area, a total of $(\frac{4+\pi}{2})$ square metres receive water from individual sprinkler nozzle within the first horizontal field strip, and this amounts to $5(\frac{4+\pi}{2})$ square metres considering the five sprinkler nozzles along the tower length (see Figure 4.3).

However, from the second horizontal field strip to the second last horizontal field strip, the areas within a 2m x 2m cell are affected as follows:

- Area within a 2m x 2m cell receiving water once = $(4 - \pi)\ m^2$
- Area within a 2m x 2m cell receiving water twice = $\pi\ m^2$

This clearly means that each section of the four square metre cells now receives water! And if it takes the machine a minute to travel past a $4\ m^2$ cell while 1000 litres of water need to be used for the entire 10m x 10m rectangular field, then a four square metre ($4\ m^2$) cell will have to receive 40 litres of water (i.e. $10\ l/m^2$). The rate of water spread in the first and second field strips may be determined as follows:

- The amount of water released in the first horizontal field strip = 10 l/m^2 x 5($\frac{4+\pi}{2}$) m^2 = 50($\frac{4+\pi}{2}$) litres.
- The rate of water spread in the first field horizontal field strip will, therefore, be 50($\frac{4+\pi}{2}$) l/minute.

The amount of water released in second horizontal field strip until the second last horizontal field strip = 10l/m^2 x 5 x 4 m^2 = 200 litres. And this then translates to a water spread rate of 200 l/minute in each of these horizontal field strips. The suggestion here seems to be that a uniform water spread for a lateral move machine only occurs in the horizontal field strips between the first and the last horizontal strips. The vegetation on rectangular fields using a linear move machine seems to support this assertion about uniformity of water spread because the crops at the part where irrigation begins are of the same approximate height as those at the other end of the field.

4.11 **Reflection and Conclusion on Continuous Single Tower Linear move Machine Data**

Though not very successful in answering the main research questions of this exploratory study, the data from the side by side arrangement of sprinkler nozzles on a single lateral move CPIS provided a solid theoretical basis for the investigative inquiry. For example, the idea of a high possibility of the swelling of the centre or water-logging could be clearly explicated in the side by side arrangement of sprinkler nozzles as testified by the “nodal points” and other outlined areas of the field that were perceived as evidence of the swelling and water-logging (receiving water twice). Figures 4.6 and 4.8 also assisted in explicating these uneven water spread within the field. For example, the field areas receive water directly from no sprinkler nozzle to a maximum of four nozzles.

The important thing to note, which is relevant and fundamental for this study though, is that general school level mathematics concepts such as determining area of rectangles and circles, number patterns (linear sequences) as well as ratio and rate of water spread can be facilitated and consolidated in this linear move irrigation machine context. The Curriculum and Assessment Policy Statement (CAPS) for Mathematical Literacy, as an integration opportunity, specifically states that learners in grade ten should be able to perform calculations of area of circles (quarters, semi- and three quarters) using known formulae (DoE, 2011a, p. 68). In the same breadth, the CAPS document for Mathematics in the Further Education and Training (FET) band of schooling (DoE, 2011b, p. 22) directs that learners should investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term or formula, therefore being linear. The single tower linear move machine scenario, theoretically explored in this chapter, therefore, explicated the afore-stated mathematics concepts or topics.

In addition, the lateral move variations explored thus far could only concur with Harrison's (2012) assertion that even if the sprinklers distribute equal amount of water regardless of its position on the tower and makes it ideal for low-pressure applications, almost 98% of the field gets wetted or irrigated. Furthermore, the major disadvantage of not having an immediate dry area ahead of the machine seems to have had a bearing on the minimal or unpopular use of lateral move machines in most farms.

CHAPTER 5

CENTRE PIVOT IRRIGATION SYSTEM

5.1 **Introduction**

This chapter purports to theoretically explore a centre pivot irrigation system in an attempt to develop a mathematics model that may be useful in explaining the irrigation machine's mobility and water spread patterns. Central to this objective is trying to answer the main research questions posed at the beginning of this research study.

Evans and Sneed (1996) indicate that a Center-pivot irrigation system (CPIS), unlike a lateral or linear move irrigation machine that has been theoretically explored in the previous chapter, is a self-propelled continuous move irrigation machine that rotates round a central pivot point.

In this study, however, it was found worthwhile to first consider possible patterns of water spread if the system was to be moved manually like is the case with systems explored earlier. In this instance, the idea is to produce pictorial patterns without necessarily exploring their associated mathematical models (that will be addressed in the future studies). Different colours are used to map different amounts of water that is received by different portions of the field.

5.2 **Getting to know the Centre Pivot Irrigation System**

The machine, as shown in figure 12 below, has become a common and popular sight in the farming fraternity to an extent that it may be regarded as an inseparable part of life in farms in most rural areas.

The diagram in Figure 5.2 is a schematic representation of a single tower CPIS on a rectangular field with a radius of five metres (5m).

Figure 5.1: A picture of CPIS in operation

5.3 **Sprinkler Nozzles**

The sprinkler nozzles are similar to those in the previously explored linear move machine and are assumed to have a metre (1m) diameter each and are still mainly arranged side by side or circumference-to-circumference, while the radii of the engraved circular tracks remain in the ratio 1:2:3:4:5. (see Figure 5.2). This, therefore, implies that R_5=5m; R_4= (R_5-1) m; R_3= (R_5-2) m; R_2= (R_5-3) m and R_1= (R_5-4) m.

5.4 **Manual Movement of a Single Tower CPIS**

There are different variations in terms of how the system can be manually moved. In a sequence of sketches below, I have shown a system moved in such a way that (a) the outer nostrils produce circumference-to-circumference arrangement whilst towards the centre circumference-to-circumference is also maintained; (b) the outer nostrils are in a circumference to centre arrangement whilst towards the centre circumference to circumference is maintained, and (c) the circumference-to-centre arrangement is done in both ways.

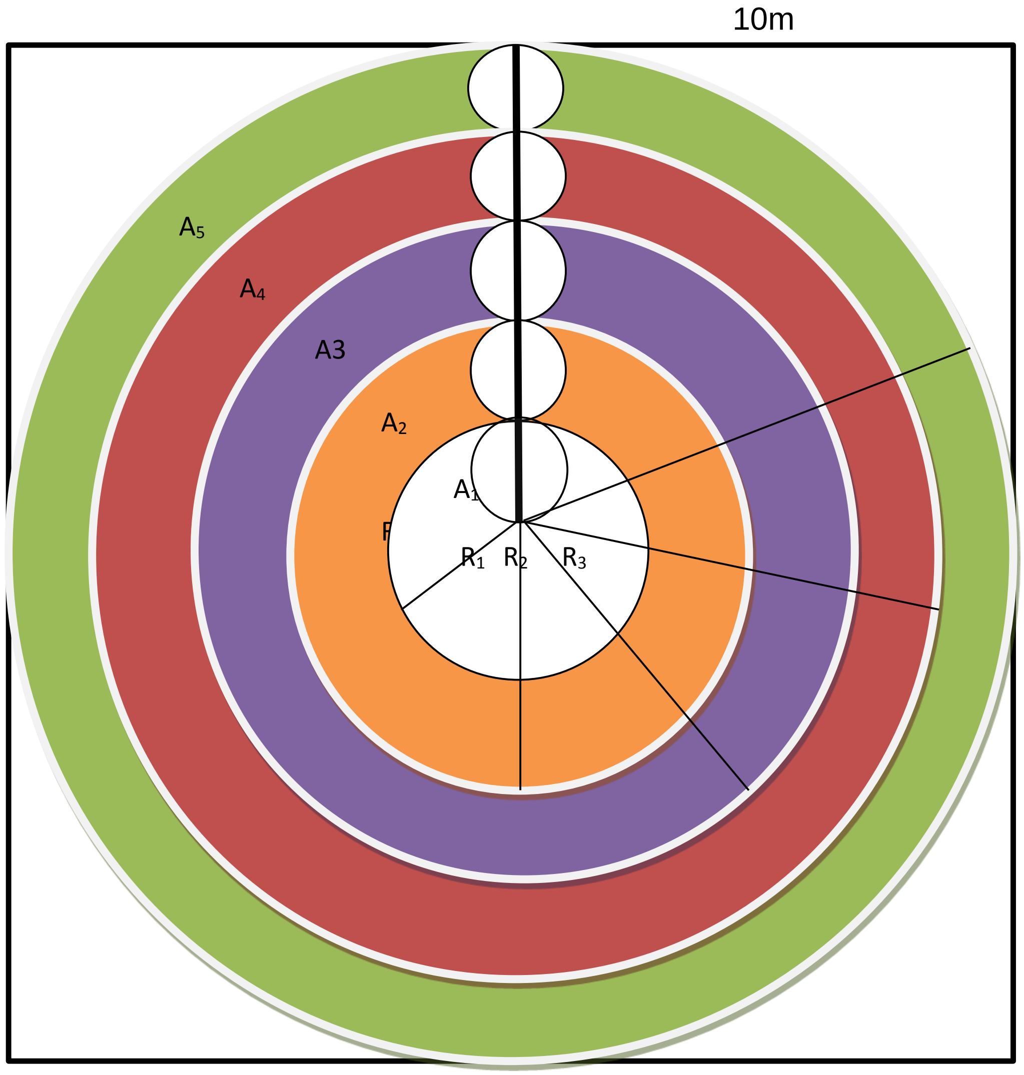
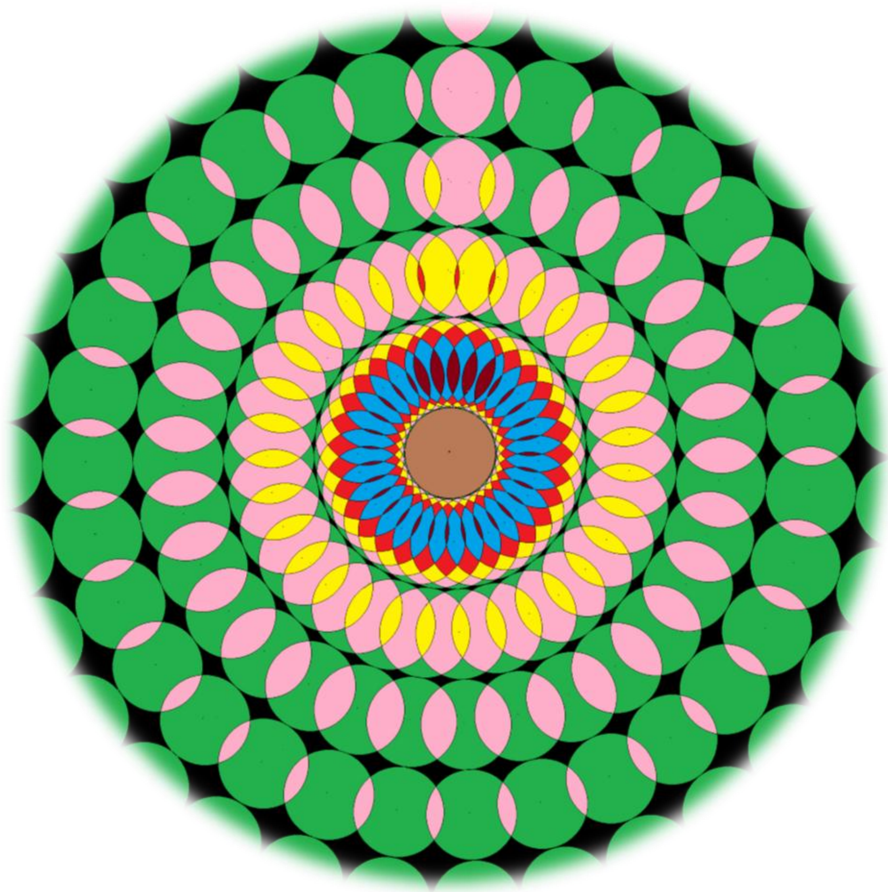


Figure 5.2: A single tower CPIS on a 5m radius irrigation field

(a) Circumference-to-circumference arrangement in both ways

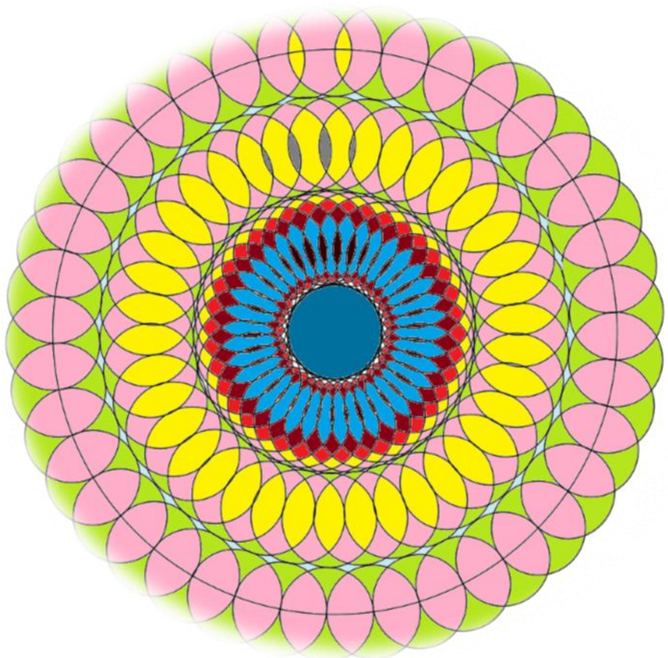


Green: One nostril	Black: No water	Pink: Two nostrils	Yellow: three nostrils
Red: four nostrils	Blue: 5 nostrils	Brown: 32 nostrils	

Figure 5.3: Water coverage in a circumference to circumference arrangement in a manually moving system

In this case, the system was moved manually 32 times to provide coverage of the field. Because of the length of the circumference, the last arrangement does not overlap into the first as the rest of the arrangement. This produces a slightly different arrangement. Interesting mathematical problem solving opportunities arises from this situation. Among those is: what are the relative water coverage as represented by the different colours.

(b) Circumference-to-centre and circumference-to-circumference arrangement

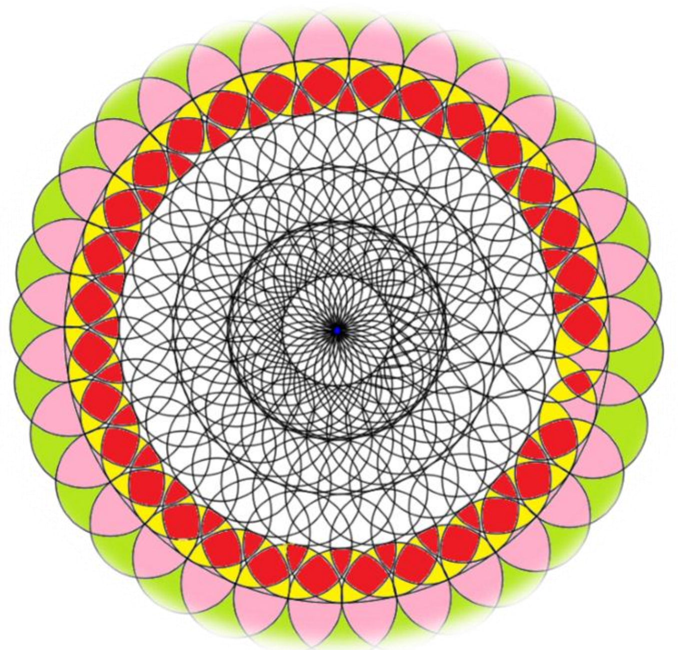


Green: 1 nostril	Pink: 2 nostrils	Yellow: 3 nostrils	Red: 4 nostrils
Dark Red: 5 nostrils	Blue: 6 nostrils	Dark Blue:	Light Blue: 0 nostrils

Figure 5.4: Circumference-to-centre and circumference-to-circumference arrangement coverage

Similar patterns like the one in Figure 5.3 are produced. However, pink and yellow emerge earlier indicating more water coverage that it was the case then. In this arrangement, the centre becomes quickly flooded that was the case earlier.

(c) Circumference to centre in both ways



Green: 1 nostril	Pink:2 nostrils	Yellow: 3nostrils	Red: 4 nostrils

Figure 5.5: Circumference-to-circumference arrangement in both ways

In this arrangement each square meter of the entire field is covered by more than one nostril. Mathematically, there are even more problem situations that could be explored. The number of nostrils covering different areas of the field is itself a worthy mathematical exploration.

In the three scenarios presented above, it is quite clear that if the nostrils were to discharge the same amount of water per time, the obviously the centre of the system will be flooded in no time whilst the outmost areas will receive little amount of water. In order to counter-act this effect, the water distribution from the inners nostrils to the outer ones should be differentiated. The how part, is the focus of this study. However, before addressing the question, we proceed to the continuous moving system.

5.5 **Continuous Moving Single Tower CPIS**

If it takes the entire single tower CPIS in Figure 5.2 an hour (60 minutes) to rotate or make one revolution around the central pivot point, then the rotational speed of the individual wheel sets carrying the machine and located on circular tracks is determined as follows:

Rotational speed for the innermost wheel set (W_1) = $(2\pi R_1/60)$

$$= 2\pi(1m)/60$$

$$= \pi/30 \text{ m/minute}$$

Similarly, the rotational speeds for the next wheel sets can be determined as captured and shown in Table 5.1.

Table 5.1: Rotational speed for the wheel sets carrying the single tower CPIS

Wheel set number	1	2	3	4	5	X
Rotational speed (m/minute)	$\pi/30$	$\pi/15$	$\pi/10$	$2\pi/15$	$\pi/6$	$\pi x/30$

The area of individual field strips, A_1 to A_5 in Figure 5.2, from the innermost field strip to the outermost field strip is determined as follows:

- The area of innermost field strip, $A_1 = \pi(R_1)^2 = \pi(R_5-4)^2 = \pi\ (5-4)^2 = \pi\ m^2$.

Similarly, areas for field strips, A_2 to A_5, can be calculated to be $3\pi\ m^2$; $5\pi\ m^2$; $7\pi\ m^2$ and $9\ \pi\ m^2$ respectively.

And if a thousand (1000) litres of water are used to irrigate the entire five metre radius field (25π m^2), then a rate of $\frac{40}{\pi}$ litres per square metre will have to be applied for the entire field. This clearly means that the amount of water per field strip will be determined as follows:

- Amount of water released in A_1 = $\frac{40}{\pi}$ L/m^2 x π m^2 = 40 L
- Amount of water released in A_2 =$\frac{40}{\pi}$ L/m^2 x 3π m^2 = 120 L
- Amount of water released in A_3 =$\frac{40}{\pi}$ L/m^2 x 5π m^2 = 200 L
- Amount of water released in A_4 = $\frac{40}{\pi}$ L/m^2 x 7π m^2 = 280 L
- Amount of water released in A_5 =$\frac{40}{\pi}$ L/m^2 x 9π m^2 = 360 L

Consequently, the water spread rate within the individual field strips, A_1 to A_5, using the assumed rotational time of 60 minutes per revolution is determined as follows:

- Water spread in the innermost field strip, A_1 = 40/60 = 2/3 L/minute
- Water spread in field strip, A_2 = 120/60 = 2 L/minute
- Water spread in field strip, A_3 = 200/60 = 10/3 L/minute
- Water spread in field strip, A_4 = 280/60 = 14/3 L/minute
- Water spread in field strip, A_5 = 360/60 = 6 L/minute

Table 5.2 below recaps and shows the data for the areas of individual field strips, the amount of water released per field strip as well as the water spread rate in the field strips on a rectangular 10m x 10 m irrigation field for a 5m radius single tower CPIS.

Table 5.2: The area, amount of water released and rate of water spread per field strip on a 100 m^2 irrigation field using a 5m single tower CPIS

Field strip number	1	2	3	4	5	X
Field strip area (m^2)	π	3π	5π	7π	9π	$\pi(2x-1)$
Amount of water released per field strip (L)	40	120	200	280	360	40(2x-1)
Water spread rate per field strip (L/minute)	2/3	2	10/3	14/3	6	4X/3 – 2/3

When the five metre single tower CPIS starts to rotate while releasing water in the various field strips, sectors of the field strips which may be labeled A_1 to A_5 as shown in Figure 5.6, which was adapted from Figure 5.2, receive fractions of water in a fraction of the rotational 60 minutes time.

For simplicity sake, if the machine covers a quarter circles from P to Q in quarter of an hour (15 minutes), the sector areas within the different field strips are determined as follows:

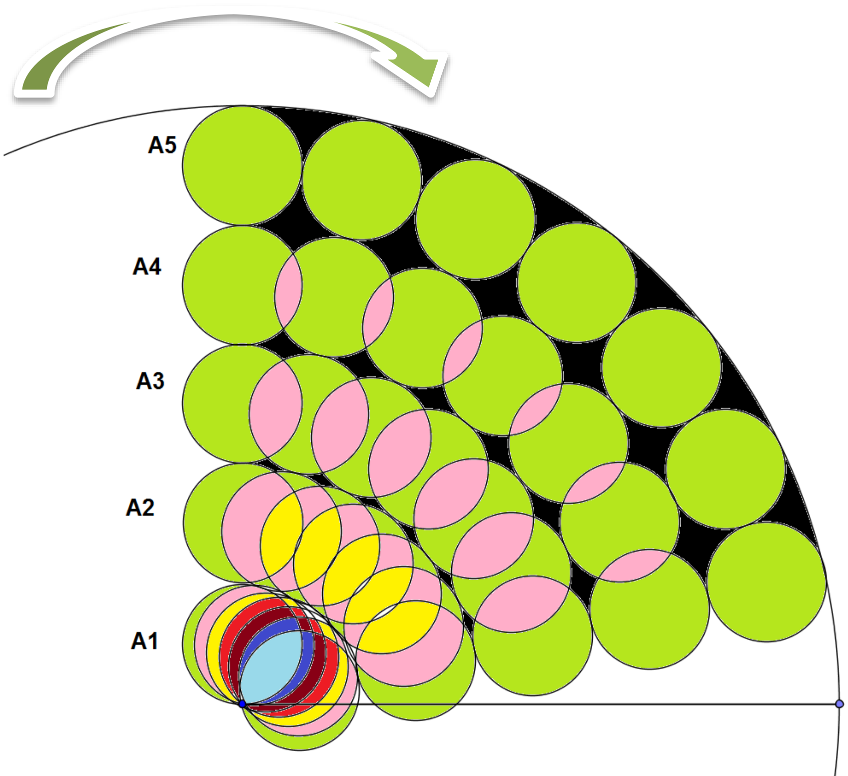
- Sector area, $a_1 = 90^0/360^0$(Area of field strip A_1)

  $= 1/4(\pi)\ m^2$

  $=\pi/4\ m^2$

- Sector area, $a_2 = 1/4(3\pi)\ m^2 = 3\pi/4\ m^2$

Similarly, the values of sector areas a_3 to a_5 may be calculated to be $5\pi/4$; $7\pi/4$ and $9\pi/4$ square metres respectively.



Green: 1 nostril	Pink: 2 Nostrils	Yellow: 3Nostrils	Red: 4 nostrils
Dark red: 5 nostrils	Dark Blue: 6 nostrils	Light Blue: 7 Nostrils	Black: 0 Nostrils

Figure 5.6: An adapted schematic representation of mobile single tower CPIS on a 5m radius irrigation field covering a quarter circle in quarter of an hour

The amount of water released by the single tower CPIS within field sector areas, a_1 to a_5, in the quarter hour (15 minutes) rotational time assumed above will be determined as follows:

- Amount of water released in sector area, $a_1 = \frac{40}{\pi}$ L/m^2 x $\pi/4$ m^2 = 10 L
- Amount of water released in sector area, $a_2 = \frac{40}{\pi}$ L/m^2 x$3\pi/4$ m^2 = 30 L
- Amount of water released in sector area, $a_3 = \frac{40}{\pi}$ L/m^2 x$5\pi/4$ m^2 = 50 L
- Amount of water released in sector area, $a_4 = \frac{40}{\pi}$ L/m^2 x$7\pi/4$ m^2 = 70 L

- Amount of water released in sector area, $a_5 = \frac{40}{\pi}$ L/m^2 x$9\pi/4$ m^2 = 90 L

It, therefore, follows that the rate of water spread within the sector areas, A_1 to A_5, during the assumed quarter hour (15 minutes) of the single tower machine's mobility can be determined as follows:

- Rate of water spread in sector area, a_1 = 10/15 L/minute =2/3 L/minute
- Rate of water spread in sector area, a_2 = 30/15 L/minute = 2 L/minute
- Rate of water spread in sector area, a_3 = 50/15 L/minute = 10/3 L/minute
- Rate of water spread in sector area, a_4 = 70/15 L/minute = 14/3 L/minute
- Rate of water spread in sector area, a_5 = 90/15 L/minute = 6 L/minute

The data in Table 5.3 recap and indicate the sector area, amount of water per sector area in a field strip as well as the rate of water spread per sector area within quarter of an hour of the single tower's operation, that is, its mobility and spread of water.

Table 5.3: Data for sector area, amount of water per sector area and water spread rate per sector area by a single tower CPIS on a 5m radius irrigation field in quarter of an hour

Field sector number	1	2	3	4	5	X
Sector area (m^2)	$\pi/4$	$3\pi/4$	$5\pi/4$	$7\pi/4$	$9\pi/4$	$\pi(2x-1)/4$
Amount of water per sector area (L)	10	30	50	70	90	10(2x-1)
Water spread rate per sector area (L/minute)	2/3	2	10/3	14/3	6	4X/3 – 2/3

Figure 5.6, like Figures 4.6, 4.8 and 5.3, depicts from a theoretical perspective the water spread pattern likely to be observed when the single tower circular move machine (CPIS), with a vertical circumference-to-circumference sprinkler arrangement, is manually lifted and moved in a clockwise direction such that the sprinkler nozzles' circumference touch. The purple areas of this quarter field do not receive water directly from any of the sprinkler

nozzles, the dark red areas receive water directly from one sprinkler nozzle and the red areas receive water directly from two sprinkler nozzles. However, the area calculations for the “lens-shaped” areas do not vary from those performed from Figure 4.7.

5.6 **Reflection and Conclusion on the Single Tower CPIS Data**

The data in Tables 5.2 and 5.3, though suggesting a “linear constant increase” water spread rate for the single tower CPIS, does not account for “corner areas” of a 10m x 10m rectangular field in Figure 4.3 which do not receive water from the sprinkler nozzles fitted along the length of the machine. A total corner area of (100 - 25π) square metres is left out without being watered when the single tower CPIS operates in a circular pattern of motion. However, Evans and Sneed (1996) concede that, with an overhand from an end gun, approximately 132 acres can be irrigated, thereby providing a solution to these corner areas which do not receive water. In addition, a corner attachment (which is operated and controlled by a signal sent through a buried electric cable) allows for the corners of square fields or other odd-shaped field areas to be watered.

The key argument for this study, as viewed from the single tower CPIS data, is that again school level mathematics concepts such as area of circles; speed of wheel sets carrying the machine; number patterns; ratio and rate can be facilitated and consolidated within the circular move irrigation machine context. According to the CAPS document for Mathematics in the Senior Phase (DoE, 2011a, p. 31), educators should define a circle and revise the terminology associated with circles, that is, centre, radius, diameter, circumference, chord, secant, tangent segment and sector. The document further emphasizes that learners should be encouraged to develop relationships between: radius, diameter and circumference, that is, d = 2r; C = πd or C = $2\pi r$. The clarification notes or teaching guidelines provided in the document categorically state that educators should invest much time explaining these relationships so that learners develop a sense of , for example, where the rational number Pi (π) is derived from in terms of the ratio of the circumference of a circle to its diameter.

However, a disconcerting limitation of the data generated from the perceived single tower CPIS, that rotates around a central pivot point, is that it overlooks the complexities of a real multi-tower CPIS's mobility and water spread mechanisms. The towers do not just rotate like a "single bar of pipe" when they operate on an irrigation field. Diener (2009) concurs with the assertion of a complex CPIS mobility when he indicates that an "overall speed" of the machine is governed by the tower on the outside, which has to travel farther and faster than the towers toward the pivot point. The diagram in Figure 5.7 shows how the towers of water pipes apes are joined using sensor joints.

Figure 5.7: A picture of sensory joint linking CPIS towers

In addition, the speed of rotation (time it takes the machine to complete one revolution of a circle) involves controlling the speed of the extreme tower, that is, does the electric motor on that tower operate continuously or is there a stop and start operation (Evans & Sneed, 1996)?

Furthermore, the notion of sprinkler nozzles having the same radii for wetted area would not suffice considering the rotational speeds of the individual wheel sets carrying the single tower CPIS. One is tempted to literally think of a situation where an empty fish tin, with pores or openings at the bottom similar to those of the assumed sprinkler nozzles, is used to spray water over a piece of ground at different speeds. Logically, with the slowest speed over the piece of ground, water-logging is more likely to result than at high speed. This clearly negates or rather puts in doubt the assumption of the sprinkler nozzles having the same wetted area size or radius from the innermost to the outermost sprinkler nozzle along the length of the single tower.

The alluded complex mobility of the real multi-tower CPIS triggers questions such as: *How much sector area is swept by the outermost tower before coming to a halt, thereby letting the next tower to catch-up in a certain time?* The diagram in Figure 5.8 illustrates a mimicked actual anti-clockwise movement of CPIS towers and shows only field strips A_1 and A_2 with their respective arcs QW and PR.

The areas of field sectors, a_1 and a_2, in Figure 5.8 can be determined when the central angle POR is known and measured either in radians or degrees using the expression $\frac{\theta}{360°}(\pi r^2)$ where θ is the central angle value in degrees while r is the radius of the sector (Russel, 2013). The calculations of the sector areas do not essentially differ from those done in the single tower CPIS scenario (Table 5.3) as they involve finding fractional parts of the entire circle area, depending on the value of the central angle.

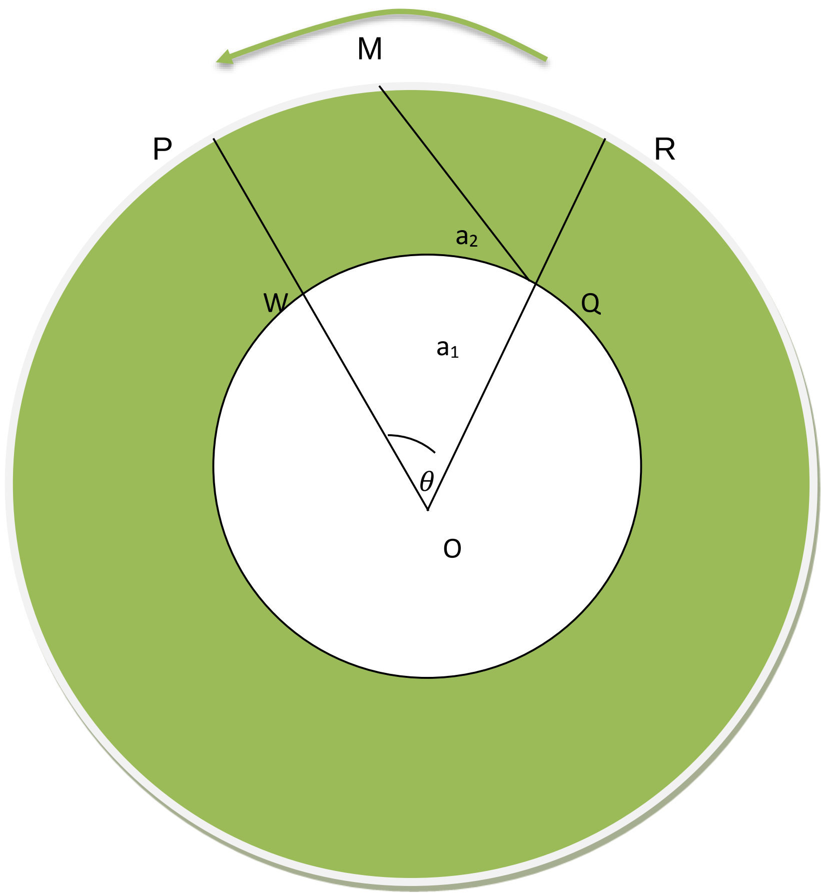


Figure 5.8: A mimicked actual movement of innermost towers of the CPIS and swept sector areas

However, the somehow “zigzag” movement of the actual multi-tower CPIS, as perceived from its stop and start mode of operation, coerces one to expect the resultant engraved track of the “entire machine’s single rotation round the pivot point” to be represented by the diagram in Figure 5.9.

The convex and concave sections of the figure represent the alternating movements of the various wheel sets carrying the towers of the actual CPIS. A closer look at Figure 5.9 makes one to perceive, from a theoretical perspective, that each wheel set except the extreme wheel set makes both concave and convex arcs and tracks “simultaneously” as they move to catch up and align with preceding wheel sets. For example, the second wheel set from the outer part of the field carrying the tower shown by the second last red dotted line makes the black solid arc as it moves to catch up and align with the outermost wheel set.

But because the outermost wheel set would already be stationary at point Q, from point P, when the black solid arc is formed by the second last wheel set, the tower between the last two wheel sets may be viewed to now have point P as its “pivot point” since it is motionless during the “perceived” formation of the green dotted concave track at that point. The same analogy may, thus, be considered for the third wheel set from the outer part of the irrigation field.

The hinted complex mobility and possible water spread mechanism of the real multi-tower CPIS warranted a practical or hands-on approach at the research site where following emergent questions needed to be addressed:

- How long does each tower take to cover a “swept” area before it comes to a halt, thereby letting the next tower to catch up?
- Are the sprinkler nozzles arranged along the lengths of the towers of the same radius as assumed in the theoretical approach chapters above?
- Does the real multi-tower CPIS release the same amount water in the various field strips within some designated areas per unit time?

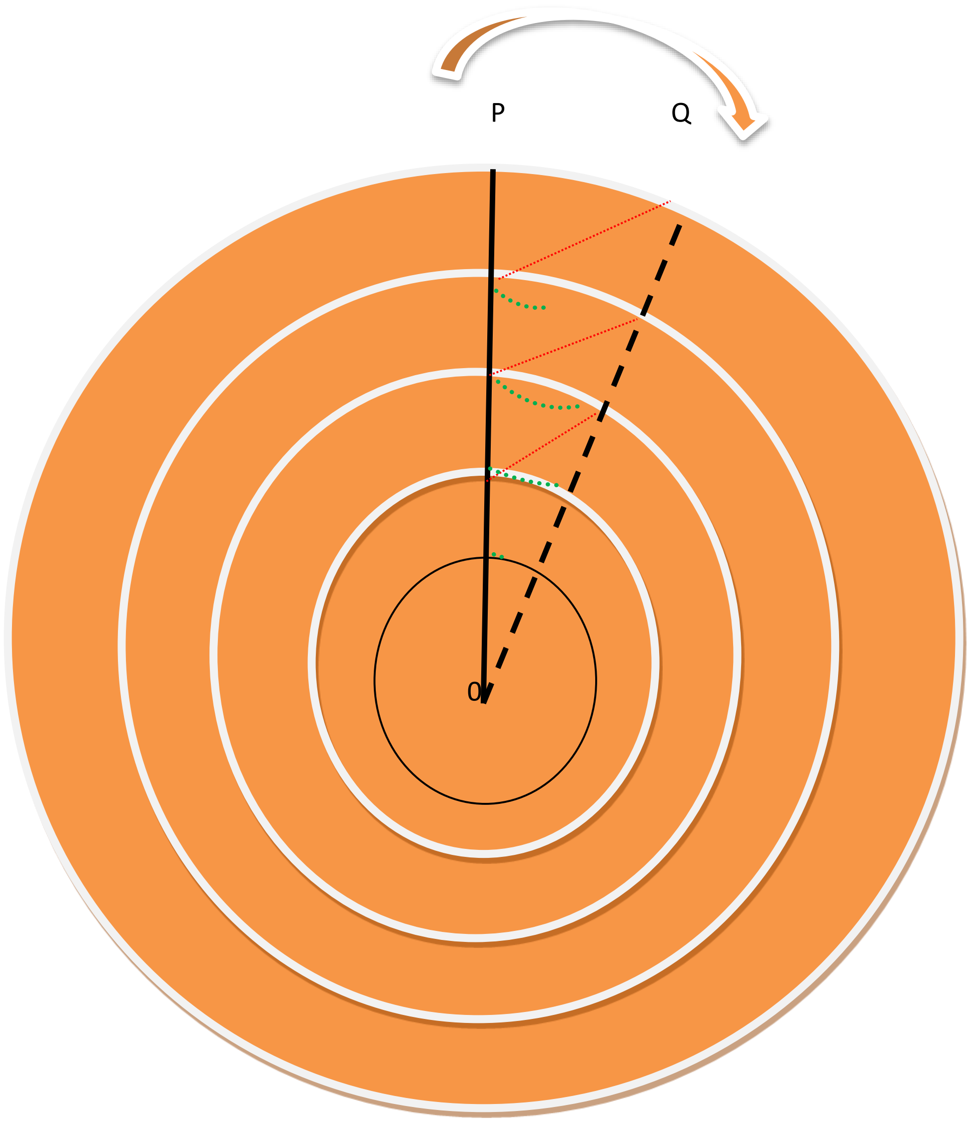


Figure 5.9: The perceived awkward looking holistic track of a single shift of the multi-tower CPIS

5.7 **Multi-Tower CPIS and Water Spread Mechanism**

This practical or hands-on stage of this intensive exploratory study focused on a five-tower CPIS at the research site, and encompassed using a stopwatch to determine the time it took individual wheel sets carrying the towers to “sweep” a sector area or arc length within a field strip before coming to a halt. The emergent questions hinted above coupled with the central research questions posed in the introductory part of the study, particularly on CPIS mobility and water spread patterns, steered and guided this stage of the project.

According to the farm manager at the research site, the observed five-tower CPIS was set at 85% of a minute on its "percentage timer" (see Figure 5.10) for the device) just to ensure enough moisture for the planted Soya beans to be easily up-rooted by the farm labourers.

Figure 5.10: A picture of a percent input/timer

In addition to determining times taken by individual CPIS towers to sweep arc lengths in field strips A_1 to A_5, the exploration involved ascertaining the uniformity of water released at identified spots within the field strips from the innermost field strip (A_1) to the outermost field strip (A_5). This was informed by Harrison's (2012) useful hint that water application uniformity under a certain centre pivot irrigation system (CPIS) is determined by setting out cans or rain gauges along the length of system, bringing it up to proper operating pressure and letting it pass over the cans or rain gauges. In this practical exploration, rain gauges were duly set out in successive field strips between the wheel sets carrying the machine towers and water was collected at such identified spots on the irrigation field. The water collection exercise was repeated at the identified field spots within the field sectors in order to maximize the accuracy of measurement. The data emerging from the practical activities mentioned above were vividly captured using Table 5.4.

Table 5.4: The data for the complex circular motion and water spread of the actual five-tower CPIS on a 5m radius irrigation field

Field strip	Water collection reading 1 (ml)	Water collection reading 2 (ml)	Water collection reading 3 (ml)	Average water collection (ml)	Time taken to sweep a field sector area (seconds)	Rate of water spread per swept field sector (ml/s)
A_1 (Innermost swept sector area)	5,7	5,9	5,8	5.80	47	0,123
A_2	5,9	5,9	5,8	5,87	48	0,122
A_3	5,9	6,0	6,5	6,13	49	0,125
A_4	6,6	6,5	6,0	6,37	50	0,127
A_5 (Outermost swept sector area)	6,5	6,0	7.0	6,50	51	0,127

5.8 **Reflection and Conclusion on Multi-Tower CPIS Data**

The data in Table 5, particularly in the “average water collection” column, seem to confirm an earlier assertion in the previous chapter that the sprinkler nozzle radii from the innermost tower span to the extreme span cannot be the same as assumed in the previous theoretical approach chapters. The seemingly slight difference in the readings across the length of the machine radius clearly confirms Frenken’s (2006) assertion that nozzle sizes of the sprinklers used in a CPIS are smallest at the inner spans and increase with distance from the pivot centre. However, these smaller sprinkler nozzle sizes used at the inner spans seem to be compensated by the slower pace or rotational speed of these towers in order for the machine to achieve uniformity of water spread. For example the wheel set closest the pivot point moves very slowly while releasing a small average water of 5,80 milliliters at the identified spot in the innermost field strip. Felck and Frenken (2005) concur by indicating that centre pivot systems require a continuously variable emitter flow rate across the radius of the machine in order to achieve uniform water application.

The intensive practical approach data in Table 5.4, as evidenced from the “rate of water spread” column, yielded a near uniform or constant water spread model across the field strips of approximately 0,12 milliliters per second. This model differs from the “constant increase” mathematical models of water spread suggested by the data in the previous chapters of the theoretical approach to the machine’s exploration. The efficient management and control aspects of the actual CPIS’ overall operation, which were not essential object of this study, clearly has an impact on application uniformity of the machine. This, therefore, begs for a thorough understanding of the system assembly aspects as well as the factors such as pressure regulations, land topography and the calculation of a “coefficient of uniformity” suggested by Harrison (2012).

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 **Introduction**

The study was inspired by the way the centre pivot irrigation system moves as well as the way it spreads water to avoid swelling or water-logging of the centre. In the provinces where farming is a major activity, pursuing this form of activity could inspire a lot of learners as it has a potential to bring mathematics closer to their environment.

6.2 **Addressing the research questions**

6.2.1 **Research Question #1: School level mathematics concepts embedded in CPIS**

This study had two research questions. The first research question was: '*What are the general mathematics concepts that are "grounded" in the CPIS' mobility and water spread patterns per sections of the irrigation field'* (Section 1.3.1)? This question was pursued throughout the different sections of this report. Geometrical and algebraic patterns were created, analysed and interpreted in the context of the centre pivot irrigation systems. That, however, did not provide complete coverage of the possible mathematics that is embedded in the system. Much more work can still be done in the area.

6.2.2 **Research Question #2:** ***"Potential" authentic tasks within school level mathematics***

The data from both theoretical and practical approaches to both linear move and a circular move irrigations machines (Tables 4.1 to 4.4 and Tables 5.1 to 5.4) make it clear that certain general school level mathematics concepts may indeed be "unearthed" or "generated" and consolidated within the CPIS setting. Mathematics concepts "grounded" in this context include, inter alia, area of circles and rectangles; number patterns with constant first difference; ratio and rate (speed of CPIS towers and rate of water spread); unit conversions and measurement in real-world setting, for example, the use of rain gauges and a stopwatch in the practical approach to the exploration of the actual CPIS (section 5.7).

The activities as well as the critical and creative aspects involved throughout the exploration are as advocated by the new mathematics curriculum documents. For example, the use of the “what if” questions in generating the data in Table 4.2 may further enhance learners understanding of the concept of area, thereby arousing the awareness campaigns for responsible water use. The design and use of tables to capture emergent data may, if left for learners to decide on, cater for the learners’ conceptual grasp “throughout the concrete-to-representational-to-abstract sequence of mathematical understanding” (DoE, 2007).

6.3 **Packaging Activities**

The package of the “possible” authentic activities involved throughout this exploratory study (see Tables 4.1 to 4.4 and Figures 4.6 and 4.7) may be used as a teaching and learning task as outlined below.

6.3.1 **Activity 1**

This may be designed to allow learners to generate the data in Tables 4.1, 4.3 and 4.4 by focusing on the following aspects:

- Considering a linear move irrigation machine, on a rectangular field, to determine the relationship between areas of the first horizontal field strip receiving and not receiving water at the start of irrigation.
- Finding theoretical mathematical models or general rules applicable in determining these areas as the field width (machine radius and number of sprinkler nozzles) changes.
- Extending the learners' thinking by introducing the "what if" questions such as the one that the entire horizontal field strip receive water when irrigation commences. The choice steps or units of translating the machine toward the outside of the field edge, but within feasible limits of the sprinkler nozzle radius, may depend on the developmental stages of learners executing the activity.
- An integration of subjects and learning areas may be catered for by soliciting from learners, through guiding questions, a response that explicates their awareness of the need to use water sparingly as well as related water policies.

6.3.2 **Activity 2**

This may involve sheer observation that at the start of irrigation within the first horizontal field strip on a rectangular field, some points or areas receive more water as a result of co-incising "water throw" from adjacent sprinkler nozzles. A mathematical model for the relationship between the number of these "nodal points" and number of sprinkler nozzles fitted along the field width may be determined by learners to this effect.

6.3.3 **Activity 3**

This may relate to finding the rate of water spread from a circular move single tower CPIS (Table 5.1). This, however, would require that learners understand that the speed of the machine's single tower as well as the amount of water used daily for irrigating a field are determined by the farmer or user depending on such factors as soil topography, type of crops and soil capacity to hold water (aeration). This clearly speaks to controlled, independent and dependent variables in an experimental set-up. The issue of measuring units for the water spread rate may, if opened for learners to decide on, pose an interesting challenge that may stretch learner thinking in this regard. In addition, the algorithms or procedural fluency required to arrive at the rate of water spread may be enhanced in this regard.

6.3.4 **Activity 4**

This may be set out in situations where the single tower CPIS has to irrigate fractions of a circular field, that is, considering quarter circles, semi-circles, and three quarter circles instead of full rotations or circles. Although the necessary algorithm or formula as suggested by Russel (2013) is hardly taught at school level, this may be handled as a useful expanded opportunity as advocated in mathematics curriculum documents.

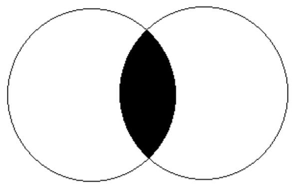
6.3.5 **Activity 5**

This may be a hands-on, field work under supervision at a research site wherein learners carry out some predetermined instructions on work sheets. This may clearly offer a mammoth of opportunities wherein mathematics process skills such as following instructions, ability to handle measuring instruments, taking measurements (e.g. taking the bottom of the meniscus when measuring amount of water collected in the rain gauges), recording using various modes of representation (e.g. tables; symbolic representation using algebraic variables) and reporting results both orally and visually using appropriate technology (e.g. power point presentations) may be facilitated and assessed. The joy and fun of having to work in an authentic, realistic setting in the field may have positive spin-offs regarding learner attitude towards mathematics, which has always been a feeling of negativity worldwide (Frykholm & Glasson, 2005). This is indeed the joy of learners having to “bang their heads against a mathematical wall and then having to discover ways of either going over or around the wall” to reach desired levels of understanding the operations of the CPIS, particularly how it moves and spread water uniformly across the entire irrigation field.

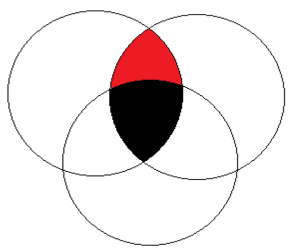
When one reverts back to Figures 4.6, 4.8, 5.3 and 5.4 about the water spread patterns suggested therein, it is crucial to realize that these are indeed mathematics exploration and learning opportunities for learners. How self-fulfilling would it be for learners to engage in the production of their own colour decorated water spread patterns of an irrigation field? The learners’ creativity and design capabilities would be aroused and nurtured if given these rare opportunities from earlier grade levels in the senior phase of schooling. After all, one of the founding principles of mathematics curriculum in South Africa is “active and critical learning” wherein an active and critical approach to learning, rather than rote and uncritical learning of given truths is encouraged ([DoE], 2011, p. 4).

6.3.6 **Activity 6**

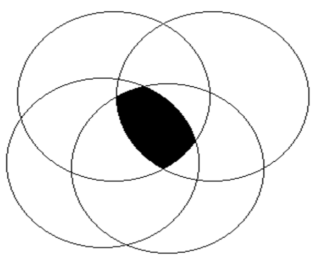
This may be a consolidation exercise for learners in the senior phase where learners may be asked to identify the various geometric shapes formed by the area of common overlap of two, three and four circles. Weisstein (1999), on one hand, refers to an area of overlap of two circles as either a “symmetrical or asymmetrical lens” depending on whether the radii of the circles are equal or not and whether a circumference-to-centre arrangement of the circles is considered or not. Fewell (2006), on the other hand, used the concepts of “circular triangle” and “circular quadrilateral” to identify the geometric shapes formed at the area of common overlap of three and four circles respectively (see Figure 6.1). In using these terms, Fewell (2006) reasoned that the sides of these shapes are actually arcs of different circles.



(a) Lens shape



(b) Circular triangles



(c) Circular quadrilateral

Figure 6.1: Geometric shapes formed at area of common overlap of two, three and four identical circles (Weisstein, 1999)

Awkward sounding as it may, learners may either draw or imagine straight lines joining the points of intersection of the circles to check if they obtain “normal” triangles and quadrilateral in these areas of common overlap.

Furthermore, learners may be asked to come up with a necessary condition that must be met in order for two circles to overlap. Weisstein (1999) indicated that for two circles to produce an area of common overlap, the distance between their centres must not be more than twice the radii of the circles.

The activity may, therefore, incorporate the "what-if" questions that lead learners to the realization of this condition. For example, how would the area of common overlap between two circles be affected if the distance between their centres is zero, equal to any value within the limit of the diameter of one of the circles? If the radii of both circles are equal in magnitude, would the shape formed by the area of common overlap be symmetrical or asymmetrical about the line joining the points of intersection of the two circles (radical line)?

The identification act in this activity may further be extended to Figure 5.6 where learners may be asked to focus on patterns that seem to be shown by the number of sprinkler nozzles and the "lens-shaped" areas within sectors of the quarter field, especially from field sector A_2 going towards the field edge. These patterns become easier to spot if learners are directed to regard sector A_2 as their point of departure or being told to assume sector A_1 as a pivot point similar to a radio dial knob for switching from one radio station to the other. This assumption would, therefore, allow learners to identify the sequences formed by the numbers of sprinkler nozzles and "lens-shaped" areas within the quarter field from the second sector to the field edge as 5; 8; 11;…. and 4; 7; 10;… respectively. These sequences would then have the general linear expressions (3x +2) and (3x +1) respectively, where x represents the sector numbers with A_2 now regarded as A_1.

6.3.7 **Activity 7**

Most learners in the senior phase and even beyond, struggle with the concepts of space as advocated in the learning outcome of “Space and Measurement”. The use of Venn diagrams in the section of “Probability calculations for dependent events” in grade eleven quickly comes to mind at this stage.

This activity may, therefore, assist learners in consolidating their identification ability regarding areas of common overlap between and among circles, and hence Venn diagrams if administered in earlier grades of high school (grades 8 and 9). In this case letters of alphabet (variables) may be randomly placed in different areas of common overlap and learners asked to tell into how many circles the variables lie. Most often learners may fail to realize that it would be easier to use “concave” and “convex” arcs surrounding the variable to tell an answer.

The calculations of areas of the common overlap of circles alluded to in Figure 6.1, though stretching beyond school level mathematics as prescribed in the CAPS documents, may serve as good expanded teaching and learning opportunities or enrichment opportunities for learners in the Further Education Training (FET) band of schooling, that is, from grade ten to grade twelve. Weisstein (1999) gave a complicated formula that may be used to determine the area of common overlap between two circles as: $A = 2R^2\text{Cos}^{-1}(d/2R) - (1/2)d \times \text{sqrt}\,(4R^2 - d^2)$, where d is the distance between the circle radii R and r of the two circles.

However, a school level expanded opportunity activity based on this area of common overlap between two circles with equal radii may still be administered in the FET band (grades 10-12).

6.3.8 **Activity 8**

This activity would require prior foundational knowledge of properties of a rhombus and equilateral triangles, the Cartesian plane, transformation (reflection of shapes across a vertical or horizontal line, Pythagorean theorem, cosine rule or cosine ratio as well as the formula for calculating area of circle segment, which may be stated in the problem or left out for learners to look for using relevant sources, together with the common value for the radii of the circles. The diagram in Figure 6.2 vividly captures the two circles and their common radius as well as the area of common overlap hinted in the scenario above.

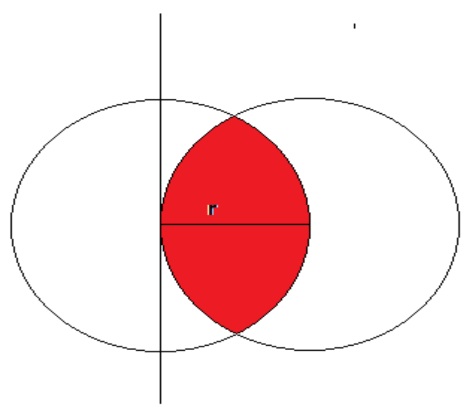


Figure 6.2: A schematic of area of common overlap of two identical circles on a Cartesian plane (Weisstein, 1999)

The diagram in Figure 6.2 may be followed by leading questions that “scaffold” learners’ understanding of the scenario and hence the calculation of the shaded area. Learners should understand and deduce that if the radii of the circles are equal, then an equilateral triangle or a rhombus may be constructed in the area of overlap by drawing straight lines to the centres to each other and then to the points of intersection of the circles, with all sides of the triangle or rhombus equal to the common radius (r). Weisstein (1999) provides the following breakdown of steps needed to finally find the total area of common overlap of circles with equal radii:

- Construction of an equilateral triangle in the shaded area of Figure 6.2 above by joining the centres together, and then joining each centre to one of the points of intersection of the circles (the cusps of the "lens" shape);
- Calculating the central angle that would assist in finding the area of a circle sector;
- Using the area formula, Area = 0,5 x r x r x Cos (central angle) or A = 0,5 x base x height, to determine the area of the equilateral triangle; and,
- Calculating the area of a sector, subtracting the area of the equilateral triangle and multiplying the result by two to get the total area of common overlap of the circles.

6.3.9 **Activity 9**

This may be a further extension activity that requires grades 11-12 learners to determine the area of common overlap of three identical circles, that is, with equal radii. Croft, Falconer and Guy (1991) liken the shape of this area of common overlap to a Reuleaux triangle, which they define as a constant width curve based on an equilateral. Weisstein (1999) gave the formula for the area of a Reuleaux triangle as: A= $0,5\pi s^2 - 0,5\text{sqrt}(3)s^2$, where s is the constant width (common radius of the circles). Weisstein (1999) further alleges that a breakdown of the steps towards finding this area of common overlap of three circles may be as follows:

- Finding the sector area and multiplying it by three (number of sectors overlapping); and
- Subtracting the area of two equilateral triangles from value of the three sector areas to account for over-count of the triangles to get an answer.

This, however, is the same as finding the total area of three sectors forming the common overlap portion and adding the area of the equilateral triangle once. The diagram in Figure 6.3 shows the Reuleaux triangle formed by the green and black areas between the arcs of the three circles. The straight line equilateral triangle, shaded in black, may be constructed within the Reuleaux triangle.

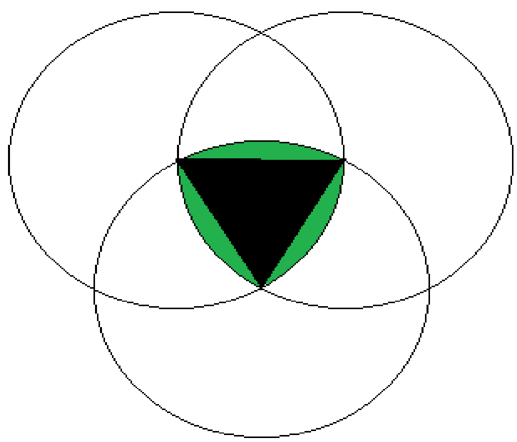


Figure 6.3: Reuleaux triangle at the area of common overlap of three identical circles (Weisstein, 1999)

It, therefore, follows that the CPIS context or setting caters for the development of the afore-mentioned mathematics concepts due to its circular pattern of movement as well as its towers which could be thought of as representing radii of various circles. In addition, the data in Tables 4.1 and 4.2 could be used to facilitate the concepts of number patterns or sequences within the FET band of the school system. For example, the theoretical model for data in various columns of Tables 4.1 and 4.2 might be useful in offering “scaffolds” necessary for conceptual understanding of sequences in the CPIS setting.

However, the concept of sector area calculations in school level mathematics is restricted to quarters and half sectors. The CAPS document for Mathematical Literacy ([DoE], 2011, p. 68) concurs by stating that learners in grade ten should be able to perform calculations of area of circles (quarters, semi and three quarters) using known formulae. Perhaps, exercises or assessment tasks involving other circle sectors than those mentioned in the document might be administered to Mathematics learners as “expanded opportunities” or “enrichment or extension exercises”.

6.4 **Linking the creative mathematics and problem-solving to the study**

According to Forbes (1995) creativity is process that involves novelty, originality, innovations and inventions in problem-solving. However, all inventions as creative solutions but not all creative solutions are innovations. This means that if one happens to know the solution to a problem without previously thinking about the problem, then that solution is not creative. During the unfolding of this study, the researcher had to think deeply about data collection techniques like generating and using sketches inspired by the mobility and water spread patterns of the CPIS using computer skills using the sketchpad program. This meant mimicking and simulating how the machine moves and spread water across the irrigation acreage, which was not an overt act but yet effective in ensuring that the research questions driving the study are addressed. Furthermore, the “possible” teaching and learning activities packaged require some measure of “out-of-the-box” thinking from learners in the identified grade levels of schooling (sections 6.3.1 to 6.3.9). The bottom line is that a real-life context known to the researcher was explored to a certain level of “understanding” how the machine moves and spread water evenly on a field. After all, creative problem-solving encompasses mental shift from working in less-authentic settings; problem reframing techniques; multiple-idea techniques; and inducing change of perspective (Forbes, 1995). These techniques are evident in solutions that have “elegant” characteristics such as using existing components of a problem without introducing any new components in the solution. Thus, a creative solution must solve the stated problem in a novel way and the solution must be reached by the learners independently.

6.5 **Recommendations**

The study finally indicated that the question of designing and packaging a “potentially” authentic task, comprising of interwoven activities, using the CPIS context for teaching and learning still needed to be thoroughly done and polished. This might, perhaps, be viewed as a “grey area” that requires to be zoomed into and evaluated under real classroom interactions in a follow-up study. These model-eliciting activities (MEAs), as advocated by Chamberlin and Moon (2005), might help to avert the “widespread teacher frustration” on the use of projects and investigations in “alternative learner assessment”. A further research study may, therefore, focus on the design and thorough packaging of authentic tasks that might be useful in actual school settings.

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APPENDIX A

Dear Farm Manager/Owner

I hereby kindly request you to allow me, [Tau ME} masters student registered with the University of Limpopo (Turf loop Campus)], to execute my research project on your farm.

The execution of the research study will focus on the mobility and water spread mechanisms of one of your irrigation technologies, the Centre Pivot Irrigation System, which I strongly believe may be used as a “rich” context in the teaching and learning of mathematics concepts at school level (grades R-12). I promise that the execution and findings of the study will in no way hamper or negatively affect your daily work as well as the integrity of the farm.

Thanking you in advance

Yours sincerely

Tau ME	Signature: ……………..
(Researcher)	Date: ……………………