REVIEW OF EXTREME VALUE THEORY WITH APPLICATION TO THE JOHANNESBURG STOCK EXCHANGE FINANCIAL MARKET DATA

by

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DISSERTATION

Submitted in fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

STATISTICS

in the

FACULTY OF SCIENCE AND AGRICULTURE
(School of Mathematical and Computer Sciences)

at the

UNIVERSITY OF LIMPOPO

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JANUARY 2023

Declaration

I, **Maashele Kholofelo Metwane**, declare that the dissertation hereby submitted to the University of Limpopo, for the degree of Master of Science in Statistics is my original work and has not been previously submitted by me before for a degree at this or other University. I further make a delcaration that the material sited or quoted herein has been duly acknowledged in the form of a comprehensive list of references.

Signature:...........*MKM*.............Date:...................09 January 2023......................
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Abstract

The financial sector is vital to the economy as it encourages economic growth. Therefore, modelling the extremes and risk management of financial stock returns and losses is vital to economic survival. However, with limited amount of information it can be difficult to estimate the parameters of the distributions due to the enormous variance and heavy tails of the financial stock returns. The problem of limited amount of information has inspired the present study to examine and model the extreme value behaviour of the Johannesburg Stock Exchange (JSE) financial market data using the extreme value theory (EVT). The study employed secondary data acquired from the JSE in South Africa, which comprises the daily total return index of the All-Share Index (ALSI) as well as the daily USD/ZAR exchange rates. The generalised extreme value distribution (GEVD), r-largest order statistics GEVD (rGEVD), generalised Pareto distribution (GPD), and the Poisson point process along with the newly proposed alternative for GEVD called blended GEVD (bGEVD) models were applied to the five-year daily JSE financial market data. The parent distribution of the maximum daily JSE financial market data was investigated and the Gamma distribution was found to be the optimal parent distribution. The block maxima method was employed in the study to fit the EVT models. The GEVD models for the USD/ZAR exchange rate and the All Share Total Return Index (ALSTRI) were developed using the weekly and monthly block maxima method. Both results of weekly and monthly maxima GEVD and the monthly rGEVD models for the ALSTRI and the USD/ZAR exchange rate can be modelled by the Weibull and/or Gumbel family. The 100-year return levels of the monthly GEVD, bGEVD, and rGEVD models are almost equal to the maximum observations of the financial markets, revealing that the ALSTRI and USD/ZAR exchange rates will exceed 10802 and R18.89 respectively, at least once in 100 years. The Poisson point process return level estimates are quite comparable with the GPD estimates, indicating that the ALSTRI and USD/ZAR exchange rates will surpass 17501.63 and R23.72 respectively, at least once in 100 years. This implies that the investors will experience higher gains in the total returns of the ALSI. The USD/ZAR exchange rate return levels suggest that the Rand will become more unstable in the long run. Instead of focusing merely on the traditional methods of block maxima, the use of advanced extreme value methods that accommodate even small datasets such as GPD, r-largest order statistics, bGEVD, and Poisson point process are encouraged. The researcher discovered that there are no studies conducted on bGEVD in the field of finance or financial markets. In the future, more studies on bGEVD, vine copulas, and r-largest order bGEVD can be conducted on the financial markets and/or finance sector. Therefore, the present study will add value to the literature and knowledge of statistics and econometrics.

Keywords: blended generalised extreme value distribution, extreme value theory, generalised extreme value distribution, generalised Pareto distribution, Johannesburg Stock Exchange, maxima, Peak-over-threshold, Poisson point process, r-largest order statistics, VaR.

Dedication

I would like to dedicate this dissertation to my son (Mr Kgalalelo Metwane), and my parents (Mr Mashishimale Patrick Metwane and Mrs Mogale Getrude Metwane).

Acknowledgments

I would like to offer my special thanks to God Almighty, for making it possible for me and giving me strength and ability to press on. I would like to express my great appreciation and special thanks to my supervisor Professor Daniel Maposa for his patience, guidance, support, and encouragements throughout the dissertation.

I would like to express my deep gratitude to the Johannesburg Stock Exchange (JSE), South Africa for providing and assisting with data for the success of this dissertation.

My special thanks are extended to my family and friends for the support, courage and prayers: my son (Kgalalelo Metwane), my mother (Mogale Getrude Metwane), my sister (Carol Makola), my brothers (Dennis Makola, and Jacky Mokadi), Makgomo Modipa, Boitumelo Kgoloko, Daniel Mashilo, and Andani Madodonga.

To my former Manager, the Director: Affordable Medicines Directorate at the National Department of Health, Ms Khadija Jamaloodien and my current Manager, the Acting Director: Organisational Performance and Reporting at the National Skills Fund, Mr Solly Baloyi; thank you so much for your constant support and encouragement. Much appreciated.

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List of Abbreviations and Acronyms

A-D	Anderson-Darling
ADF	Augmented Dickey-Fuller
AIC	Akaike Information Criterion
ALSI	All Share Index
ALSTRI	All Share Total Return Index
ARMA	Autoregressive-Moving Average
bGEVD	Blended Generalised Extreme Value Distribution
BIC	Bayesian Information Criterion
BRICS	Brazil, Russia, India, China and South Africa
Canada-S&P/TSX	Standard and Poor's / Toronto Stock Exchange
CDF	Cummulative Density Function
China-SSE	China - Shanghai Stock Exchange
COVID-19	Coronavirus of 2019
CSI	Company Security Index
CVaR	Conditional Value-at-Risk
EGARCH	Exponential Generalised Autoregressive Conditional Heteroskedasticity
ES	Expected Shortfall
Europe-Euro Stoxx	Europe - Eurozone Stock Index
EVA	Extreme Value Analysis

EVM	Extreme Value Machine
EVT	Extreme Value Theory
FIAGARCH	Fractionally Integrated Asymmetric power Generalised Autoregressive Conditional Heteroskedasticity
FIGARCH	Fractionally Integrated Generalised Autoregressive Conditional Heteroskedasticity
France-CAC	France - Cotation Assistée en Continu
FTSE	Financial Times Stock Exchange
GARCH	Generalised Autoregressive Conditional Heteroskedasticity
GED	Generalised Error Distribution
Germany-DAX	German - Deutscher Aktein Index
GEVD	Generalised Extreme Value Distribution
GEVDC	Generalised Extreme Value Distribution Classifier
GPD	Generalised Pareto Distribution
GPDC	Generalised Pareto Distribution Classifier
GSV	Google Search Volume
Hong Kong-HIS	Hong Kong - Hang Seng Index
HyGARCH	Hyperbolic Generalised Autoregressive Conditional Heteroskedasticity
i.i.d	independent and identically distributed
India-BSESN	India - Bombay Stock Exchange Sensitive Index
INLA	Intergrated Nested Laplace Approximation
Japan-N	Japan - Nikkei Index
J-B	Jaque-Bera
JSE	Johannesburg Stock Exchange
KPSS	Kwiatkowski, Phillips, Schmidt and Shin
K-S	Kolmogorov-Smirnow
MCMC	Markov Chain Monte Carlo
MEVT	Multivariate Extreme Value Theory

M-K	Mann-Kendall
MLE	Maximum Likelihood Estimation
MoM	Moments of Moments
NASDAQ	National Association of Securities Dealers Authority Quotation
NICD	National Institute for Communicable Diseases
OLS	Ordinary Least Squares
P^3C	Property Preserving Penalised Complexity
PDF	Probability Density Function
POT	Peak-Over-Threshold
P-P plot	Probability-Probability plot
p-value	Probability Value
PWM	Probability Weighted Moments
Q-Q plot	Quantile-Quantile plot
rGEVD	r-largest Generalised Extreme Value Distribution
SAWS	South African Weather Services
Sensex	Sensitive Index
South Korea-KOPSI	South Korea - Korea Composite Stock Price Index
S&P	Standard and Poor's
S-W	Shapiro-Wilk
Switzerland-SMI	Switzerland - Swiss Market Index
UK	United Kingdom
UK-FTSE	United Kingdom - Financial Times Stock Exchange
USA	United States of America
USD/ZAR	United States Dollar against the South African Rand
US-SPX	United States - Standard and Poor's Index
VaR	Value-at-Risk

Chapter 1

Introduction and background

1.1 Introduction and background

The primary engine of the economy that fosters economic growth and development is the financial system. Financial infrastructure enables economic development and growth by distributing funds from financial entities to potential investors (Darškuvienė, 2010). In general, financial markets are frequently referred to be the global economy's "barometer" of the nation (Wei and Han, 2021). There are three main components of the financial system of an economy namely; financial regulators, financial intermediaries, and financial markets. The financial markets comprise amongst others; exchange rates, stock markets, government bonds, credit default swap markets, equity markets, debt markets, and derivatives markets (Darškuvienė, 2010; Wei and Han, 2021). According to Su (2020), the risk in one stock market is very likely to be diffused to another stock market because the global markets are not regulated and integrated.

There are rare events that contribute towards the negative impact on the financial markets such as pandemics and financial crises, among others. Due to the instability and unpredictability of the financial markets, the manifestation of the financial crises and pandemics can put investors, financial economists, and risk experts in an unease situation by not knowing whether the investments will return gains or losses, as well as the unknown impact of these rare events on the stock and financial markets.

Several global financial crises worldwide emerged as early as the nineteenth century. Bordo and Landon-Lane (2010) identified the five global crises which occurred in the years 1880, 1890-1891, 1907-1908, 1913-1914, 1931-1932, and 2007-2008. According to Chikobvu and Jakata (2020), other international crises include but are not limited to; the United States of America (USA) recession from 1937-1938, the 1971 Brazilian Stock market crash, the Japanese "asset price bubble" for the period 1986 to 1991, as well as the 1997 Asian financial crises. These financial crises caused great recession around the globe which affected both emerging and advanced markets. This kind of trend in the financial markets is a cause for concern to the investors and such concern has become vital in financial risk management as risk managers are mainly concerned with measuring market risk as to what possible extent can be the cause of market losses with the main interest in the tails (Makatjane et al., 2021; McNeil and Frey, 2000). The volatility in the financial markets leads to criticism about the existing risk management systems and motivates the search for more appropriate methodologies that can be able to cope with rare extreme events with heavy consequences (Gilli et al., 2006).

In accordance with Zhang and Hamori (2021), one of the examples of rare events instigated by health crisis was the Spanish flue outbreak which occurred in the year 1918. The most recent pandemic worldwide is the coronavirus of

2019 (COVID-19). Since the year 2020, the COVID-19 pandemic has been raging globally and has had a traumatic impact on the global economy, trade and other aspects.

The Johannesburg Stock Exchange (JSE) is the South African stock exchange located in Sandton, Johannesburg city; Gauteng province of South Africa. The JSE was established on 08 November 1887 by Benjamin Wollan and is currently the largest stock exchange in Africa. The JSE provides investors a return on investment in the form of dividends while also channeling funds back into the economy. In addition, it offers a technique for managing price risk and a facility for efficient pricing determination, among others. According to Makhwiting et al. (2014), the JSE is a licensed frontline regulator under the Securities Services Act, 2004, which offers the equity markets, debt/interest rate market, commodities, and currencies, among others.

According to Iyke and Ho (2021), one of the leading currencies in Africa is the South African rand. Due to disrupted financial markets during the COVID-19 pandemic, the South African rand suffered even more. However, some sectors have benefited from exchange rate exposure which includes among others, personal and consumer goods, tobacco, beverages, and technology (Iyke and Ho, 2021). South Africa is the dominant player in the southern African export markets in the sectors and industries such as mining, tobacco, and beverages but tends to import electric/electronic equipment, automobile and car parts. Generally, the sectors and industries that are import or export-dependent experience an increase or decline in stock returns if the local currency depreciates. According to Takyi and Bentum-Ennin (2021), as the pandemic prolongs economies of many countries are exposed to a great threat posed to economic growth and development.

Evaluating the probability of the occurrence of rare and extreme events is crucial in the financial industry. The Extreme value theory (EVT) is the best-suited statistical method to assist with the evaluation and modelling of extreme events. In accordance with Gencay and Selcuk (2004); Williams et al. (2018), EVT is able to model the left and the right tails independently which is important because risk and reward are not equally likely, especially in emerging markets. EVT has become a fairly robust framework for the tail behaviours of distribution with regard to the probability of extreme events in the financial markets (Andreev et al., 2012).

1.2 Problem statement

The changing economic environment has put the financial industry in distress. The financial industry includes among others, insurance companies, commercial banks, investment banks, and financial markets. The most recent pandemic worldwide, COVID-19 has affected the economic development and growth of various economies around the globe. The COVID-19 pandemic has already caused economic damage that supersedes the 2007-2008 global financial crises (Wei and Han, 2021; Takyi and Bentum-Ennin, 2021). The 2007-2008 financial recession and the COVID-19 pandemic are examples of rare disasters that affect and contribute towards the volatile market behaviour, currency collapse, stock market crises, financial distress, and bankruptcy, among others; which have a direct impact on the banking system, financial and stock markets, as well as the stock returns and losses.

An important matter to investment analysts, risk experts, financial portfolios, and financial regulators is the assessment of the probability of rare and extreme events, and modelling of the unknown outcomes and occurrences of these probabilities (Këllezi et al., 2000; Gilli et al., 2006; Makhwiting et al., 2014; Wachter, 2020). In this situation, EVT is an essential methodology to estimate and build statistical models for events with low probabilities and high impact. Extreme value analysis is important in modelling the extremes and risk management of financial stock returns and losses. However, the limited amount of information used for parameter estimation makes it difficult to frame the extreme values and extreme return levels due to the enormous variance and heavy tails of the extremes (Da Silva and do Nascimento, 2019; Coles et al., 2001). The problem of the limited amount of information has inspired the present study to examine the extreme value behaviour of the JSE financial data using the r-largest order statistics and peaks-over-threshold (POT) approaches. Rather than making use of a single maximum value within a block, the r-largest order statistics and POT retains more data and therefore makes maximum use of available information.

1.3 Rationale

Several studies have been conducted about EVT in various disciplines which include, but are not limited to, financial risk measure and assessment, financial markets, engineering, environmental sciences, and meteorology (Këllezi et al., 2000; Gencay and Selcuk, 2004; Soukissian and Kalantzi, 2007; Nemukula and Sigauke, 2018; Nemukula, 2018; Maposa et al., 2021; Mashishi, 2020). According to Coles et al. (2001), EVT has been used for more than 50 years as one of the essential statistical methodologies in the field of applied sciences. In the financial industry, EVT has recently been applied in South Africa and abroad (Makatjane et al., 2021; Takyi and Bentum-Ennin, 2021; Wachter, 2020; Makhwiting et al., 2014).

Gencay and Selcuk (2004) applied EVT to examine the performance and the value-at-risk (VAR) of the emerging markets. The study aimed at investigating the non-linear estimation and forecasting of daily stock market returns of the emerging markets. The study focused on block maxima and minima. The maximum likelihood estimation (MLE) method was used to estimate the parameters of the generalised Pareto distribution (GPD). The results indicated that the GPD is the perfect fit for the left and the right tails of daily stock market returns. Makhwiting et al. (2014) applied the generalised extreme value distribution (GEVD) to model the tail behaviour of daily share returns of the JSE markets over the period 2002 to 2011. The MLE method was used to estimate the parameters of the GEVD. The outcome of the study indicated that the daily share returns of the JSE follow the Weibull class distribution, implying that the GEVD gives a better fit for the daily share returns. Shin et al. (2020) applied the r-largest four-parameter Kappa distribution (rK4D) in order to compare with the r-largest GEVD (rGEVD) to model the 10 largest sea-level data in Venice over the period 1931-1981. The MLE approach was used for parameter estimation. The results demonstrated that the rK4D gives a better fit or less bias compared to the rGEVD.

Bader et al. (2017) employed the GEVD based on the r-largest order for sequential testing. In the study, two specification tests namely; parametric bootstrap and multiplier bootstrap were employed to select r through sequential hypothesis testing. The extreme sea level and precipitation data were used to exemplify the procedure. The parametric bootstrap was used to assess the significance of the observed statistics, whereas the multiplier bootstrap was applied to differentiate the entropy estimated between the rGEV and $(r-1)$GEV models to establish the statistical difference.

Da Silva and do Nascimento (2019) employed the Bayesian parameter estimation approach to estimate parameters of the r-largest order statistics applied to the economic and environmental data. The study used daily maximum temperature data as well as the mean daily return index of the Sao Paulo Stock Exchange (BOVESPA). In addition, the GEVD parameter estimation and return levels were compared to the r-largest order statistics. The simulation findings showed that the Bayesian method performed comparably the same as the MLE in terms of parameter estimation. However, the Bayesian approach was more accurate compared to other estimators. The results indicated that r-largest order statistics is an appropriate method for analysing enormous data with reduced observations.

Several authors in South Africa applied EVT in the financial industry to model the tail behaviour of returns and financial risk using the JSE financial market data concentrating on the GPD and GEVD (Makatjane et al., 2021; Makatjane, 2019; Makhwiting, 2014). Few studies have been conducted using advanced EVT methods in the financial sector such as the r-largest order statistics, rGEVD, and blended GEVD (bGEVD). The literature on studies that apply advanced EVT methods in the financial industry is scarce in South Africa. Thus, this study will employ advanced EVT methods such as the rGEVD, bGEVD and Poisson point process to model the behaviour of extremes of the JSE daily financial market data.

1.4 Aim and objectives

1.4.1 Aim

The study aims to examine and model the extreme value behaviour of the JSE financial market data using EVT.

1.4.2 Objectives

The objectives of the study are to:

1. Investigate the parent distribution of the maximum daily JSE financial market data and select the best fitting distribution using the Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC) values.

2. Use the rGEVD to model the daily JSE financial market data using the MLE and Bayesian estimation methods and compare the variability of the parameter estimates.

3. Use the Bayesian method to select the r-optimal of the largest order and apply the Markov chain Monte Carlo (MCMC) approach.

4. Employ the bGEVD and Poisson point process in modelling daily JSE financial market data.

5. Perform a comparative analysis of the EVT models in order to select the best models that fit the daily JSE financial market data.

1.5 Significance of the study

In South Africa, several studies have been conducted using EVT in the fields of hydrology, environmental sciences, and climatology with few studies conducted using advanced EVT methods such as the r-largest order statistics, rGEVD, and bGEVD in the financial sector. In the present study, the behaviour of extremes with limited data is modelled through the r-largest order statistics, which is an expansion of the block maxima approach. Therefore, the present study will add value to the literature and knowledge of statistics and econometrics. The results of this study will assist investors, investment analysts, risk experts, financial economists, financial analysts, and financial regulators to assess the probability of rare events in modelling and predicting the uncertainties and occurrences using limited available data.

1.6 Structure of the dissertation

The dissertation is organised as follows: Chapter 2 presents the literature review; which entails the previous studies conducted by various authors. Chapter 3 provides the methodology used to model the data, which encompasses the introduction of the methodology, data source, and analytical procedures. Chapter 4 comprises data analysis and discussions of the research findings and Chapter 5 outlines the conclusion of the findings and recommendations.

Chapter 2

Literature review

2.1 Introduction

Several studies have been conducted about EVT and financial stock markets around the globe as well as in South Africa. Thus, this chapter focuses on the existing literature on studies that have been conducted by various authors on EVT, financial stock markets as well as the models applied.

2.2 Global stock market trends during the COVID-19 pandemic

The COVID-19 pandemic which began as an epidemic in December 2019 in China, has affected both emerging and developed economies around the globe. As a result of the pandemic, strict quarantine policies and lockdowns were implemented in several countries worldwide (Zhang and Hamori, 2021), in order to prevent further spread. However, this has affected the economy as some industrial productions were put to a standstill, instigating job losses, affecting the supply and demand; resulting in a global economic recession (Guo et al., 2021). According to Muktadir-Al-Mukit (2021), the COVID-19 pandemic had already severely damaged the global economy with an expected loss of about 7% of the world's Gross Domestic Product (GDP) in the year 2020. According to Smales (2021), the COVID-19 confirmed cases had a strong negative influence on stock returns and has significantly impacted the liquidity of firms that are exposed to China. The average stock markets of the G7 countries namely; Canada, France, Germany, Italy, Japan, the United Kingdom (UK), and the USA had dropped by 36,3% which was almost similar to the G20 countries with a fall of 35,4%. Comparing the economic contribution of the G7 and the G20 countries towards the global GDP; the G7 contributes 40% while the G20 contributes 90% of the global GDP with the stock market capitalisation of $46 trillion and $58 trillion respectively.

Zhang and Hamori (2021) conducted a study titled "crude oil market and stock markets during the COVID-19 pandemic". The study focused on the crude oil and stock markets in three countries namely; the USA, Japan, and Germany. The study aimed at investigating the impact of the COVID-19 pandemic on financial markets by modelling and analysing the return and volatility spillover between the COVID-19 pandemic in 2020, the crude oil market, and the stock market. The data used in the study was for the period 4 January 2006 to 31 August 2020. The data was employed to compare the COVID-19 pandemic for the period 2020 with the 2008 global crisis. It was found that the investors suffered losses in a short period due to the USA stock market circuit breaker which was triggered four times within ten days and the prices surged by 20% in 3 days. International oil prices saw a rare plunge. In March 2020, the main stock markets in Germany dropped by more than 10% whereas the Japanese main stock markets collapsed by more than 20%. The Value at Risk (VAR) model was applied to compute the return and volatility spillover. The result also indicated that the impact of COVID-19 pandemic on the volatility of the oil and stock markets exceeded the volatility of the 2008 global market crisis. The return spillover mainly transpired in the short term whereas in the long term the results indicated an occurrence of volatility spillover.

Wei and Han (2021) studied the impact of the COVID-19 pandemic transmission of monetary policy to financial markets. The study used a sample of 37 countries based on the severity of the pandemic to explore the impact of the COVID-19 pandemic on the transmission of monetary policy to financial markets of both emerging and developing countries. The study employed panel data of 37 countries, from 1 January 2011 to 30 April 2020. The countries include among others; China, South Africa, the UK and, the USA. The data also included each country's rate of exchange against the USA dollar in order to estimate the transmission of monetary policy to exchange rate markets. The findings indicated that the rise of the pandemic has significantly deteriorated the transmission of monetary policy to the financial market.

A study administered by Takahashi and Yamada (2021) focused on factors affecting the Japanese stock market performance during the COVID-19 pandemic. The stock market reaction on the Japanese firms and the impact of the global value chain during the COVID-19 pandemic was explored. It was found that the stock returns for companies with exposure to China and USA were lower. The results indicated that whereas the number of infections was lower in Japan as compared to other developed countries such as China and USA, the Japanese stock market plummeted at the same rate as China and the USA.

Smales (2021) carried out a research titled: "investor attention and global market returns during the COVID-19 crisis". The study was administered with the aim of examining the market response to changes in investor attention. The Google Search Volume (GSV) for the coronavirus keyword was used as a proxy for retail investors' attention. The outcomes of the study indicated that the GSV was a proxy for the retail investor's attention and has confirmed a negative influence on global stock returns during the COVID-19 pandemic. During the COVID-19 crisis period, an increased number of internet searches included a rapid rate of information associated with higher volatility flow into the financial markets. Thus, the stock returns are lower with higher price volatility when investor attention increases.

Muktadir-Al-Mukit (2021) studied COVID-19 information consumption and stock market returns. The study examined the impact of COVID-19 information attention on stock market returns using the GSV as the proxy focusing on COVID-19 cases, lockdown, death and vaccines. The study aimed at examining how COVID-19 information is incorporated into stock prices by stock market investors. The study employed the stock market index return of three developed countries namely; the USA, the UK, and Australia. These three countries' major stock markets combined share was about 62% of the total world equity market value. The pooled ordinary least squares (OLS) regression and multivariate analysis were performed on the panel data to explore the impact of COVID-19 information attention on stock market returns. The authors found that the COVID-19 vaccine and lockdown information has a positive influence on investors' sentiment reflected by increased stock market returns, whereas the COVID-19 death information indicated a negative association with stock market returns. There was a relationship between investor information attention and asset pricing during economic uncertainty and particularly during the pandemic crisis period. Thus, the findings showed a significant positive relationship between the stock market returns and lockdown due to rejuvenation in investor confidence.

Guo et al. (2021) employed the time-varying financial network model and the factor-adjusted regularised model selection to analyse the tail risk contagion between the international financial market of 19 countries during the COVID-19 pandemic. It was found that the COVID-19 pandemic has a significant influence on the tail risk contagion of the financial markets. The global spread of the pandemic directed negative signals to the financial markets which affected stock prices and negatively influenced investment confidence.

The international exchange markets or exchange rates were also influenced by the COVID-19 pandemic. Iyke and Ho (2021) studied the exchange rate exposure in the South African stock market before and during the COVID-19 pandemic to explore the exposure of the exchange rate before and during the COVID-19 pandemic. The daily data on sectoral and industry stock markets, market returns, exchange rate i.e. the ZA Rand/US Dollar exchange rate, and economic factors were used to compare the exposure of the exchange rate risk in various sectors and industries. The exponential generalised autoregressive conditional heteroscedastic (EGARCH) (1,1) model and the multifactor arbitrary pricing model were applied to the daily data for the period 1 January 2019 to 19 November 2020. The results showed that stock markets responded both good and bad, and were largely exposed to the exchange rate before or during the pandemic. There was a significant drop in the stock market index. Sectors like personal goods, beverages, and tobacco have benefited from the exchange rate exposure as well as the few industries like materials, consumer goods, and technology that also benefited while others suffered. Thus, as the rand depreciates the import and export-dependent industries and sectors have experienced a decline.

Takyi and Bentum-Ennin (2021) applied the Bayesian structural time series approach to assess the impact of COVID-19 on stock market performance in thirteen African countries. Daily stock market time series data for the period 01 October 2019 to 30 June 2020 was employed. The Bayesian posterior estimates indicated a significant decline in the African stock market performance ranging between -2.7% and -21%. The findings indicated that out of 19 African countries sampled in the study, the COVID-19 pandemic has significantly affected the stock markets of about 10 African countries; five countries experienced a short-lived negative impact on the stock markets, whereas there was a very significant impact on stock markets in countries like Cote D'Ivoire, Uganda, and South Africa. The authors concluded that there is a poor prospect of a positive effect on African stock market performance due to the pandemic.

Financial markets were impacted by the COVID-19 pandemic around the globe. This massive influence created an unexpected risk ambiguity for investors inspired Omar et al. (2020) to conduct a study titled "Forecasting value-at-risk of financial market under the global pandemic of COVID-19 using conditional extreme value theory". To examine the extreme tail behaviour of the stock indices from twelve major economies, namely; the US-Standard and Poor's Index (SPX), Standard and Poor's (S&P) 500, the UK-Financial Times Stock Exchange (FTSE) 100, Germany- Deutscher Aktein Index (DAX), France- Cotation Assistée en Continu (CAC) 40, Switzerland- Swiss Market Index (SMI), Europe- Eurozone Stock Index (Euro Stoxx) 50, Canada- Standard and Poor's / Toronto Stock Exchange (S&P/TSX) Composite, Japan- Nikkei Index (N) 225, South Korea- Korea Composite Stock Price Index (KOPSI) 200, Hong Kong-Hang Seng Index (HIS), China- Shanghai Stock Exchange (SSE) Shanghai Composite, and India- Bombay Stock Exchange Sensitive Index (BSESN) Sensitive Index (Sensex). The EVT and conditional heteroscedastic models were combined. The daily return data of the stock market indices used in the study covers the period before and during the COVID-19 pandemic. The dynamic forecasting approach of one-day ahead VaR was applied. The back-testing results showed that the conditional EVT methods performed better than the conditional models with asymmetric probability distributions in terms of the one-day VaR for return innovations and estimation of VaR.

2.3 Value-at-Risk and EVT

Financial time series comprises uncertain prices of assets such as stock, foreign currencies, or commodities (Beirlant et al., 2006). The characteristics of financial time series include but are not limited to; volatility, time-varying, excess kurtosis, skewness, and leptokurtic behaviour also known as the heavy tail (Makhwiting et al., 2014; Vee et al., 2014). Risk management in the financial industry such as insurance companies, commercial banks, investment banks, and financial markets is intended to guard against risks of loss due to the drop of foreign exchange, stock market prices, and the occurrence of rare events (Beirlant et al., 2006). There are various types of financial risks which include amongst others; credit risk, financial or liquidity risk, and market risk (Woods and Dowd, 2008). One of the most essential and frequently used methods to measure, assess and evaluate financial risk management is Value-at-Risk (VaR), which is extensively used by risk managers or investors in an attempt to assess or predict the impact of unfavourable events.

Williams et al. (2018) applied VaR EVT to calculate VaR to compare with the traditional VaR methods using the JSE data. The study aimed at determining whether the application of EVT in computing VaR for South African equity markets will provide similar good results as compared to developed markets and examine the best model suited to calculate FTSE/JSE Total Return of All Share Index (ALSI) across a range of quantiles. The study examined and compared the seven traditional VaR models classified as parametric, semi-parametric, and non-parametric as well as the VaR EVT models on FTSE/JSE Total Return ALSI. The daily FTSE/JSE Total Return ALSI data from 30 June 1995 to 17 November 2014 was used in the study. The findings of the study indicated that the VaR semi-parametric method filtered historical simulation was found to be the perfectly suitable model to apply when computing VaR for the South African equity market at 5%, 1%, and 0.5% quantiles although the conditional GPD VaR model performed very well when calculating quantiles 0.1% and smaller.

A study undertaken by McNeil and Frey (2000) concentrated on the EVT to estimate and measure the tail-related risk for heteroscedastic financial time series. A pseudo-maximum likelihood approach was used to fit the generalised autoregressive conditional heteroskedasticity (GARCH) model to estimate the volatility, and EVT was used to estimate the tail of innovation distributions of the GARCH models of the return data of the five historical series namely; the Standard and Poor's index, the DAX index, the BMW share price, the US dollar/British pound exchange rate and the price of gold. The threshold values based on the Hill approach and maximum likelihood GPD parameter estimates were fitted to the data to model both tails of losses and gains. As the GPD approach was compared with other methods, it was found that the GPD was the best preferable fit as it is capable of dealing with the asymmetries in the tails. As compared to the standard GARCH model, it was found that the innovation distribution should be fitted by the fat-tailed distribution preferably using EVT. Thus, EVT outperformed other proposed methods such as standard GARCH models and traditional VaR estimation methods.

The daily losses from 1997 to 2015 of the second-largest pension fund in Uruguay called AFAP SURA were modelled by Magnou (2017) using the GPD and POT in order to investigate the tail distribution of financial losses. These EVT methods were compared with the traditional methods for risk measures such as the expected shortfall and return level. The model parameters were estimated through the MLE approach. It was found that traditional models for risk measures do not consider the volatility of financial markets that cause extreme values. The results show that the POT model is best suited for measuring the size of extreme events. As a result, EVT can be applied to model financial risk measures such as VaR, expected shortfall, and return levels.

In Iran, Tabasi et al. (2019) conducted research entitled: "estimating the conditional value at risk in the Tehran Stock Exchange based on extreme value theory using GARCH models" over the period 2009 to 2015. The aimed to estimate the market risk in the Tehran Stock Exchange by applying the VaR and conditional VaR (CVaR) through EVT. The MLE approach was used for parameter estimation and GARCH models were fitted to model the volatility-clustering feature. The findings of the study indicated that the estimation of model parameters, assuming the T-student distribution function, gave better results than the normal distribution function. The peak-over-threshold (POT) model showed a positive impact on the estimation of risk in the financial markets. The application of EVT based on expected shortfall was recommended by the authors, for the measurement and management of market risk.

Vee et al. (2014) administered a study whereby EVT was applied as a risk measuring tool for the daily closing prices of the indices from six stock exchanges of Frontier markets, namely; the Colombo Stock Exchange, Croatia Stock Exchange, Karachi Stock Exchange, Kazakhstan Stock Exchange, Mauritius Stock Exchange, and Tunisia Stock Exchange. The authors used POT to compute the VaR, and the coverage tests and loss functions were employed to evaluate the performance of the model index-wise. The research findings indicated that the indices for the five Stock Exchanges were fat-tailed excluding the Karachi stock exchange. The GPD was found to be the best fit for estimating the VaR of the indices. However, the EVT VaR did not properly estimate VaR for the Karachi Stock Exchange index. Within the evaluation sample, the returns were found to be significantly less volatile.

Li (2017) conducted a comparative study, comparing EVT and GARCH to model the VaR of the daily loss of stock markets in the USA, the UK, China, and Hong Kong. The daily stock market return data employed in the study was from the US-S&P500, UK-FTSE 100, US-National Association of Securities Dealers Authority Quotation (NASDAQ) composite, Hong Kong-HSI as well as China-Company Security Index (CSI) 300 for the period 2006 to 2015. The data included extreme events such as the China economic slowdown as well as the 2008 global financial crisis. The aim of the study was to understand the strengths and limits of the GARCH and EVT under a less restrictive but more realistic environment. An asymmetric shock of volatility was incorporated in the financial time series and relative forecasting performance was compared. The generalised error distribution (GED) was used to determine the skewed fat-tailed return distribution. The dynamic backtesting method was used to assess the performance of VaR. The MLE parameter estimation method was applied in the study. The results of the back testing technique illustrated that the GARCH and conditional EVT models performed equally well under the GED. The exponential GARCH model was found to be the best in VaR forecasting as the model identifies future extreme losses as well as independent occurrences.

Szubzda and Chlebus (2019) used two EVT-based approaches namely; the BM and POT to assess VaR for market risk in numerous economic conditions. The daily closing prices of markets for the ten European countries which comprised seven emerging countries and three developed countries for the period 2000 to 2009 were applied. Three various economic conditions namely; the pre-2007-2008 global financial crisis, during the 2008 global financial crisis, and the post-2007-2008 global financial crisis were considered in the study. The outcomes of the study demonstrated that the unconditional EVT models were not found to be best suited to give adequate quality VaR forecasts, particularly during the 2007-2008 economic crisis due to the inability to account for the dynamics of variation in conditional variance. It was concluded that it is possible to combine the concepts of EVT and models that take into account the GARCH process in the ranges of return on financial instruments, and it seems that these models allow for obtaining better VaR forecasts as supported in the study conducted by (Li, 2017).

In Canada, Ren and Giles (2010) administered a study entitled “Extreme Value Analysis of daily Canadian crude oil”. EVT was applied to the Canadian spot market, daily returns of crude oil prices between 1998 to 2006. The POT approach was utilised to determine the threshold exceedance to estimate the tail risk measure such as the expected shortfall (ES) and VaR. The study was intended to model the price fluctuations of crude oil in order to provide an effective energy price risk measurement tool. In conclusion, it was found that the GPD was the best fit for the positive daily return series as compared to the negative daily return series. The POT technique provided an effective and simple means for choosing thresholds and estimating parameters. The EVT-based VaR method provided qualitative information for potential extreme risk analysis in the energy market mainly the crude oil. The risk measure estimates calculated under various high quantile levels demonstrated strong stability across thresholds.

Marimoutou et al. (2009) applied EVT to model the VaR of the oil markets namely, the West Texas International (WTI) crude oil and the Brent. The WTI crude oil data covers the period April 1983 to April 2007 whereas the Brent dataset covers the period May 1987 to January 2006. The VaR for short and long trading positions in oil markets was modelled by applying the conditional and unconditional EVT models for forecasting VaR. The EVT models were compared to the modelling methods such as historical simulation, filtered historical simulation, and GARCH. The results have shown that the conditional EVT and filtered historical simulation techniques offered a major improvement over the traditional parametric methods. The GARCH(1,1)-t model yielded good positive results for the left tail and the two combined approaches.

In a study carried out by Youssef et al. (2015) the GARCH-EVT technique was used to estimate the VaR of energy commodities markets. The daily closing spot prices of four energy commodities namely; the WTI crude oil, Europe Brent crude oil, the USA Gulf coast conventional Gasoline Regular (GCCGR), and the New York Harbor Conventional Gasoline Regular (NYHCGR) over the period January 2003 to December 2012. The authors implemented three models namely; the hyperbolic generalised autoregressive conditionally heteroskedasticity (HyGARCH), the fractionally integrated generalised autoregressive conditionally heteroskedasticity (FIGARCH) as well as the fractionally integrated asymmetric power generalised autoregressive conditionally heteroskedasticity (FIAPGARCH) to forecast, and EVT was also considered with the main focus on the tail distribution. The outcomes of the research illustrated that the FIAGARCH model along with the EVT produced better results in forecasting the one-day ahead VaR.

Through the use of stable parameter estimation, Naradh et al., (2022) investigated the fit of stable distributions on the stock exchange risk of the three

FTSE/JSE indices, namely the FTSE/JSE All Share Index, FTSE/JSE Banks Index, FTSE/JSE Mining Index, and USD/ZAR as well as the USD/ZAR exchange rate in the South African financial markets. The stable parameter estimation takes into account the frequent skewness and high-tail behavior in financial data. The univariate Nolan's S0 parameterisation stable distribution was fitted using the maximum likelihood parameter estimation approach. The Kupiec likelihood test and VaR estimates were used to analyse the market risk and the extreme value behaviour of the fitted stable model, respectively. This study demonstrates the applicability of stable parameter estimation, and diagnostics demonstrate that stable models accurately capture large-scale financial data with high tails and skewness, as demonstrated by the Anderson-Darling goodness-of-fit test.

2.4 EVT in other sectors globally

EVT has been commonly applied in different fields by various researchers as early as the nineteenth century with problems associated with hydrology and engineering (Fuller, 1914; Griffith, 1921; Gnedenko, 1948). Astronomy was recorded as the first sector to apply EVT. In the 1970s EVT was applied by (Pickands III, 1975; Balkema and de Haan, 1974).

Hasan et al. (2012) employed GEVD to model extreme temperatures in Penang, Malaysia. The daily maximum temperatures data for the period of 10 years was employed in the study to quantify and describe the behaviour of extreme temperatures for the period 2000 to 2009. Five selection periods namely; weekly, bi-weekly, monthly, quarterly, and half-yearly were considered in the study. The L-Moments method was used to estimate the parameters. The stationary test was conducted through the Augment Dickey-Fuller (ADF) and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) in order to detect trends over various selection periods. The Mann-Kendall (MK) test was executed to determine the presence of a monotonic trend. The study revealed that all the selection periods are stationary whereas the trend test indicated a decreasing trend. The Kolmogorov-Smirnov and Anderson Darling goodness of fit test revealed that modelling using different selection periods derived almost similar fits. However, modelling using weekly maximum provided the best convergence to the GEVD. Thus, GEVD appeared to be equitable for modelling maximum temperatures.

In Canada, An and Pandey (2007) carried out a study to model and estimate the extreme wind speed using the r-largest order statistics approach. The joint GEVD of the r-largest order statistics derived from the theory of the Poisson process was employed to model the daily maximum hourly wind speed data of 38 years and the parameters were estimated through the MLE method. The findings of the study indicated that the Gumbel distribution is best suited to model the r-largest order statistics of the wind speed.

Soares and Scotto (2004) applied the limiting joints GEVD for the r-largest order statistics to model the occurrences of the wave high sea states and estimate the return values of the extreme wave height. The authors carried out a comparative study using the northern North Sea wave time series data for the period 1976 to 1999. These advanced EVT techniques were compared with the classical annual maxima (AM) approach. The authors demonstrated that the advanced EVT techniques are using more data and are more advantageous in the sense that parameter and quantile estimates are more accurate as compared to the estimates computed using the AM approach.

In a study carried out by Sandoval and Raynal-Villasenor (2008), the trivariate method of multivariate extreme value distribution with GEVD marginal was applied to the regional at-site flood frequency estimates obtained from 21 gauging stations in north Mexico. The multivariate constrained optimisation algorithm was used to obtain the parameter estimates through the MLE. The fitted trivariate extreme value distribution was compared with the univariate distributions, the bivariate approach of the logistic model, and the L-moments. The minimum value of the standard error of fit was employed to select the best combination of the analysis for each station. The results revealed a reduction in the standard error of fit when estimating the marginal distribution with trivariate distribution as compared to the univariate and bivariate distribution, which indicated that the proposed trivariate model is the perfect fit to be considered when performing regional flood frequency analysis.

In New Zealand, Ailliot et al. (2011) used mixed methods i.e. the generalised mixed maximum likelihood and L-moments estimations approach to fit the GEVD applied on the maximum daily rainfall for Wellington over the period 1940 to 1999. The researchers carried out the study for theoretical development and practical application of mixed estimation methods for fitting the GEVD using a mixture of two methods namely the MLE and methods of moments (MoM). A simulation study was conducted to verify the asymptotic outcomes. The findings of the research revealed that the estimation techniques were found to be numerically vigorous, computationally efficient, and converged for all the simulations undertaken in the New Zealand data sets of annual maximum rainfall for varying durations and locations.

Bezak et al. (2014) compared the POT approach with the annual maximum (AM) approach applied in the flood frequency analysis for the period 1895 to 2010; from the Litija 1 gauging station on the Sava River in Slovenia. The three-parameter estimation methods, namely; the MoM, L-moments, and MLE were evaluated and compared. The goodness-of-fit test was explored in order to investigate the adequacy of the tested distributions which provides a suitable fit. The main outcomes of the study indicated that the difference between the MOM and MLE is not unambiguous, whereas, in some distributions such as Gumbel and the probability density functions (P3), the MLE method yielded better results than the MOM and ML methods. The POT approach provided better results than the AM method.

The newly developed EVT approach called the blended GEVD (bGEVD) was employed by Vandeskog et al. (2021b) to model the annual extremes of the short-term precipitation of south Norway. This approach comprises the right tail of the Fréchet distribution and the left tail of the Gumbel distribution. The yearly precipitation maxima were modelled in order to generate better return levels of spatial maps. The Bayesian hierarchical model along with the latent Gaussian field was applied to model the annual short-term precipitation maxima over the period 1967 to 2020. The integrated nested Laplace approximation (INLA) was used to perform inference. The research results indicated that the bGEVD performed better than the traditional block maxima models. Thus, the bGEVD generated good estimates for the large returns of the short-term precipitation.

Vandeskog et al. (2021a) used bGEV distribution to model block maxima. The main aim of the study focused on the newly advanced two-step hierarchical method which considered the block maxima and the POT as well as testing the performance of the bGEVD in a simulation study. The research findings illustrated that the bGEVD outperformed the traditional GEVD because it generates accurate estimates, caters for smaller and faster inference, and estimates good return levels. The bGEVD is considered to be the promising alternative of the GEVD for modelling block maxima. The two-step model was found to be having the potential to improve the inference by using more data.

In Bakersfield, a city in Kern County, California, a study was conducted by Castro-Camilo et al. (2021). The authors fitted bGEVD to the monthly maximum concentration of nitrogen dioxide (NO_2) for the period of January 2000 to March 2016. The study's authors proposed three contributions to the regression setting: the bGEVD approach, which smoothly integrates the left tail of a Gumbel distribution and the right tail of a Fréchet distribution; a principled method called property preserving penalised complexity P^{3}C before deciding whether to consider the first and second moment priors of the GEVD; and the reparametrisation of the GEVD. The new proposed bGEVD parameterisation and P^{3}C approaches were implemented using the R package known as integrated nested Laplace approximation (R-INLA). Comparing the effectiveness of the GEVD with the bGEVD in estimating the return levels was done through simulation studies. The study's results showed that when estimating the right tail with a large sample size, GEVD and bGEVD are more comparable, according to the simulation study approach. The bGEVD fit was discovered to marginally outperform the GEVD fit in the case of small sample settings. As a result, it was determined that bGEVD demonstrated the stability of the proposed parameterisation and the P^{3}C prior method in the Bayesian framework when applied to the NO_2 pollution levels.

The yearly maxima of sub-daily precipitation from the southern part of Norway were modelled by (Vandeskog et al., 2022) using the bGEVD. The yearly precipitation maxima were modelled using a Bayesian hierarchical model with a latent Gaussian field and bGEVD instead of the traditional GEVD. The bGEVD's scale parameter was modelled utilizing the two-step method employing the POT to draw conclusions. Along with the stochastic partial difference equation technique, R-INLA was employed to perform inference. In order to create the spatial maps of return level estimates, the model was fitted with the annual sub-daily precipitation maxima from South of Norway. It was discovered that while modelling the yearly maxima of the sub-daily precipitation data with bGEVD, the two-step proposed method offered a better model fit than the traditional inference method.

A study carried out by Seenoi et al. (2020) applied the Bayesian analysis of the four-parameter Kappa distribution (K4D) on the yearly maximum wind speed in Udon Thani, Thailand for the period 1990 to 2015 as well as the yearly maximum sea-level in Fremantle, Australia from 1891 to 1989. The authors argue that the MLE of the K4D is very sensitive to outliers and rarely stable for small sample sizes. Thus, the MLE, L-Moments, and the Bayesian parameter estimation approaches with the five priors were employed to fit the K4D and then compared. A random walk Metropolis-Hastings algorithm was used for simulating the posterior distribution. It was found that the Bayesian parameter estimation outperformed the MLE and L-moments thus, the Bayesian inference for the K4D was the best fit and useful to model extremes.

Pan et al. (2022) conducted a scoping review study and employed the POT approach to model the flood frequency analysis. The annual maxima (AM) and the POT methods were used to model the flood frequency analysis. A scoping review was conducted in order to close the gap by utilising the POT technique which was found by the author to be under-utilised in the flood frequency analysis. Among others; the complexity of selecting the threshold contributes to the under-utilisation of the POT within the flood frequency analysis field. Whereas, the POT methods offer greater flexibility as compared to the AM approach. The sampling process of the POT approach extracted a greater number of data points from the historical records as compared to the AM model. The outcomes of the scoping review illustrated that the POT was more flexible than the AM the reason being that the process for extracting data varies. The POT based on the flood frequency analysis was found to be providing less bias. Thus, POT was found to be a more suitable approach for modelling the flood estimation.

Rudd et al. (2017) introduced the extreme value machine (EVM). EVM is an algorithm that uses classifiers motivated by EVT joint with a machine learning algorithm designed for anomaly detection. In Switzerland, Vignotto and Engelke (2020) administered a study entitled "Extreme value theory for anomaly detection – the GPD classifier". In order to differentiate between the normal and abnormal test data points that are extremely far, training data with classes was considered. The GPD classifier (GPDC) and GEVD classifiers (GEVDC) were the alternative methods employed to detect the data anomalies. These approaches are based on the GPD and GEVD approximation from EVT and were compared with the EMV methodology. The results of the methods indicated that the GPDC and GEVDC were found to be computationally faster as compared to the EVM during the evaluation phase.

2.5 EVT in the financial and stock markets globally

Gilli et al. (2006) applied EVT to measure the tail risk and the related confidence intervals for major stock market indices namely; Dow Jones Euro Stoxx 50, FTSE 100, Hang Seng, Nikkei 225, Swiss Market Index, and S&P 500. The aimed to quantify the market risk by demonstrating the tail distribution estimation using daily returns of financial series. The MLE parameter estimation method was employed. The block maxima approach was compared with the POT approach in order to model the tail-related risk measure, expected shortfall, and return level. In conclusion, it was found that EVT can be utilised to assess extreme events, and the POT was considered as the best method compared to the block maxima.

Cotter (2001) used EVT in order to determine the unconditional optimal margin levels for selecting stock index futures traded on the European exchanges focusing on tail returns. In the study, the non-parametric Hill index was employed to compute the influence of the extreme returns and it was required that the margin should not exceed a range of probability levels. After comparing the extreme value and Gaussian margins, it was found that the common margin requirements are adequate for each contract in providing unbiased costs for traders.

Andreev et al. (2012) carried out a study in Russia and utilised EVT, POT and GPD to model the Russian stock market. The study utilised daily log losses (negative returns) of the largest Russian stock namely, the RTS index over a 15-year period from 1995 to 2009. The proposed methods were employed to evaluate and model the tail-related risk and for testing the volatility of risk management tools. The MLE approach was used to estimate the parameters. The two most widely used methods in EVT namely the block maxima and the POT were applied to model the tail returns in order to integrate the EVT estimates into the risk measures. The Hill plot was used to determine the optimal threshold. The outcomes of the study illustrated that GPD performed better and fits the tail distribution of financial products more accurately compared to the traditional methods of risk measures.

In the USA, Longin and Pagliardi (2016) employed POT and EVT to investigate the tails correlation between the transaction volume and returns associated with crashes and booms in the USA stock markets. The authors utilised the daily S&P 500 for the period 3 January 1950 to 30 September 2015. The GPD for each marginal distribution as well as the Gumbel copula models were fitted to assess model dependence. The POT approach was used to extract extreme returns and volumes in order to select returns the threshold exceedances for returns that lie above or below the extremes. The Gumbel copula function was used to model the dependence between bivariate exceedances. The parameter estimation method employed in the study was MLE. The findings of the study indicated that there is a very low significant correlation between the return and volume in the left and the right tail of the return distribution during the crashes and the boom of the stock markets. This implies that the returns do not depend on the transaction volumes.

Rydman (2018) applied the POT and GPD to the automobile property insurance data in the USA; aiming at setting an appropriate threshold, analysing, and modelling the data. In order to set a threshold, the rule of thumb and the graphical methods such as the mean residual life plot and the parameter stability plot were employed. The MLE and the probability weighted moments (PWM) were explored for estimating the parameters. It was found that the PWM parameter estimation approach was more efficient for small samples, thus the MLE approach was selected as the best parameter estimation approach to fit the GPD using the automobile insurance claims data.

The extreme behavior stock market returns for the five BRICS countries which comprise; Brazil, Russia, India, China, and South Africa for the period 1995 to 2015 was studied by Afuecheta et al. (2020). The study was carried out in order to measure financial risk using EVT. The MLE parameter estimation method was used to fit the parameters of the distributions. Various copula models were fitted to determine the tail dependence of the markets. The Gumbel copula was found to be the best suitable model with significant relationships for all pairs of the markets for BRICS countries and the GEVD was found to be the best fit.

In a study conducted by Khan et al. (2021), EVT was employed to perform a risk analysis of gold prices in Pakistan. The daily gains and losses in the gold prices of the Pakistan Bullion market for the period 1 August 2011 to 30 June 2021 was used in the study. The EVT methods adopted in the study were BM and POT whereas the risk measures used were VaR, return levels as well as ES. The GPD model was also fitted and the parameters were estimated using the MLE parameter estimation technique. The POT was utilised to estimate the threshold, So, the threshold was estimated through the shape parameter plot as well as the mean excess plot. The outcomes of the study illustrated that EVT is a powerful method to estimate the effects of extreme events on the financial markets.

2.6 Application of EVT in other sectors in South Africa

In South Africa, studies have been conducted by various authors where EVT has been applied in several sectors like meteorology and hydrology.

Extremal dependence of monthly maximum temperatures of the Limpopo province in South Africa for the period 1994 to 2009 was modelled by Maposa et al. (2021). The authors employed two modelling approaches in the study, namely; the bivariate conditional extremes model and time-varying threshold. The study was conducted in four meteorological stations located in the Limpopo province namely; Mara, Messina, Polokwane, and Thabazimbi. The research results showed significant positive and negative extremal dependence in some pairs of meteorological stations. The Thabazimbi meteorological station revealed a strong positive extremal dependence on high temperature at Mara meteorological station whereas; the Polokwane meteorological station indicated a significantly strong negative extremal dependence on high-temperature values at Messina meteorological station.

Bhagwandin (2017) carried out a study entitled “multivariate extreme value theory with an application to climate data in the Western Cape province”. The study aimed at evaluating the effectiveness of univariate and multivariate EVT models on the Western Cape’s climate data. Data from five weather stations namely, George Airport, Cape Town International Airport, Langebaanweg, Plettenberg Bay and Vredendal from the year 1965 to 2015 was analysed and modelled. The author employed the block maxima, threshold excess, and the Point Process methods on the weather data mainly focusing on the maxima of the wind speed, temperature, and rainfall. The research results indicated that in both univariate and multivariate cases, the block maxima models were not best suited in modelling the weather variable of the five weather stations. The threshold excess and point process methods produced better models for the weather extremes. In the multivariate case, it turned out that the point process method generated estimates with better accuracy. Thus, the point process model demonstrated an extremely weak dependence on wind and rainfall maxima along with the maxima of speed and temperatures.

Diriba and Debusho (2021) employed the multivariate EVT (MEVT) to model extreme rainfall indices and to examine the behaviour of joint dependence of the extreme rainfall indices. Daily aggregated rainfall data for selected weather stations from the South African Weather Services (SAWS) ranged over the period January 1991 to May 2019 was utilised. In order to investigate the intensities and frequencies of rainfall events, 1-day to 5-day indices of the rainfall were produced. The conditional multivariate model was fitted in order to explore the dependencies between the series of extreme rainfall events using the Laplace marginal transformations. It was found that the multivariate models provided dependence which indicates that all the selected weather stations are not similarly extreme.

Summer daily maximum temperatures of Bisho weather station in the Eastern Cape province of South Africa was modelled by Debusho and Diriba (2016) using Bayesian modelling. The GPD was applied for data analysis. For parameter estimation, the Bayesian inference was used and the results were compared with the estimates generated through the MLE approach. The Bayesian method using Markov chain Monte Carlo (MCMC) for the GPD was used. The study indicated that the informative prior constructed using the East London daily summer data was greater than the informative prior using the Queenstown data. The outcomes of the return level demonstrated an increase in the median return as the length of the return period increased. Thus, the Bayesian analysis produced more accurate parameter estimates as compared to the MLE parameter estimates.

Nemukula and Sigauke (2018) utilised the r-largest order statistics to model the average maximum daily temperature from 2000 to 2010 excluding the non-winder season. The aim of the study focused on reducing the risk of disasters occurring due to heatwaves and extremely high temperatures using the SAWS and Eskom data. The GEVD for the r-largest order was fitted to the yearly temperature maxima. The MLE parameter estimation approach was utilised by the authors in the study. The research results showed that for all r values, the estimates of the shape parameter were found to be negative, this denotes that the GEV distribution of the annual maximum daily temperatures converges to the Weibull distribution family. Out of the 10 order statistics, $r = 4$ was considered to be the best fit. Thus, it was concluded that the Weibull family distribution was the best fit to model the behaviour of the r-largest order statistics which can be used to model the South African annual maximum daily temperatures.

The r-largest order statistics based on the joined GEVD was employed by Kajambeu et al. (2020) to model the probability of the extreme return levels of the flood heights of the Limpopo river at the Beitbridge. The MLE parameter estimation approach was used. The researchers used the monthly flood height data for the period 1992 to 2014. The r-largest order statistics method was regarded to be a more effective approach for modelling the flood heights as compared to the traditional block maxima approach. Therefore, the Gumbel class distribution was considered to be a suitable model for fitting the flood heights of the Limpopo river.

Mashishi et al. (2020) employed the EVT to model the average monthly 100-year i.e for the period 1949 to 2017, return level rainfall for South Africa, and in order to compare the extreme value distribution with the parent distribution. The MLE parameter estimation approach was used in the study to estimate the parameters of the models. The Weibull distribution was found to be the suitable parent distribution to model the data. The parent distribution i.e the Weibull distribution was compared with the GEVD and the Poisson point process. The results indicated that the Weibull distribution was lower in comparison with the Poisson point process and the GEVD. Whereas the EVT provided a higher quantile estimation of the 100-year average monthly rainfall. Thus, the study revealed some evidence that EVT models can play a vital role in reducing the risk of occurrence of disasters and the construction of dams and bridges.

Sikhwari et al. (2022) fitted GEVD to extreme high rainfall in Limpopo Province, South Africa. Data on the daily and monthly maximum rail falls were collected between 1960 and 2020. The extreme rainfall dataset was fitted using the yearly block maxima and the r-largest order statistics approach in the study. In the investigation, the MLE parameter estimate approach was used. The study also made use of the POT methodology. For the situation of $r > 1$, the automatic selection algorithm approach was employed to choose the maxima. The study's conclusions showed that $\text{GEVD}_{r=8}$ was determined to be the most suitable fit for the data.

2.7 Application of EVT in financial sector and the JSE in South Africa

Merwe et al. (2018) administered a study titled: "Bayesian extreme value analysis of stock exchange data". The authors explored the use of EVT in financial modelling using the share losses on the daily JSE Top 40 Index over a period of 10 years. Visual methods such as the mean excess plot, Hill plot, and the Pareto quantile plot were applied to select the threshold. The MLE, PWM, and MOM parameter estimation methods were employed and compared with the Bayes method. A simulation study was executed in order to compare the various estimators. The GPD above the threshold was combined with the non-parametric technique below the threshold. The Bayes method was the best estimation technique and it was used in order to fit the POT model. The authors indicated that the Bayes method can improve parameter estimation and computation of risk measures.

The South African financial index (J580) for the period 1995 to 2018 was analysed and modelled by Chikobvu and Jakata (2020) through the GEVD to estimate the extreme gains and losses. The quarterly block maxima/minima of the monthly returns were fitted to the GEVD and comparative analysis with

GPD was conducted. The main findings of the research demonstrated that EVT is a proficient approach for forecasting potential high risks in advance. Naradh et al. (2021) employed the GPD and the stable distributions to estimate and model the VaR of the JSE indices as well as the South African exchange rate. The hybrid GARCH-type combined with the GPD as well as Nolan's S0-parameterisation stable distribution (SD) were employed to fit the daily closing prices of the USD/ZAR exchange rate as well as the three FTSE/JSE return indices namely; All-Share Index (ALSI), Mining Index and Banks Index. The Kupiec likelihood ratio test was used to estimate and back-test the VaR estimates. The research outcomes indicated that the hybrid exponential GARCH (1,1) model along with the EGARCH (1,1)-SD model of the FTSE/JSE ALSI performed better than the GARCH-GPD model of the FTSE/JSE ALSI at 2,5% VaR level. The outcomes of the FTSE/JSE Bank Index returns showed that the GARCH (1,1)-SD model was a better fit than the GARCH (1,1)-GPD at 95% and 95,5% VaR levels. Furthermore, the results of the returns of the FTSE/JSE Mining Index illustrated that the GARCH (1,1)-SD was the best-suited model in comparison with the GARCH (1,1)-GPD at 5% and 97.5% VaR levels.

Sigauke et al. (2014) used the GPD to model the conditional heteroscedasticity of the JSE stock returns. The daily ALSI of the JSE from 2002 to 2011 was used for the study. The two-stage modelling framework considered in the study is; the autoregressive–moving-average (ARMA)-GARCH model was fitted to the stock return series and then the residuals from the ARMA-GARCH were filtered. The upper tail of the residual series was fitted through the GPD which led to a hybrid called the ARMA-GARCH-GPD. The research results indicated that the Weibull class of the distribution was the best fit for the daily returns data. The ARMA-GARCH-GPD outperformed the ARMA-GARCH model and provided accurate estimates.

A study carried out by Makatjane et al. (2021) focused on EVT to predict the tail behaviours of the financial times stock exchange for the banking closing indices FTSE/JSE. The closing banking indices for the five major South African banks namely; ABSA, Capitec, FNB, Nedbank, and Standard Bank were considered to investigate the tail behaviour of the stock returns and relative risk. The MLE approach was used to estimate the parameters of the GEVD and GPD. The GEVD for the block minima was utilised as well as the GPD for the POT. It was found that all the parameters of the GEVD were significantly positive for all the five major banks in South Africa and the shape parameter was greater than zero. This implied that all the closing banking indices for the FTSE/JSE could be fitted with a Frèchet family of distribution.

Jakata and Chikobvu (2022) applied EVT to model the monthly returns of the South African industrial Index (J520) for the period 1995 to 2018. The authors used EVT, VaR, and ES as methods for extreme financial risk management. The POT approach was used to assess the tail-related risk measure of the returns. The parameter estimation method employed in the study was MLE. The graphical assessment of goodness-of-fit of the models such as Probability-Probability (P-P) plots, Quantile-Quantile (Q-Q) plots, residual plots, scatter plots, return level and density plots were conducted. The findings of the study illustrated that the GDP provided a suitable fit for extreme gains and losses. Extreme high or low returns with low frequency had an impact on investment decisions. As a result, the prospect of potential extreme losses is less than the prospect of potential gains for investors in the South African Industry Index (J520).

2.8 Summary of the chapter

This chapter has reviewed the previous research conducted by various authors in various sectors such as meteorology, hydrology, insurance, and financial markets in several countries around the globe including South Africa. Parameter estimation methods and models used by the researchers were reviewed and outlined in this chapter. The most broadly used parameter estimation method was MLE. EVT and advanced EVT methods have been widely used in other fields such as hydrology and meteorology in South Africa and globally. However, there is limited research in South Africa on modelling the JSE financial market data and the exchange rates using advanced EVT techniques.

Chapter 3

Methodology

3.1 Introduction

To examine and model the extreme value behaviour of the JSE financial markets data, the EVT is used in the study to offer good results in the solution of research problem. Thus, this chapter mainly outlines the statistical techniques and analytical procedures that will be employed to analyse the JSE financial market data.

3.2 Research methodology

3.2.1 Data source and study area

The study will employ secondary data acquired from the JSE in South Africa for the analysis of the financial markets. The data consists of total return of the All-Share Index and the USD/ZAR exchange rates. The two datasets are recorded daily starting from 01 February 2016 to 26 April 2021. The five-year datasets do not contain data for Saturdays, Sundays as well as the public holidays of South Africa.

3.3 Parent distributions

This section confers the probability models namely; the Gamma, normal, log-normal, and Weibull distributions examined in determining the parent distribution that governs the tail behaviour of the JSE daily financial market data.

3.3.1 Gamma distribution

The Gamma distribution's cummulative density function (CDF) is given by:

$$F(x) = \frac{\Gamma_{(x-\gamma)}/\beta^{(\alpha)}}{\Gamma(\alpha)}, \tag{3.1}$$

where Γ is the gamma function, and $\gamma, \beta(> 0)$ and α are the continuous location, scale, and shape parameters respectively (Mashishi, 2020; Beirlant et al., 2004).

3.3.2 Normal distribution

The normal distribution is one of the continuous case of the standard families of probability distributions (Coles et al., 2001). The normal density function is given by

$$f(x|\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(\frac{-(x-\mu)^2}{2\sigma}\right), x \in \mathbb{R}, \tag{3.2}$$

where x is the sample mean, μ is the mean, σ is the standard deviation, (μ and $\sigma > 0$), and are also referred to as the location and scale parameters, respectively

(Coles et al., 2001; Seimela, 2021).

3.3.3 Log-normal distribution

The log-normal distribution follows the normal distribution with logarithm random variables that are normally distributed (Donegan et al., 2013; Masingi, 2021).The probability density function (PDF) is given by

$$f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(\frac{-(logx-\mu)^2}{2\sigma}\right). \tag{3.3}$$

3.3.4 Weibull distribution

The Weibull distribution PDF is given by

$$f(x|\alpha,\beta) = \left(\frac{\alpha}{\beta^{-\alpha}}\right)\left(x^{\alpha-1}\right)\exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right), \tag{3.4}$$

where α is the shape parameter and β is the scale parameter.

3.4 Model selection

To select the best suitable model for the parent distribution, the goodness-of-fit of the model should be assessed. The most commonly used criterion for assessing the goodness-of-fit of the parent distributions is Akaike's information criterion as well as the Bayesian information criterion (Masingi, 2021).

3.4.1 Akaike's Information Criterion (AIC)

The best-fit distribution or model will be selected on the basis of the least AIC value. The minimum AIC value denotes both the fewer explanatory variables

and the best fit (Langat et al., 2019).
The AIC is denoted by:

$$AIC = 2k - 2log(L), \tag{3.5}$$

where
k is the number of parameters in the model,
L is the likelihood function for the fitted model.

3.4.2 Bayesian Information Criterion (BIC)

An additional criterion for evaluating the goodness-of-fit of a model is the BIC. Similar to the AIC, model selection for the BIC is based on the likelihood function. Thus, the distribution or model with the least BIC is selected as the best fit (Langat et al., 2019).

The BIC is denoted by:

$$BIC = -2log(L) + klog(n), \tag{3.6}$$

where
L is the maximum likelihood estimate of the model,
k is the number of parameters in the model,
n is the sample size.

3.5 Parameter estimation

Parameter estimation methods that will be employed to estimate the parameters for the parent distributions are maximum likelihood estimation (MLE) and Bayesian parameter estimation.

3.5.1 Maximum likelihood estimation (MLE) method

Under the assumption that $X_1, ..., X_m$ are independent identical distributed (iid) random variables having the GEVD, the log-likelihood for the GEV parameters with $\xi \neq 0$ is

$$l(\mu,\sigma,\xi) = -m\ log\sigma - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{m} log\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right] - \sum_{i=1}^{m}\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}, \tag{3.7}$$

provided that

$$1+\xi\left(\frac{z_i-\mu}{\sigma}\right) > 0, \text{ for } i = 1, ..., m.$$

In the case where $\xi - 0$, the log-likelihood is given by

$$l(\mu,\sigma) = -m\ log\sigma - \sum_{i=1}^{m}\left(\frac{z_i-\mu}{\sigma}\right) - \sum_{i=1}^{m} exp\left\{-\left(\frac{z_i-\mu}{\sigma}\right)\right\}. \tag{3.8}$$

The maximisation of (3.7) and (3.8) with respect to the parameters (μ, σ, ξ) leads to the MLE with respect to the whole GEV family (Seimela, 2021).

3.5.2 Bayesian estimation method

Extremes are scarce in nature, so adding other sources of information through a prior distribution may improve the statistical inference on extremes (Debusho and Diriba, 2016; Coles et al., 2001). The Markov chain Monte Carlo (MCMC) method will be applied to the JSE financial stock market data.

Let $f(\theta)$ denote the density of the prior distribution for θ where by f denote an arbitrary density function, then the likelihood for θ is given by $f(x|\theta)$. If the x_i are independent, then

$f(x|\theta) = \prod_{i=1}^{n} f(x_i; \theta)$.

Baye's theorem states that

$$f(x|\theta) = \frac{f(\theta) f(x|\theta)}{\int_{\Theta} f(\theta) f(x|\theta) d\theta}, \tag{3.9}$$

where by $f(x|\theta)$ is the prior distribution and $f(x|\theta)$ is the posterior distribution.

MCMC is one of the popularised method that uses the Bayesian techniques (Coles et al., 2001). The technique of MCMC stimulates a sequence $\theta_1, \theta_2, ...$ as follows:

1. Set the initial value θ_1, and specify an arbitrary probability rule $q(\theta_{i+1}|\theta_i)$ for all simulation of successive values.
2. Possibilities include $(\theta_{i+1}|\theta_i) \sim N(\theta_i, 1)$ or $(\theta_{i+1}|\theta_i) \sim Gamma(1, 1)$.

This procedure produces a first Markov chain since, given θ_i, the stochastic properties of θ_{i+1} are independent of the earlier history $\theta_1, ..., \theta_{i-1}$.

3.6 Tests for stationarity

Augmented Dickey-Fuller (ADF) and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) are performed in the data in order to detect trends over various selection periods. Both tests are used to test the null hypothesis against the alternatie hypothesis. The null hypothesis indicates that the trend is stationary whereas the alternative hypothesis states that the trend is difference stationary (Hasan et al., 2012).

3.6.1 Augmented Dickey-Fuller test

The ADF is given by:

$$y_t = \Phi y_{t-1} + v_t, \tag{3.10}$$

where

y_t is the maximum financial stock market data,

v_t is the zero-mean term that is iid.

3.6.2 Kwiatkowski, Phillips, Schmidt and Shin test

The KPSS is used in order to detect trends over various selection periods. The KPSS test is utilised to test the null hypothesis against the alternatie hypothesis. The null hypothesis states that the trend is stationary whereas the alternative hypothesis states that the trend is difference stationary. (Hasan et al., 2012; Seimela, 2021)

3.7 Test for normality

The Shapiro-Wilk (S-W) and the Jarque-Bera (JB) tests will be used to test whether the the JSE financial market data is normally distributed or not.

3.7.1 Shapiro-Wilk test

S-W test is used in the data to test the null hypothesis of a sample $X_1, ..., X_n$ that is normally distributed.

The test statistic is given by

$$W = \frac{(\sum_{i=1}^{n}(a_i)(x_i))^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}, \tag{3.11}$$

where x_i is the i^{th} order statistics of the i^{th} smallest number in the sample,

$$\bar{x} = \frac{(x_i + ... + x_n)}{n}. \tag{3.12}$$

The constants $a_i s$ in (3.11) are given by

$$(a_1, ..., a_n) = \frac{m^t(V^{-1})}{(m^t(V^{-1}(V^{-1})m)^{1/2}}, \tag{3.13}$$

where

$$m = (m_1, ..., m_n)^T, \tag{3.14}$$

$m_1, ..., m_n$ are the iid random variables.

3.7.2 Jarque-Bera test

The JB test is given by:

$$JB = \frac{n-k+1}{6}\left(s^2 + \frac{1}{4}(c-3)^2\right), \tag{3.15}$$

where

n is the number of observations,

s is the sample skewness,

c is the sample kurtosis,

k is the number of regressors .

Additionally, the sample skewness s and sample kurtosis c are given by

$$s = \frac{\hat{\mu_3}}{\hat{\sigma_3}}, \tag{3.16}$$

$$c = \frac{\hat{\mu_4}}{\hat{\sigma_4}}, \tag{3.17}$$

respectively, where $\hat{\mu_3}$ and $\hat{\mu_4}$ are the estimates of the third and fourth central moments .

3.8 Trend analysis

Mann-Kendall test

The non-parametric Mann-Kendall is used to quantify the monotonic trend of the JSE financial market data (Masingi, 2021).

Suppose $x_1, x_2, ..., x_n$ is a sequence of random variables, then the M-K statistic can be computed as:

$$S = \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} sign(x_j - x_k), \tag{3.18}$$

where x_j is the number of extremes in the data, n is the number of extreme values and S is the M-K statistic. A positive value of S indicates an increasing trend and a negative S shows a decreasing trend.

The sign$(x_j - x_k)$ expression in (3.18) is denoted by:

$$sign(x_j - x_k) = \begin{cases} 1 & \text{if } x_j - x_k > 0, \\ 0 & \text{if } x_j - x_k = 0, \\ -1 & \text{if } x_j - x_k < 0. \end{cases} . \tag{3.19}$$

3.9 GEVD models

3.9.1 Block maximum approach

The two most commonly used distributions for modelling extreme values are the GEVD and GPD (Ferreira and de Haan, 2015). The GEVD is based on the block-maxima realisation, whereas the GPD focuses on the POT or exceedances above the threshold. The GEVD is made up of the Fréchet, Weibull and Gumbel parametric distributions (Coles et al., 2001).

Let $X_1, X_2, ..., X_n$ be a sequence of independent and identically distributed (i.i.d) random variables with distribution function F. The model focuses on statistical behaviour of

$$M_n = Max(X_1, ..., X_n), \tag{3.20}$$

where
X_i denotes values of a process measured on a regular time-scale,
M_n represents the maximum of the process over n times units of the observations.
The distribution of M_n is

$$Pr\{M_n \leq x\} = Pr\{X_1 \leq x, ..., X_n \leq x\} = Pr\{X_1 \leq x\} \times ... \times Pr\{X_n \leq x\} = F(x)^n. \tag{3.21}$$

If there exists a sequence of constants $\{a_n > 0\}$ and $\{b_n\}$ such that

$$P_r = \left\{ \left(\frac{M_n - b_n}{a_n} \right) \leq x \right\} \rightarrow G(x), \text{ as } n \rightarrow \infty, \tag{3.22}$$

for a non-degenerated distribution of G, then G belongs to one of the following families:

$$TypeI : G(x) = exp\left\{exp\left[-\frac{x-b}{a}\right]\right\}, \ -\infty < x < \infty, \tag{3.23}$$

$$TypeII : G(x) \quad = \quad \begin{cases} 0, & x \leq b, \\ exp\{-\left(\dfrac{x-b}{a}\right)^{-\alpha}, & x \ > \ b, \end{cases} \quad , \tag{3.24}$$

$$TypeIII : G(x) \quad = \quad \begin{cases} exp\left\{-\left[-\left(\dfrac{x-b}{a}\right)^{\alpha}\right]\right\}, & x \ < \ b, \\ 1, & x \geq b, \end{cases} \quad , \tag{3.25}$$

for parameters $a > 0$ and b. In the case of families II and III, $a > 0$ (Coles et al., 2001). The GEVD is made up of three extreme value distributions clasiffied as types I, II and III also known as the Gumbel, Fréchet and the Weibull distributions, respectively (Coles et al., 2001). The cummulative probability distribution for the GEVD is of the form:

$$G(x, \mu, \sigma, \xi) \quad = \quad \begin{cases} exp - [1 + \xi(\dfrac{x-\mu}{\sigma})]^{\dfrac{-1}{\xi}} \ ; & \xi \neq 0, \\ exp(-exp(-\dfrac{x-\mu}{\sigma})); & \xi = 0, \end{cases} \quad , \tag{3.26}$$

where

x are the extreme values from the blocks,

μ is the location parameter,

σ is the scale parameter,

ξ is the shape parameter.

Thus, when $\xi > 0$ the distribution is considered as Fréchet, when $\xi = 0$ the didtribution is Gumbel and when $\xi < 0$ the distribution is considered as the Weibull distribution (Masingi, 2021).

3.9.2 Parameter estimation

Maximum likelihood estimation (MLE)

Under the assumption that $X_1, ..., X_m$ are iid random variables having the GEVD, the log-likelihood for the GEV parameters with $\xi \neq 0$ is

$$l(\mu, \sigma, \xi) = -m\ log\sigma - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^{m} log\left[1 + \xi\left(\frac{z_i - \mu}{\sigma}\right)\right] - \sum_{i=1}^{m}\left[1 + \xi\left(\frac{z_i - \mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}, \tag{3.27}$$

provided that

$$1 + \xi\left(\frac{z_i - \mu}{\sigma}\right) > 0, \text{ for } i = 1, ..., m.$$

In the case where $\xi = 0$, the log-likelihood is given by

$$l(\mu, \sigma) = -m\ log\sigma - \sum_{i=1}^{m}\left(\frac{z_i - \mu}{\sigma}\right) - \sum_{i=1}^{m} exp\left\{-\left(\frac{z_i - \mu}{\sigma}\right)\right\}. \tag{3.28}$$

The maximisation of (3.27) and (3.28) with respect to the parameters(μ, σ, ξ) leads to the MLE with respect to the whole GEV family (Seimela, 2021).

3.9.3 Inference for return levels

By substituting the maximum likelihood estimates of the the GEV parameters into (3.26), the MLE of z_p for $0 < p < 1$, the $\frac{1}{p}$, (Coles et al., 2001) return level is acquired by

$$\hat{z_P} = \begin{cases} \mu - \dfrac{\hat{\mu}}{\hat{\xi}}\left[1 - y_p^{-\hat{\xi}}\right], & \text{for } \hat{\xi} \neq 0, \\ \hat{\mu} - \hat{\sigma}\ log\ y_p\ , & \text{for } \hat{\xi} = 0, \end{cases} \quad , \tag{3.29}$$

where $y_p = log(1-p)$.

3.9.4 Model checking

The probability plots and the quantile plots for model checking will be used.

Probability plot

Coles et al. (2001) explains the probability plot as a comparison of the emperical fitted distributions. With ordered block maximum data $z_{(1)} \leq ... \leq z(m)$, the empirical distribution function evaluated at $z_{(i)}$ is denoted as $\hat{G}(Z_{(i)}) = \dfrac{i}{m+1}$.

By substiluting the parameter estimates into (3.26), the corresponding model-based estimates are:

$$\hat{G}(Z_{(i)}) = exp\left\{ -\left[1 + \hat{\xi}\left(\frac{z_{(i)} - \mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}. \tag{3.30}$$

The GEV model is considered to be functioning well if $\hat{G}(Z_{(i)}) \approx \hat{G}(Z_{(i)})$ for each i, so a probability plot $(\hat{G}(Z_{(i)}), \hat{G}(Z_{(i)})),\ i = 1, ..., m$, should lie close to the unit diagonal. A weakness of the probability plot for extreme value models is that both $\hat{G}(Z_{(i)})$ and $\hat{G}(Z_{(i)})$ are bound to approach 1 as z_i increases, while it is usually the accuracy of the model for larger values of z that is of greatest concern (Coles et al., 2001).

Quantile plot

The quantile plot comprises the points

$$\left\{\left(\hat{G}^{-1}\left(\frac{i}{m+1}\right)\right),\ i\ =\ 1,...,m\right\}, \tag{3.31}$$

where

$$\hat{G}^{-1}\left(\frac{i}{m+1}\right) = \hat{\mu} - \frac{\hat{\sigma}}{\hat{\xi}}\left[1 - \left\{-log\left(\frac{i}{m+1}\right)\right\}^{-\xi}\right]. \tag{3.32}$$

Depatures from linearity in the quantile plot also indicate model failure.

3.9.5 Model selection

Return level estimate

The return level plot consists of a graph of

$$z_p = \mu - \frac{\sigma}{\mu}[1 - \{log(1-p)\}^{-\xi}], \tag{3.33}$$

against $y_p = log\ (1-p)$ on a logarithmic scale, and is particularly convenient for interpreting extreme value models. The tail of the distribution is compressed, so that return level estimates for long return periods are displayed, while the linearity of the plot in the case $\xi = 0$ provides a baseline against which to judge the effect of the estimated shape parameter. Thus, the return level plot of the fitted model comprises the locus points $\{(log\ y_p, \hat{z}_p) :\ 0 < p < 1\}$, where $\hat{z}_p$ is the MLE of z_p (Coles et al., 2001).

3.10 Blended GEV distribution (bGEVD) models

Daniella Castro-Camilo has developed and suggested an enhanced model called the blended GEVD (bGEVD) as an alternative for the standard GEVD when the tail parameter is positive i.e., $\xi > 0$ (Vandeskog et al., 2021b). The inherited artificial limiting constraints of the GEVD model are addressed by the bGEVD model, which was developed. For purposed of practical application, it is assumed that the GEVD is a decent approximation of the distribution of maxima over blocks, and the model is fitted in accordance with assumption (Rue et al., 2009).

To provide a more informative parameter interpretation, an alternative parameterisation for the GEVD is introduced. In terms of the distribution's mean and standard deviation, statistically, the location-spread parameterisation is rather common (Rue et al., 2009). The GEVD parameterisation in terms of the location $q_\alpha \in \mathbb{R}$, the spread $s_\beta \in (0, \infty)$ and the tail parameter $\xi \in \mathbb{R}$ is defined as:

$$F(y|q_\alpha, s_\beta, \xi) = \exp\left\{ - \left[\left(\frac{y - q_\alpha}{s_\beta (l_{1-\beta/2} - l_{\beta/2,\xi})^{-1}} \right) \right]_+^{-1/\xi} \right\}, \tag{3.34}$$

where $a_+ = \max(a, 0)$ and for any $a > 0, l_{\alpha\xi} = \{-log\ a\}^{-}\xi$. In the case whereby $\xi = 0$, it implies that

$$F(y|q_\alpha, s_\beta, \xi) = \exp\left\{ - \left[\left(\frac{y - q_\alpha}{s_\beta (l_{1-\beta/2} - l_{\beta/2,\xi})^{-1}} \right) \right] \right\}, \tag{3.35}$$

with $l_a = log\{-log\ a\}$. There is a one to one mapping between (q_α, s_β, ξ) and the usual location, scale and the shape GEVD parameters, (μ, σ, ξ) (Coles et al., 2001; Castro-Camilo et al., 2021). In the case where $\xi = 0$, the mapping is defined as:

$$\mu = q_\alpha - \frac{s_\beta(l_{\alpha,\xi} - 1)}{\xi(l_{1-\beta/2,\xi} - l_{\beta/2,\xi})}, \sigma = \frac{s_\beta}{(l_{1-\beta/2,\xi} - l_{\beta/2,\xi})}, \xi = \xi. \tag{3.36}$$

In the case where $\xi = 0$, then

$$\mu = q_\alpha + \frac{s_\beta l_\alpha}{l_{\beta/2} - l_{1-/\beta/2}}, \sigma = \frac{s_\beta}{l_{\beta/2} - l_{1-/\beta/2}}. \tag{3.37}$$

With this new parameterisation when the mean and variance are undefined, the location and spread parameters are still interpretable. The user should define the fixed hyperparameters of the probabilities α and β that determines the quatiles q_α, $q_{\beta/2}$, and $q_{1-\beta/2}$ (Castro-Camilo et al., 2021).

Assume that the limiting distribution of the standardised block maximum $(Y_k - b_k)/a_k$ in non-degenerate, where $Y_k =$ max $X_1, X_2, ..., K_k$ is the maximum over k random variables from stationary stochastic process, and b_k and $a_k > 0$ are some appropriate sequences of standardising constants.Then, for large enough block sizes k, the distribution of the block maximum Y_k is approximately equal to the GEV distribution (Coles et al., 2001; Vandeskog et al., 2022; Fisher and Tippett, 1928; Vandeskog et al., 2021a).

The GEVD function is given by

$$F(x, \mu, \sigma, \xi) = exp\{-(1 + \xi(x - \mu)/\sigma)_+^{-1/\xi}\}, \tag{3.38}$$

with a probability function given by

$$P(Y_k \le y) \;\approx\; \begin{cases} \exp\{-[1+\xi(y-\mu)/\sigma_k]_+^{-\frac{1}{\xi}} & , \quad \text{if} \quad \xi \neq 0, \\ \exp\left[\exp[-(y-\mu)/\sigma]\right] & , \quad \text{if} \quad \xi = 0, \end{cases} , \tag{3.39}$$

where $(a)_+ = \max 0, a$ which implies that the support of F is dependent on its parameters. An alternative distribution of the GEVD in settings where the tail parameter ξ is non-negative (i.e $\xi \ge 0$) is referred to as the bGEV distribution (Vandeskog et al., 2021a).

The bGEVD function is given by

$$H(y, \mu, \sigma, \xi, a, b) = F(y, \mu, \sigma, \xi, a, b)^{p(y,a,b)} G(y, \tilde{\mu}, \tilde{\sigma})^{1-p(y,a,b)}, \tag{3.40}$$

where F is a GEVD with $\xi \ge 0$ and G is a Gumbel distribution. The weight function p is equal to

$$P(Y_k \le y) = F_\beta\left(\frac{y-a}{b-a}; c_1, c_2\right), \tag{3.41}$$

where $F_\beta(.; c_1, c_2)$ is the distribution function of a beta distribution with parameters $c_1 = c_2 > 1$, which leads to a symmetric and computationally efficient weight function. The weight $p(y, a, b)$ is zero for $y \le a$ and one for $y \ge b$, which implies that the left tail of the bGEVD equals to the left tail in G, whereas the right tail equals to the right tail in F. (Vandeskog et al., 2022). Thus, H is fully defined by the parameters (q_α, s_β, ξ) and the hyperparameters $(\alpha, \beta, p_a, p_b, c_1, c_2)$ (Castro-Camilo et al., 2021).

3.10.1 bGEVD models

Let $y_t(s)$ denote the maximum JSE stock market at location $s \in S$ during month $t \in T$ where S is the study area and T is the period in focus. The bGEVD for the monthly maximum is given by:

$$[y_t(s)|q_\alpha(s), s_\beta(s), \xi(s)] \sim \text{bGEVD}(q_\alpha(s), s_\beta(s), \xi(s)), \tag{3.42}$$

where all observations are assumed to be conditionally independent given the parameters $q_\alpha(s), s_\beta(s)$ and $\xi(s)$.

To describe the structure of $q_\alpha(s)$ and $s_\beta(s)$, the two competing models namely; the joint model and the two-step model will be contructed.

3.10.2 The joint model

In the joint model, both parameters are modelled using the linear combinations of explanatory variables. Furthermore, a Gaussian random field is added to the location parameter. The joint model is denoted as:

$$[y_t(s)|q_\alpha(s), s_\beta(s), \xi(s)] \sim \text{bGEVD}(q_\alpha(s), s_\beta(s), \xi(s)), \tag{3.43}$$

$$q_\alpha(s) = x_q(s)^T \beta_q + \mu_q(s), \tag{3.44}$$

$$log(s_\beta(s)) = x_s(s)^T \beta_s, \tag{3.45}$$

where

x_q and x_s are vectors comprising an intercept plus the explanatory variables.

β_q and β_s are vectors of regression coefficients.

$\mu_q(s)$ is a zero-mean Gaussian field with Matérn covariance function denoted as:

$$Cov(\mu_q(s_i), \mu_q(s_j)) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}\left(\sqrt{8\nu}\frac{d(s_i, s_j)}{\rho}\right)^{\nu} K_\nu\left(\sqrt{8\nu}\frac{d(s_i, s_j)}{\rho}\right), \quad (3.46)$$

where

$d(s_i, s_j)$ is the Euclidean distance between s_i and s_j, ρ is the range parameter, σ^2 is the marginal variance and ν is the smoothness parameter. The function K_ν is the modified Bessel function of the second kind and order ν.

3.10.3 The two-step model

The two-step model is specifically tailored for sparse data with large block sizes (Vandeskog et al., 2022). In order to put a flexible model on the spread while avoiding such issues, Vandeskog et al. (2021b) propose a model which borrows strength from the peaks over threshold method for separate modelling of $s_\beta(s)$. For some large enough threshold $\mu(s)$, it is assumed to follow a GPD

$$P(X(s) > \mu(s) + x | X(s > \mu(s))) = \left(1 + \frac{\xi x}{\tilde{\sigma(s)}}\right)^{-\frac{1}{\xi}}, \quad (3.47)$$

where

ξ is the tail parameter,

the scale parameter $\tilde{\sigma}(s) = \sigma(s) + \xi(\mu(s) - \mu(s))$.

3.10.4 Model evaluation

The bGEVD model performane will be assessed by employing the continuos ranked probability score (CRPS) given by

$$CRPS(F, y) = \int_{-\infty}^{\infty} (F(t) - I(t \geq y))^2 dt = 2\int_0^1 \rho_p(y - F^{-1}(p))dp, \tag{3.48}$$

where

F is the forecast distribution,

y is an observation,

$\rho_p(\mu) = \mu(p - I(\mu < 0))$ is the quantile loss function,

$I(.)$ is an indicator function.

To allow for emphasis on specific areas of the forecast distribution, the CRPS approach was then modified to the threshold weighted CRPS (twCRPS), given by

$$twCRPS(F, y) = 2\int_0^1 \rho_p(y - F^{-1}(p))w(p)dp, \tag{3.49}$$

where $w(p)$ is a non-negative weight function. A possible choice for $w(p)$ for focusing on the right tail is the indicator function $w(p) = I(p > p_0)$.

3.10.5 R-INLA package

The package that will be used to model and test the performance of the bGEVD is called R-INLA. The R-INLA package is a recent addition to the extreme value framework, which is relatively new. Both the joint and the two step models falls within the latent Gaussian models when the Gaussian prior β_μ is included. This has the advantage of enabling INLA inference using the R-INLA library (Vandeskog et al., 2022).

3.11 r-largest order GEVD models

In analysing extreme values, one of the limitations in framing the extreme values is the limited amount of data available for parameter estimation especially since extremes are rare and may result in massive variability (Da Silva and do Nascimento, 2019). The GEVD is then expanded to provide an asymptotic model for the joint distribution for the r-largest order statistics within annual blocks for fixed values of r (Nemukula and Sigauke, 2018). Thus, the r-largest order statistics is an alternative for analysing extreme events in blocks of size n and is defined as

$$M_n^k = \text{k-th largest value of } (X_1, ..., X_n), \tag{3.50}$$

and the identity of the limiting behaviour of these variables for fixed k, as $n \to \infty$ is given as:

$$P_r\{\frac{M_n^{(k)} - b_n}{a_n} \leq x\} \to G_k\left(x\right) \text{ as } n \to \infty, \tag{3.51}$$

defined on

$$\{x : 1 + \xi\left(\frac{x-\mu}{\sigma}\right) > 0\}, \tag{3.52}$$

where:

$$G_k\left(x\right) = exp\{-\tau\left(x\right)\}\sum_{s=0}^{k-1}\frac{\tau(x)^s}{s}, \tag{3.53}$$

with

$$\tau(x) = \left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}. \tag{3.54}$$

The r-largest order statistics

$$M_n^{(r)} = \left(\frac{M_n^{(1)} - b_n}{a_n}, \frac{M_n^{(2)} - b_n}{a_n}, ..., \frac{M_n^{(r-1)} - b_n}{a_n}, \frac{M_n^{(r)} - b_n}{a_n} \right), \tag{3.55}$$

then the PDF is given as follows:

$$f(x^{(1)}, ..., x^{(r)}|\mu, \sigma, \xi) = exp\left\{ -\left(1+\xi\left(\frac{(x^{(r)} - \mu)}{\sigma} \right) \right) \right\} \times \prod_{k=1}^{r} \sigma^{-1} \left(1 + \xi \left(\frac{(x^{(r)} - \mu)}{\sigma} \right) \right)^{-\frac{1}{\xi}-1}, \tag{3.56}$$

valid for $-\infty < \mu < \infty,\ \sigma > 0$ and $-\infty < \xi < \infty; x^{(r)} \leq x^{(r-1)} \leq ... \leq x^{(1)}$; and: $x^{(k)} : 1 + \xi \left(\frac{x^{(k)} - \mu}{\sigma} \right) > 0\ for\ k = 1, 2, ..., r.$

3.11.1 The Likelihood fuction for the r-largest order statistics

The likelihood function for the r-largest order statistics model when $\xi = 0$ is denoted by (Nemukula and Sigauke, 2018):

$$L(\mu, \sigma, \xi) = \prod_{i=i}^{m} \left(exp\left\{ -\left[1 + \xi(\frac{x_i^{(r_i)} - \mu}{\sigma}) \right]^{-\frac{1}{\xi}} \right\} \times \prod_{k=1}^{r_i} \frac{1}{\sigma} \left[1 + \xi(\frac{x_i^{(k)} - \mu}{\sigma}) \right]^{-\frac{1}{\xi}-1} \right), \tag{3.57}$$

defined for $1 + \xi(\frac{x_i^{(k)} - \mu}{\sigma}) > 0, = k\ 1, ..., r_i, i\ \ 1, ..., m.$

3.11.2 Prior distribution

The prior distribution of each component of the parametric vector (μ, σ, ξ) for the parameters of the rGEVD is proposed to be independent of the prior distribution in the other segment (e Silva et al., 2020). For σ, the parameter is necessarily positive, thus, according to (Da Silva and do Nascimento, 2019) the $\gamma(a, b)$ prior distribution will be considered. However, μ and σ are considered to be normally distributed based on the assumption that these parameters may have negative values. Consequently, the prior distributions $N(\mu_0, \sigma_\mu^2)$ and $N(\xi_0, \sigma_\xi^2)$ are normal. Therefore, the joint distribution of the parameters is given by the following expression:

$$p(\mu, \sigma, \xi)\alpha\sigma^{a-1}exp(-b\sigma)exp\left[-\frac{(\mu-\mu_0)^2}{2\sigma_\mu^2}\right]exp\left[-\frac{(\xi-\xi_0)^2}{2\sigma_\xi^2}\right]. \tag{3.58}$$

3.11.3 Posterior distribution

Considering the likelihood function given in (3.41) and the prior distribution illustrated by (3.42), taking the log-likelihood function as the product of the densities the conditional distribution of the vector (ξ, σ, μ) when $\xi \neq 0$ is given by (Da Silva and do Nascimento, 2019)

$$l(\mu\sigma\xi) = \sum_{i=1}^{m} -log(\sigma) - \left(\frac{1}{\xi}+1\right)\sum_{k=1}^{r} log\left(1+\frac{\xi(z_i^{(r_i)}-\mu)}{\sigma}\right) - \left[1+\xi\left(\frac{z_i^{(r_i)}-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}. \tag{3.59}$$

Stating that $1+\xi\left(\frac{z_i^{(r_i)}-\mu}{\sigma}\right) > 0$, for $k = 1, ..., r_i, i = 1, ..., m$; otherwise, the likelihood is zero. Finally, the proportional function of the posterior distrubution was obtained by taking $\tau(\mu, \sigma, \xi)\alpha p(\mu, \sigma, \xi)e^{i(\mu,\sigma,\xi)}$.

3.11.4 Selection of value of r

The selection of the value of r is vital. Nemukula and Sigauke (2018) argues that the legitimacy of the rGEVD is dependent on the choice of the number of the r order statistics. High values of r are likely to violate the model and lead to the occurance of bias whereas too low and few values of r can lead to high variance of the estimator (Bader et al., 2017; Nemukula and Sigauke, 2018). The Bayesian-adapted criteria employed by Bader et al. (2017); Da Silva and do Nascimento (2019) will be employed in the study for hypothesis testing. The null hypothesis of the r-largest order statistics is given by

$H_0^{(r)}$: The rGEVD fits the sample of the r-largest order statistics well

The Fisher information matrix $I(\theta)$ was based on the work of Tawn (1988). Owing to the instability of the maximun likelihood estimators, the matrix is necessary for $\xi > -0.5$. Therefore, the statistic punctuation is given by

$$V_n = \frac{1}{n} S^T(\hat{\theta_n}) I^{-1} S(\hat{\theta_n}). \tag{3.60}$$

3.11.5 Parametric bootstrap: (PB Score)

The parametric bootstrap is the first solution for determining the r-optimal selection. In accordance with Bader et al. (2017), the hypothesis test procedure for $H_0^{(r)}$ is given by:

1. Compute $\hat{\theta_n}$ under H_0 with the observed data.
2. Compute the testing statistic V_n.
3. For every $k \in \{1, ..., L\}$ with a large number L, repeat:
(a) Generate a bootstrap sample of size n for the r-largest statistics from rGEV with parameter vector $\hat{\theta_n}$.
(b) Compute the $\hat{\theta_n^{(k)}}$ under H_0 with the bootstrap sample.
(c) Compute the score test statistic $V_n^{(k)}$.
4. Return an approximate p-value of V_n as
$L^{-1} \sum_{k=1}^{L} 1(V_n^{(k)} > V_n)$.

3.11.6 Entropy difference test (EDT)

The entropy differential test (EDT) based on the work of Bader et al. (2017); Da Silva and do Nascimento (2019) will be employed to assess the goodness-of-fit of the r-largest order and $r-1$-largest order statistics. Considering a continuous random variable X with density function f, the entropy difference is

$$E[-lnf(x)] = -\int_{-\infty}^{\infty} f(x)logf(x)dx, \tag{3.61}$$

which is the average of the negative log-likelihood estimated by the sample mean of the contribution to the log-likelihood from the observed data or simply the log-likelihood from the observed data. Assumming $r-1$ top order statistics for the $(r-1)$GEVD, then the difference in the log-likelihood between $(r-1)$GEVD and rGEVD provides a measure of deviation from the null hypothesis $H_0^{(r)}$. However, the large deviation from the expected difference under $H_0^{(r)}$ proposes possible misspecification of the null hypothesis (Bader et al., 2017; Nemukula and Sigauke, 2018). From the log-likelihood contribution, the difference in log-likelihood for the i-th block:

$$X_i r(\theta) = l_i^{(r)} - l_i^{(r-1)}, \tag{3.62}$$

is given by:

$$X_i r(\theta) = -log\sigma - (1+\xi z_{ri}) + (1+\xi z_{ri-1})^{-\frac{1}{\xi}} - \left(\frac{1}{\xi}+1\right) log\left(1+\xi z_{ri}\right), \tag{3.63}$$

where $z_i = \xi\left(\frac{x_i^{(r_i)}-\mu}{\sigma}\right)$.

3.11.7 Hypothesis testing procedure

Suppose there are R hypotheses, $H_0^{(r)},\ r = 1, ..., R$ tested in sequence for the proposed procedure. Thus, the hypothesis will be rejected based on the condition that: if $H_0^{(r)}$ is rejected, $r < R$ then $H_0^{(k)}$ will be rejected for all $r < k \leq R$ (Da Silva and do Nascimento, 2019).

3.11.8 Return level for the r-largest order statistics

According to Da Silva and do Nascimento (2019), it is challenging to find the cumulative distribution of r-largest order statistics, thus the accumulated values of the GEVD will be used to obtain the maximum return levels.

The parameter estimation for the proposed model allowed for the estimation of the expected levels in t time periods, which are represented by the quantile $p = 1\frac{1}{t}$ of the GEVD quantile formula. In the situation where the parameter estimation $\ddot{\xi} < 0$, the distribution is upper bounded. Thus, the estimate of the maximum value which the data will assume is given by

$$\ddot{z}_0 = \ddot{\mu} - \frac{\ddot{\sigma}}{\ddot{\xi}}. \tag{3.64}$$

3.11.9 Simulation

The R package called EVA of Bader et al. (2017) will be used to demonstrate the efficiency of the parameters of the estimation point of the r-largest order statistics. The Bayesisan approach through the MCMC algorithm will be applied (Da Silva and do Nascimento, 2019).

3.12 Generalised Pareto distribution (GPD) models

3.12.1 GPD Model

Let $X_1, X_2, ...$ be a sequence of iid random variables, having a marginal distribution function F. Denoting an arbitrary term in the X_i sequence by X, it follows that a description of the stochastic behaviour of extreme events is given by the conditional probability

$$Pr\{X > u + y|X > u\} = \frac{1 - F(u + y)}{1 - F(u)}, \quad y > 0, \tag{3.65}$$

where

u is the threshold,

$y = x - u$ are excesses. The main results for GDP is given by the GEVD theorem as follows:

Theorem: Let $X_1, X_2, ...$ be a sequence of iid random variables with common distribution function F, and let $M_n = max\{X_1, ..., X_n\}$. Denote an arbitrary term in the X_i sequence by X, and suppose that F satisfies

$$P_r = \left\{ \left(\frac{M_n - b_n}{a_n} \right) \leq x \right\} \rightarrow G(x), \text{ as } n \rightarrow \infty, \tag{3.66}$$

where

$$G(z) = exp \left\{ - \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\}, \tag{3.67}$$

for some $\mu, \sigma > 0$ and ξ. Then, for large enough μ, the distribution function of $(X - \mu)$, conditional on $X > \mu$, is approximately

$$H(y) = 1 - \left(1 + \frac{\xi y}{\tilde{\sigma}} \right)^{-\frac{1}{\xi}}, \tag{3.68}$$

defined on $\{y : \; y > 0$ and $\left(1 + \frac{\xi y}{\tilde{\sigma}} \right) > 0\}$, where $\tilde{\sigma} = \sigma + \xi(u - \mu)$. The family of distributions defined by (3.67) is called the generalised Pareto family.

The duality between the GEVD and generalised Pareto families means that the shape parameter ξ is dominant in determining the qualitative behaviour of the GPD, just as it is for the GEVD (Coles et al., 2001). If $\xi < 0$ the distribution has no upper limit. The distribution is $\left(\mu - \frac{\tilde{\sigma}}{\xi}\right)$; if $\xi > 0$ the distribution has no upper limit. The distribution is also unbounded if $\xi = 0$, which should again be interpreted by taking the limit $\xi \to 0$ in (3.67), leading to

$$H(y) = 1 - exp\left(-\frac{y}{\tilde{\sigma}}\right), \quad y > 0, \tag{3.69}$$

corresponding to an exponential distribution with parameter $\frac{1}{\tilde{\sigma}}$.

3.12.2 Parameter estimation

Maximum likehood estimation

Assume that $y_1, ..., y_m$ be a random sample of m excesses of a threshold u, the log-likehood function of a GDP is denoted by

$$l(\sigma_u, \xi) = m\ log(\sigma_u) - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^{m} log\left(1 + \frac{\xi y_i}{\sigma_u}\right), \quad \xi \neq 0, \tag{3.70}$$

if $\left(1 + \frac{\xi y_i}{\sigma_u}\right) > 0$ for $i = 1, ..., m$. Otherwise, $l(\sigma_u, \xi) = -\infty$. Thus, likelihood can also be reduced to

$$l(\sigma_u) = -m\ log\sigma_u - \frac{1}{\sigma_u} \sum_{i-1}^{m} y_i, \quad \xi = 0. \tag{3.71}$$

Return levels

Suppose that the GPD with parameters σ and ξ is a suitable model for exceedances of a threshold u by a variable X (Seimela, 2021). That is, for $x > u$,

$$P_r\{X > x|X > u\} = \left[1 + \xi\left(\frac{x-u}{\sigma}\right)\right]^{-\frac{1}{\xi}}. \tag{3.72}$$

It follows that

$$P_r\{X > x\} = \zeta_u\left[1 + \xi\left(\frac{x-u}{\sigma}\right)\right]^{-\frac{1}{\xi}}, \tag{3.73}$$

where $\zeta = P_r\{X > u\}$. Hence, the level x_m that is exceeded on average once every m observations is given by

$$\zeta_u\left[1 + \xi\left(\frac{x_m-u}{\sigma}\right)\right]^{-\frac{1}{\xi}} = \frac{1}{m}, \tag{3.74}$$

which can be simplified as:

$$x_m = \begin{cases} u + \frac{\sigma}{\xi}[(m\zeta_u)^{\xi} - 1] & \xi \neq 0, \\ u + \sigma log(m\zeta_u) & \xi = 0, \end{cases}, \tag{3.75}$$

provided m is sufficiently large to ensure that $m_m > u$ (Coles et al., 2001; Seimela, 2021). An estimate of ζ_u is also given by:

$$\hat{\zeta_u} = \frac{k}{n}, \tag{3.76}$$

where

k is the number of exceedances,

n is the sample size.

The number of exceedances of u follows a binomial Bin(n, ζ_u) distribution and $\hat{\zeta}_u$ is the MLE of ζ_u (Coles et al., 2001).

3.12.3 Model checking

To assess the quality of the fitted GPD model, the probability plots, return level plots, density plots and quantile plots will be employed. Suppose a threshold u, threshold excess $y_1 \leq ... \leq y_{(k)}$ and an estimate $\hat{H}$, the probability plot consists of the pair

$$\{(i/(k+1), \hat{H}(Y_{(i)}));\ i = 1, ..., k\}, \tag{3.77}$$

where

$$\hat{H(y)} = 1 - \left(1 + \frac{\hat{\xi} y}{\hat{\sigma}}\right)^{-\frac{1}{\hat{\xi}}}, \tag{3.78}$$

provided $\hat{\xi} \neq 0$. Suppose $\hat{\xi} \neq 0$, the quantile plot consists

$$\{(\hat{H}^{-1}(i/(k+1)), y_{(i)}),\ i = 1, ..., k\}, \tag{3.79}$$

where

$$\hat{H}^{-1} = u + \frac{\hat{\sigma}}{\hat{\xi}}[y_{-\hat{\xi}} - 1]. \tag{3.80}$$

A return level plot consists of the locus of points $(m, \hat{x}_m)$ for large values of m, where $\hat{x}_m$ is the estimated $m-$observation return level:

$$\hat{x}_m = u + \frac{\hat{\sigma}}{\hat{\xi}}\left[(m\hat{\zeta_u})^{\hat{\xi}} - 1\right], \tag{3.81}$$

modified if $\hat{\xi} = 0$.

3.13 Poisson point process models

3.13.1 Poisson process limit for extreme values

The framework for the Poisson point process provides an elegant way to formulate extreme value limit results (Coles et al., 2001).

Theorem: Assume that $X_i, ..., X_n$ is a series of iid random variables, and let

$$N_n = \left\{ \left(\frac{1}{n+1}, \frac{X_i - b_n}{a_n} \right) \; : \; i = 1, ..., n \right\},$$

and $A_z = (0, 1) \times [z, \infty)$.

Then, for sufficiently large u, on regions of the form $(0, 1) \times [u, \infty)$, N_n is approximately a Poisson process, then the intensive measure on $A = [t_1, t_2] \; \times \; (z, \infty)$ is defined as

$$\Lambda(A_z) = (t_2 - t_1) \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}}. \tag{3.82}$$

3.13.2 Parameter estimation

Maximum likelihood method

The Poisson point process likelihood corresponds to the GEVD,

$$\Lambda(A_z) = n_y(t_2 - t_1) \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}}, \tag{3.83}$$

where n_y is the number of years of observations (Coles et al., 2001).
When $[t_1, t_2] = [0, 1]$ is replaced in (3.83), the likelihood Poisson point process takes the form:

$$L_A(\mu,\sigma,\xi;x_1,...,x_n) = exp\{-\Lambda(A)\}\prod_{i=1}^{N(A)}\lambda(t_i,x_i)\times\alpha exp\left\{n_y\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}$$

$$\prod_{i=1}^{N(A)}\sigma\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}-1}. \tag{3.84}$$

3.13.3 Return Level

Suppose z_m denotes the $m-$year return level, and let n denote the number of observations in year z_m (Coles et al., 2001),

$$1-\frac{1}{m} = Pr\{\mathbf{max}(X_1,...,X_n)\} \leq z_m \approx \prod_{i=1}^{n} pi,$$

where

$$p_i = \begin{cases} 1-n^{-1}\left[1+\xi\left(\frac{z_m-\mu_i}{\sigma_i}\right)\right]^{-\frac{1}{\xi_i}}, & \text{if}\left[1+\xi\left(\frac{z_m-\mu_i}{\sigma_i}\right)\right] > 0, \\ 1, & \text{otherwise}, \end{cases} \tag{3.85}$$

and (μ_i,σ_i,ξ_i) are the parameters of the point process model for observation i. Taking the logarithms,

$$\sum_{i=1}^{n} log\ p_i = log\left(1-\frac{1}{m}\right), \tag{3.86}$$

which can easily be solved for $z_m$using standard numerical methods for non-linear equations.

3.13.4 Poisson process of the r-largest order

Let $M_n^{(k)}$ be the point process of the kth largest iid variables of $X_1, ..., X_n$. Then

$$N_n = \left\{ \left(\frac{1}{n+1}, \frac{X_i - b_n}{a_n} \right) \; : \; i = 1, ..., n \right\},$$

and $A_z = (0,1) \times [z, \infty)$,

$$Pr\left\{ \frac{M_n^{(k)} - b_n}{a_n} \leq z \right\} = Pr\{N_n(A_z) \leq k-1\} = \sum_{s=0}^{k-1} Pr\{N_n(A_z) = s\}. \qquad (3.87)$$

By the Poisson process limit, $N_n(A_z)$ converges, as $n \to \infty$, the Poisson variable mean is given by

$$\Lambda(A_z)\left[1 + \xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}.$$

Taking the limits in (3.87),

$$Pr\left\{ \frac{M_n^{(k)} - b_n}{a_n} \leq z \right\} \to \sum_{s=0}^{k-1} e^{-\tau(z)} \frac{\tau(z^s)}{s!},$$

where $\tau(z) = \Lambda(A_z)$.

3.13.5 Parameter estimation for the r-largest order statistics

Maximum likelihood method

Let $(z^{(1)}, ...z^{(r)})$ indicates the obsered value of M_n, and subtituting $u = z^{(r)}$ as well as replacing x_i with $z^{(i)}$ in (3.84), it gives the likelihood for the r–largest order statistics.

$$f(z^{(1)},...,z^{(r)}) = exp\left\{ -\left[1+\xi\left(\frac{z^{(r)}-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}\prod_{k=1}^{r}\sigma^{-1}\left[1+\xi\left(\frac{z^{(k)}-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}-1}, \tag{3.88}$$

where $-\infty < \mu < \infty,\ \sigma > 0$ and $-\infty < \xi < \infty;\ z^{(r)} \leq z^{(r-1)} \leq ... \leq z^{(1)}$; and $z^{(k)};\ 1 + \xi\left(\frac{z^{(k)}-\mu}{\sigma}\right) > 0$ for $k = 1, ..., r$.

Chapter 4

Results and discussions

4.1 Introduction

This chapter presents analysis and discussions of results using the methods outlined in Chapter 3. The five EVT models were employed in conducting the data analysis. R programming data analysis software was utilised to analyse the financial market data.

4.2 Data description

The study will employ secondary data acquired from the JSE in South Africa for the analysis of the financial markets. The data consists of the total return index of the All-Share Index (ALSI) and the currency of the United States of America called the US Dollar against the South African Rand (USD/ZAR) exchange rates. All the datasets are recorded daily starting from 01 February 2016 to 26 April 2021. The total return index of the All Share Index comprises

1307 observations whereas the USD/ZAR exchange rate data consists of 1307 as well. The five-year datasets do not contain data for Saturdays, Sundays as well as the public holidays of South Africa.

4.3 Descriptive statistics

Descriptive statistics analysis was conducted in order to obtain the minimum, maximum, mean, kurtosis, and the skewness of the financial market data. Thus, this section presents the summary statistics of the datasets.

4.3.1 All-Share Total Return Index (ALSTRI)

Financial markets are known to be heavy tailed (Makhwiting et al., 2014; Sigauke et al., 2014). The financial time series data is characterised by volatility, time-varying, and leptokurtic behaviour amongst others (Makhwiting et al., 2014). The time series plot of the daily total return index of the ALSI is illustrated in Figure 4.1. The data employed is the daily data from 01 February 2016 to 26 April 2021, giving $n = 1307$ observations. The characteristics of the financial time series data are displayed in Table 4.1. The pattern of the time series plot in Figure 4.1 indicates seasonal components and volatility within the considered data. The time series plot shows that the data is not stationary which implies that the variance is not stable

On 05 March 2020, the National Institute for Communicable Diseases (NIDC) of South Africa, reported the first case of COVID-19 in South Africa (NICD, 2020). As a result, the financial markets experienced a market shock. There was a decline in the total return index of the ALSI on 19 March 2020. A nationwide lockdown was announced to commence on 26 March 2020. Thus, there was a steep decline in the total return index during March 2020 as a result of the COVID-19 pandemic.

Several studies were conducted on extremes of share returns by various authors globally. Gettinby et al. (2004) conducted a study entitled “An analysis of the distribution of extreme share returns in the UK from 1975 to 2000”. The authors employed daily returns for the UK’s FT All Share Index for the period 1975 to 2000 and weekly maxima and minima were considered to model the extreme returns. In 2006, Gettinby et al. (2004) studied the “analysis of the distribution of extremes in indices of share returns in the USA, UK, and Japan from 1963 to 2000”. The daily share return indices for the three countries were used in the study.

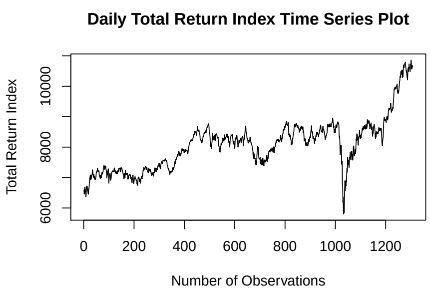

Figure 4.1: Time series plot of the total return index of the ALSTRI from 01 February 2016 to 26 April 2021.

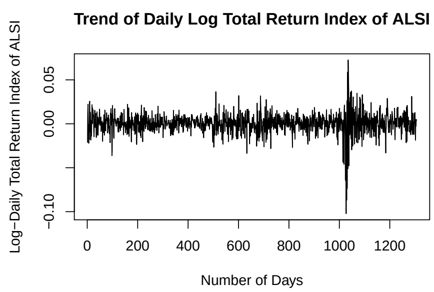

Figure 4.2: Time series plot of log-returns for the ALSTRI from 01 February 2016 to 26 April 2021

Figure 4.2 displays the daily log total return index of the ALSI. The log returns were computed in order to stabilise the variance. The log returns data in Figure 4.2 is centred around zero, which implies that the variance is stabilised and the data is stationary. The instability of the markets with a day-to-day change in prices exemplified positive spikes, showing daily gains, and negative spikes showing daily losses. There were significant losses during March 2020 after the first reported COVID-19 case and the first national lockdown in South Africa, similar to the findings of Hailu and Vural (2021).

Table 4.1 illustrates the summary statistics of the ALSTRI. The results in Table 4.1 indicates that the total return index of the ALSI ranges from 5 802 recorded on 19 March 2020 to 10 860 recorded on 16 April 2021.

Table 4.1: Descriptive statistics of the ALSTRI.

	Min	1Q	Median	Mean	3Q	Max	kurt	Skew
RI	5802	7336	8162	8068	8518	10860	4.175	0.7622
LR	-0.102	-0.005	0.0008	0.0003	0.006	0.0726	11.108	-0.955

Key: 1Q = 1st Quantile, 3Q = 3rd Quantile, Kurt = Kurtosis, LR = Log Returns, RI = Return Index, and Skew = Skewness.

In accordance with Table 4.1, the minimum and maximum indices are far from the mean which demonstrates the presence of extreme events such as the COVID-19 pandemic. The mean is less than the median and the skewness of the log returns is negative, this infers that the presence of extreme values lies more in the left tail. The kurtosis value of 4.175 which is greater than 3, demonstrates that the distribution of the returns is leptokurtic and fat-tailed.

4.3.2 USD/ZAR exchange rate

The time series plot of the USD/ZAR exchange rate is displayed in Figure 4.3. The data used is the daily data from 01 February 2016 to 26 April 2021, giving $n = 1307$ observations. The trend in mean and variance of the USD/ZAR exchange rate is non-stationary and unstable similar to the findings of (Naradh et al., 2021). Figure 4.4 displays the daily log of the USD/ZAR log exchange rate. The log returns were computed in order to stabilise the variance. The log returns data in Figure 4.4 is centered around zero, which implies that the variance is stabilised and the data is stationary. The instability of the markets with a day-to-day change in prices exemplified positive spikes which suggests daily gains while the negative spikes represents daily losses. The Rand was significantly high during March 2020 and was at its peak in April 2022 after the first reported COVID-19 case and the first national lockdown in South Africa.

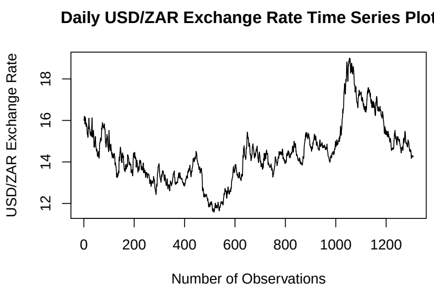

Figure 4.3: Time series plot of the USD/ZAR exchange rate from 01 February 2016 to 26 April 2021.

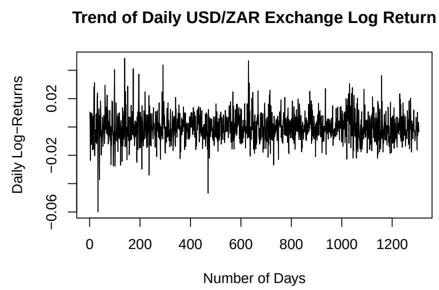

Figure 4.4: Log-returns plot of the USD/ZAR exchange rate from 01 February 2016 to 26 April 2021

Table 4.2: Descriptive statistics of the USD/ZAR exchange rate.

	Min	1Q	Mdn	Mean	3Q	Max	Kurt	Skew
USD/ZAR	11.58	13.49	14.31	14.43	15.05	18.99	0.716	0.719
LR	-0.060	-0.007	-0.0008	<0.001	0.006	0.049	2.099	0.246

Key: 1Q = 1st Quantile, 3Q = 3rd Quantile, Kurt = Kurtosis, LR = Log Return, Mdn = Median, USD/ZAR = USD/ZAR exchange rate, and Skew = Skewness.

Table 4.2 presents the summary statistics of the USD/ZAR exchange rate of the daily data from 01 February 2016 to 26 April 2021. The USA currency is denoted by Dollars ($) whereas the South African currency is represented in Rands (R). The lowest exchange rate was R11.58 recorded on 23 February 2018 and the highest exchange rate was R18.99 recorded on 23 April 2020, during the second month following the initial reported COVID-19 cases in South Africa.

The minimum and maximum USD/ZAR exchange rates of R11.58 and R18.99, respectively, are far from the mean which confirms the presence of extreme events. The mean is greater than the median and the skewness of the log returns is positive, this indicates the existence of extreme values in the right tail. The kurtosis value of 0.719 is less than 3, suggesting that the distribution of the returns is platykurtic.

4.3.3 Results for stationarity test

The ADF and KPSS tests were conducted to determine the stationarity of the financial market data at a 5% level of significance. Table 4.3 presents the results of the test statistics together with the p-value. The ADF test p-value of the ALSTRI is 0.461 which is greater than the significance level (p-value > 0.05), which implies that we fail to reject the null hypothesis (H_0) and conclude that the ALSI returns are difference stationary. The ADF p-value of the USD/ZAR exchange rate is 0.01 which is less than the significance level (p-value < 0.05) recommending that the null hypothesis (H_0) is rejected and concluding that the USD/ZAR exchange rates are stationary.

Table 4.3: Stationarity test results.

	Test name	Test statistic	p-value
All Share Total Return Index	ADF	-2.2767	0.4612
	KPSS	9.1863	0.01
USD/ZAR exchange rate	ADF	-2.6565	0.3004
	KPSS	6.2917	0.01

The KPSS p-value for the ALSTRI and the USD/ZAR exchange rates are less than the significance level (p-value < 0.05) and it implies that the null hypothesis (H_0) is rejected. Therefore, we conclude that the financial market trends are non-stationary.

4.3.4 Results for normality test

The normality test for the ALSTRI and the USD/ZAR exchange rates was conducted and the normality test results are shown in Table 4.4 at $\alpha = 0.05$ significance level. It was found that the p-value produced for both the ALSTRI and the USD/ZAR exchange rates were less than the significance level (p-value < 0.05), indicating that the data is not normally distributed.

Table 4.4: Normality test results.

	Test name	Test statistic	p-value
All Share Total Return Index	Jarque Bera	201.8	<0.001
	Shapiro-Wilk	0.93932	<0.001
USD/ZAR exchange rate	Jarque Bera	141.37	<0.001
	Shapiro-Wilk	0.96021	<0.001

4.4 Parent distribution

The four parent distributions namely; the Normal distribution, Log-normal distribution, Weibull distribution, and Gamma distribution were fitted to the entire dataset in order to determine the best fit for the financial market data.

4.4.1 Parent distributions and diagnostic plots

The parent distributions were fitted on the daily ALSTRI and the USD/ZAR exchange rate. The MLE parameter estimation approach was used to estimate the parameters of the parent distributions.

Figures 4.5, 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, and 4.12 present the results of the empirical and theoretical density plot, Q-Q plot, empirical and theoretical cumulative distribution function (CDF) plot and the P-P plot of the daily ALSTRI and the USD/ZAR exchange rate. These figures illustrate the goodness-of-fit diagnostic statistics for the main body. The empirical and theoretical quantiles of the model are indicated by the Q-Q plots whereas the empirical probabilities and the theoretical probabilities are shown by the P-P plots.

Normal distribution

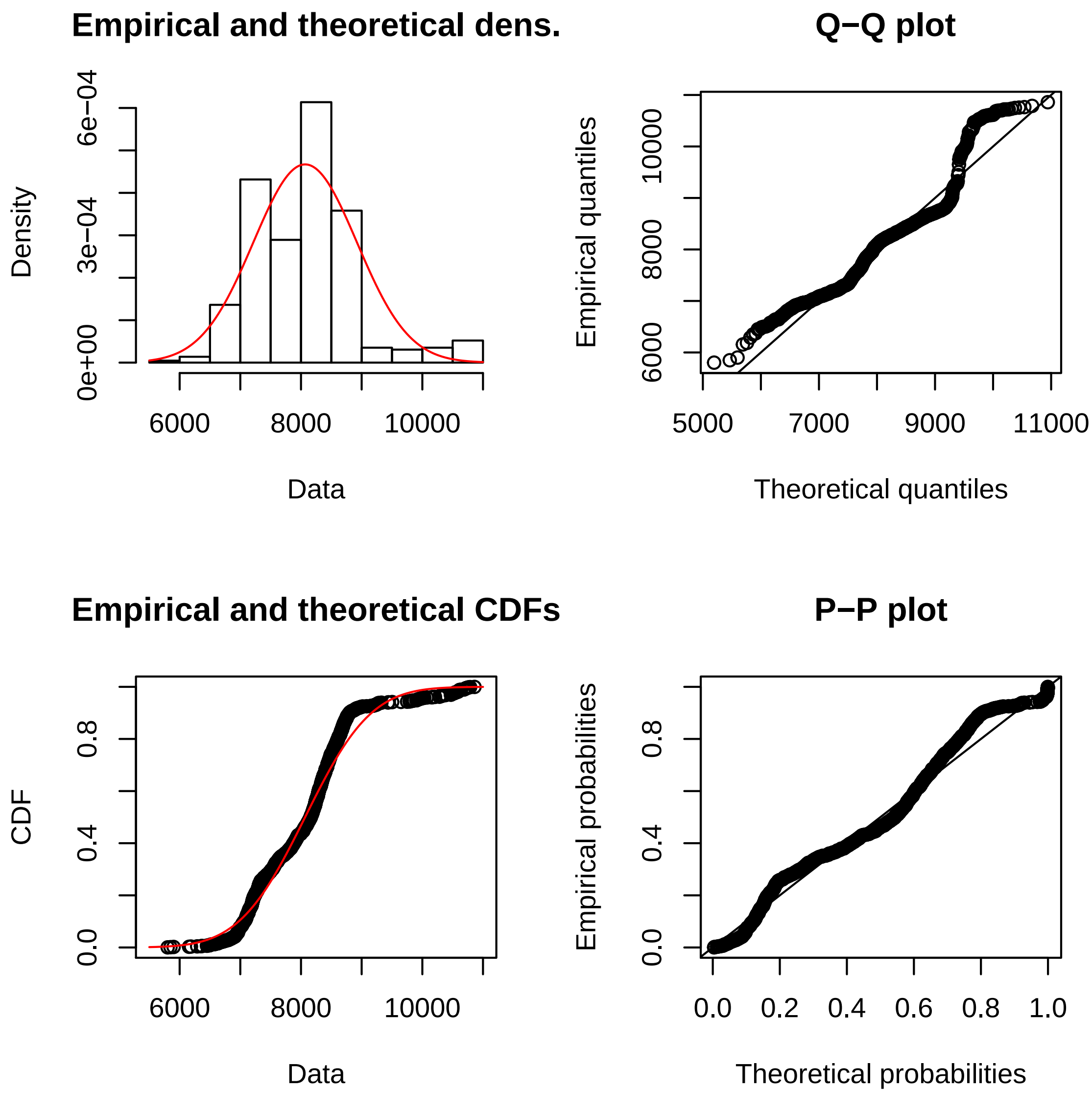

Figure 4.5: Normal distribution diagnostic plots of the All-Share Total Return Index.

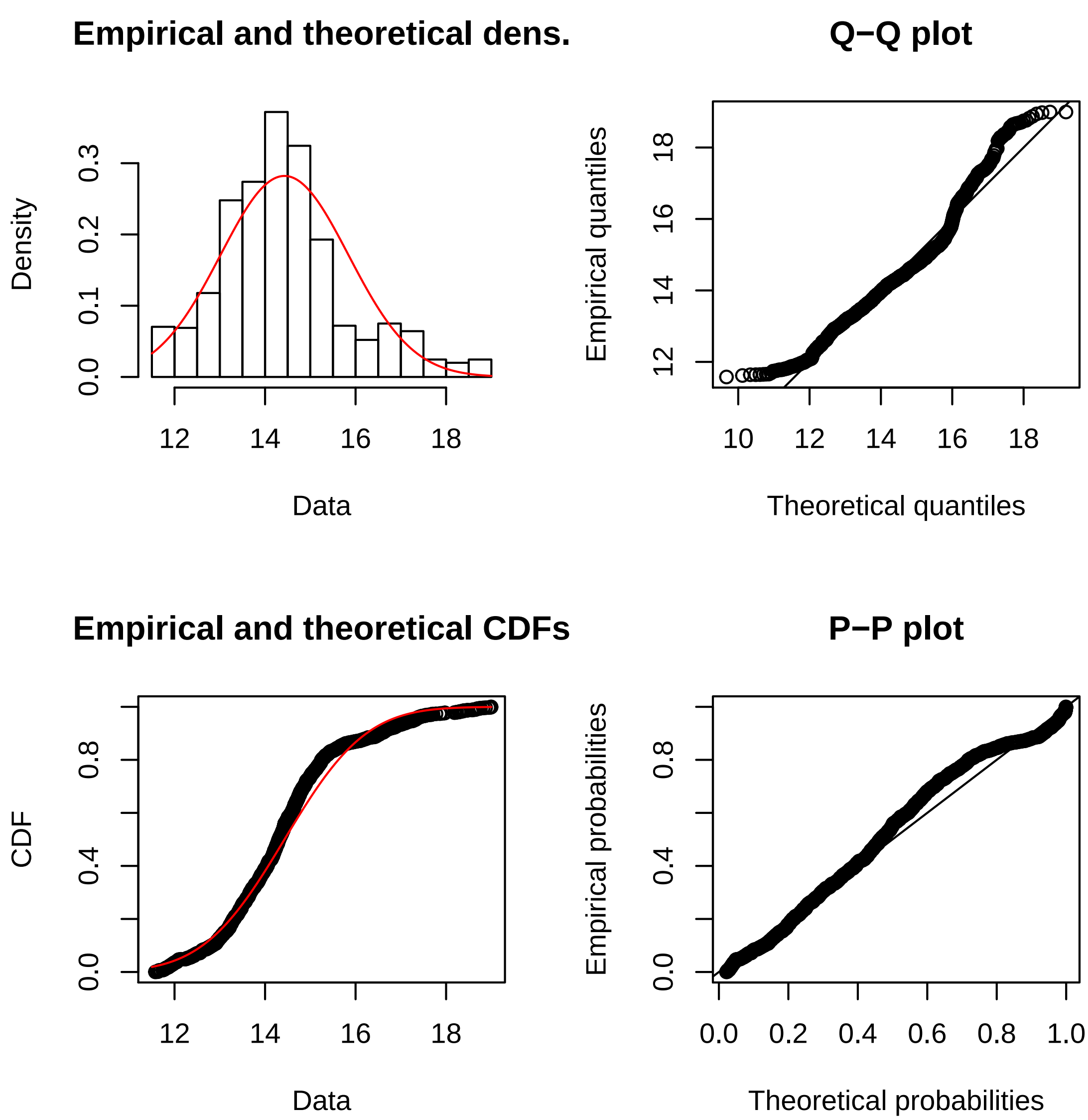

Figure 4.6: Normal distribution diagnostic plots of the USD/ZAR exchange rate.

Log-normal distribution

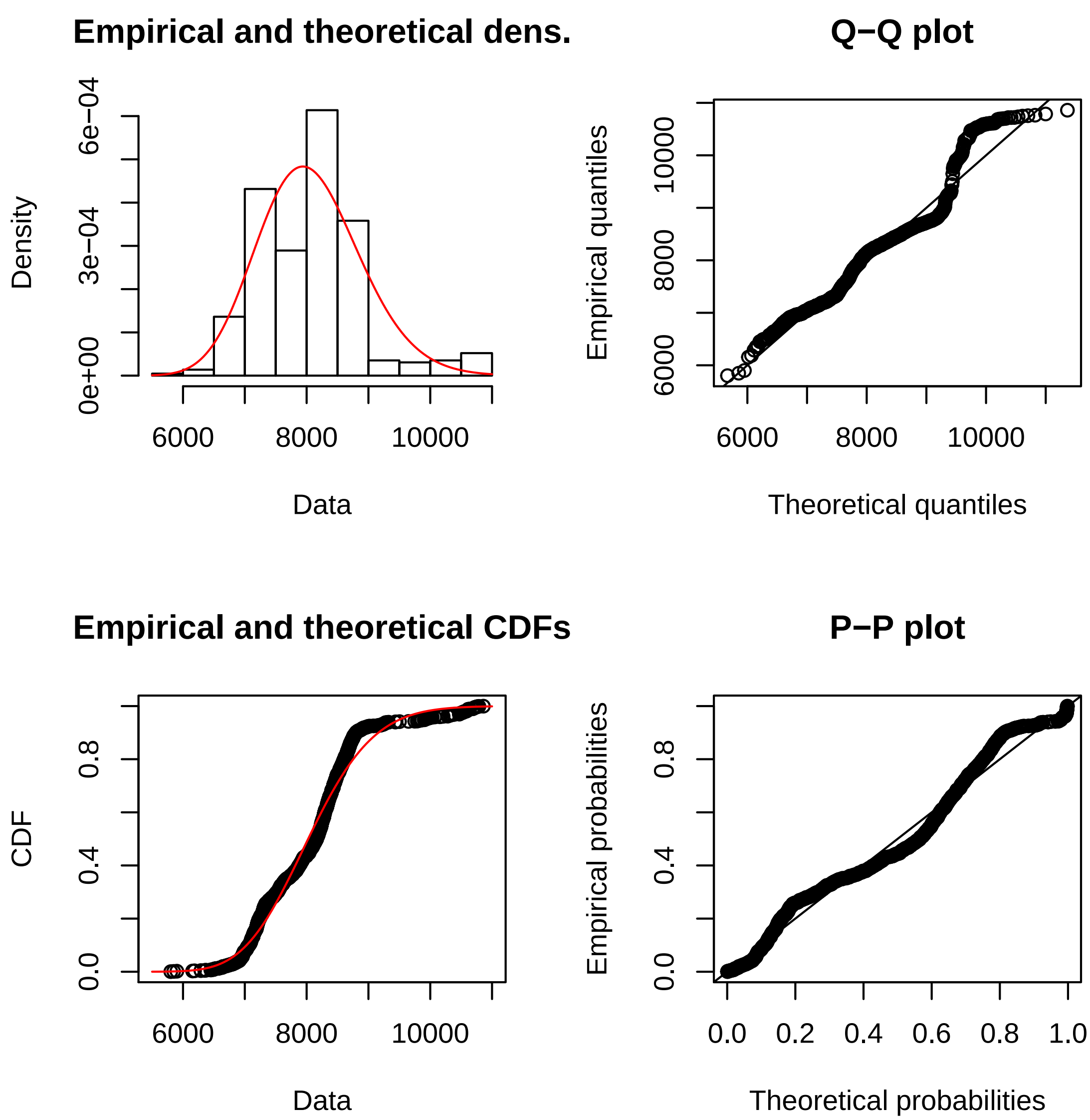

Figure 4.7: Log-normal distribution diagnostic plots of the All-Share Total Return Index.

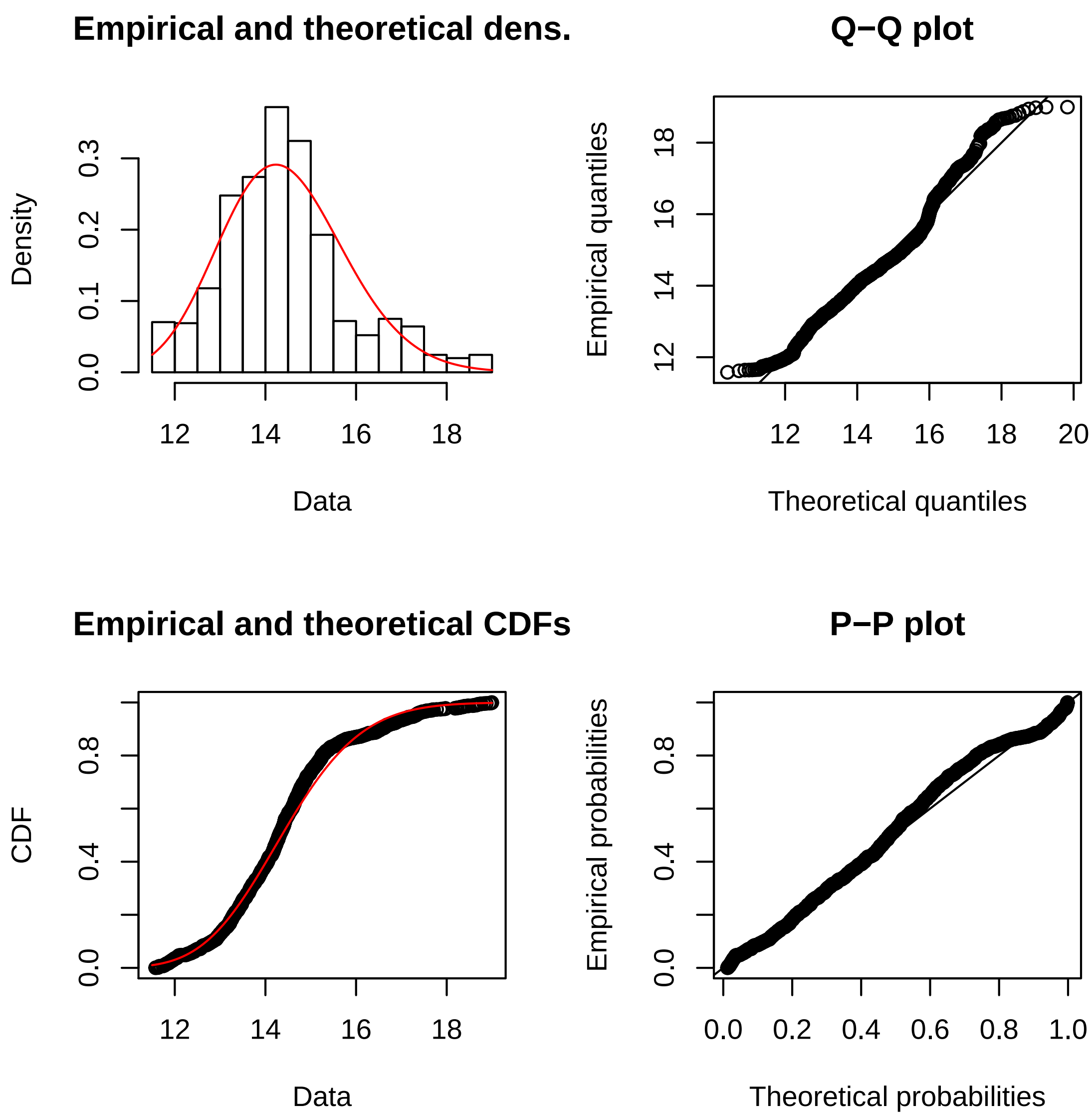

Figure 4.8: Log-normal distribution diagnostic plots of the USD/ZAR exchange rate.

Weibull Distribution

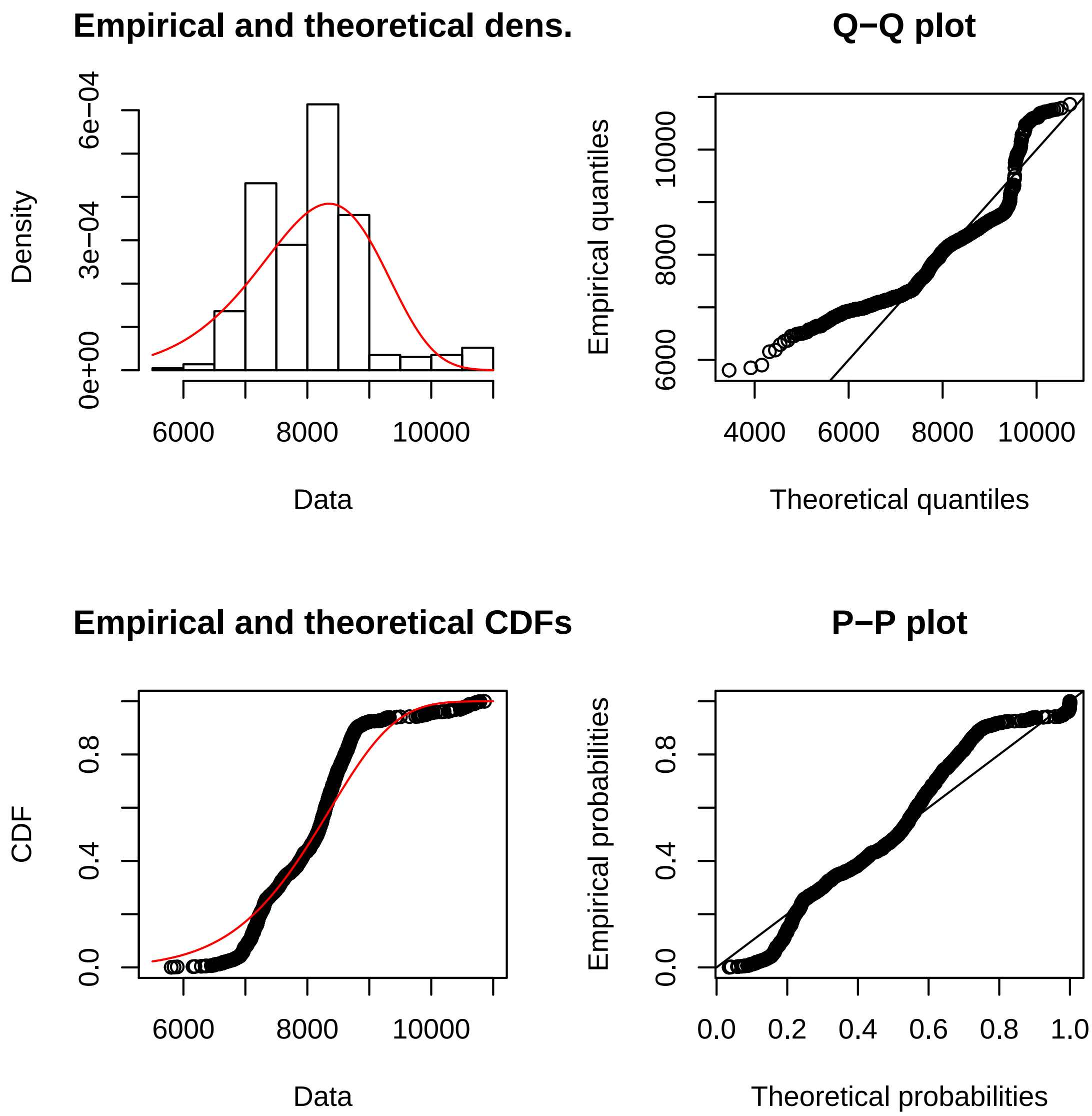

Figure 4.9: Weibull distribution diagnostic plots of the All-Share Total Return Index.

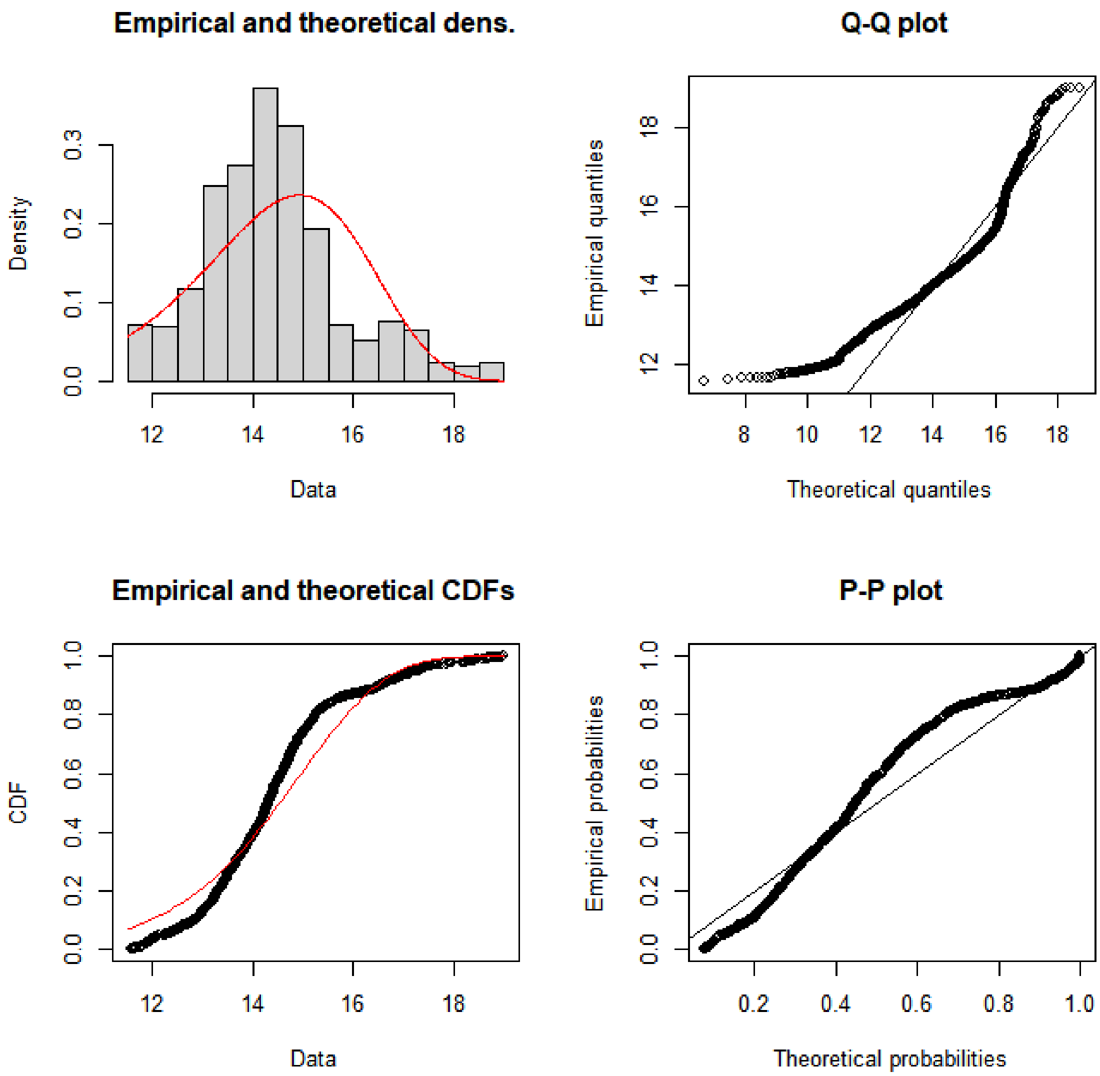

Figure 4.10: Weibull distribution diagnostic plots of the USD/ZAR exchange rate.

Gamma distribution

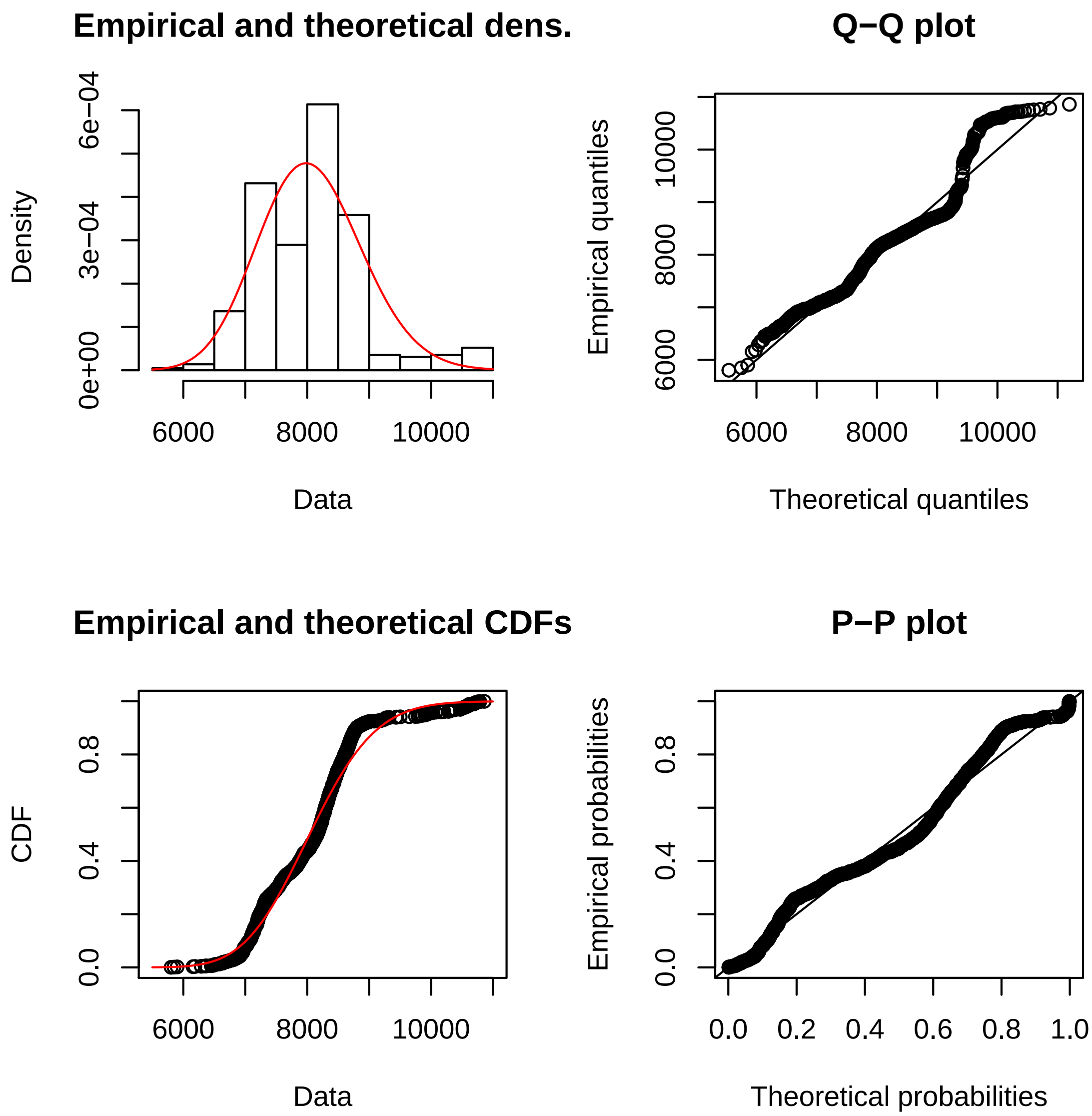

Figure 4.11: Gamma distribution diagnostic plots of the All-Share Total Return Index.

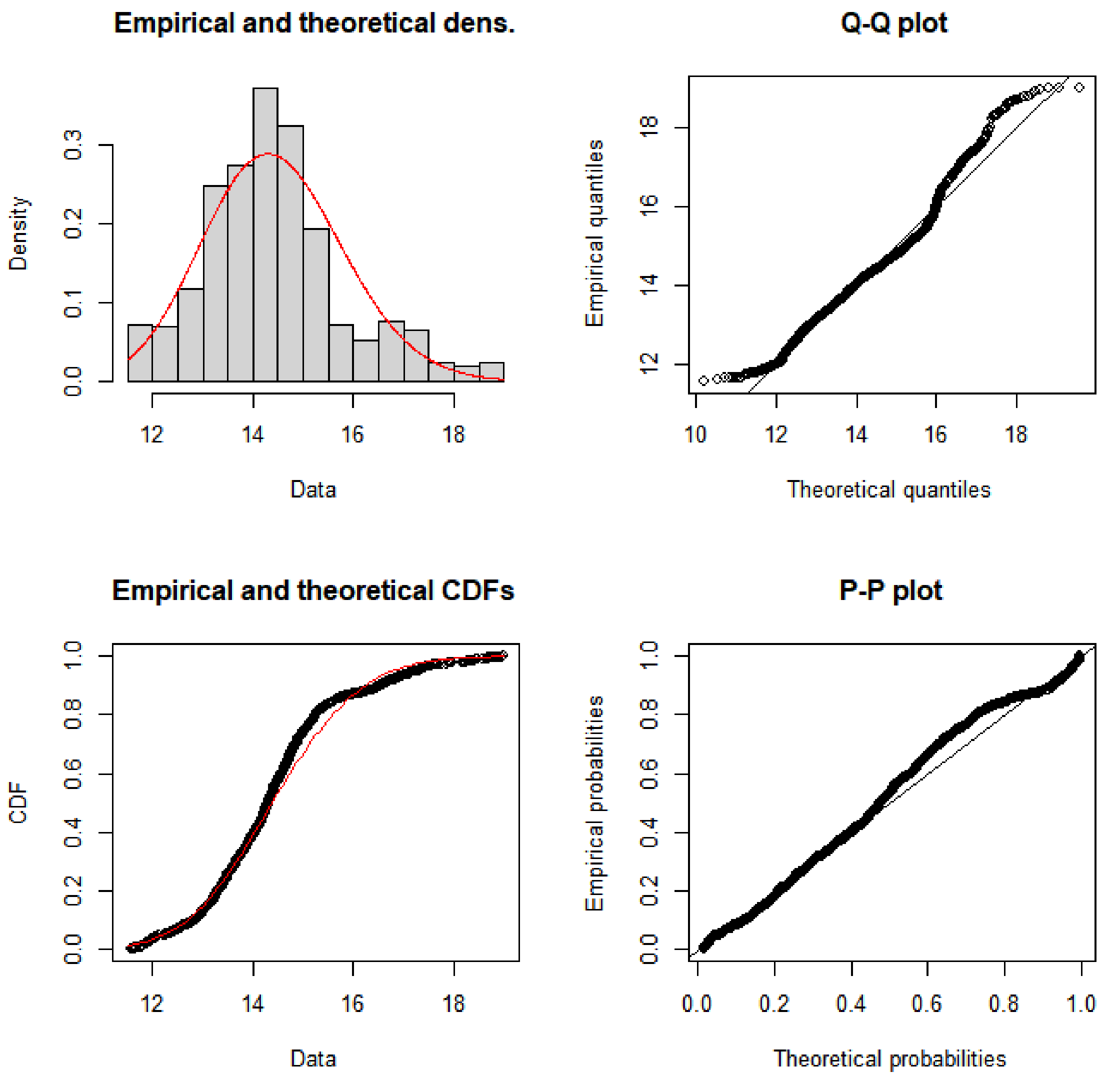

Figure 4.12: Gamma distribution diagnostic plots of the USD/ZAR exchange rate.

4.4.2 AIC and BIC of the ALSTRI and USD/ZAR exchange rate

To determine where the data is more concentrated, the parent distribution assists in fitting the data to detect extreme values and this gives rise to extreme value analysis. To select the best fit of the parent distribution, the AIC and the BIC tests were conducted and the distribution with the least AIC and BIC was selected as the best fit for the daily ALSTRI and the USD/ZAR exchange rate data.

Table 4.5: Summary of parent distributions AIC and BIC.

Financial Market	Distribution	AIC Value	BIC Value
All Share Index Total Return Index	Normal	21357.91	21368.26
	Log-Normal	21281.74	21292.09
	Weibull	21668.78	21679.13
	Gamma	**21301.81**	**21312.16**
USD/ZAR exchange rate	Normal	4618.805	4629.156
	Log-Normal	4548.321	4558.672
	Weibull	4917.611	4927.962
	Gamma	**4567.584**	**4577.935**

Table 4.5 presents the goodness-of-fit results for the parent distributions for both the ALSTRI and the USD/ZAR exchange rate. The results reveal that the best parent distribution fit is the Log-normal when fitted for ALSTRI and the USD/ZAR exchange rate. However, when fitting the Log-normal using the ALSI log-returns, the Log-normal cannot fit the log-returns because they consist of negative numbers. Log-normal can only fit non-negative numbers. Thus, the best fit for the ALSTRI log-returns is the Gamma distribution.

4.4.3 Trend analysis

Trend analysis was conducted through the Mann-Kendall (M-K) test. The results of the trend analysis are shown in Table 4.6. The p-values of the ALSTRI and the USD/ZAR exchange rate are less than the significance level of $\alpha = 0.05$ ($p < 0.05$). The results suggest that there is a highly, highly significant trend in both the ALSI and the USD/ZAR exchange rate.

Table 4.6: Summary of trend analysis.

Financial Market	M-K Test Statistic (S)	Kendall's τ	Var (S)	p-value
ALSTRI	523155	0.6129748	248360100	< 0.001
USD/ZAR exchange rate	281525	0.3298863	248359900	< 0.001

Key: ALSTRI = All Share Total Return Index.

4.5 Extreme Value Analysis

This section presents the EVT models. The extreme values are fitted at the tails of the data. The block maxima approach is employed. The section is divided into five sub-sections for fitting the EVT models namely; GEVD, bGEVD, rGEVD, GPD, and the Poisson point process.

4.5.1 GEVD models

The weekly block maxima approach was used to model the total return index of the ALSTRI and the USD/ZAR exchange rate. The MLE and the Bayesian parameter estimation methods were used to estimate the location (μ), scale (σ), and shape (ξ) parameters.

The parameter estimates for the GEVD weekly maxima of the ALSTRI and the USD/ZAR exchange rates are represented in Tables 4.7 and 4.8. The estimate of shape parameter is negative, which indicates that both datasets can be modelled by the Weibull family or domain of attraction. The confidence intervals at 95% of the shape parameter does not include zero, which implies that the data can be modelled by the Weibull family of distributions because $\xi < 0$.

In a study conducted by Makhwiting et al. (2014), the ALSI of the JSE was modelled, and the findings of the study illustrated that the underlying daily returns can be modelled by the Weibull class of distributions.

Table 4.7: MLE parameter estimates and standard errors (in parantheses) for the GEVD weekly maxima.

	Location (μ)	Scale (σ)	Shape (ξ)	95% CI of (ξ)	NLL (λ)
ALSTRI	7797.29	726.99	-0.07	(-0.14,1.50)	2128.33
	(50.11)	(35.62)	(0.04)		
USD/ZAR	13.99	1.26	-0.099	(-0.18,-0.02)	457.80
	(0.085)	(0.059)	(0.039)		

Key: ALSTRI = All Share Total Return Index, USD/ZAR = USD/ZAR exchange rate, CI = Confidence Intervals, and NLL = Negative Log-Likelihood.

Table 4.8: Bayesian parameter estimates for the GEVD weekly maxima.

	Location (μ)	log.scale (σ)	Shape (ξ)	95% CI of (ξ)	PM of (ξ)
ALSTRI	4.0409	1522.71	0.0020	(-0.1484,0.0264)	-0.0662
USD/ZAR	0.0093	0.0045	0.0021	(-0.174,0.006)	-0.0893

Key: ALSTRI = All Share Total Return Index, CI = Confidence Intervals, PM = Posterior Mean, and USD/ZAR = USD/ZAR exchange rate.

The monthly block maxima approach was used to model the total return index of the All Share Total Return Index and the USD/ZAR exchange rates. The MLE and the Bayesian parameter estimation methods were used to estimate the location (μ), scale (σ), and shape (ξ) parameters.

The MLE parameter estimates for the GEVD monthly maxima of the ALSTRI and the USD/ZAR exchange rates are presented in Table 4.9. The estimate of shape parameter is negative, which indicates that both datasets can be modelled by the Weibull family or domain of attraction. The confidence intervals at 95% of the shape parameter do not include zero, which implies that the data can be modelled by the Weibull family of distributions because $\xi < 0$.

Table 4.9: MLE parameter estimates and standard errors (in parantheses) for the GEVD monthly maxima.

	Location (μ)	Scale (σ)	Shape (ξ)	95% CI of (ξ)	NLL (λ)
ALSTRI	7960.54	743.68	-0.083	(-3.02,0.007)	503.0
	(104.56)	(75.43)	(0.084)		
USD/ZAR	14.27	1.292	-0.115	(-0.267,0.037)	111.25
	(0.179)	(0.077)	(0.124)		

Key: ALSTRI = All Share Total Return Index, USD/ZAR = USD/ZAR exchange rate, CI = Confidence Intervals, and NLL= Negative Log-Likelihood.

Makatjane et al. (2021) administered a study titled “Predicting Extreme Daily Regime Shifts in Financial Time Series Exchange/Johannesburg Stock Exchange - All Share Index”. The Bayesian MCMC parameter estimation approach was employed to estimate the parameters for the GEVD. According to Makatjane et al. (2021), complex likelihood functions with higher dimensions are difficult to estimate, thus, the Bayesian methods have densities to solve complex model estimation. The shape parameter was found to be negative, suggesting that FTSE/JSE-ALSI returns conform to a Weibull class of distribution.

The Bayesian parameter estimates for the GEVD monthly maxima of the ALSTRI and the USD/ZAR exchange rates are presented in Table 4.10. The estimate of shape parameter is negative, which indicates that both datasets can be modelled by the Weibull family or domain of attraction. The confidence intervals at 95% of the shape parameter do not include zero, which implies that the data can be modelled by the Weibull family of distributions.

Table 4.10: Bayesian parameter estimates for the GEVD monthly maxima.

	Location (μ)	log.scale (σ)	Shape (ξ)	95% CI of (ξ)	PM of (ξ)
ALSTRI	3.9385	7187.27	0.0089	(-0.2137,0.1498)	-0.0545
USD/ZAR	0.0553	0.0270	0.0097	(-0.266,0.111)	-0.093

Key: ALSTRI = All Share Total Return Index, CI = Confidence Intervals, PM = Posterior Mean, and USD/ZAR = USD/ZAR exchange rate.

Diagnostic plot

The diagnostic plot for assessing the GEVD monthly maxima are displayed in Figures 4.13, and 4.14 respectively. The P-P plot, Q-Q plot, return level, and density plot assess whether the dataset follows the GEVD. The ALSTRI and the USD/ZAR exchange rates are plotted on the theoretical distribution which indicates that the plotted points are linear, implying that the GEVD is a good fit for the financial market data. The return level plot shows the return period against the return level together with an estimated 95% confidence interval (Makhwiting, 2014).

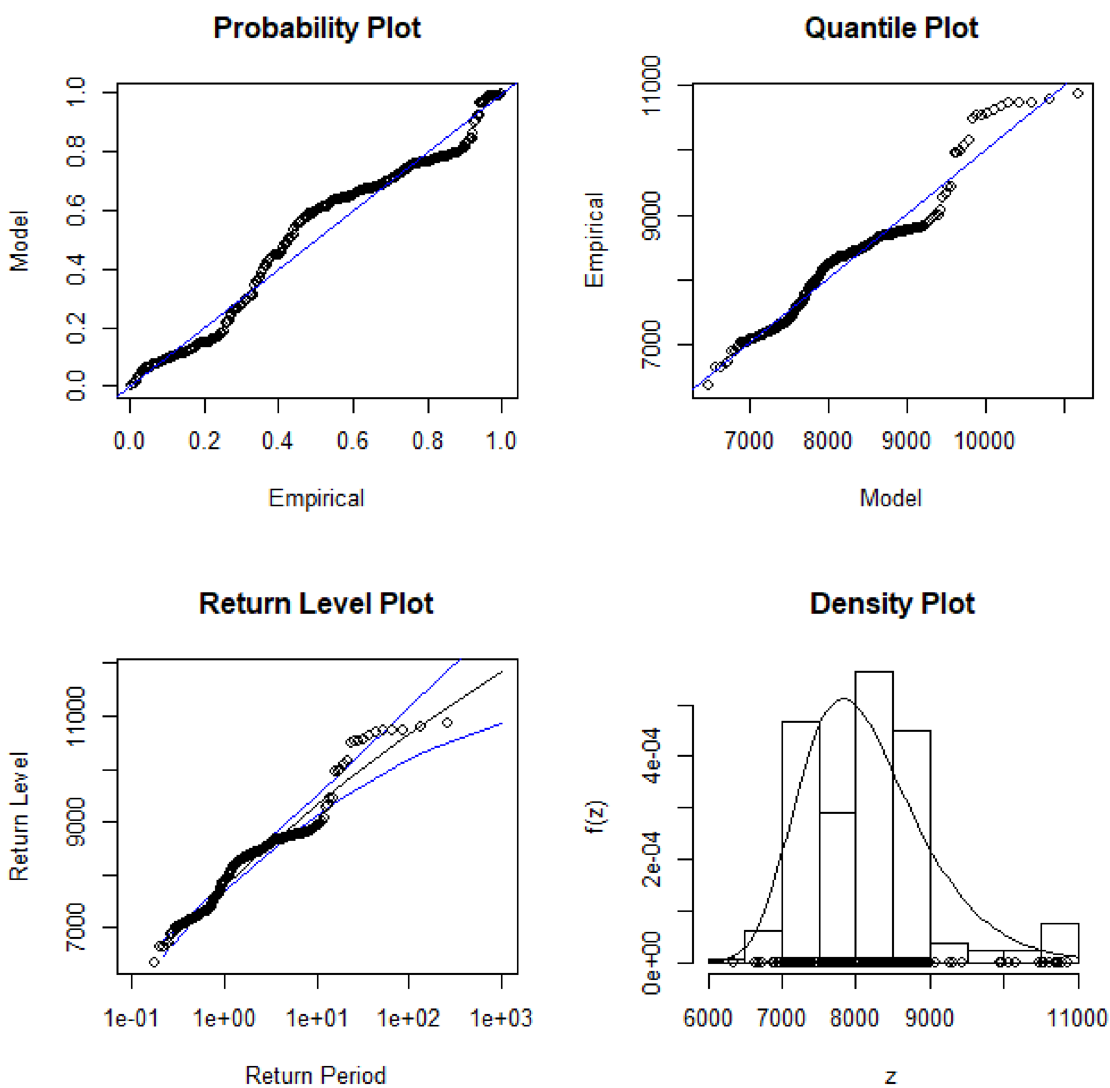

Figure 4.13: GEVD monthly maxima diagnostic plots for the All-Share Total Return Index.

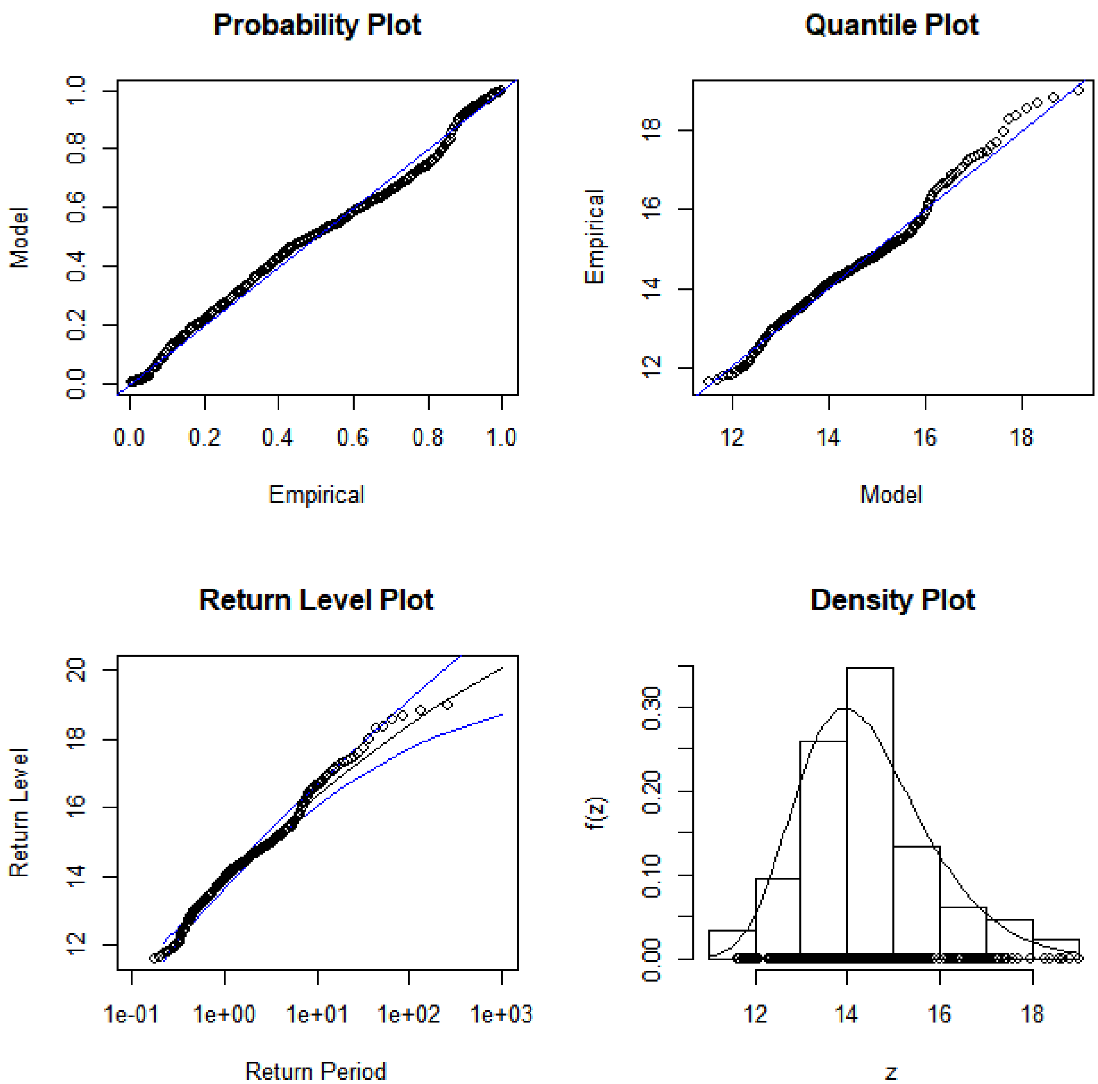

Figure 4.14: GEVD monthly maxima diagnostic plots for the USD/ZAR exchange rates.

Return levels

Return levels together with the corresponding return periods for 5−years, 10−years, 20−years, 50−years, and 100−years were computed as employed by Nemukula (2018) and the results of the return levels and corresponding return periods are shown in Tables 4.11, 4.13, and 4.14. There is an increase in return levels as the return period rises. Also, results reveal that the monthly block maxima return levels are slightly higher than the corresponding weekly block maxima return levels. The Bayesian monthly block maxima return levels are slightly higher than the corresponding MLE monthly block maxima return levels.

Table 4.11: MLE weekly block maxima return levels for the GEVD model.

Financial Market	5−years	10−years	20−years	50−years	100−years
ALSTRI	8835.25	9317.06	9757.22	10296.63	10679.55
USD/ZAR	15.56	16.33	17.03	17.85	18.43

Key: ALSTRI = All Share Total Return Index, and USD/ZAR = USD/ZAR exchange rate.

Table 4.12: Bayesian weekly block maxima return levels for the GEVD model.

Financial Market	5−years	10−years	20−years	50−years	100−years
ALSTRI	8872.18	9360.90	9807.46	10354.83	1074.47
USD/ZAR	15.58	16.37	17.08	17.93	18.53

Key: ALSTRI = All Share Total Return Index, and USD/ZAR = USD/ZAR exchange rate.

Table 4.13: MLE monthly block maxima return levels for the GEVD model.

Financial Market	5−years	10−years	20−years	50−years	100−years
ALSTRI	9009.11	9486.51	9917.22	10437.70	10802.07
USD/ZAR	16.05	16.83	17.52	18.33	18.89

Key: ALSTRI = All Share Total Return Index, and USD/ZAR = USD/ZAR exchange rate.

Table 4.14: Bayesian monthly block maxima return levels for GEVD.

Financial Market	5−years	10−years	20−years	50−years	100−years
ALSTRI	9075.05	9595.61	10076.19	10672.10	11100.08
USD/ZAR	16.13	16.97	17.71	18.61	19.23

Key: ALSTRI = All Share Total Return Index, and USD/ZAR = USD/ZAR exchange rate.

In conclusion, the monthly block maxima return levels reveal that the ALSTRI and USD/ZAR exchange rate will exceed 11100.08 and R19.23 respectively, at least once in 100 years.

4.5.2 bGEVD models

The bGEV distribution (bGEVD) was proposed by Castro-Camilo in (2020), as a substitute for the GEVD in phenomena where the tail parameter ξ is non-negative (Vandeskog et al., 2021b). The bGEVD smoothly incorporates the left tail of a Gumbel distribution which indicates a GEVD with $\xi = 0$ and a right tail of a Frèchet distribution with $\xi > 0$ (Castro-Camilo et al., 2021). In accordance with Vandeskog et al. (2022), the bGEVD is only applicable for modelling heavy-tailed or exponential phenomena where $\xi > 0$.

There are no studies conducted on bGEVD in the field of finance or financial markets. However, there are limited studies conducted on bGEVD in other fields of application (Vandeskog et al., 2021a,b; Castro-Camilo et al., 2021; Vandeskog et al., 2022).

In a study conducted by Vandeskog et al. (2021a) block maxima was used to model the data using bGEVD, whereas Vandeskog et al. (2022) modelled the yearly maxima of the sub-daily precipitation using bGEVD. The monthly maxima of the ALSTRI and the USD/ZAR exchange rate were used to model the bGEVD. Prior to modelling the JSE financial market data with the bGEVD model, the explanatory variables were standardised to have a mean of zero and a standard deviation of 1 (Vandeskog et al., 2021b). Thus, Figures 4.15 and 4.16 displays the standardised monthly maxima of the ALSTRI and the USD/ZAR exchange used in modelling the bGEVD model.

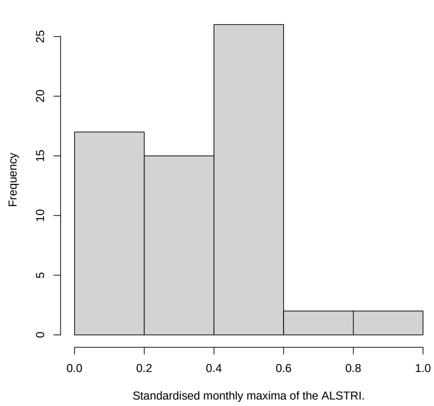

Figure 4.15: Histogram displaying the standardised monthly maxima of the ALSTRI.

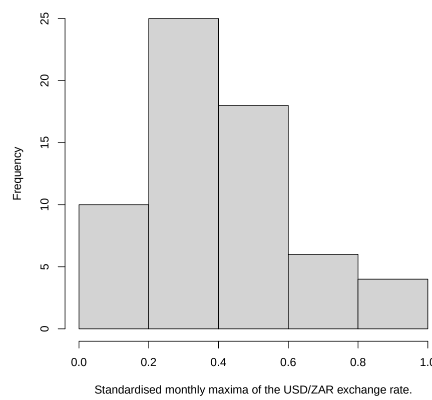

Figure 4.16: Histogram displaying the standardised monthly maxima of the USD/ZAR exchange rate.

Parameter estimates

Statistically, the location-spread parameterisation predominates when compared to the distribution's mean and standard deviation (Rue et al., 2009). Castro-Camilo et al. (2021) assert that the new parameterisation allows for the interpretation of the location and spread parameters even when the mean and the variance are unknown. The results of the bGEVD for the ALSTRI and the USD/ZAR exchange rate monthly maxima are presented in Table 4.15. The R-INLA package was used to model the bGEVD.

Table 4.15: Posterior means, standard deviation (SD (μ_q)) and quantiles for the estimated linear regression coefficients β_q, β_s, and hyperparameters of the bGEVD INLA fits for the monthly maxima of the JSE financial market data.

	Explanatory Variable	Mean	SD	2.5% Q	50% Q	97,5%
ALSTRI	bGEVD intercept	0.947	0.088	0.765	0.95	1.112
	Monthly ALSTRI maxima	0.387	0.212	-0.008	0.38	0.823
	s_β	1.143	0.121	0.925	1.135	1.402
	ξ	0.060	0.053	0.002	0.044	0.190
	β_q	0.105	0.046	0..14	0.105	0.197
	β_s	0.005	0.058	-0.109	0.005	0.119
USD/ZAR	bGEVD intercept	1.162	0.116	0.954	1.155	1.408
	Monthly USD/ZAR maxima	0.392	0.258	-0.189	0.421	0.815
	s_β	1.380	0.155	1.105	1.370	1.715
	ξ	0.112	0.028	0.067	0.108	0.176
	β_q	0.067	0.048	-0.030	0.068	0.160
	β_s	0.008	0.058	-0.106	0.008	0.122

Key: SD = Standard Deviation, Q = Quantile, s_β = Spread for bGEVD observations, ξ = Tail for bGEVD observations, β_q = (Spread) for bGEVD observations, and β_s = (Tail) for bGEVD observations.

Table 4.15 display the bGEVD intercept, explanatory variables which are the monthly maxima of the ALSTRI, and USD/ZAR exchange rate as well as the regression coefficients β_q and β_s. There is a positive linear trend term indicating an increase in the total returns of the ALSI and the USD/ZAR exchange rate over time. The explanatory variables have a greater effect on the location parameter. Thus, an increase in the total return of the ALSI and the USD/ZAR exchange rate is expected with more variance in the distribution of the extremes.

Return levels

Tables 4.16 and 4.17 displays the standardised and actual return levels of the bGEVD model for the ALSTRI and the USD/ZAR, respectively.

Table 4.16: Monthly block maxima standardised return levels for the bGEVD model.

Financial Market	5−years	10−years	20−years	50−years	100−years
ALSTRI	1.2630	1.0985	1.0023	1.0000	1.0000
USD/ZAR Exc Rate	1.2312	1,0774	1.0013	1.0000	1.0000

Key: ALSTRI = All Share Total Return Index, and USD/ZAR = USD/ZAR exchange rate.

Table 4.17: Monthly block maxima actual return levels for the bGEVD model.

Financial Market	5−years	10−years	20−years	50−years	100−years
ALSTRI	8688.45	9886.61	10834.82	10860	10860
USD/ZAR Exc Rate	16.83	17.63	18.96	18.99	18.99

Key: ALSTRI = All Share Total Return Index, and USD/ZAR = USD/ZAR exchange rate.

As the return period increases, the return levels substantially increase in Table 4.17. The 100−year return levels are slightly equivalent to the maximum observed values for both the ASTRI and the USD/ZAR exchange rate, respectively. Thus, the monthly block maxima return levels reveal that the ALSTRI and the USD/ZAR exchange rate will exceed 10860, and R18.99, respectively. In conclusion, the findings of the return levels for the bGEVD and GEVD models suggest that the two models result in similar return level outcomes. This suggests that bGEVD can be a suitable alternative for the GEVD for modelling block maxima (Castro-Camilo et al., 2021; Vandeskog et al., 2021a).

4.5.3 rGEVD models

The r-largest order statistics was employed with the aim to model the characterisation behaviour of extremes of the JSE financial data, rather than making use of a single maximum value within a block in order to model the JSE financial data. The r-largest order approach is usually used in cases where there is limited data. The monthly block maxima approach was employed to model the 5$-$year daily data. In the case where $r = 1$, it is similar to the results of the GEVD, i.e., the values are sampled from the monthly block maxima. The rGEVD is fitted in this section. The MLE and Bayesian parameter estimation methods are employed to fit the parameters of the rGEVD. The estimated parameters are location (μ), scale (σ), and shape (ξ), confidence interval of the shape parameter and, the negative log-likelihood (λ_i).

MLE parameter estimaton method

The MLE parameter estimation approach was employed by various authors on rGEVD which includes among others, (Mashishi, 2020; Nemukula and Sigauke, 2018; Kajambeu et al., 2020).

Table 4.18 presents the rGEVD model parameter estimates for the location ($\hat{\mu}$), scale ($\hat{\sigma}$) and, shape ($\hat{\xi}$) using the MLE parameter estimation technique for $r = 1, ..., r = 10$. The parameter estimates of the shape parameter for $r = 1, ..., r = 10$ are all negative for both the total returns of the ALSI and the USD/ZAR exchange rate. It implies that a distribution with a Weibull domain of attraction may be used to model the total returns of the ALSI and the USD/ZAR exchange rates data because $\xi < 0$.

With regards to the ALSTRI, the standard errors for all parameters decrease as r increases. The confidence intervals of the shape parameter for the total return index of the ALSI in Table 4.18 for $r = 1$ to $r = 5$ comprises zero suggesting that the data may also be best modelled by the Gumbel distribution. The confidence intervals of the shape parameter for $r = 6$ to $r = 10$ does not contain zero implying that it is significantly different from zero (Chikobvu and Chifurira, 2015), which suggests that the data can be modelled by the Weibull family of distribution (Nemukula and Sigauke, 2018).

The findings of the USD/ZAR exchange rates in Table 4.18 indicate that the standard errors for all parameters decrease as r increases. The confidence intervals of the shape parameter for the USD/ZAR exchange rates when $r = 1$ contains zero suggesting that the Gumbel distribution may also be best suited to model the data. The confidence intervals of the shape parameter when $r = 2$ to $r = 10$ does not contain zero and this indicates that it is significantly different from zero (Chikobvu and Chifurira, 2015).

Table 4.18: MLE parameter estimates and standard errors (in parantheses) for the rGEVD monthly maxima.

	r	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\xi}$	CI(95%) of $\hat{\xi}$	$\hat{\lambda}_i$
	1	7960.54(104.56)	743.68(75.43)	-0.083(0.084)	(-3.02,0.007)	503.70
	2	7907.17(76.30)	771.05(56.70)	0.128(0.059)	(-0.25,8.15)	503.697
ASI	3	7921.10(59.16)	722.34(41.37)	-0.074(0.048)	(-0.16,0.0198)	1509.62
	4	**7909.55(51.26)**	**724.97 (36.34)**	**-0.073(0.042)**	**(-0.16,8.23)**	**2012.28**
	5	7895.81(45.68)	721.93(32.38)	0.071(0.03)	(-0.14,0.0028)	2514.90
	6	7881.46(41.65)	720.20(29.53)	-0.068(0.03)	(-0.13,-0.0005)	3017.98
	7	7871.31(38.49)	717.21(27.15)	-0.066(0.03)	(-0.13,-0.0035)	3520.82
	8	7855.15(36.01)	718.95(25.76)	-0.06(0.03)	(-0.12,-0.004)	4023.53
	9	7848.58(33.96)	716.84(24.06)	-0.064(0.03)	(-0.12,-0.007)	4527.09
	10	7836.89(32.27)	718.14(22.92)	-0.064(0.027)	(-0.12,-0.01)	5030.87
	1	14.27(0.179)	1.292(0.124)	-0.115(0.077)	(-0.267,0.037)	111.25
	2	14.23(0.126)	1.285(0.087)	-0.112(0.055)	(-0.221,-0.005)	221.983
$/R	3	14.20(0.11)	1.28(0.07)	-0.107(0.04)	(-0.20,-0.02)	332.21
	4	**14.17(0.09)**	**1.27(0.06)**	**-0.11(0.03)**	**(-0.18,-0.03)**	**442.12**
	5	14.15(0.08)	1.27(0.05)	-0.105(0.03)	(-0.17,-0.04)	552.13
	6	14.12(0.072)	1.267(0.049)	-0.1048(0.031)	(-0.167,-0.043)	661.95
	7	14.10(0.066)	1.264(0.046)	-0.1038(0.029)	(-0.161,-0.046)	771.54
	8	14.08(0.062)	1.260(0.043)	-0.1027(0.028)	(-0.157,-0.049)	880.83
	9	14.06(0.058)	1.258(0.040)	-0.1017(0.026)	(-0.153,-0.051)	990.02
	10	14.04(0.055)	1.254(0.038)	-0.101(0.024)	(-0.149,-0.052)	1098.61

Key: ASI = All Share Total Return Index, $/R = USD/ZAR exchange rate, and CI = Confidence Intervals.

Bayesian parameter estimaton method

Da Silva and do Nascimento (2019) used the Bayesian MCMC method to model the r-largest order in EVT. The results revealed that the Bayesian method produced more accurate estimates as compared to other parameter estimation approaches such as MLE. The MCMC Bayesian parameter estimation method was employed by e Silva et al. (2020) to model the r-largest order statistics of the environmental and financial data.

The Bayesian approach was used to estimate the parameters of the r-largest order statistics of the GEVD for r = 1, . . . , r = 10, with MCMC (Da Silva and do Nascimento, 2019). As r increased, estimator accuracy improved. Table 4.19 presents the Bayesian parameter estimates for the total return index of the ALSI and the USD/ZAR exchange rates.

For both financial market data, the shape parameter for all r order statistics is positive. However, the entire posterior mean of the shape parameter is negative. The confidence intervals of the shape parameter for the total return index of the ALSI in Table 4.19 for $r = 1$ to $r = 6$ include zero suggesting that the data may also be best modelled by the Gumbel distribution. Likewise, the confidence intervals of the shape parameter for the USD/ZAR exchange rates for $r = 1$ to $r = 3$ comprise zero suggesting that the data may also be best modelled by Gumbel distribution.

In accordance with Tawn (1988); Bader et al. (2017); Nemukula and Sigauke (2018), the high values of r are likely to violate the model and lead to the occurrence of bias whereas too low and few values of r can lead to a high variance of the estimator, thus, r must be as small as possible. According to the findings of Mashishi (2020), the order statistics of $r = 5$ was found to be the perfect fit for the data. The results of Nemukula and Sigauke (2018); Nemukula (2018) revealed that $r = 4$ was the best fit for the data. Kajambeu et al. (2020) found that the suitable fit order statistics for the data was $r = 6$.

Table 4.19: Bayesian MCMC parameter estimates for rGEVD monthly maxima with r = 1, . . . , r = 10, posterior mean and confidence interval of the shape parameter with lower limit (2.5%) and upper limit (97.5%) .

	r	$\hat{\mu}$	log- $\hat{\sigma}$	$\hat{\xi}$	CI of $\hat{\xi}$	PM of $\hat{\xi}$
ASI	1	3.9385	7187.27	0.0089	(-0.2137,0.1498)	-0.0545
	2	3.5876	3510.50	0.0047	(-0.1851,0.08219)	-0.0680
	3	5.1359	2164.15	0.0027	(-0.1651,0.0369)	-0.0692
	4	**12.5749**	**1516.26**	**0.0019**	**(-0.1550,0.0167)**	**-0.0734**
	5	3.1170	1189.84	0.0015	(-0.1448,0.008247)	-0.0728
	6	3.2037	982.35	0.0013	(-0.1387,0.004181)	-0.0702
	7	5.2620	854.73	0.0010	(-0.1309,-0.003485)	-0.0705
	8	6.1561	798.04	0.0009	(-0.1256,-0.009892)	-0.0694
	9	32.413	665.25	0.0009	(-0.1238,-0.0049)	-0.0653
	10	11.1110	552.88	0.0008	(-0.1196,-0.00919)	-0.0671
$/R	1	0.0553	0.0270	0.0097	(-0.266,0.111)	-0.093
	2	0.0227	0.0116	0.0045	(-0.2286,0.0389)	-0.1049
	3	0.0139	0.0065	0.0027	(-0.1961,0.0032)	-0.1035
	4	**0.0101**	**0.0047**	**0.0019**	**(-0.1833,-0.0142)**	**-0.1038**
	5	0.0078	0.0039	0.0015	(-0.175,-0.0252)	-0.1029
	6	0.0063	0.0030	0.0012	(-0.1662,-0.0327)	-0.1001
	7	0.0052	0.0027	0.0011	(-0.1620,-0.0329)	-0.1008
	8	0.0048	0.0021	0.0009	(-0.1594,-0.0408)	-0.1014
	9	0.0039	0.0019	0.0008	(-0.1564,-0.0431)	-0.1000
	10	0.0035	0.0017	0.0007	(-0.1489,-0.0445)	-0.0993

Key: ASI = All Share Total Return Index, $/R = USD/ZAR exchange rate, CI = Confidence Intervals, and PM = Posterior Mean.

According to Tables 4.18, and 4.19, the shape parameter decreases as r increases. In conclusion, the suitable model for the ALSTRI was found to be GEVD$_{r=4}$ and for the USD/ZAR exchange rate data, the best model is GEVD$_{r=4}$.

Entropy difference test results

Testing the rGEVD distribution's goodness of fit using a sequence of the null hypothesis is fundamental (Bader et al., 2017; Nemukula and Sigauke, 2018).The entropy difference test is used in this study to test the null hypothesis H_0^r: the rGEVD distribution fits the sample of the r-largest order statistics well for $r = 1$,...R, where R is the maximum of the specified number of the top order statistics test (Nemukula and Sigauke, 2018).

Table 4.20: Entropy difference test for the rGEVD monthly maxima.

	r	p-value	FS	SS	Statistic	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\xi}$
	2	< 0.001	0.1022	0.2011	5.0119	37.8250	13.8850	-0.4403
	3	< 0.001	0.1159	0.0585	4.2664	42.9786	11.6131	-0.4402
ASI	**4**	**0.0007**	**0.1314**	**0.0239**	**3.3734**	**46.1198**	**10.2285**	**-0.4405**
	5	0.0400	0.1531	0.0107	2.0533	48.2916	9.2697	-0.4404
	2	0.0039	0.1510	0.5396	2.8879	38.4297	14.1052	-0.4405
	3	0.0604	0.1694	0.4273	1.8780	43.6701	11.8000	-0.4403
$/R	**4**	**0.0108**	**0.1847**	**0,2557**	**2.5498**	**46.8554**	**10.3947**	**-0.4404**
	5	0.0067	0.2137	0.1296	2.7106	49.0684	9.4226	-0.4403

Key: ASI = All Share Total Return Index, $/R = USD/ZAR exchange rate, FS = ForwardStop, and SS = StrongStop.

Table 4.20 presents the entropy difference test results. The forward and the strong Stop are also provided.

Diagnostic plot

The diagnostic plot for assessing the GEVD$_{r=4}$ model for the ALSTRI and the USD/ZAR exchange rates are displayed in Figures Figures 4.17 and 4.18 respectively. The P-P plot, Q-Q plot, return level, and density plot assess are displayed.

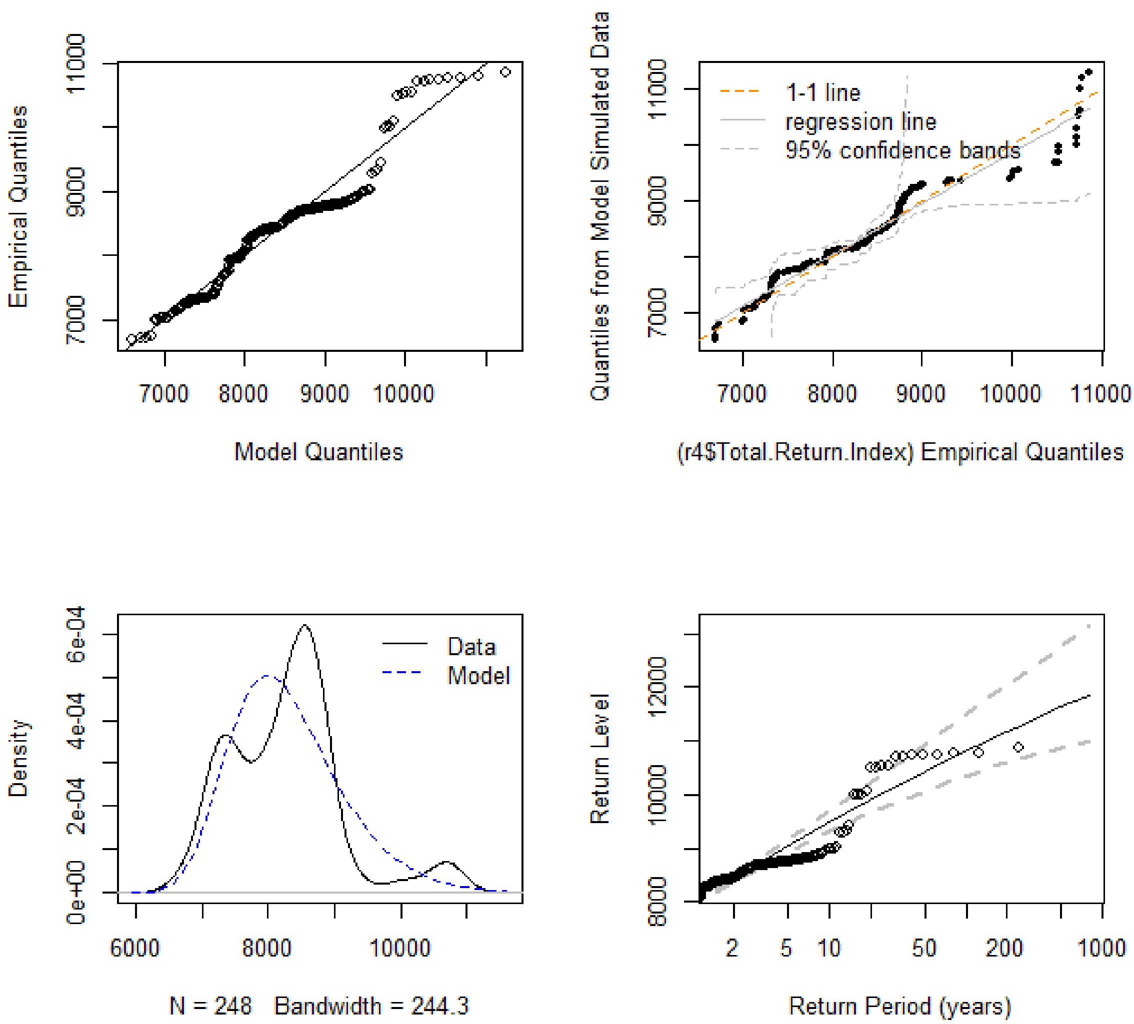

Figure 4.17: $\text{GEVD}_{r=4}$ diagnostic plots of the ALSTRI.

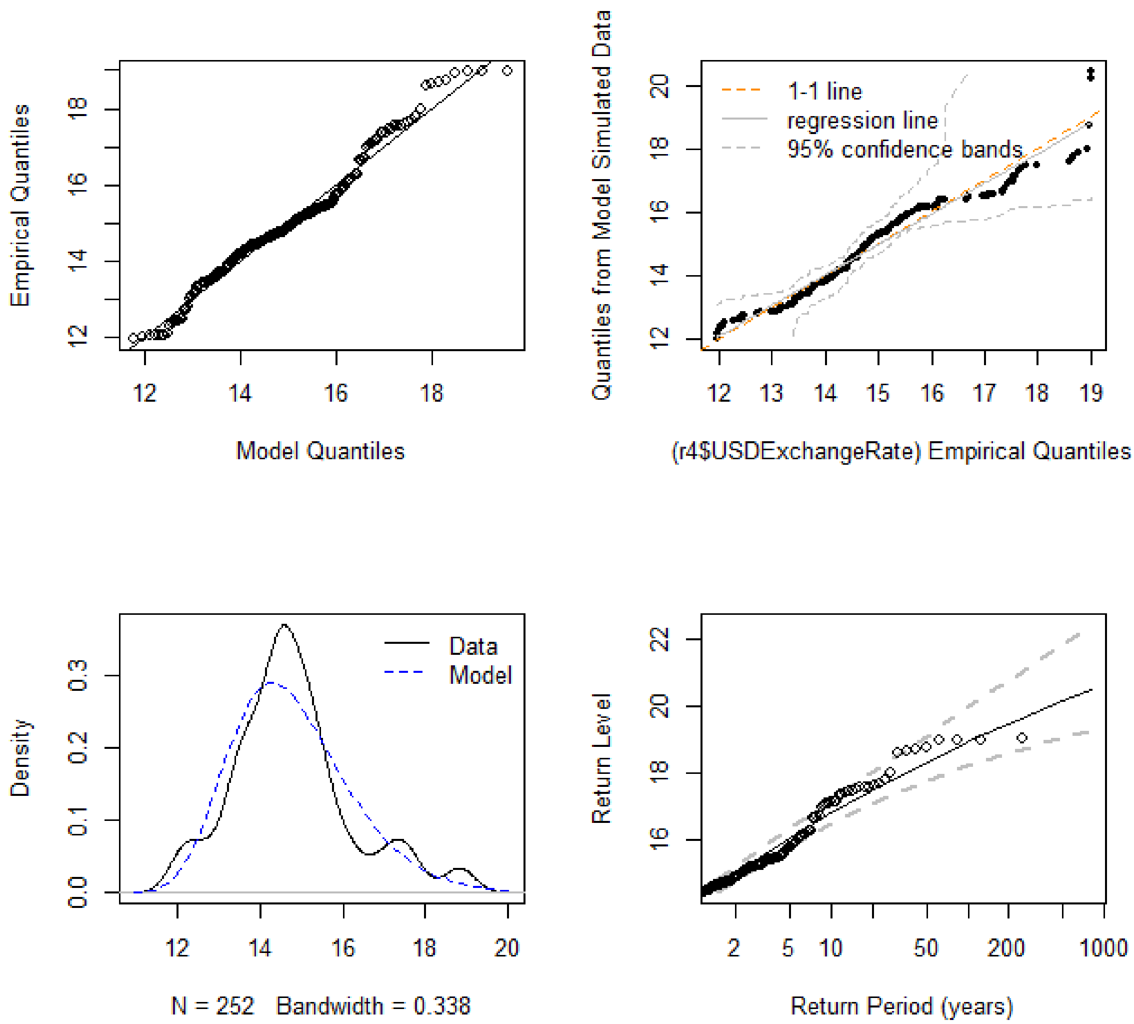

Figure 4.18: $\text{GEVD}_{r=4}$ diagnostic plots of the USD/ZAR exchange rates.

Return levels

Return levels in accordance with return periods for 5−years, 10−years, 20−years, 50−years, and 100−years were computed as employed by Nemukula (2018) and the results are shown in Tables 4.21 and 4.22, respectively. There is an increase in return levels as the return period increases. The Bayesian return levels are slightly higher as compared to the corresponding MLE return levels. This implies that the Bayesian parameter method performed better in comparison with the MLE method (Seenoi et al., 2020; Debusho and Diriba, 2016; Takyi and Bentum-Ennin, 2021). The return level results for the Bayesian approach suggest that the ALSTRI will exceed 10796.5 at least once in 100 years, while the USD/ZAR exchange rate will surpass R18.87 at least once in 100 years. The rGEVD Bayesian return levels are closely comparable with the GEVD, and bGEVD monthly block maxima estimates.

Table 4.21: MLE return levels for rGEVD.

	r	5−years	10−years	20−years	50−years	100−years
ALSTRI	4	8938.88	9412.61	9842.10	10367.16	10736.97
USD/ZAR	4	15.93	16.72	17.41	18.23	18.80

Key: ALSTRI = All Share Total Return Index, and USD/ZAR = USD/ZAR exchange rate.

Table 4.22: Bayesian return levels for rGEVD.

	r	5−years	10−years	20−years	50−years	100−years
ALSTRI	4	8967.12	9447.35	9884.68	10418.73	10796.50
USD/ZAR	4	15.95	16.75	17.45	18.29	18.87

Key: ALSTRI = All Share Total Return Index, and USD/ZAR = USD/ZAR exchange rate.

4.5.4 GPD models

This section presents the results of the analysis of the generalised Pareto distribution (GPD). The peak-over-threshold (POT) method is employed to fit the GPD model. To fit the GPD model, a subjective choice of a suitable threshold value should be obtained. In order to select the suitable thresholds for the JSE financial data, the mean excess plot, and the threshold stability plot are used (Das and Halder, 2016; Hakim, 2018). Das and Halder (2016); Hakim (2018) used the mean excess plots to select the appropriate threshold.

Zeng et al. (2022) fitted POT to the daily average return of Chinese, Shanghai stock market in different periods. The results of the study indicated that the POT model was found to be an effective fit for the thick tail distribution of the Shanghai stock market. Thus, it was concluded that the POT method is an effective method to measure extreme risk.

Figures 4.19 and 4.20 display the mean residual life plots for the ALSTRI and the USD/ZAR exchange rates, respectively. The mean residual life of the ALSTRI and the USD/ZAR exchange rates in Figures 4.19 and 4.20 displays the average excess values over given thresholds for a series of thresholds. In order to model and select a valid GDP model, the mean residual life plot should be approximately linear above the threshold (Coles et al., 2001).

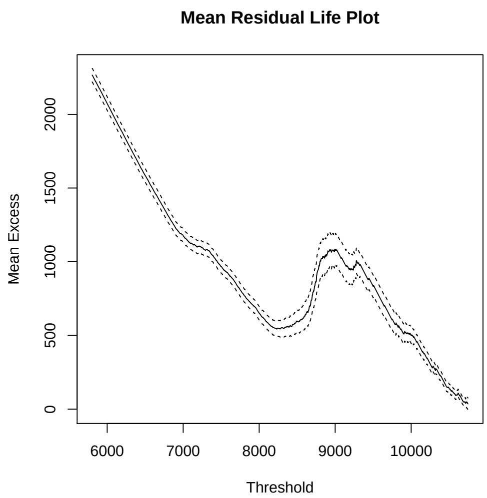

Figure 4.19: Mean residual life plot for ALSTRI.

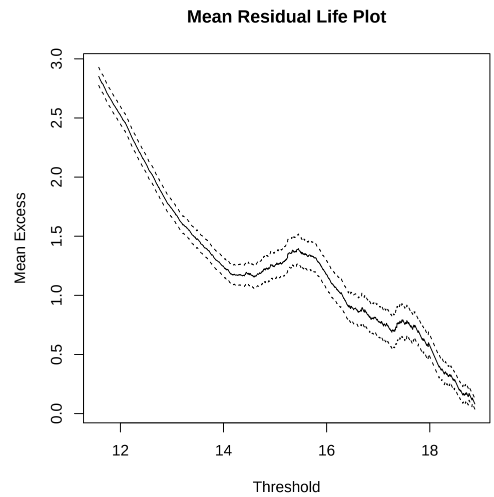

Figure 4.20: Mean residual life plot for USD/ZAR exchange rates

The mean residual life plot of the ALSTRI in Figure 4.19 shows the evidence of linearity above u = 8 500 for the ALSTRI and Figure 4.20 provides evidence that the USD/ZAR exchange rate is approximately linear above u = 14.25.

MLE parameter estimates

Table 4.23 displays the findings of the GPD parameter estimates of the total return ALSI and the USD/ZAR exchange rates. The shape parameter for the ALSTRI is positive ($\xi > 0$) and the confidence intervals are significantly different from zero. This implies that the ALSTRI is equivalent to the Pareto distribution. The shape parameter for the USD/ZAR exchange rates is negative ($\xi < 0$) and the confidence intervals contain zero. This means the ALSTRI has a light tail as compared to the exponential distribution. Thus, the tail of the USD/ZAR exchange rate is equivalent to the exponential distribution.

Table 4.23: MLE parameter estimates and standard errors (in parantheses) for the GPD Model.

	Scale (σ)	Shape (ξ)	95% CI of (ξ)	NLL (λ)
ALSTRI	478.528 (28.774)	0.153(0.046)	(0.064,0.242)	4800.515
USD/ZAR	1.2363(0.0774)	-0.05358(0.049)	(-0.1511,0.0439)	795.886

Key: ASTRI = All Share Total Return Index, USD/ZAR = USD/ZAR exchange rate, CI = Confidence Intervals, and NLL = Negative Log-Likelihood.

Return levels

According to Coles et al. (2001), the threshold approach is more efficient than the block maxima method and consider POT as a better alternative to block maxima or block minima approach due to its capability to use as much as possible of the available information.

Table 4.24 presents the results of the GPD return periods and the corresponding return levels. In accordance with the findings of Table 4.24, the return levels of the financial markets increase as the return period increases. This suggests that the ALSTRI will exceed 19092.42 at least once in 100 years, while the USD/ZAR exchange rates will surpass R23.72 at least once in 100 years. Thus, the financial markets will exceeds the maximum observations in the future. These findings further reveal that the GPD return levels estimates are much higher than both the GEVD, bGEVD, and rGEVD return levels.

Table 4.24: Return levels of the GPD model.

Financial Market	5−years	10−years	20−years	50−years	100−years
ALSTRI	13917.65	14913.01	16019.93	17675.85	19092.42
USD/ZAR	21.35	21.93	22.50	23.21	23.72

Key: ASTRI = All Share Total Return Index, and USD/ZAR = USD/ZAR exchange rate.

4.5.5 Poisson point process models

In accordance with Coles et al. (2001); Mashishi (2020), the Poisson point method is similar to the POT approach, this implies that the mean residual life plot for the ALSTRI and the USD/ZAR exchange rates are the same for the Poisson point process as those for the GPD.

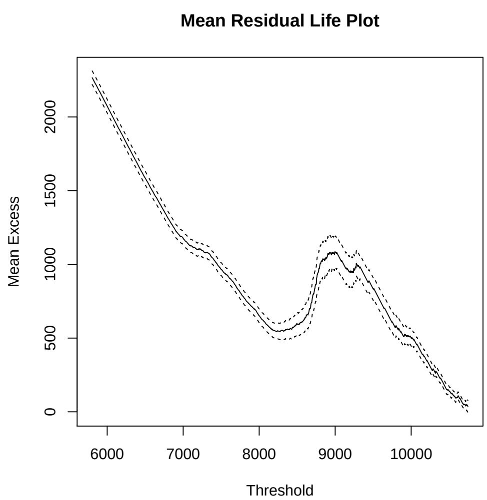

Figure 4.21: Mean residual life plot for ALSTRI.

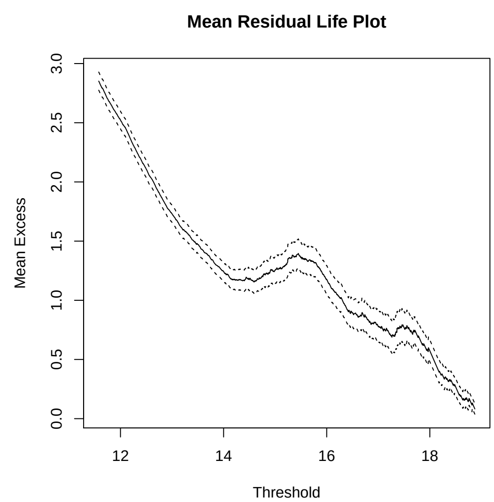

Figure 4.22: Mean residual life plot for USD/ZAR exchange rates

Figures 4.21 and 4.22 show the mean residual plots for the financial market data. Similarly to the GPD outcomes, the mean residual life plot of the ALSTRI in Figure 4.19 provides evidence of linearity above u = 8 500 for the ALSTRI and Figure 4.20 provides evidence that USD/ZAR exchange rate is approximately linear above u = 14.25 for the Poisson point process.

The threshold stability plot for the ALSTRI and the USD/ZAR exchange rates are presented in Figures 4.23 and 4.24, respectively. The parameter estimates are presented in Table 4.25.

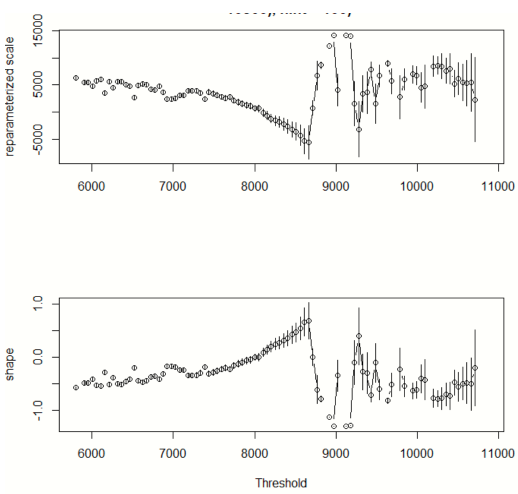

Figure 4.23: Threshold stability plot for ALSTRI.

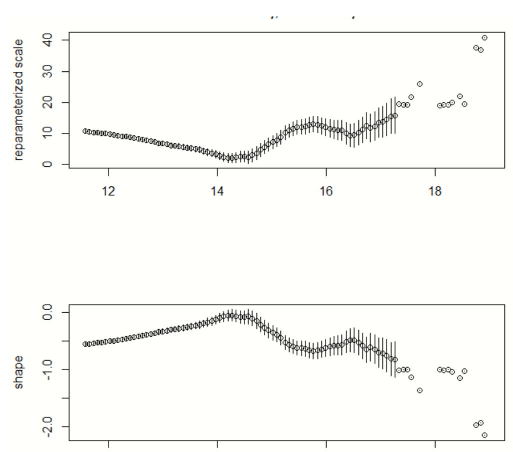

Figure 4.24: Threshold stability plot for USD/ZAR exchange rates

MLE parameter estimation

Table 4.25 displays the results of the Poisson point process parameter estimates of the total return ALSI and the USD/ZAR exchange rates. The shape parameter for the ALSTRI is positive ($\xi > 0$) and the confidence intervals are significantly different from zero. This implies that the ALSTRI can be modelled according to the Frèchet family of distribution. Whereas, the shape parameter for the USD/ZAR exchange rate is negative ($\xi < 0$) and the confidence intervals contains zero. This means that the USD/ZAR exchange rate data can be fitted by the Gumbel family of distribution.

Table 4.25: Parameter estimates and standard errors (in parantheses) for the Poisson point process.

	Location (μ)	Scale (σ)	Shape (ξ)	95% CI of (ξ)	NLL (λ)
ALSTRI	11,714.95	929.65	0.125	(0.048,0.203)	2128.33
	(309.14)	(159.75)	(0.039)		
USD/ZAR	19.915	0.933	-0.053	(-0.150,0.043)	-2128.96
	(0.462)	(0.198)	(0.049)		

Key: ASTRI = All Share Total Return Index, USD/ZAR = USD/ZAR exchange rate, CI = Confidence Intervals, and NLL = Negative Log-Likelihood.

Return levels

Table 4.26 shows the results of the Poisson point process return levels and the corresponding return periods. In accordance with the results from Table 4.26 the 100-year return level for the ALSTRI exceeds the maximum. This implies that the total return of the ALSI are likely to exceed the observed maximum total returns. With regards to the USD/ZAR exchange rate, the 100-year return levels of the exchange rates exceeds the maximum of R18.99. This suggests that the maximum exchange rate is likely to be exceeded at least once in 100 years. In actual fact, the ALSTRI and USD/ZAR exchange rates will surpass 17501.63 and R23.72 respectively, at least once in 100 years. The Poisson point process return level estimates are quite comparable with the GPD estimates.

Table 4.26: Return levels of the Poisson point process model.

Financial Market	5 years	10 years	20 years	50 years	100 years
ALSTRI	13249.21	14132.22	15061.02	16394.82	17501.63
USD/ZAR Exc Rate	21.26	21.89	22.47	23.20	23.72

Key: ASTRI = All Share Total Return Index, and USD/ZAR = USD/ZAR exchange rate.

Chapter 5

Conclusion

5.1 Introduction

The financial sector is essential to the economy as it encourages economic growth. The financial markets include, among other things, government bonds, exchange rates, stock markets, credit default swap markets, equity markets, debt markets, and derivatives markets. The study focused on the JSE financial market data, which comprises the total return index of the All-Share Index as well as the USD/ZAR exchange rate for the period from February 2016 to April 2021. The study utilised secondary data obtained from South Africa's Johannesburg Stock Exchange. Pandemics and financial crises are just two examples of infrequent occurrences that have a detrimental impact on the financial markets. In the financial sector, determining the likelihood that unusual and extreme occurrences will occur is essential. The most effective approach for aiding in the evaluation and modelling of extreme occurrences is the EVT.

Thus, in this chapter, the findings of the data analysis work from Chapter 4 are provided. Also provided in this chapter are the limitations of the study as well as the recommendations for future research directions.

5.2 Conclusion

The aim of the study was to examine and model the characterisation of extreme value behaviour of the JSE financial market data using EVT. Chapter 1 of the study outlined five objectives. The log returns data of the All-Share Index are centred at zero, suggesting that the variance is stable, and the data are stationary. The volatility of the markets and day-to-day fluctuation in prices are highlighted by positive spikes displaying daily profits and negative spikes showing daily losses. During the first national lockdown in South Africa and the first known COVID-19 case, there were significant losses of the ALSTRI during March 2020. The log returns of the USD/ZAR exchange rate are centred around zero, indicating that the variance is stable, and the data are stationary. Positive spikes displaying daily profits, and negative spikes showing daily losses, are examples of market instability with daily changes in prices. The first recorded COVID-19 case and the first national lockdown in South Africa caused the Rand to a spike in March 2020 and reach its peak in April 2022.

The Jarque Bera and Shapiro-Wilk tests were used in the study to determine the normality of the ALSTRI and USD/ZAR exchange rate data. The findings showed that the USD/ZAR exchange rate and the ALSI are not normally distributed.

To ascertain the stationarity of the financial market data, the ADF and KPSS tests were run. According to the results, neither the ALSTRI nor the USD/ZAR rate trends were stationary. As a result, it can be inferred that the patterns in the financial markets are not stationary. Thus, it can be concluded that both the ALSTRI and the USD/ZAR exchange rate data are neither stationary nor normally distributed.

The daily ALSTRI and the USD/ZAR exchange rate were used in fitting the parent distributions. The findings show that, when adjusted for ALSTRI and the USD/ZAR exchange rate, the log-normal was the most suitable parent distribution for the USD/ZAR exchange rates. The log-normal, however, is unable to fit the ALSTRI log-returns since they contain negative integers. Only non-negative values are suitable for log-normal. Consequently, the Gamma distribution was found to be the optimal parent distribution for the ALSTRI log-returns. The Mann-Kendall (M-K) test was used to analyse trends of the ALSTRI and the USD/ZAR exchange rate. According to the findings, both the ALSTRI and the USD/ZAR exchange rate exhibit a highly significant upward trend.

The block maxima method was employed in the study. The GEVD models for the USD/ZAR exchange rate and the ALSTRI were developed using the weekly and monthly block maxima method. Both results of weekly and monthly maxima GEVD models for the ALSTRI and the USD/ZAR exchange rate can be modelled by the Weibull and/or Gumbel family or domain of attraction. Due to the limited amount of data i.e., the five−year daily data for the ALSTRI and the USD/ZAR exchange rate, the monthly maxima approach was used to model the r-largest order GEVD (rGEVD). The findings of the rGEVD suggested that the distribution of the data belongs to the Weibull or Gumbel domain of attraction.

Instead of using a single highest value within a block to model the JSE financial data, the r-largest order statistics was used. Its goal was to represent the categorisation behaviour of the extremes of the JSE financial data. A technique using monthly block maxima was used. When $r = 1$, the findings are similar to those of the GEVD in that the values are taken as samples from the monthly block maxima. To fit the parameters of the rGEVD, the MLE and Bayesian parameter estimation methods were used.

The weekly and monthly block maxima approach was used to model the total return index of the ALSTRI and the USD/ZAR exchange rate using the GEVD. There is an increase in return levels as the return period rises. Also, results reveal that the monthly block maxima return levels are slightly higher than the corresponding weekly block maxima return levels. In conclusion, based on the GEVD outcomes, it will be expected that South Africa will experience an extreme increase in the USD/ZAR exchange rate. There won't be any unexpectedly high total returns for the ALSTRI, due to the fact that the monthly maxima's 100-year return of the ALSI's total return index is roughly comparable to the maximum observation. In conclusion, the monthly block maxima return levels reveal that the ALSTRI and USD/ZAR exchange rate will exceed 11100.08 and R19.23 respectively, at least once in 100 years.

The monthly maxima was used to model the ALSTRI and the USD/ZAR exchange using the bGEVD model which is an alternative of GEVD model. The bGEVD smoothly incorporates the left tail of a Gumbel distribution and a right tail of a Frèchet distribution. The results revealed that there is a positive linear trend term indicating an increase in the total returns of the ALSI and the USD/ZAR exchange rate over time. The explanatory variables have a greater effect on the location parameter. The $100-$year return levels are slightly equivalent to the maximum observed values for both the ASTRI and the USD/ZAR exchange rate. Thus, the monthly block maxima return levels reveals that the ALSTRI and the USD/ZAR exchange rate will exceed 10860, and R18.99, respectively. In conclusion, the findings of the return levels for the bGEVD, and GEVD models suggest that the two models result in similar outcomes. This implies that bGEVD can be a feasible alternative to the GEVD for modelling block maxima.

The 100-year return levels of the monthly GEVD, bGEVD and rGEVD models are almost equal to the maximum observations of the financial markets. While the 100-year return levels for the GPD and the Poisson point process models were extremely higher than the maximum observations of the financial market data. The Poisson point process return level estimates are quite comparable with the GPD estimates. In actual fact, the ALSTRI and USD/ZAR exchange rates will surpass 17501.63 and R23.72 respectively, at least once in 100 years. This implies that the investors will experience higher gains in the total returns of the ALSI. The USD/ZAR exchange rate return levels suggest that the Rand will become more unstable in the long run.

As the Rand goes higher it will affect the inflation and the economy of South Africa. The USD will become much stronger, and this will affect the ZAR as the USD appreciates or strengthens, and this will depreciate or weaken the South African currency.

5.3 Recommendations

There are rare events that contribute towards the negative impact on the financial markets such as pandemics and financial crises, among others. Due to the instability and unpredictability of the financial markets, the manifestation of the financial crises and pandemics can put investors, financial economists and risk experts in an unease situation by not knowing whether the investments will return gains or losses, as well as the unknown impact of these rare events on the stock and financial markets.

Financial market volatility can result in risk, whether big or small, that can be concealed in the tails rather than the mean. This drives the current risk management systems to include cutting-edge techniques such as advanced EVT methods which comprise among others POT, point process, r-largest order statistics, blended GEVD for anticipating the uncommon extreme occurrences with significant ramifications that could be harmful to the financial industry. These rare extreme events with high consequences can affect the economy of South Africa. This can be mitigated by using the advanced EVT to identify and examine the characteristics of extreme value behaviour and thereby preventing the risk.

The insufficient amount of data available for model estimation is an inherent challenge in any extreme value investigation. Since extremes are by definition rare, the model estimates particularly those of extreme return levels have a high degree of variance. Instead of focusing merely on the traditional methods of block maxima, the use of advanced extreme value methods that accommodates even small datasets such as GPD, r-largest order statistics, bGEVD, and Poisson point process is encouraged. The use of statistical methods that can predict the occurrence of extremes, focusing on the tails of the distribution to predict the future events is recommended.

5.4 Future studies

The researcher discovered that there are no studies conducted on bGEVD in the field of finance or financial markets. However, there are limited studies conducted on bGEVD applied to other fields. More studies on bGEVD and r-largest order bGEVD can be conducted on the financial markets and/or sector. Other areas for future studies may include multivariate extreme value analysis, and vine copulas in the financial markets, where the frequency of occurrence of the extreme returns in the financial sector is studied to assist in the improvement of risk management systems.

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Appendix

Some selected R codes

R codes for fitting EVT

```
# Installing packages for Extreme Value Analysis

library(extRemes)
library(ismev)
library(“evd”)
library(fitdistrplus)
library(fExtremes)
library(nortest)
library(MASS)
library(aTSA)
library(actuar)
```

R codes for fitting GEVD model

```
# The GEVD model
#Importing ALSTRI data to R
ALSI_GEVD < − read.csv2(“r1_ALSI.csv”)
```

```
GEVD.fit1 < − fgev(ALSI_GEVD$Total.Return.Index)
dev.new()
plot(GEVD.fit1)
```

R codes for fitting bGEVD model

```
library(INLA)
library(evd)
library(caret)
#Data simulation
n = nrow(r1_ALSI)
x1 = xk‘r1_ALSI$Total.Return.Index’
eta.x = 1 + 0.4*x1
#The spread and tail parameter can be simulated as follows
x2 = xk‘r1_ALSI$Total.Return.Index’
s.x = exp(0.1 + 0.3*x2)
x3 = runif(n,-0.5,2)
t.x = 0.1 + 0.2*x3
tail.intern = map.tail(t.x, tail.interval, inverse=TRUE) # internal ξ
#Obtaining the usual GEV parameters to generate the samples
par = giveme.gev.par(q = eta.x, sbeta = s.x, alpha = p.alpha, beta = p.beta, xi = t.x)
#INLA fit
ALS_INLAfit = inla(formula, family = “bgev”, data = data.bgev, control.family = list(hyper = hyper.bgev, control.bgev = control.bgev), control.predictor = list(compute = TRUE), control.compute = list(cpo = TRUE), control.inla = list(int.strategy = “eb”), verbose=FALSE)
return_levels = return_level_bgev(return_period, mu, sigma, xi, p_a, p_b, s)
```

R codes for fitting rGEVD model

#Fitting rGEVD model

#GEVD$_{r=4}$

r4_RI $<-$ read.csv2("r4_ALSI.csv")

RI4_fit $<-$ fevd(r4_RI$Total.Return.Index, type = "GEV", method = "MLE")

RI_fit4 $<-$ fevd(r4_RI$Total.Return.Index, type = "GEV", method = "Bayesian")

summary(RI4_fit)

summary(RI_fit4)

ci(RI4_fit, type = "parameter")

ci(RI_fit4, type = "parameter")

dev.new()

plot(RI4_fit)

plot(RI_fit4)

return.level(RI4_fit, return.period = c(5, 10, 20, 50, 100))

return.level(RI_fit4, return.period = c(5, 10, 20, 50, 100))

R codes for fitting GPD model

mrlplot(ALSI_data$Total.Return.Index, tlim = c(5802, 11000))

ALSI_GPD $<-$ fevd(ALSI_data$Total.Return.Index, threshold = 8500, type = "GP", method = "MLE")

plot(ALSI_GPD)

return.level(ALSI_GPD, return.period = c(5, 10, 20, 50, 100))

R codes for fitting Poisson Point Process model

threshrange.plot(ALSI_data$Total.Return.Index, r = c(5802, 11000), nint = 100)

mrlplot(ALSI_data$Total.Return.Index, tlim = c(5802, 11000))

tcplot(ALSI_data$Total.Return.Idex,tlim = c(10,5000))

ALSI_PP < − fevd(ALSI_data$Total.Return.Index, threshold = 8500, type = "PP")

plot(ALSI_PP)

ci(ALSI_PP, type = "parameter")

ci(ALSI_PP, return.period = c(5, 10, 20, 50, 100)